A Stochastic Approach to Model Household Re-occupancy in A Community Following A Natural Hazard

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ABSTRACT: The re-occupancy of displaced households in a community following a hazard event is a complex social process driven collectively by the functionality states of community building portfolios and supporting lifelines. This study presents a novel approach for household re-occupancy (HRO) modeling using discrete state, Discrete Time Markov Chain (DTMC). Our hypothesis is that the re-occupancy state of a displaced household at a post-event time is collectively determined by the joint functionality status (JFS) (of its related school(s), workplace(s) and pre-event dwelling unit) and by the resourcefulness of the household largely determined by its income level. Accordingly, we construct a one-step transition probability matrix of the DTMC modeling the household’s JFS as a function of the functionality states of its school(s), workplace(s) and pre-event dwelling unit; additionally, based on available social science studies, we further define a set of time-dependent conditional re-occupancy probability functions (CRPFs) that give the probability of a household re-occupying its pre-event dwelling units at any time conditional on the change in household’s JFS and its income level. Finally, the time-variant household-level re-occupancy probability is derived by solving the DTMC with partially absorbing boundary conditions described by the CRPFs. The community-level HRO is then obtained through aggregating the household-level re-occupancy state across the community over the recovery time horizon. The model will be further calibrated by data collected in ongoing field studies with an ultimate goal of supporting further researches on community resilience planning.

1. INTRODUCTION
In recent years, disaster research has recognized the needs to go beyond physical damage to capture the social and economic impact of a community caused by natural or man-made disaster events. For example, the on-going efforts in the NIST-funded Center for Risk-based Community Resilience Planning are targeted at the development of measurement science and technology to integrate science-based models of community socio-economic systems and supporting interdependent infrastructure. While loss estimation models and recovery prediction models are expanding, overcoming the significant gap between the engineering and the social
In this study, a novel stochastic approach is developed to model HRO in a community following a natural hazard event. Our hypothesis is that the re-occupancy state of a displaced household at a post-event time is collectively determined by the joint functionality status (JFS) (of its related school(s), workplace(s) and pre-event dwelling unit) and by the resourcefulness of the household largely determined by its income level. Accordingly, we simulate the HRO through constructing a discrete-state, discrete time Markov Chain (DTMC) subjected to partially absorbing boundary conditions. The next section discusses the overall simulation framework and mathematical formulation, which is followed by conclusions summarized in Section 3.

2. METHODOLOGY
This study is aimed at predicting stochastic re-occupancy process for pre-event household reoccupying their pre-event dwelling unit (as opposed to a dwelling unit occupied by a new household). Examining existing studies on population recovery, it is found that population demographics (e.g., family income) and the severity of damage to the built infrastructure are important predictors of whether evacuees returned to their houses (Groen & Polivka, 2010; Fussell et al., 2010). Moreover, a household’s decision to return in most cases is strictly dependent on whether the dwelling unit is safe to occupy, the residents can go to work, and whether children can go to school (FEMA/NIBS, 2003; Peacock et al., 2008; Lin, 2009; Xiao & Van Zandt, 2012; Masoomi et al., 2017).

Accordingly, our hypothesis is that the re-occupancy state of a displaced household at a post-event time is collectively determined by the JFS of its dwelling unit, school(s), and workplace(s) and by its income level. At household-level, a unique household’s post-event re-occupancy state is simulated by three analysis modules: (1) a discrete state DTMC that simulates the JFS of school(s), workplace(s), and pre-event dwelling unit associated with a unique household is developed; (2) a set of time-dependent conditional re-occupancy probability
functions (CRPFs) are defined that give the probability of a household re-occupying its pre-event dwelling unit at any time conditional on the change in household’s JFS and its income level; (3) the time-variant household-level re-occupancy probability is derived by solving the DTMC with partially absorbing boundary conditions described by the CRPFs. A flowchart of the stochastic model to predict household-level re-occupancy state is illustrated in Figure 1 with aforementioned three steps colored in yellow. Lastly, the community-level HRO recovery curve is obtained through aggregating the re-occupancy states of all households across the community over the recovery time horizon.

![Flowchart of the stochastic model to predict household-level re-occupancy state](image)

Figure 1: Flowchart of the stochastic model to predict household-level re-occupancy state

2.1 DTMC modeling JFS of dwelling unit, workplaces, and schools

A household consisting of children, employees and perhaps elderly, are geographically linked to a dwelling unit, a school (or schools), and workplaces. The post-disaster functionality states of these buildings, as stated above, serve as key driving factors of the HRO model. For instance, a household would not return to their house unless the severely damaged building is repaired, or the destroyed building is reconstructed; a household would probably choose to shelter in place before utility service at the building site is restored.

2.1.1 Functionality states of individual buildings

In the literature, a building’s functionality is defined as the capability of that building to serve its intended purpose and can be categorized into five functionality states —Restricted Entry (RE), Restricted Use (RU), Re-Occupancy (RO), Baseline Functionality (BF) and Full Functionality (FF) (Lin & Wang, 2017a). Based on this definition, in the present study the dwelling unit (i.e., residential building)’s functionality status is classified into three groups: (1) G1: RE and RU, in which case buildings are not safe to be occupied; (2) G2: RO, in which case buildings can be occupied but lack utility supply; (3) G3: BF and FF, in which case buildings both are re-occupiable and have utility supply. Compared with dwelling units, schools and workplaces have less influences on HRO, thus their functionality status is defined binary for simplicity: Functional (denoted as F, buildings are re-occupiable and have utility supply) or Non-Functional otherwise (denoted as NF).

Mathematically, the functionality states of dwelling units, schools, and workplaces can be simulated using discrete state, DTMC process. Taking dwelling unit (D) unit as an example. Let us consider a DTMC, \( \{S^D(k), k \in T\} \), modeling time-variant functionality state of a dwelling unit. Given discrete time steps \( T = \{1, 2, \ldots, k, \ldots\} \), the \( \lambda^D_1(k) \) is defined as the one-step transition probability of residential buildings upgrading from functionality state G1 to G2 at k-th time step; \( \lambda^D_2(k) \) is defined as the one-step transition probability of residential buildings upgrading from G2 to G3 at k-th time step. Accordingly, the one-step transition probability matrix of \( S^D(k) \) is described by

\[
P^D(k) = 
\begin{pmatrix}
1 - \lambda^D_1(k) & \lambda^D_1(k) & 0 \\
0 & 1 - \lambda^D_2(k) & \lambda^D_2(k)
\end{pmatrix}
\]
Similarly, for constructing DTMC of schools and workplaces, \( \lambda^S(k) \) and \( \lambda^W(k) \) are defined as the time-variant one-step transition probabilities of school (\( S \)) and workplace (\( W \)) upgrading from \( NF \) state to \( F \) state at \( k \)-th time step, respectively. These one-step transition probabilities can be either obtained from empirical study through field survey or calculated from analytical approach using building-level restoration functions (Lin & Wang, 2017a).

### 2.1.2 JFS of dwelling unit, schools, and workplaces

With the above functionality state categorizations, we further construct 12 JFSs (ranging from J1 to J12) of dwelling unit, workplace(s), and school(s) associated with a specific household, as shown in Table 1. In the table, the schools (workplaces) are considered \( F \) only if all schools (workplaces) linking to that household are functional. Note that this assumption can be relaxed by introducing more JFSs to include different combinations of multiple schools and workplaces.

### Table 1: Definition of JFSs of school(s), workplace(s), and dwelling unit

<table>
<thead>
<tr>
<th>JFS</th>
<th>School(s)</th>
<th>Workplace(s)</th>
<th>Dwelling unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>NF</td>
<td>NF</td>
<td>RE, RU</td>
</tr>
<tr>
<td>J2</td>
<td>F or NA</td>
<td>NF</td>
<td>RE, RU</td>
</tr>
<tr>
<td>J3</td>
<td>NF</td>
<td>F or NA</td>
<td>RE, RU</td>
</tr>
<tr>
<td>J4</td>
<td>F or NA</td>
<td>F or NA</td>
<td>RE, RU</td>
</tr>
<tr>
<td>J5</td>
<td>NF</td>
<td>NF</td>
<td>RO</td>
</tr>
<tr>
<td>J6</td>
<td>F or NA</td>
<td>NF</td>
<td>RO</td>
</tr>
<tr>
<td>J7</td>
<td>NF</td>
<td>F or NA</td>
<td>RO</td>
</tr>
<tr>
<td>J8</td>
<td>F or NA</td>
<td>F or NA</td>
<td>RO</td>
</tr>
<tr>
<td>J9</td>
<td>NF</td>
<td>NF</td>
<td>BF, FF</td>
</tr>
<tr>
<td>J10</td>
<td>F or NA</td>
<td>NF</td>
<td>BF, FF</td>
</tr>
<tr>
<td>J11</td>
<td>NF</td>
<td>F or NA</td>
<td>BF, FF</td>
</tr>
<tr>
<td>J12</td>
<td>F or NA</td>
<td>F or NA</td>
<td>BF, FF</td>
</tr>
</tbody>
</table>

*NA— not available

A household’s JFS, as a function of the functionality states of its school(s), workplace(s) and pre-event dwelling unit, is simulated using a discrete state, DTMC. This DTMC is denoted as \( \{JFS(k), k \in T\} \), with its time-variant probability vector, \( X(k), k = 0, 1, 2, \ldots \), taking the following form,

\[
X(k) = [x_1(k), \ldots, x_{12}(k)]
\]

in which \( X(0) \) is the initial JFS probability vector after occurrence of the hazard event; \( P^X(k) \) is the one-step transition probability matrix with its element defined as

\[
p_{m,n}^X(k) = P[JFS(k) = J_m \mid JFS(k - 1) = J_n]
\]

in which \( m = 1, 2, \ldots, 12; n = 1, 2, \ldots, 12 \).

Neglecting the stochastic correlation among functionality states of dwelling unit, school, and workplace, as well as neglecting higher order probabilities (which is valid when the time step is small), \( P^X(k) \) is expressed as

\[
P^X(k) = \begin{pmatrix}
A_1(k) & B_1(k) & 0 \\
0 & A_2(k) & B_2(k) \\
0 & 0 & A_3(k)
\end{pmatrix}
\]

in which \( A_q, q = 1, 2, 3 \) are 4×4 upper triangular matrices, with their elements tabulated in Table 2; \( B_q = \lambda^D_q(k) \cdot I_{4 \times 4}, q = 1, 2, \) in which \( I_{4 \times 4} \) is the 4×4 identify matrix.

### Table 2. Element of \( A_q, q = 1, 2, 3 (\lambda^D_q(k) \equiv 0) \)

<table>
<thead>
<tr>
<th>( a_{ij} )</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{11} )</td>
<td>( 1 - \lambda^S(k) - \lambda^W(k) - \lambda^D_q(k) )</td>
</tr>
<tr>
<td>( a_{12} )</td>
<td>( \lambda^S(k) )</td>
</tr>
<tr>
<td>( a_{13} )</td>
<td>( \lambda^W(k) )</td>
</tr>
<tr>
<td>( a_{14} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( a_{22} )</td>
<td>( 1 - \lambda^W(k) - \lambda^D_q(k) )</td>
</tr>
<tr>
<td>( a_{23} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( a_{24} )</td>
<td>( \lambda^W(k) )</td>
</tr>
<tr>
<td>( a_{33} )</td>
<td>( 1 - \lambda^S(k) - \lambda^D_q(k) )</td>
</tr>
<tr>
<td>( a_{34} )</td>
<td>( \lambda^S(k) )</td>
</tr>
<tr>
<td>( a_{44} )</td>
<td>( 1 - \lambda^D_q(k) )</td>
</tr>
</tbody>
</table>
2.2 Conditional re-occupancy probability functions (CRPFs)

With the pre-defined 12 JFSs associated with a household, the effect of restoring buildings’ functionality on HRO is reflected in the CRPFs defined in this section. The CRPFs is defined as the probability of a household re-occupying its pre-event dwelling unit at any post-event time conditional on the status change in household’s JFS and its income level.

Let a household’s re-occupancy status be $\Theta(k)$, which is a two-state Markov chain (namely, occupancy, $O$ and dislocation, $\bar{O}$). For a certain group of income level (IL), the CRPF, $\gamma_{m,n}(k)$, is the re-occupancy probability of a displaced household at $k$-th time step given that the displaced household is upgrading from JFS $J_m$ at previous time step $k − 1$ to JFS $J_n$ at current time step $k$, i.e. (the subscript IL of $\gamma_{m,n}$ is not present in the following equations for simplicity purpose),

$$
\gamma_{m,n}(k) = P[E^1(k)|E^2_{m,n}(k)]
$$

$$
E^1(k) = [\Theta(k) = O | \Theta(k-1) = \bar{O}]
$$

$$
E^2_{m,n}(k) = [JFS(k) = J_n | JFS(k-1) = J_m]
$$

in which $m = 1,2, \ldots, 12; n = 1,2, \ldots, 12$.

In this study, it is assumed that the household won’t return as long as their dwelling unit is un-occupiable (functionality state $RE$ and $RU$), therefore, $\gamma_{m,n} = 0$ for all $m \leq 4, n \leq 4$. Moreover, $\gamma_{m,n} = 0$ for all $n \leq m$ since a household’s JFS is always non-decreasing. The CRPF curves can be collected based on anecdotal evidence, expert opinion, and field work in multiple hazards over several decades (Peacock et al., 2008; Lin, 2009; Xiao & Van Zandt, 2012).

2.3 Stochastic household re-occupancy process

To derive the two-state Markov Chain $\{\Theta(k), k \in T\}$, modeling a household’s post-event re-occupancy state, we first examine the joint events $\Lambda_{m}(k) = [JFS(k) = J_m, \Theta = \bar{O}], m = 1,2, \ldots, 12$.

Define a stochastic process $\{\Lambda(k), k \in T\}$, in which $\Lambda(k)$ is assumed to take one of the 12 joint events $\Lambda_1(k), \Lambda_2(k), \ldots, \Lambda_{12}(k)$. The time-variant state probability of $\Lambda(k)$ is

$$
Y(k) = [y_1(k), \ldots, y_{12}(k)] = Y(0) \ast P^Y(1) \ast \ldots \ast P^Y(k)
$$

in which $Y(0) = [y_1(0), \ldots, y_{12}(0)]$ is the initial state probability vector of $\Lambda(k)$ determined by household dislocation model. The element of $P^Y$ is

$$
p^Y_{m,n}(k) = [\Lambda_n(k)|\Lambda_m(k)] = P\left[\frac{E^1(k)}{E^2_{m,n}(k)}\right] = P\left[\frac{E^1_{m,n}(k)}{E^2_{m,n}(k)}\right] = [1 - \gamma_{m,n}(k)] \cdot P^Y_{m,n}(k)
$$

Eq. (7) is equivalent to solving the DTMC process modeling JFS of a household subjected to the following partially absorbing boundary conditions (as illustrated in Figure 2),

$$
P\left[\frac{E^1(k)}{E^2_{m,n}(k)}\right] = 1 - \gamma_{m,n}(k)
$$

![Figure 2: Illustration of applying partially absorbing boundary condition to DTMC modeling JFS of a household](image)

Accordingly, the time-variant post-event HRO probability is obtained using the total probability theorem

$$
[\Theta(k) = O] = 1 - P[\Theta(k) = \bar{O}] = 1 - \sum_{m=1}^{12} \gamma_{m}(k)
$$

Lastly, considering a community with $N_H$ households, the community-level household re-
occupancy curve (CHRO), defined as the expected percentage of households reoccupying their pre-event dwelling unit, is obtained by

\[
CHRO(k) = \frac{1}{N_H} \sum_{l=1}^{N_H} P[\theta^{(l)}(k) = 0]
\]

in which \(\theta^{(l)}\) is the occupancy state of the \(l\)-th household in the community.

3 CONCLUSIONS
The re-occupancy of dislocated households in a community following a hazard event is a stochastic process largely determined by the functionality recovery of community building portfolios as well as households’ socioeconomic characteristics. In this study a novel stochastic approach is proposed to model post-hazard HRO process, through integration of physical and social infrastructure systems within a community. The coupling of existing engineering models (building functionality recovery model) and social science models (population dislocation model) is fulfilled via imposing partially absorbing boundary conditions (described by the CRPFs in Section 2.2) to the discrete state, DTMC modeling JFSs of dwelling unit, schools, and workplaces associated with a household. Such a stochastic approach has enabled those key community socioeconomic characteristics to be included in the HRO model while to a great extent capturing the fundamental mechanisms of the post-hazard HRO process. The model will be further calibrated by data collected in ongoing field studies with an ultimate goal of supporting further researches on community resilience planning.

Overall, the approach presented herein is greatly distinguished from those conventional social science models, which have utilized regression analysis to quantify hazard-induced social losses using expected building damage as the major input variable along with other demographic characteristics. The HRO process is highly intricate and its stochastic nature can hardly be captured solely using conventional social science models. The merit of our approach lies in its capability to inherit consistent uncertainty treatment from existing engineering models, which can provide insight into interdisciplinary research on coupling engineering and social science models.

4 ACKNOWLEDGEMENT
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5 REFERENCES
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