A new method for load and resistance factor design

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ABSTRACT: The reliability-based load and resistance factors design (LRFD) has been widely used in the structural design codes. In almost all of the current reliability methods for the determination of the load and resistance factors, the basic random variables are assumed to have known probability distributions. The third-moment method has been proposed to overcome the shortcomings (e.g., requirement of probability density functions (PDF) of random variables, inevitable iterative computation, requirement of design points) of other methods. However, in the existing third-moment method, the iterative computation is required. In this paper, the application of the existing third-moment method is inspected and a simpler method is proposed. In the computation of the target mean resistance, one-time iteration in the existing third-moment method is further simplified to no iteration, by changing the equation of the target reliability to another one. In addition, there is not any mathematical limitation in the new equation of the target reliability. From the proposed method in this paper, it can be concluded that: 1) the present method gives good improvement upon the method based on the third-moment method; 2) the computation of the existing third-moment method is further simplified to no iteration; 3) the limitations of applicable range in exiting third-moment methods are avoided; 4) the accuracy of the proposed method is proved to be higher than the exiting third-moment methods. With several examples, the comparison of the existing third-moment method, the ASCE method and the proposed method is given. The results show that the proposed method is accurate, simple and safe.

1. INTRODUCTION

In the past three decades, the reliability-based load and resistance factors design (LRFD) has been widely used in the structural design codes (2010). The load and resistance factors are generally determined using the first order reliability method (FORM) in which, the design point must be firstly determined and then derivative-based iterations have to be used. Some simplified methods (2002; Zhao et al., 2000 and 2001; Lin et al., 2001; Lin et al., 2003; Nowak et al., 2000; Branco et al., 2009) are proposed in order to avoid iteration computation. However, in almost all of the current methods, the basic random variables are assumed to have known distribution function. In reality, the CDF/PDFs of some of the basic random variables are often unknown due to the lack of the statistical data. (Mori et al., 2002; Ugata et al., 2000)

Then the third-moment (3M) method, with no need for the design point or any assumption of the PDFs of random variables, was proposed. (Zhao et al., 2011 and 2012). In this method the computation of the target mean resistance is simplified to one iteration.

In this paper, the application of the existing 3M method is reviewed and a simpler method without iteration is proposed. In the computation of the target mean resistance, the iteration in the existing 3M method is eliminated. Moreover, there is not any mathematical limitation in the new equation of the target reliability. Compare the
2. COMPUTATION PROCEDURE OF LOAD AND RESISTANCE FACTORES

2.1. Determination of Load and Resistance Factors

The LRFD format may be expressed as the follows:

\[ \Phi R_n \geq \sum \gamma_i Q_{ni} \]  

where \( \Phi \) is the resistance factor, \( \gamma_i \) is the partial load factor to be applied to load \( S_i \), \( R_n \) is the nominal value of the resistance, \( Q_{ni} \) is the nominal value of load \( Q_i \).

In reliability-based design, the load and resistance factors \( \Phi \) and \( \gamma_i \) should be determined with a specified reliability, called the target reliability. Therefore, Eq. (1) should be probabilistically to the following equations:

\[ G(X) = R - \sum Q_i \]  

where \( R \) and \( Q_i \) are random variables representing uncertainty in the resistance and load effects, respectively.

For a given target reliability \( \beta_T \) or target probability of failure \( P_f \), Eq. (2) can be expressed in terms of probability:

\[ \beta \geq \beta_T \text{ or } P_f \leq P_{fT} \]  

where \( \beta \) and \( P_f \) are the reliability and the probability of failure, respectively.

If \( R \) and \( Q_i \) are mutually independent normal random variables, the second-moment (2M) method is correct and the design formula is expressed as:

\[ \beta_{2M} \geq \beta_T \]  

where

\[ \beta_{2M} = \frac{\mu_Z}{\sigma_Z} \]  

\[ \mu_z = \mu_R - \sum \mu Q_i \]  

Substituting Eq. (5) in Eq. (4), the load and resistance factors can be expressed as:

\[ \mu_r (1 - \alpha_R V_R \beta_T) \geq \sum \mu_i (1 + \alpha Q_i V_i \beta_T) \]  

Comparing Eq. (6) with Eq. (1), the load and resistance factors can be expressed as:

\[ \phi = (1 - \alpha_R V_R \beta_T) \frac{\mu_R}{R_n} \]  

\[ \gamma_i = (1 + \alpha Q_i V_i \beta_T) \frac{\mu_0}{Q_{ni}} \]  

where \( V_R \) and \( V_i \) are the coefficient of variation for \( R \) and \( Q_i \), respectively; and \( \alpha_R \) and \( \alpha Q_i \) are the sensivity coefficients of \( R \) and \( Q_i \), respectively, where

\[ \alpha_R = \frac{\sigma_R}{\sigma_Z}, \alpha_S_i = \frac{\sigma_{Q_i}}{\sigma_z} \]

As introduced above, the 2M method is based on the assumption of all the variables obey normal distribution and are independent of each other. In the case of \( R \) and \( Q_i \) are other random variables, the 2M reliability in Eq. (5) is incorrect. Therefore, other methods were proposed, typically, the FORM. [11] The load and resistance factors can be obtained as:

\[ \phi = \frac{R^*}{R_n}, \gamma_i = \frac{Q_i^*}{Q_{ni}} \]  

where \( R^* \) and \( Q_i^* \) are the values of the variables \( R \) and \( Q_i \), respectively, at the design point of the FORM.

2.2. Existing 3M Method for the Computation of Load and Resistance Factors

In the existing 3M method based on 3P-lognormal
distribution the two steps recursive optimization is used to avoid the iteration computation:

\[ \mu_{RT} = \sum \mu_Q + \beta_{2T} \sigma_z \]  
(10)

\[ \mu_{R0} = \sum \mu_Q + \sqrt{\frac{3}{5}} \sum \sigma_{Q_i}^2 \]  
(11)

where \( \mu_{RT} \) is the target mean resistance; \( \mu_{R0} \) is the original target mean resistance; \( \sigma_z \) is the standard deviation of \( G(X) \); and \( \beta_{2T} \) is the target 2M reliability, which is obtained by the 3M method

\[ \beta_{3M} = -\frac{\alpha_{3Z}}{6} - \frac{3}{\alpha_{3Z}} \ln(1 - \frac{1}{3} \alpha_3, \beta_{2M}) \]  
(12)

The inverse function of Equation (12) is expressed as:

\[ \beta_{2T} = \frac{3}{\alpha_{3Z}} \left\{ 1 - \exp \left[ \frac{\alpha_{3Z}}{3} (-\beta_{3T} - \frac{3 \alpha_{3Z}}{6}) \right] \right\} \]  
(13)

\[ \alpha_{3Z} = \frac{1}{\sigma_Z} \left( \alpha_{3R} \sigma_R^3 - \sum \alpha_{3n} \sigma_{Q_n}^3 \right) \]  
(14)

The steps for determining the load and resistance factors using this method are as follows:

a. Calculate \( \mu_{R0} \) using Eq. (11).

b. Calculate \( \sigma_Z \), \( \alpha_{3Z} \) and \( \beta_{2T} \) using Eq. (5), Eq. (14) and Eq. (13), respectively.

c. Calculate \( \mu_{RT} \) with Eq. (10).

d. Repeat step b with \( \mu_{RT} \). Then with the values of \( \sigma_Z \), \( \alpha_{3Z} \) and \( \beta_{2T} \), calculate \( \alpha_R \) and \( \alpha_{S_i} \) with Eq. (8).

e. Determine the load and resistance factors with Eq. (7).

The shortcoming of the existing 3M method is that one iteration calculation of \( \sigma_Z, \alpha_{3Z} \) and \( \beta_{2T} \) is inevitable. And Equation (13) is complicated. When Equation (12) is used for the calculation of 3M reliability, there is a mathematical limitation in as:

\[ 1 - \frac{1}{3} \alpha_{3Z} \beta_{2M} > 0 \]

which is

\[ \alpha_{3Z} < \frac{3}{\beta_{2M}} \]

3. PROPOSITION OF THE NEW METHOD

3.1. Computation Process of the Proposed Method

In order to overcome the shortcomings of Equation (12) and (13), a new model is proposed to replace Equation (12):

\[ \beta_{3M} = \frac{1}{3} \beta_{2M} \left[ 2 + e^{\frac{1}{3} \alpha_{3Z} \left( \beta_{3M} - \frac{1}{\beta_{3M}} \right)} \right] \]  
(15)

Regard \( \beta_{3M} \) and \( \beta_{2M} \) as the target 3M reliability index \( \beta_T \) and target 2M reliability index \( \beta_{2T} \). \( \beta_{2T} \) can be expressed as the inverse function of Equation (15):

\[ \beta_{2T} = 3.8 \left\{ 1 - e^{-\frac{3.8}{\alpha_{3Z}}} \right\} \]  
(16)

For the mathematic inverse function of Equation (15) is inexistent, here Equation (16) is an approximate inverse function, which is used to replace Equation (13). Obviously, Equation (16) is simpler than Equation (13). In existing research Equation (15) is proved more applicable and accurate than the inverse function of Equation (13). Moreover, there is not any mathematical limitation in Equation (15).

The convergence of the two steps recursive optimization Eq. (10) and Eq. (11) is inspected and then the following formula, with better convergence, is proposed to replace Eq. (10) and Eq. (11)

\[ \mu_{RT \text{new}} = \sum \mu_i + \left( \beta_T + \frac{1}{2 \beta_T} \right)^{1.7} \sqrt{\sum \sigma_{S_i}^2} \]  
(17)

The steps for determining the load and resistance factors using the new method are as follows:

1. Calculate \( \mu_{RT} \) using Eq. (17).

2. Calculate \( \sigma_Z, \alpha_{3Z} \) and \( \beta_{2T} \) using Eq. (5), Eq. (14) and Eq. (16), respectively. Then calculate \( \alpha_R \) and \( \alpha_{S_i} \) with Eq. (8).

3. Determine the load and resistance factors with Eq. (7).
4. COMPARISON OF ASCE METHOD, THE EXISTING 3M METHOD, AND THE PROPOSED METHOD

In order to compare the application of the proposed method, the following example is considered (ASCE 7-10, C2.3.6)

\[ G(X) = R - (D + L + S) \]  \hspace{1cm} (18)

where \( R, D, L \) and \( S \) are the resistance, dead load, live load and snow load, respectively.

The load combination is the same as ASCE 7-10, combination 2 of Section 2.3.2. The details (the mean value, coefficient of variation and third moment) of the basic variables are shown in Table 1. Although the distribution types of loads and resistance are not necessary in the calculation of load and resistance factors, the distribution types are given in Table 1 for the Monte-Carlo (MC) simulation.

<table>
<thead>
<tr>
<th>RVs</th>
<th>( \mu_{Qi}/D_n )</th>
<th>( V_i )</th>
<th>( \sigma_{Qi} = \sigma_{Qi}/D_n \cdot V_i )</th>
<th>( \alpha_3i )</th>
<th>( \mu_R/R_n ) or ( \mu_{Qi}/Qi_n )</th>
<th>( Qi_n/D_n )</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>-</td>
<td>0.09</td>
<td>0.27</td>
<td>1.06</td>
<td>Lognormal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D )</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
<td>1.0</td>
<td>1</td>
<td>Normal</td>
</tr>
<tr>
<td>( L )</td>
<td>0.175</td>
<td>0.59</td>
<td>0.103</td>
<td>1.18</td>
<td>0.35</td>
<td>0.5</td>
<td>Gamma</td>
</tr>
<tr>
<td>( S )</td>
<td>0.6874</td>
<td>0.21</td>
<td>0.144</td>
<td>1.14</td>
<td>0.982</td>
<td>0.7</td>
<td>Gumbel</td>
</tr>
</tbody>
</table>

The results of load and resistance factors in different methods are listed in Table 2. The results show that the resistance factors \( \phi \) of three methods are in great agreement. And the results of the live load factor \( \gamma_L \) and the snow load factor \( \gamma_S \) are also close, while the dead load factor \( \gamma_D \) in ASCE is slightly greater than that in existing and proposed 3M method.

<table>
<thead>
<tr>
<th></th>
<th>( \phi )</th>
<th>( \gamma_D )</th>
<th>( \gamma_L )</th>
<th>( \gamma_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASCE method</td>
<td>0.877</td>
<td>1.600</td>
<td>0.598</td>
<td>1.229</td>
</tr>
<tr>
<td>Existing 3M method</td>
<td>0.865</td>
<td>1.439</td>
<td>0.500</td>
<td>1.191</td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.848</td>
<td>1.383</td>
<td>0.481</td>
<td>1.164</td>
</tr>
</tbody>
</table>

In order to compare the accuracy of three methods, with the load and resistance factors in Table 2, MC simulation (100,000 times) is used to calculate the reliabilities. For MC simulation, the calculation of \( \mu_R/D_n \) is necessary (e.g. the proposed method):

\[
\mu_R = \frac{\mu_R}{D_n} \cdot \frac{R_n}{D_n} = \frac{\mu_R}{R_n} \cdot \frac{1}{\phi} \left( \gamma_D \cdot \frac{D_n}{D_n} + \gamma_L \cdot \frac{L_n}{D_n} + \gamma_S \cdot \frac{S_n}{D_n} \right) = 3.05
\]

With the values of \( \mu_R/D_n, \mu_D/D_n, \mu_L/D_n \) and \( \mu_S/D_n \), the results of MC simulation are shown in Table 3, reliability 3.01 of the proposed method is closest to the target reliability 3.0. Therefore, the proposed method is considered accurate enough. The existing 3M method is also accurate and safe, but in this method, there are a lot of limitations and iteration is inevitable. The ASCE method is much simple, but the reliability by this method is much greater than the target reliability, which is safe but waste of structural materials.

<table>
<thead>
<tr>
<th></th>
<th>ASCE method</th>
<th>Existing 3M method</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>3.53</td>
<td>3.08</td>
<td>3.01</td>
</tr>
</tbody>
</table>
The above example is based on the assumption of $\beta_T = 3$. In ASCE 7-10 Section 1.5.1, building and other structures are classified to four different risk categories based on the risk to human life, health, and welfare associated with their damage or failure by nature of their occupancy or use. For different risk category and damage type, in ASCE 7-10 C1.3.1 the acceptable reliability indexes ($\beta_T = 2.5 - 4.5$) are provided for a 50-year service period. In this paper, $\beta_T = 1.0 - 4.5$ is chose to analyze the application of three different methods.

As shown in Figure 1, load and resistance factors calculated with the existing method, ASCE method and the proposed method are inconsistent. And the difference increases with the increase of the target reliability $\beta_T$. For resistance factor, the difference of three methods is slight.

![Figure 1. Load and resistance factors calculated with different methods for $\beta_T = 1.0 - 4.5$](image)

5. CONCLUSION

a. The proposed method for load and resistance factors is simpler than the existing 3M method. The iteration in the computation of the target mean resistance in the existing 3M method is eliminated in the proposed method.

b. There is no mathematical limitation in the computation of the target reliability in the proposed method, while in the existing 3M method, the mathematical limitation is inevitable.

c. Compared with the ASCE method, the proposed method is considered safe and saving material.

6. REFERENCES


