

How heuristic behavior can affect SHM-based decision problems?

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ABSTRACT: The main purpose of structural health monitoring (SHM) is to provide accurate and real-time information about the state of a structure, which can be used as objective inputs for decision-making regarding its management. However, SHM and decision-making occur at various stages. SHM assesses the state of a structure based on the acquisition and interpretation of data, which is usually provided by sensors. Conversely, decision-making helps us to identify the optimal management action to undertake. Generally, the research community recognizes people tend to use irrational methods for their interpretation of monitoring data, instead of rational algorithms such as Bayesian inference. People use heuristics as efficient rules to simplify complex problems and overcome the limits in rationality and computation of the human brain. Even though the results are typically satisfactory, they can differ from results derived from a rational process; psychologists call these differences cognitive biases. Many heuristic behaviors have been studied and demonstrated, with applications in various fields such as psychology, cognitive science, economics and finance, but not yet to SHM-based decision problems. SHM-based decision making is particularly susceptible to the representativeness heuristic, where simplified rules for updating probabilities can distort the decision maker's perception of risk. In this work, we examine how this heuristic affects the interpretation of data, providing a deeper understanding of the differences between a heuristic method affected by cognitive biases and the classical approach. Our study is conducted both theoretically through comparison with formal Bayesian methods as well as empirically through the application to a real-life case study in the field of civil engineering. With this application we demonstrate the heuristic framework and we show how this cognitive bias affects decision-making by distorting the representation of information provided by SHM.

1. INTRODUCTION

Structural health monitoring (SHM) is commonly seen as a powerful tool that allows bridge managers to make decisions on maintenance, reconstruction and repair of their assets. The logic of making decision based on SHM is formally stated in Cappello et al. (Cappello, et al., 2016), under the assumption that the decision maker is an

ideal rational agent, who judges using Bayes' theorem (Bolstad, 2010), and decides consistently with Neumann-Morgenstern's Expected Utility Theory (EUT) (Neumann & Morgenstern, 1944). However, we often observe real-life decision makers departing from this ideal model of rationality, judging and deciding using common sense and privileging fast and frugal heuristics to rational analytic thinking. Hence, if we wish to

describe mathematically and predict the choices of a real world bridge manager, we have to accept that their behavior may not be necessarily fully rational. Biased judgement and decision making have been widely reported and investigated starting the 1970s in the fields of cognitive sciences, social sciences and behavioral economics: key papers include the fundamental works by Kahneman and Tversky (Tversky & Kahneman, 1974) (Kahneman & Tversky, 1972) (Kahneman & Tversky, 1973); Kahneman's famous textbook (Gilovich, et al., 2002) is an exhaustive reference for those approaching the topic for the first time. Apparent irrational behavior in SHM-based bridge management is reported in (Zonta, et al., 2014) and suggested in (Cappello, et al., 2016). Another typical example of cognitive bias frequently observed in bridge management, is the confusion between condition state and safety of a bridge, as reported for example in (Zonta, et al., 2007). We remind here for clarity that safety is about the capacity of a bridge to withstand the traffic loads and the other external actions without collapsing, while the condition state expresses the degree of deterioration of a bridge respect to its design state. The condition state is usually appraised through a combination of routing visual inspections, non-destructive evaluation and SHM. It is expressed in the form of a condition index that depends on the particular management system. For example, bridge management systems based on AASHTO Commonly Recognized (CoRe) Standard Element System, such as the APT-BMS reported in (Zonta, et al., 2007), classify the state of an element on a scale from 1 to 5, where 1 means 'as per design' and 5 corresponds to the most severe observable deterioration state. On the contrary, the safety of a bridge is typically encoded in its probability of failure P_F , reliability index β , or safety factor γ , evaluated through formal structural analysis. Condition state and safety are obviously correlated (logically, the load-carrying capacity of a deteriorated bridge is equal or lower than that of the same bridge in undamaged condition) but are not the same thing. For example, an old bridge can

be unsafe, regardless its preservation state, simply because designed to an old code, which does not comply with the current load demand. As a counterexample, we may have the case of bridge, severely deteriorated, but still with enough capacity to safely withstand all the external loads, either because overdesigned or simply because its deterioration does not affect its load-carrying capacity. In principle, rational bridge management should target the safety of the bridge stock, and therefore prioritize retrofit of unsafe bridges, regardless they degree of deterioration. In practice we frequently observe that bridge managers tend to delay retrofit of substandard bridges which do not show sign of deterioration, while repair promptly deteriorated bridges as soon as the damage is observed, regardless their actual residual load-carrying capacity. The biased rationale behind this apparent behavior is that undamaged bridges 'look' safe, while damaged bridges 'look' unsafe, simply because we know that deterioration negatively affects safety.

The ambition of this paper is to tackle mathematically this observed biased judgement, a condition that, we will show, is broadly described by Kahneman and Tversky's representativeness heuristic (Kahneman & Tversky, 1972). We begin reminding, in Section 2, the formal framework of rational decision based on SHM information. We introduce the concept of heuristic in Section 3, focusing in detail on the representativeness. In Section 4 we use representativeness models to reproduce the biased evaluation of the safety of a bridge concrete slab, based on the condition state appraised through visual inspections. Concluding remarks are presented at the end of the paper.

2. SHM-BASED DECISION MAKING RATIONAL FRAMEWORK

We refer to the problem of optimal decision based on data provided by visual inspection or SHM. Generally speaking, this is a two-step process, as shown in Figure 1, which includes the judgement of the state h based on the observations \mathbf{y} , and the decision of the optimal action a_{opt} based on the uncertain knowledge of the state.

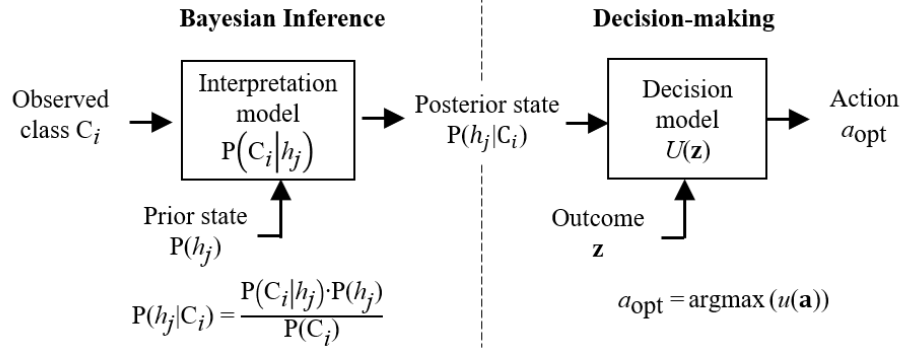


Figure 1: The rational process of an SHM-based decision problem.

Assume that the safety state of the bridge is described by one of n mutually exclusive and exhaustive state hypothesis $\mathcal{H} = \{h_1, h_2, \dots, h_j, \dots, h_n\}$ (e.g.: h_1 = 'safe', h_2 = 'failure', ...). Further assume that observing the bridge, or element, either through visual inspection or SHM, ultimately consists of assessing its condition out of a number of m possible classes $C_1, C_2, \dots, C_i, \dots, C_m$ which express its degree of damage or deterioration (e.g. C_1 = 'not damaged', C_2 = 'moderately damaged', C_3 = 'severely damaged'). Therefore, the value of an observation y_i , is one of the possible condition classes: $y_i \in \{C_1, C_2, C_3, C_4, C_5\}$. Multiple independent observations on the same bridge may occur because of repeated inspections by different inspectors, or redundant independent measurements by the monitoring system. We indicate with vector \mathbf{y} the full set of observations $\mathbf{y} = \{y_1, y_2, \dots, y_k, \dots, y_N\}$. The likelihood of condition C_i for a bridge/element in state h_j is encoded in the probabilistic distribution $P(C_i|h_j)$. If we restrict the problem to a single-observation case, the first step of the process consists of judging the state of a structure h_j based on the i -th class observed C_i . In the presence of uncertainty, the state of the structure after observing the class C_i is probabilistically described by the posterior $P(h_j|C_i)$, and the inference process followed by a rational agent is mathematically developed in the Bayes' rule (Bolstad, 2010):

$$P(h_j|C_i) = \frac{P(C_i|h_j)P(h_j)}{P(C_i)}, \quad (1)$$

where $P(h_j|C_i)$ is the posterior knowledge of the structural state and represents the best estimation after the acquisition of SHM observation; it depends on the likelihood $P(C_i|h_j)$ and on the prior knowledge $P(h_j)$, which is our estimate of h_j before the acquisition of the observation. $P(C_i)$ is simply a normalization constant, called evidence. The second step starts after the assessment of the posterior probability of the structure, and concerns choosing the 'best' action based on Expected utility theory (EUT) axioms. EUT, introduced by von Neumann and Morgenstern in 1944 (Neumann & Morgenstern, 1944) and later developed in the form that we currently know by Raiffa and Schlaifer in 1961 (Raiffa & Schlaifer, 1961), describes the analysis of decision making under risk and is considered as a normative model of rational choice (Parmigiani & Inoue, 2009).

In conclusion, Bayes theorem and EUT provide a rational method to solve respectively the two steps of a classical SHM-based decision process, as shown in Figure 1. However, most people use heuristics (Tversky & Kahneman, 1974) (Gilovich, et al., 2002) to determine their action which does not coincide with the rational decision. Therefore, we will investigate the impact of the representativeness heuristic on classical SHM-based decision problems.

3. THE REPRESENTATIVENESS HEURISTIC

The concept of heuristic has been subject to several definitions during the history and everyone who made use of the term seemed obliged to give his own interpretation of it. A very

important contribution is the work of Daniel Kahneman and Amos Tversky in the early 1970s, which revolutionized the academic research on human judgment (Tversky & Kahneman, 1974) (Kahneman & Tversky, 1973). They developed the so-called heuristics and biases approach, challenging the dominance of strictly rational models. The main innovation lays in the analysis of the descriptive adequacy of ideal models of judgment and in the proposal of a cognitive alternative that explained human error without invoking motivated irrationality. In this paper we want to focus on the representativeness, which seems the most affecting heuristic in judgments under uncertainty: events are ranked according to their representativeness and people consistently judge the more representative event to be the more likely, whether it is or not (Kahneman & Tversky, 1972). Its definition was: “A person who follows this heuristic evaluates the probability of an uncertain event, or a sample, by the degree to which it is: (i) similar in essential properties to its parent population; and (ii) reflects the salient features of the process by which it is generated”. This means that an event A is judged more probable than an event B whenever A appears more representative than B, that is, the ordering of events by their subjective probabilities coincides with their ordering by representativeness. Therefore, to be representative an uncertain event should not only be similar to its parent population, but it should also reflect the properties of the uncertain process by which it is generated, i.e. it should reflect the idea of randomness. There are various models attempting to explain this heuristic from a mathematical perspective, see for example (Edward, 1968), (Grether, 1980), (Grether, 1992), (Gigerenzer, 1995), (Barberis, et al., 1998), (Tenenbaum & Griffiths, 2011), (Bordalo, et al., 2016). While introducing the models, we want to point and analyze the two main aspects regarding the definition of representativeness and its application: what is the representativeness and how is defined among the different authors? To what extent and how does the representativeness affect the final judgment according to the Bayes’ rule?

3.1. Definition of Representativeness

All models described in the following propose representativeness as the ratio between the likelihood of the reference hypothesis h_j and its negation $-h_j$, or a set of alternative hypotheses. This agreement on the representativeness formulation is in line with Tversky and Kahneman definition of representativeness (Tversky & Kahneman, 1983), they write that “an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in the relevant reference class.” Bordalo et al. state the representativeness that a class C_i observed from a set of data \mathbf{y} , such that $y_k \in \{C_1, \dots, C_i, \dots, C_m\}$, provides for the reference hypothesis h_j , as:

$$R(C_i, h_j) = \frac{P(C_i|h_j)}{P(C_i|-h_j)}. \quad (2)$$

Therefore, they assume that a class C_i is representative for a hypothesis h_j , relative to an alternative hypothesis $-h_j$, if it scores high on the likelihood ratio described by Eq. (2). Edward and Gigerenzer agree on Eq. (2). Tenenbaum and Griffiths and Grether measure it with the same likelihood ratio, but adjusted with a logarithm scale to have a natural measure of how good a class C_i is in representing a hypothesis h_j :

$$R(C_i, h_j) = \log \frac{P(C_i|h_j)}{P(C_i|-h_j)}. \quad (3)$$

3.2. Representativeness in judgments

To evaluate how representativeness affects the final judgment of a hypothesis, we must understand how to calculate the posterior probability of the hypothesis h_j , by considering a possible distortion in the likelihood term due to this heuristic. This issue is clearly defined by the above-mentioned authors. In general, they provide a specific definition about what representativeness is, but they do not explain how it affects the posterior probability, i.e. how the standard Bayes’ rule, which reflects the judgment of a rational thinker, must be adjusted to consider

representativeness instead. Just Bordalo et al. and Grether try to explain how to calculate this distorted posterior probability. Bordalo et al. define this distorted Bayesian likelihood due to representativeness:

$$P(C_i|h_j)^{st} = P(C_i|h_j) \cdot (R(C_i, h_j))^\theta, \quad (4)$$

where θ is a subjective parameter that has to be calibrated with cognitive tests and could vary considerably among different people. Consequently, according to Bayes' theorem, the posterior probability of h_j becomes:

$$P(h_j|C_i) = \frac{P(C_i|h_j)^{st} P(h_j)}{P(C_i)}. \quad (5)$$

A different approach is provided by Grether; he suggests a model that provides the final judgment of h_j , by considering the representativeness:

$$\log O(h_j|C_i) = \alpha + \beta_1 \cdot R(C_i, h_j) + \beta_2 \cdot \log O(h_j), \quad (6)$$

where $\log O(h_j|C_i)$ is the posterior odds, $R(C_i, h_j)$ is the representativeness, $\log O(h_j)$ is the prior odds, while α , β_1 and β_2 are subjective parameters that must be calibrated. Thus, the interpretation of Kahneman and Tversky's representativeness heuristic suggested by the author is that individuals place greater weight on the likelihood ratio than on the prior odds. Consequently, the author proposed $\beta_1 > \beta_2 \geq 0$ for the representativeness model, in contrast with $\alpha = 0, \beta_1 = \beta_2 = > 0$ of the Bayes' rule.

4. RELIABILITY-BASED BRIDGE MANAGEMENT

The Autonomous Province of Trento (APT) has the ownership and the management of approximately 936 bridges. Consequently, APT committed the realization of a Bridge Management System (BMS) to University of Trento in 2004. The aim was to develop a management tool which could enable a systematic determination of the present and future need for maintenance, rehabilitation and replacement of bridges using various scenarios, along with a prioritization system which would provide

guidance in the effective utilization of available funds. To combine simplicity and efficiency, the bridge is broken down into Structural Units (SU), such as deck, piles, abutments, which include a set of Standard Elements (SE), specified in terms of quantity and Condition State (CS). CS are evaluated based on scheduled inspections according to the APT evaluation manual, which in turn partially refers to AASHTO (1997) Commonly Recognized (CoRe) Standard Element System (American Ass. State Highway and Transportation Off, 1997). Since 1995, the CoRe element standard has been adopted by FHWA and AASHTO to measure bridges condition on a single scale that reflects the most common processes of deterioration, to provide performance-based decision support that includes economic considerations. Five deterioration levels, called CS, have been defined, among which each bridge element is allocated based on the visual observations of an inspector. Based on the outcomes of a special inspection, the System Manager can stop the evaluation procedure, activate a safety assessment procedure or directly proceed with an intervention (Zonta, et al., 2007). The goal of this work is to estimate how big is the error committed by a biased manager in judging the bridge state when this is presented under low frequency CS, i.e. very representative of failure.

4.1. Application: SP65 bridge on the Maso River
The SP65 bridge on the Maso river (Figure 2), is a common type of bridge in the APT stock. The bridge was formally evaluated during the start-up phase, through the full application of the five-step assessment procedure (Zonta, et al., 2007). To analyze representativeness in visual inspections, we limit our analysis to a single SU, the slab. Table 1 reports slab state descriptions and the related possible actions for every CS, available from the website of the APT-BMS. We want to assess how much representativeness distorts manager's judgment compared to the Bayesian approach and to investigate how much high CS, as CS₅, are representative for two possible state of the slab: "SAFE= h_S " and "FAIL= h_F ".

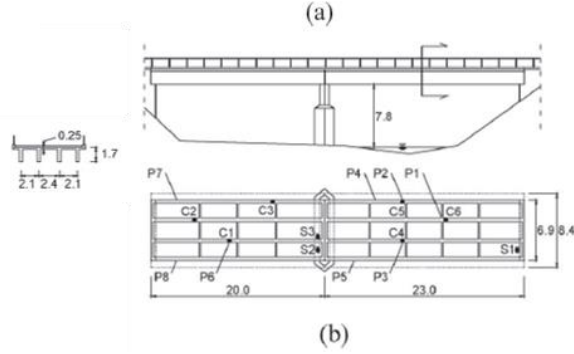


Figure 2: SP65 bridge: (a) overview; (b) plan view, elevation and cross section of the deck.

Table 1: SU SLAB - State description and action for each Condition State (CS).

	Slab surface state description	Action
1	No delamination, spalling or water infiltration	No intervention
2	Possible delamination, spalling or water infiltration. Possible segregation and consequently reinforcement exposure	No intervention Repairing affected areas
3	Previously repaired or subjected to delamination or spalling. Segregation and consequently reinforcement exposure. Limited water infiltration	No intervention Repairing affected areas Protection system
4	Extended parts previously repaired or subject to delamination or spalling; deep segregation phenomena with extended exposure of reinforcement. Extended water infiltration	No intervention Repairing affected areas Protection system
5	Deep deterioration or anomalies. Reinforcement corrosion and cross-section loss require a deep analysis to verify the struct. safety of the element	Interventions Repairing affected areas Protection system Slab replacement

We know that, in average, among all the APT bridges stock, the percentage of possible failure is very low compared to the safe condition. Consequently, we chose a prior probability $P(h_F)=0.001$ for the state hypothesis “FAIL” and $P(h_S)=0.999$ for the state hypothesis “SAFE”. We want to answer, from a mathematical point of view, the frequent questions: “How much CS₅ is representative of bridge failure?”, “How distorted could be the judgment of a biased inspector that observe the bridge classified in CS₅”, “Is his judgment coherent with the Bayes’ rule?” We have first to define a proper likelihood distribution for each hypothesis $P(CS_i|h_F)$ and $P(CS_i|h_S)$. According to (Melchers, 1999), we employ II level probabilistic methods, which allows to calculate the reliability index $\beta=-\Phi(P_{h_F})$, where Φ is a cumulative normal distribution function. Two normal distributions are considered: the loads effect S and the starting resistance R_0 of the bridge. We assume that the structure will not maintain its mechanical characteristics in the years, i.e. we have to consider the deterioration of construction material through a probabilistic degradation model $R = R_0(1 - \delta(CS_i))$ (Zonta, et al., 2007), where $\delta(CS_i)$ is a probabilistic capacity degradation function, depending only on the CS_i of the SE that control the capacity of the SU at the limit state. Typically, low values of CS are not associated with any loss of capacity, and in this case δ_i coincides with a Dirac delta function. Higher CSs are associated with distributions that reflect the uncertainty of the system in correlating the actual loss in capacity. CS₄ is associated with a uniform distribution δ_4 of loss in capacity, for values of $\delta = [0,5\%]$. In the same way, the system associates CS₅ with a triangular distribution, for $\delta=[5\%,70\%]$. It is convenient to define a normalized capacity $r = R_0/\mu_S$, with mean value $\mu_r = \mu_{R_0}/\mu_S$, equal to the central safety factor γ_0 associated with the limit state Z , and a normalized demand $s = S/\mu_S$ with mean value $\mu_s = 1$. The coefficients of variations of the normalized variables are equal to those of R and S . The failure probability P_{h_F}

associated with normalized limit state equation $z = r - s$ is $P_{h_F}(CS_i) = P(Z < 0) = P(z < 0)$. According to Eurocode 0, if we employ II level probabilistic methods, the target reliability index β for Class RC2 structural member in the ULS and with a reference time of 1 year is equal to $\beta = 4.75$. Using $V_R = 0.05$ and $V_S = 0.10$, from the equation $\beta = (\gamma_0 - 1) / (\sqrt{V_R^2 \cdot \gamma_0^2 + V_S^2})$, we can obtain $\gamma_0 = 1.96$. Then $P_{h_F}(CS_i)$ is calculated through Montecarlo by computing the cumulative-time failure probability of the normalized limit state z , by using the normalized Gaussian distribution for the demand $f_S(r) = \text{norm}(s, 1, V_S)$ and a normalized non-Gaussian distribution for the reduced capacity $f_r(r, CS_i) = \text{norm}(\gamma_0, 1, V_R) \cdot (1 - \delta(CS_i))$. For each Condition State CS_i , we obtain: $P_{h_F}(CS_i) = [6.12 \cdot 10^{-5}, 2.68 \cdot 10^{-6}, 6.47 \cdot 10^{-6}, 6.61 \cdot 10^{-4}, 2.04 \cdot 10^{-1}]$. Assuming a priori $P(CS_i) = [50, 20, 15, 10, 5]\%$, we obtain $P_{h_F} = 0.0103$ and consequently $P_{h_S} = 0.9897$. Then, according to Bayes' rule, for both hypothesis "S=SAFE" and "F=FAIL", we can calculate the relative likelihood distributions for each Condition State CS_i (Figure 3). To be consistent in our case study with these outcomes, we choose the following likelihood distributions: $P(CS_i|h_F) = [0, 0, 0, 2, 98]\%$, $P(CS_i|h_S) = [50, 20, 15, 10, 5]\%$. Once we know the likelihood distributions, we can calculate how much CS_5 is representative of the hypothesis h_F through all the representativeness models of Section 3; we can also calculate the posterior odds of the inspector distorted by the representativeness heuristic.

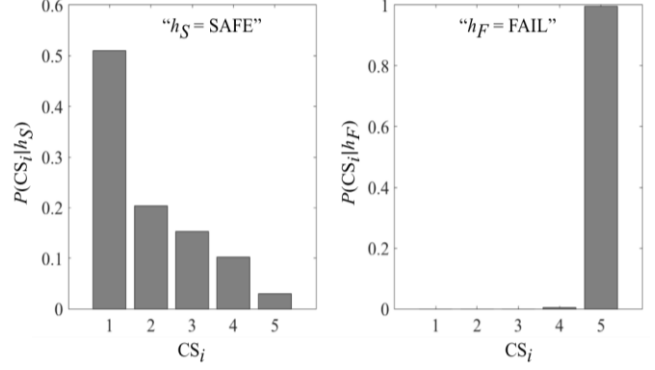


Figure 3: Likelihood distributions for each h_i .

Table 2 reports all the results: as we can observe, rational managers, in line with Bayes' rule and after observing CS_5 , would judge the possibility that the bridge could fail as very unlikely, i.e. $P(h_F|CS_5) = 1.92\%$ and $P(h_S|CS_5) = 98.08\%$. Contrary, representativeness models provide a significantly distorted probability. For instance, according to Bordalo et al., the failure probability of the bridge $P(h_F|CS_5) = 69.74\%$, is double the probability of a safe state, i.e. $P(h_S|CS_5) = 30.26\%$. According to Grether's model, the posterior odds of the failure condition against the safe condition, given CS_5 , are clearly higher than one. CS_5 is very representative for the failure condition and all models agree on that, $R(CS_5|h_F) \gg R(CS_5|h_S)$. Consequently, when irrational managers judge the state of a bridge by observing a high CS, as CS_5 , they are biased by representativeness: in their posterior judgments they tend to neglect the prior probability of the failure condition and to weight too much the likelihood of the observations; so, their final judgments are distorted.

Table 2: Achieved results for each model.

Model	Likelihood $P(C_i h_j)$ or Representativeness $R(C_i h_j)$	Posterior (distorted) probability $P(h_j C_i)$	Posterior odds $P(h_j C_i)/P(h_{-j} C_i)$
Bayes	$P(CS_5 h_F) = 98$ $P(CS h_S) = 5$	$P(h_F CS_5) = 1.92\%$ $P(h_S CS_5) = 98.08\%$	$\frac{P(h_F CS_5)}{P(h_S CS_5)} = 0.20$
Grether 1980-1992 ($\alpha=0$; $\beta_1=0.8$; $\beta_2=0.2$)	$R(CS_5 h_F) = 2.98$ $R(CS_5 h_S) = -2.98$	/	$\frac{P(h_F CS_5)}{P(h_S CS_5)} = 2.73$
Bordalo et al. 2016 ($\theta=0.8$)	$R(CS_5 h_F) = 19.6$ $R(CS_5 h_S) = 0.05$	$P(h_F CS_5) = 69.74\%$ $P(h_S CS_5) = 30.26\%$	$\frac{P(h_F CS_5)}{P(h_S CS_5)} = 2.30$

5. CONCLUSIONS

Nowadays managers are faced with many competing priorities and must rely on computerized data processing when managing large infrastructure assets. This “management by data” is only possible when there is an understanding of what the data represents and a trust in the quality of the data. For collecting bridge data, it can be used the “Commonly Recognized (CoRe) Elements for Bridge Inspection”, which allow to classify bridges in a limited number of Condition States (CS). However, questions as: “How is a single CS representative of the real state of the bridge under exam?”; “Are decision makers biased by heuristics when they face highest CS?”; “How are posterior probabilities distorted by representativeness if people behave irrationally?”, have still no answer. We tried to answer by analyzing how representativeness, the main heuristic by Kahneman and Tversky, influences the interpretation of data, leading different results in comparison to those achieved with the classical rational method of Bayes’ rule. After defining and contextualizing representativeness from a mathematical point of view, we applied existing representativeness models from literature to the judgment of the state of a concrete bridge, based on visual inspections. Our results demonstrate the consequential distortions from rational decision making of this heuristic.

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