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Smartphone Based Cycle Slip Detection for Wheeled Mobile Robot Navigation

바퀴형 이동로봇의 항법을 위한 스마트폰 기반 사이클 슬립 검출

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Abstract

Smartphone Based Cycle Slip Detection for Wheeled Mobile Robot Navigation

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With the rapid development of autonomous navigation technology, there is growing interests in precise navigation for a variety of applications. Accordingly, methods for calculating precise positions by utilizing and integrating various sensors have been proposed. Among them, satellite navigation system is the most widely used method for performing navigation in outdoor environment. For cm–level precise navigation, positioning using carrier measurements has been studied and adopted. However, it is necessary to cope with the cycle slip issue when using the carrier measurement, and many studies have been carried out for this purpose. The methods that have been proposed have used low–cost devices for application in various fields and cost reduction, but high cost equipment such as using survey–grade GPS antenna was partially used.
On the other hand, smartphones, which have been widely distributed in recent decades, have become a multi sensor platform with popular usage. Since Google has made it possible to access raw measurements of GPS on smart devices that support Android 7.0 or higher, studies are being actively conducted using GPS measurements of smartphones. Although studies on using carrier measurements of smartphones for precise navigation are being carried out, studies to detect and compensate for cycle slips have not been treated enough.

In this study, in order to implement smartphone-based GPS / INS integrated navigation system with cycle slip detection ability, the low-cost GPS patch antenna and receiver are firstly combined with the IMU of the smartphone. 2D GPS / INS navigation is applied to a small wheeled mobile robot that requires cm precision navigation. The effect of the IMU sensor error to the cycle slip detection performance within 2D navigation context is analyzed, and IMU error calibration scheme is suggested to overcome the limitations caused by the low-cost smartphone and the characteristics of the robot.

As a result, cycle slip detection performance is improved up to 40% compared to conventional modeling/calibration method. With proposed method, as small as 1 cycle slip detection was possible, while using smartphone IMU and additional low-cost GPS receiver. Additionally, the applicability of the proposed technique to smartphone raw GPS measurements is also treated.

**Keywords**: Cycle slip detection, Smartphone IMU, Smartphone raw
GPS measurement, 2D GPS/INS integration, Extended Kalman Filter, INS error calibration

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I. Introduction

I.1. Motivation and background

Wheeled Mobile Robot (WMR) is a simple kind of system, which is usually used with restricted mission area. Many applications using WMR for missions with low dynamics movement. Though it may seem to be relatively simple, most applications using WMR requires centimeter–level high precision navigation. For the reason, most of the outdoor application uses Global Navigational Satellite System (GNSS) for the basic positioning method. Especially for high–precision navigation, high cost GNSS equipment is usually adopted to use positioning method based on carrier phase measurement as it is shown in Figure I–1.
Carrier phase-based positioning can provide centimeter-level high precision navigation solution. However, it requires continuous tracking of carrier phase signal without any discontinuity. If Phase Lock Loop (PLL) of GNSS receiver temporarily loses its lock due to signal block, cycle slip occurs which damages the positioning accuracy. Cycle slip takes places as instantaneous jump in the integer ambiguity term of the carrier phase measurement, and this results in bias error to the measurement, continuously degrading the position solution. Therefore, cycle slip detection must be carried out when using carrier phase measurement-based positioning. This is part of the reason why high cost equipment is used for many applications.

Meanwhile, Google started to provide GNSS measurements from smartphones supporting Android 7.0 or higher since 2016. Previously, only the location calculated from the smartphone’s GNSS chipset was accessible, but as the smartphone raw GNSS measurement became accessible, smartphone-based positioning technology received a lot of attention. However, if a centimeter-level positioning capable of detecting cycle slip can be achieved based on such a widely used smartphone, it can have a tremendous cost efficiency, and the field using it can be further expanded.

In this study, in the process of developing a smartphone navigation system, a study combining the low-cost GPS antenna and receiver with the smartphone’s IMU is conducted before using the
smartphone's raw GNSS measurement. Small sized WMR, which requires precise navigation, was used as test bed. IMU calibration was performed to improve cycle slip detection performance to maintain robust cm-class navigation even with low-cost equipment which tends to be more susceptible to cycle slip.

I.2. Trends of Research

There has been steady research on carrier phase cycle slip detection for a few decades [1, 2]. Various approaches on cycle slip detection was treated, including the ones using multi-frequency receiver [3–5], and the ones single frequency receiver [6–7]. Detection approach using only GPS equipment have some limitations. While multi-frequency method has high performance in detecting cycle slips, it requires expensive multi-frequency receiver. Method using single frequency alone has performance degradation especially for dynamic user due to the low data rate of GPS receiver.

IMU based cycle slip detection schemes were also suggested [8–10]. This approach has benefits that it is adoptable to dynamic user using single-frequency GPS receiver. However, those approaches used complicated high-order 3D navigation system for vehicle environment user which has relatively high dynamics. Since WMR
usually has restricted mission area, simplified 2D navigation can be adopted. Even though there was an approach utilizing 2D navigation for pseudolite cycle slip detection [11], it used high pseudolite data rates which is not a common case for GPS.

On the other hand, research is being actively conducted utilizing GNSS measurements on smartphones. Studies that analyze GNSS measurements on smartphones report the impact of duty cycling, which is temporarily turning off GNSS chipset to lower power consumption [12–14]. Thus, most studies are based on pseudorange measurements. This is because the duty cycle is fatal for carrier phase measurements, which requires continuous tracking. Some smartphones do not use duty cycling, so some methods have been proposed to utilize carrier phase measurements [15–20]. However, either it was limited to meter-level navigation using pseudo-range smoothing, or the cycle slip issue was hardly treated.

I.3. Research Contents and Method

This study performs smartphone-based cycle slip detection for precise navigation of WMR. Simple 2D navigation is applied to the characteristics of WMR with low dynamics movement in limited environment. However, because the centimeter-class navigation is
required, the position is calculated by CDGPS, a carrier phase based differential GPS technique. The low-cost patch antenna and receiver were used, and as the use of low-cost equipment is more susceptible to cycle slip, the study on cycle slip detection was conducted.

The cycle slip detection based on GPS/INS integrated is applied to 2D navigation. The cycle slip monitoring value was re-derived from CDGPS context, and the relationship between relative position error was analyzed. Unlike the previous study that used high data rates in 2D navigation for detecting pseudolite cycle slips, cycle slip detection with only a low GPS date rate of 1 Hz is attempted. For this purpose, modeling & calibration of smartphone IMU was conducted.

Through experiments, the effectiveness of the proposed IMU calibration technique is examined. 2D GPS / INS navigation combined with a low-cost antenna, receiver and smartphone IMU. In addition, the effectiveness of the proposed method was verified through the absence of false alarms and the level of monitoring value errors. CDGPS solution quality of smartphone raw GPS measurement is also presented to show the applicability of the presented method to smartphone GPS system.
I.4. Contribution

This study applied and analyzed the cycle slip detection method studied in the complex three-dimensional environment into the two-dimensional environment. The monitoring value equation was re-derived to match the CDGPS navigation and the relationship with the relative position error was confirmed. In addition, it was verified mathematically how the IMU error affects the monitoring value in a two-dimensional environment.

IMU calibration was performed to successfully detect the cycle slip even with a low date rate of 1 Hz. Through experiments, the IMU error was modeled. Dynamic constraints were applied for better estimation of the IMU error. Through experiments, the proposed technique is validated. Finally, one-cycle slip detection was possible while using low-cost GPS equipment and smartphone IMU for simplified 2D navigation.
II. GPS/INS integrated WMR navigation

In this chapter, overall outline of 2D GPS/INS integrated system for wheeled mobile robot (WMR) navigation is described. Brief explanation of WMR is introduced, followed by configuration and features of Global Positioning System (GPS) and Inertial Navigation System (INS). Also, integration of GPS and INS within 2D navigation context is provided.

II.1. Wheeled Mobile Robot System

In this section, simple kind of WMR is introduced. WMR used here is the kind which has two wheels with nonholonomic constraints. This implies that it cannot have velocity in certain direction. Using simple 2-dimensional coordinate, the dynamics of WMR can be expressed as follows:

\[
\begin{bmatrix}
\dot{x} \\ \dot{y} \\ \dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\nu \cos \theta \\ \nu \sin \theta \\ \omega
\end{bmatrix}
\]

\[
x = f(x, u)
\]

\[
x = [x \ y \ \theta]^T, \quad u = [\nu \ \omega]^T
\]
In (II.1), the dynamics of WMR is expressed with velocity input. Using velocity input dynamics, motion of WMR is easily defined. Therefore, WMR manufacturers use this kind of dynamics when designing motion controller. Since this kind of WMR has simple motion characteristic, this system is being widely used for various kind of research purposes; especially for GNC (Guidance, Navigation, and Control) applications. In this research, WMR is used for 2D navigation platform. 2D GPS/INS integrated navigation is utilized for the platform. Since the goal of the research is to implement smartphone-alone navigation system, odometer information, which is normally used with WMR navigation, is not utilized.
II.2. Global Positioning System (GPS)

II.2.1 GPS configuration

GPS is the representative satellite navigation system, which began development in 1970’s and established to start Full Operation Capability (FOC) [21–23]. A satellite navigation system is a system that broadcasts navigation messages from satellites and calculates user position by receiving this signal through a receiver. Satellite navigation system with global capacity is called Global Navigation Satellite System (GNSS), and there are several systems operating globally other than GPS. Even though each system has significant differences if treated in detail, basic principle and configuration are the same. Therefore, the explanation in this paper is based on GPS.

GPS can be classified as three segments: space segment, control segment, and user segment. Space segment includes GPS satellites that transmits the navigation signal and message which are used for position calculation of user. GPS satellites are designed so that at least 4 satellites are visible from every position on the Earth. Navigation message is transmitted through carrier wave, which is set to be L-band frequency by high precision atomic clock of each satellite. Each satellite has its own PRN (Pseudo Random Number) which is uncorrelated to other PRNs so that they are distinguishable.
Control segment consists of master control stations, monitor stations, and ground antennas, which are globally distributed. They play roll of continuous monitoring and updating new information for each satellite so that GPS satellite can transmit navigation signal consistently.

User segment includes GPS antenna, receiver, and processor that can decode and handle the L-band frequency. User segment provides information to calculate position or velocity of user such as
pseudorange measurement, Doppler measurement, carrier phase measurement, navigation message, etc. Depending on the algorithm and quality of the equipment, user segment can provide position solution with accuracy varying from meter-level to centimeter-level.

II.2.2 GPS measurement and error factor

There are three representative measurements provided by GPS system: pseudorange, Doppler, and carrier phase. These measurements provide distance or distance change rate information between the transmitter (GPS satellite) and the receiver (User), which includes various error factors. This information is used for calculating the position and velocity of the user.

Among the three measurements, pseudorange is the most widely used one for positioning. The reason it’s called pseudo-range is because it measures the range between the satellite and user by counting from transmitted time to received time; it naturally includes some error sources other than true range. Considering error factors, pseudorange measurement can be stated as follows:

$$\rho^i = d^i - b^i + B + I^i + T^i + m^i_{\rho} + \epsilon^i_{\rho}$$  \hspace{1cm} (II.2)
where $d$ stands for the distance between satellite and user, $b$ for satellite clock bias, $B$ for receiver clock bias, $I$ for ionospheric delay, $T$ for tropospheric delay, $M$ for multipath error, and $\varepsilon$ for noise of the measurement. The superscript $i$ indicates $i^{th}$ satellite and the subscript $\rho$ pseudorange.

Doppler measurement is the one used for velocity calculation. It measures the distance change rate from the value of Doppler shift, based on the principle of Doppler effect. Similarly, Doppler measurement can be stated including error factors as follows:

$$\dot{\rho}^i = \dot{d}^i - \dot{b}^i + \dot{B} + \dot{I}^i + \dot{T}^i + \varepsilon^i_{\rho} \quad (\text{II.3})$$

All the terms except for multipath error and measurement noise is expressed as time derivative of the terms in (II.2).

Carrier phase measurement is another measurement used for position calculation. Rather than measuring the distance itself, GPS receiver cumulates the value of Doppler shift while phase tracking loop is locked. Therefore, carrier phase includes ambiguity term and this term may be changed, if the receiver temporarily loses the signal and then locked again. Carrier phase measurement with its error factor can be expressed as follows:
\[ \phi^i = d^i - b^i + B - I^i + T^i + m^i + N^i \lambda + \varepsilon^i \phi \]  

(II.4)

where \( N \) is integer ambiguity, \( \lambda \) is wavelength of carrier phase. The subscript \( \phi \) stands for carrier phase. Since carrier phase measurement has smaller multipath error and smaller noise characteristic, it is used for precise navigation. For the case, the integer ambiguity must be resolved. The change of the ambiguity due to previously mentioned short-term signal loss is called ‘cycle slip’ and it also must be carefully treated when implementing carrier phase-based positioning.

II.2.3 Position and velocity calculation

There are many ways to determine position and velocity using GPS measurements. In this study, CDGPS technique, which applies the DGPS technique to the carrier phase measurement, is utilized to perform the cm-level precise navigation. Since the multipath error of the carrier phase is known to be small, it is neglected. Describing the measurement models of (II.4) for the user and reference station are as follows.
\[
\phi_i^r = d_i^r - b_i^r + B_r - I_i^r + T_i^r + N_i^r \lambda + \varepsilon_i^r
\]
\[
\phi_u^i = d_i^u - b_i^u + B_u - I_u^i + T_u^i + N_u^i \lambda + \varepsilon_u^i
\]

where the subscript \( r \) indicates reference station, and \( u \) indicates user. The difference between the satellites and the difference between the receivers is a widely used method to remove the satellite clock error and the receiver clock error, respectively. These differencing schemes are expressed as follows:

\[
\nabla^i_\phi_i^r \equiv \phi_i^r - \phi_i^u = \nabla^i d_r - \nabla^i b - \nabla^i I_r + \nabla^i T_r + \nabla^i N_r \lambda + \nabla^i \varepsilon_r
\]
\[
\Delta_u^i \phi_i^r \equiv \phi_i^r - \phi_i^u = \Delta_u^i d_i^r + \Delta_u^i B - \Delta_u^i I_i^r + \Delta_u^i T_i^r + \Delta_u^i N_i^r \lambda + \Delta_u^i \varepsilon_i^r
\]

where the superscript \( i \) indicates reference satellite, and \( j \) indicates another satellite. Using the difference between satellites and the difference between receivers together eliminates clock errors completely. This method is called the double difference technique. It is expressed as follows.

\[
\nabla^i_\Delta \nabla^j_\phi_i^r \equiv \nabla^i_\phi_i^r - \nabla^i_\phi_j^u
\]
\[
= \Delta_u^i \phi_i^r - \Delta_u^j \phi_j^u
\]
\[
= \nabla^i_\Delta \nabla^j_d - \nabla^i_\Delta \nabla^j b + \nabla^i_\Delta \nabla^j I - \nabla^i_\Delta \nabla^j T + \nabla^i_\Delta \nabla^j N \lambda + \nabla^i_\Delta \nabla^j \varepsilon
\]
In the case of a short baseline, or short distance of the user and a reference station, the double-differenced ionospheric delay and tropospheric delay are almost eliminated. Then the equation is simplified as follows.

\[ i \Delta V_{\phi}^i = \Delta V_{\delta}^i d + \Delta V_{\zeta}^i N \lambda + \Delta V_{\epsilon}^i \]  

(II.8)

In order to compute position through the double-differenced carrier phase, the integer ambiguity term must be resolved. One solution to this problem is to collect several measurements and calculate integer ambiguity and position in batch form at once. Another approach uses pseudorange-based position to start, and then search through bounded search space by comparing each possible solution. However, if the initial position is known, one can easily determine the integer ambiguity. If the user's initial position is known correctly, the distance term of (II.8) can be calculated using the known reference station position and the satellite position calculated from ephemeris. Therefore, to eliminate the effects of noise, n measurements are collected and then determine the integer ambiguity as follows:
\[ \dot{i}_t \Delta V_u^j N = \text{round} \left[ \sum_{k=1}^{n} \left( i_t \Delta V_u^j \phi(k) - i_t \Delta V_u^j d(k) \right) \right] / \lambda \]  

(II.9)

Applying the integer ambiguity obtained in (II.9) to (II.8), the equation can be rearranged for the user's position as follows.

\[
\dot{i}_t \Delta V_u^j \phi = i_t \Delta V_u^j d + i_t \Delta V_u^j N \lambda \\
= \dot{i}_t V^j d_r - \{(r^i - r_u^i) \cdot \hat{e}_u^i - (r^j - r_u^j) \cdot \hat{e}_u^j\} + i_t \Delta V_u^j N \lambda \\
\left( \hat{e}_u^i - \hat{e}_u^j \right) \cdot r_u^i = i_t \Delta V_u^j \phi - i_t \Delta V_u^j N \lambda - i_t \Delta V_u^j d_r + r^i \cdot \hat{e}_u^i - r^j \cdot \hat{e}_u^j 
\]

(II.10)

where \( \mathbf{r} \) indicates the position vector and \( \hat{e} \) indicates the line of sight (LOS) unit vector of the corresponding superscript or subscript. If there are at least four satellites with determined integer ambiguity, it is possible to calculate the user's position through least square and numerical iteration by constructing matrix from (II.10) for each satellite [24].

Doppler measurements are used to calculate the velocity of the user. To remove the error factors, double difference technique is used again.

\[ \dot{i}_t \Delta V_u^j \dot{\rho} = i_t \Delta V_u^j \dot{d} + i_t \Delta V_u^j \epsilon_{\dot{\rho}} \]  

(II.11)
Just like (II.10), the user velocity can be summarized as follows.

\[
^i_r \Delta \nabla^j_u \hat{\rho} = \hat{^i_r \nabla^j_u} \hat{d} \\
= \hat{^i_r \nabla^j_u} \hat{d} - \{(\hat{\mathbf{r}}^i - \hat{\mathbf{r}}^i_u) \cdot \hat{\mathbf{e}}^j_u - (\hat{\mathbf{r}}^j - \hat{\mathbf{r}}^j_u) \cdot \hat{\mathbf{e}}^j_u\}
\]

(II.12)

\[
(\hat{\mathbf{e}}^i_u - \hat{\mathbf{e}}^j_u) \cdot \hat{\mathbf{r}}^i_u = \hat{^i_r \Delta \nabla^j_u} \hat{\rho} - \hat{^i_r \nabla^j_r} \hat{d} + \hat{^i_r} \cdot \hat{\mathbf{e}}^j_u - \hat{\mathbf{e}}^j_u
\]

Here, the LOS vector uses the value obtained when calculating the position, and the velocity of the satellite can be obtained through ephemeris. If there are at least four satellites, it is possible to calculate the user’s velocity \( \hat{\mathbf{r}}^j_u \) through least square and numerical iteration by constructing matrix from (II.12) for each satellite.

### II.3 Inertial Navigation System (INS)

INS is a navigation system developed in the early 1950s by MIT and is widely used for various kind of applications currently. By integrating acceleration and angular velocity measurement, relative position, velocity, and heading can be calculated from INS. If initial pose, i.e. initial position and attitude, is known, absolute positioning is also possible. Unlike GPS which usually has low data rates (1Hz most popularly), INS can provide navigation solution with very high
data rates (usually 100Hz). However, error or noise cumulation is inevitable due to its navigating principle [25–26]. Therefore, the error source of INS should be treated carefully. Usually, it is combined with other system to overcome its limitations.

II.3.1 Coordinate system definition

1) Earth Centered Earth Fixed (ECEF) Frame

As the name implies, ECEF frame is fixed on and rotates with the Earth. Its origin is located at the center of mass of the Earth, and z-axis lies along with the Earth’s rotation axis. Its x-axis is in the equatorial plane directing through the Greenwich Meridian, and y-axis is determined by the right-hand rule [27–28]. Even though ECEF frame is not directly used when 2D navigation is implemented, it is essential for positioning with GPS. For example, position and vector of user or satellite used in section II.2.3 is normally described in EFEC frame.
2) East North Up (ENU) Frame

ENU frame is commonly used navigation frame and is essential for 2D navigation. Since differential GPS (DGPS) technique is used for this research, the origin is located at the position of the reference station unlike common case where the origin of ENU frame is located at the position of user. Similar North East Down (NED) frame is also widely used, and the difference comes from the order of North and East, and Down is opposite direction of Up.
3) Body Frame

Body frame is a coordinate system fixed to wheeled mobile robot, whose center is located at center of mass of WMR. It is assumed that body frame is aligned with sensor frame. Although it is almost impossible to align GPS and INS altogether with the body frame especially for the z-axis, alignment to the horizontal xy-plane is enough for 2D navigation. Note that x-axis is the heading direction of WMR, and y-axis is along with wheel direction to the right.
II.3.2 IMU measurement model

INS can be classified into gimbaled system and strapdown system [26]. Since INS using smartphone is assumed for this research, the scope is fixed on strapdown system.

Inertial Measurement Unit (IMU) sensor which consists of 3-axis accelerometer and 3-axis gyroscope is used in INS. Accelerometer measures specific force and gyroscope measures angular velocity. Assuming other error sources such as scale factor is well calibrated, remaining main error comes from the sensor bias ($b$) and sensor noise ($w$). This can be expressed as follows:
\[
\bar{f} = f + b_f + w_f
\]
\[
\bar{\omega} = \omega + b_\omega + w_\omega
\]

(II.13)

where \( f \) and \( \omega \) indicates true acceleration (specific force) and angular velocity, \( \bar{f} \) and \( \bar{\omega} \) their measurement, respectively. The sensor bias term can be decomposed as constant term, which varies every time the sensor is turned off and turned on but remains constant afterward (random constant), and time varying term which is sometimes referred as bias drift. For the case where bias drift critically affects the navigation solution, it should be estimated and eliminated by filter though modeling and calibration. Sometimes the drift is modeled as 1\textsuperscript{st} order Gauss–Markov process [25]. For simpler modeling, rate random walk modeling is possible. This can be expressed as follows:

\[
\dot{b}_f = w_{bf}
\]
\[
\dot{b}_\omega = w_{b_\omega}
\]

(II.14)

where \( w_b \) implies the noise for bias drift. The measure of how bias drift can get larger is described by standard deviation of the drift noise. Since only 2–axis accelerometer and 1–axis gyroscope is used for simplified 2D navigation, characteristic of sensor noise and
bias drift can be expressed as follows:

\[
\begin{align*}
\text{var}(w_f) &= \sigma_f^2 \\
\text{var}(w_\omega) &= \sigma_\omega^2 \\
\text{var}(w_{hr}) &= \sigma_{hr}^2 \\
\text{var}(w_{ha}) &= \sigma_{ha}^2
\end{align*}
\] (II.15)

II.3.3 2D INS mechanization

Unlike complicated 3D INS mechanization, 2D INS mechanization can be expressed simply as (II.16).

\[
\begin{align*}
\dot{p}_N &= v_N, \quad \dot{p}_E = v_E \\
\begin{bmatrix} \dot{v}_N \\ \dot{v}_E \end{bmatrix} &= \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} \\
\psi &= \omega
\end{align*}
\] (II.16)

where \( p \) is position, \( v \) is velocity, and \( \psi \) is heading in the North–East horizontal plane of NED frame. Subscript \( N \) indicates North–axis, \( E \) East–axis, \( x \) body \( x \)–axis, and \( y \) body \( y \)–axis. Note that heading is measured from North to East since the horizontal plane comes from NED frame. This is shown in Figure II–6.
Since IMU measurement comes from smartphone which is assumed to be aligned with WMR body frame, direction cosine matrix must be multiplied to accelerometer measurements for computing North and East velocity.

II.4 2D GPS/INS integration

II.4.1 2D GPS/INS integration overview

GPS and INS are two different navigation systems, and because
they have opposite advantages and disadvantages, they are often combined to complement each other. GPS is most commonly used in outdoor navigation because even low-cost receivers have an error characteristic that remains constant over time all over the Earth. However, if there are obstructions that block satellites, the position cannot be calculated if the number of visible satellites is less than four. Also, it is vulnerable to jamming because navigation is performed by receiving a propagated signal. In addition, the navigation period is very low, such as 1 ~ 5Hz. On the other hand, in the case of the INS, safe navigation is possible since the navigation is performed independently without transmitting or receiving any signal. Furthermore, position solution with 100 Hz data rate can be provided. However, since the navigation is performed by integrating the measured values, the error accumulates with time and the navigation solution diverges after a long time period.

Combining GPS and INS allows robust navigation complementing for each other's shortcomings. When the navigation is not available in GPS due to lack of visible satellite, the INS can be used for continuous navigation. When the visibility is restored and the navigation is performed in GPS, the error factors of INS can be effectively bounded. The measurements of GPS can be used to estimate and eliminate the previously mentioned INS error. It also has the advantage of detecting and compensating for cycle slip using
the position estimated by INS in the case of navigation using GPS carrier phase measurements. The advantages of GPS/INS integrated navigation are illustrated in the following figure.

![Integration of GPS and INS - Satellite outage](image)

**Figure II-7 Characteristics of GPS/INS integrated navigation**

**II.4.2 Loosely coupled Extended Kalman Filter**

EKF is a method of applying state estimation theory through linearization of nonlinear equations. To do this, nonlinear equation must be derived first. The following nonlinear equations can be obtained by applying the IMU measurement model identified in II.3.2
to the 2D mechanization identified in II.3.3.

\[
\dot{x} = \begin{bmatrix}
\dot{p}_N \\
\dot{p}_E \\
\dot{v}_N \\
\dot{v}_E \\
\dot{\psi} \\
\dot{b}_f \\
\dot{b}_f \\
\dot{b}_{\omega}
\end{bmatrix} = \begin{bmatrix}
v_N \\
v_E \\
(f_x - b_{f_x} - w_{f_x}) \cos \psi - (f_y - b_{f_y} - w_{f_y}) \sin \psi \\
(f_x - b_{f_x} - w_{f_x}) \sin \psi + (f_y - b_{f_y} - w_{f_y}) \cos \psi \\
\alpha - b_{\omega_x} - w_{\omega_x} \\
w_{b_{\omega_x}} \\
w_{b_{\omega_y}} \\
0
\end{bmatrix} = f(x, w)
\]

(II.17)

\[
x = \begin{bmatrix}
p_N \\
p_E \\
v_N \\
v_E \\
\psi \\
b_f \\
b_f \\
b_{\omega}
\end{bmatrix}^T
\]

\[
\omega = \begin{bmatrix}
w_{f_x} \\
w_{f_y} \\
w_{\omega_x} \\
w_{b_{f_x}} \\
w_{b_{f_y}}
\end{bmatrix}^T
\]

Note that for gyroscope, it is assumed that sensor bias drift is negligible. Compared to 3D navigation equation, it only has 8 states and expressed in much simpler way.

For nominal points, (II.17) is assumed to be perfect. From the nominal points, the nonlinear equation can be linearized as follows:

\[
\dot{x}_n = f(x_n, w_n)
\]

\[
\dot{x}_n + \delta \dot{x} = f(x_n + \delta x, w_n + \delta w)
\]

(II.18)

\[
\dot{x}_n + \delta \dot{x} = f(x_n, w_n) + \left. \frac{\partial f}{\partial x} \right|_{x=x_n, w=w_n} \delta x + \left. \frac{\partial f}{\partial w} \right|_{x=x_n, w=w_n} \delta w + h.o.t.
\]
\[
F = \frac{\partial f(x, w)}{\partial x} \bigg|_{x=\hat{x}, w=0}
\]
\[
G = \frac{\partial f(x, w)}{\partial w} \bigg|_{x=\hat{x}, w=0}
\]

\[
\delta \dot{x} = F \delta x + G w
\]  

\[
\frac{\partial f(x, w)}{\partial w} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\cos \psi & \sin \psi & 0 & 0 & 0 \\
-\sin \psi & -\cos \psi & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\frac{\partial f(x, w)}{\partial x} =
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(\bar{f}_s - b_f - w_i) \sin \psi - (\bar{f}_r - b_r - w_i) \cos \psi & -\cos \psi & \sin \psi & 0 \\
0 & 0 & 0 & 0 & (\bar{f}_s - b_f - w_i) \cos \psi - (\bar{f}_r - b_r - w_i) \sin \psi & -\sin \psi & -\cos \psi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

This linearized system is expressed in continuous time. For discretization, Van loan’s algorithm is used.
where $Q$ is power spectral density of the process noise vector $w$.

With this linearized and discretized system, EKF estimates the state including position, velocity, heading, and IMU biases by following order shown in Figure II–8 [11].

\[
C = \begin{bmatrix}
-F & GQG^T \\
0 & F^T
\end{bmatrix}
\]

\[
e_{CM}^c = \begin{bmatrix}
F_2 \\
G_2 \\
F_3
\end{bmatrix}
\]

\[
F_d = F_3^T, W_d = F_3^T G_2
\]
III. GPS cycle slip detection

In this chapter, cycle slip detection & compensation through 2D GPS/INS integration is introduced. Monitoring value for detecting cycle slip is explained and analyzed within 2D navigation context.

III.1 GPS carrier phase cycle slip

As explained previously, cycle slip of GPS carrier phase measurement is the jump on integer ambiguity term when Phase Lock Loop (PLL) loses its lock due to temporary signal block. This results in bias error to carrier phase measurement and defects the positioning accuracy. Figure III–1 shows the position error caused by 1 cycle slip.

Figure III–1 Position error from 1 cycle slip
As shown in the figure above, just 1 cycle slip can result in decimeter-level position error. Therefore, to have consistent centimeter-level position accuracy robustly, cycle slip must be detected and compensated.

### III.2 GPS/INS integrated cycle slip detection

Overall system block diagram of cycle slip detection using GPS/INS integration is shown in Figure III–2.

![Figure III–2 GPS/INS integrated cycle slip detection block diagram](image)

As indicated in the figure, upper bar stands for measurement from
IMU sensor and GPS solutions. Upper hat stands for estimates. Superscript + indicates a posteriori estimates, or estimates after EKF measurement update, and – indicates a priori estimates, estimates before EKF measurement update. From smartphone IMU mounted on WMR, 2-axis accelerometer and 1-axis gyroscope is used for the INS propagation. The propagated states are used to estimate relative position among consecutive GPS epochs, which is used for cycle slip detection. Detailed method for cycle slip detection is explained below.

From (II.7), time difference among GPS epoch can reduce atmospheric errors to negligible size. This results in the following expression:

$$
\Delta_i \left( \hat{\nu} \Delta_i^d \phi \right) \approx \Delta_i \left( \hat{\nu} \Delta_i^d d \right) + \Delta_i \left( \hat{\nu} \Delta_i^d N \right) \lambda + \Delta_i \left( \hat{\nu} \Delta_i^d \varepsilon \right)
$$

(III.1)

where $\Delta_i$ stands for time difference among consecutive GPS epochs.

From (III.1), the distance term of user can be estimated by INS as shown in (III.2).

$$
\Delta_i \left( \hat{\nu} \Delta_i^d \hat{d} \right) \Delta_i \left( \hat{\nu} \Delta_i^d d \right) - \Delta_i \left( \hat{\nu} \Delta_i^d \hat{d} \right)
$$

(III.2)

Estimated distance in (III.2) can be subtracted from (III.1). This gives the following monitoring value equation.
\[
M \triangleq \Delta_t \left( \mathbf{r}_u^i \mathbf{\Delta}_u^{i} \phi \right) - \Delta_t \left( \mathbf{r}_u^i \mathbf{\Delta}_u^{i} \hat{d} \right) \\
= -\Delta_t \left( \mathbf{r}_u^i \mathbf{\Delta}_u^{i} \delta d \right) + \Delta_t \left( \mathbf{r}_u^i \mathbf{\Delta}_u^{i} N \right) \hat{\lambda} + \Delta_t \left( \mathbf{r}_u^i \mathbf{\Delta}_u^{i} \varepsilon \phi \right)
\]  

(III.3)

This cycle slip detection scheme is expressed in the following figure.

![Figure III–3 Cycle slip detection method](image)

Figure III–3 Cycle slip detection method

Time differenced measurement is used, and this implies that incremental amount of double differenced carrier phase measurement and that of estimated distance is compared to detect the amount of cycle slip.
If (III.3) is divided by the carrier wave length $\lambda$, the monitoring value can be expressed in cycle unit. Note that since double differenced carrier phase is used for CDGPS positioning, cycle slip term in (III.3) is also double differenced cycle slip. Every cycle slip mentioned in this thesis afterward will always means double differenced cycle slip.

### III.3 Cycle slip monitoring value analysis

From the bottom righthand side of (III.3), it is shown that the monitoring value consists of distance estimation error term, cycle slip term, and carrier phase measurement noise term. If there is no cycle slip, the cycle slip term is zero. Since carrier phase measurement noise is mm-level, the noise term is considered to be negligible. Therefore, the distance estimation error term is the main source of the monitoring value residual.

Since the double differenced distance estimation error term includes a term related to reference station, it can be simplified as the following equation.

$$
\Delta_i \left( \nabla \Delta_i^j \delta d \right) = \Delta_i \nabla^j \delta d_r - \Delta_i \nabla^j \delta d_u = - \Delta_i \nabla^j \left( \hat{d}_u - d_u \right) \quad (III.4)
$$
Note that $\delta d_i = 0$ since exact position of reference station is assumed to be known. Before considering time difference of the very righthand side of (III.4), just satellite differenced distance estimation error is analyzed as follows:

$$i\nabla^j\left(\hat{d}_i^j - d_i^j\right) = \left(\hat{d}_i^j - d_i^j\right) - \left(\hat{d}_i^j - d_i^j\right) = (\hat{r}_u^i - r_u^i) \cdot e_u^i - (\hat{r}_u^i - r_u^i) \cdot e_u^i = (\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i$$ (III.5)

It is shown that the distance estimation error is related to the position estimation error. Time differencing (III.5) gives (III.6), assuming that LOS vector has little change for consecutive GPS epoch.

$$\Delta_i i\nabla^j\left(\hat{d}_i^j - d_i^j\right) = \Delta_i \left\{(\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i\right\} = \left\{(\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i\right\}_k - \left\{(\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i\right\}_{k-1} = \Delta_i (\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i = \left\{(\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i\right\}_k - \left\{(\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i\right\}_{k-1} \approx \Delta_i (\hat{r}_u^i - r_u^i) \cdot i\nabla^j e_u^i$$ (III.6)

Combining (III-4) and (III-6) gives the following equation.
\[ \Delta_i \left( \iota^j \nabla \Delta^i_u \delta d \right) = - \iota^j \mathbf{e}_u \cdot \delta \Delta_i \mathbf{r}_u \]  

(III.7)

Using (III.7), the monitoring value residual can be expressed as follows:

\[ \delta M = \iota^j \mathbf{e}_u \cdot \delta \Delta_i \mathbf{r}_u + \Delta_i \left( \iota^j \nabla \Delta^i_u \phi \right) \]  

(III.8)

As mentioned before, relative position error among consecutive GPS epoch is the main source of the monitoring value residual. Note that satellite differenced LOS vector can be assumed to be less than \( \sqrt{2} \), since reference satellite will be chosen with the highest elevation angle. Therefore, how small that relative position error can be reduced determines the cycle slip detection performance. If relative position error is too large, so will be the monitoring value residual, and small cycle slips will be hard to detect.

Figure III-4 Monitoring value residual and cycle slip
The size of monitoring value residual affects false alarm probability as well as miss detection probability. For 1 cycle slip detection and compensation, false alarm and miss detection is equally important. Therefore, monitoring value threshold is set to be 0.5 cycle as shown in Figure III–5.

![Figure III–5 False alarm, miss detection, and M.V. threshold](image)

Assuming that monitoring value residual follows the Gaussian distribution, False alarm and miss detection probability can be calculated as follows

\[
P_{FA} = 2 \int_{0.5}^{\infty} f(x \mid 0, \sigma_{MV}^2) dx
\]

\[
P_{MD} = 2 \int_{-\infty}^{0.5} f(x \mid 1, \sigma_{MV}^2) dx
\]

(III.9)

where \( f(x \mid \text{mean}, \text{var}) \) is probability density function following
Gaussian distribution with corresponding mean and variance. $\sigma_{MV}$ is standard deviation of the monitoring value residual in cycle unit. It is clear from (III.9) that smaller $\sigma_{MV}$ will lower both false alarm and miss detection probability.

III.4 Monitoring value residual and GPS Hz

Relative position among consecutive GPS epoch is estimated by propagating INS mechanism from previous GPS epoch. Since data rate of GPS and INS differs, $n$ INS epoch is propagated among consecutive GPS epoch. To analyze how much error is cumulated during the propagation, continuous time system is discretized with appropriate assumptions.

For the purpose of simplicity, it is assumed that bias error of IMU sensor is estimated and eliminated. Then, (II.17) can be simplified as follows:

$$\begin{bmatrix}
\dot{p}_N \\
\dot{p}_E \\
\dot{v}_N \\
\dot{v}_E \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
v_N \\
v_E \\
(f_x - w_{f_x}) \cos \psi - (f_y - w_{f_y}) \sin \psi \\
(f_x - w_{f_x}) \sin \psi + (f_y - w_{f_y}) \cos \psi \\
\dot{\omega}_z - w_{oz}
\end{bmatrix} = f(x, w) \quad (III.10)$$
(III.10) can also be linearized with the same method previously used.

\[
\begin{bmatrix}
\delta p_N \\
\delta p_E \\
\delta v_N \\
\delta v_E \\
\delta \psi
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & \sin \psi \\
0 & 0 & 0 & 0 & \cos \psi \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta p_N \\
\delta p_E \\
\delta v_N \\
\delta v_E \\
\delta \psi
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
-\sin \psi & -\cos \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
w_f \\
w_i \\
w_f \\
w_i \\
w_i
\end{bmatrix}
\]  

(III.11)

\[\delta \mathbf{x} = F \delta \mathbf{x} + G \mathbf{w}\]

(III.11) can be discretized using the linear system theory [29].

\[\delta \mathbf{x}_{k+1} = F_d \delta \mathbf{x}_k + G_d \mathbf{w}_k\]

(III.12)

(III.12) is error propagation for 1 INS epoch. Propagating \( n \) INS epochs for position gives the following equation.

\[
\begin{align*}
\delta p_{N,k+1} &= \delta p_{N,k} + nT \delta v_{N,k} + \frac{1}{2} n^2 T^2 \left(-\bar{f}_i\right) \delta \psi_k - \frac{T^2}{2} \sum_{i=k}^{n+k-1} w_{f_i,j} + \frac{1}{6} T^3 \left(\bar{f}_i\right) \sum_{i=k}^{n+k-1} w_{w_{i,j}} \\
\delta p_{E,k+1} &= \delta p_{E,k} + nT \delta v_{E,k} + \frac{1}{2} n^2 T^2 \left(\bar{f}_i\right) \delta \psi_k - \frac{T^2}{2} \sum_{i=k}^{n+k-1} w_{f_i,j} - \frac{1}{6} T^3 \left(\bar{f}_i\right) \sum_{i=k}^{n+k-1} w_{w_{i,j}}
\end{align*}\]  

(III.13)

In (III.13), it is assumed that body frame is aligned to NED frame for the purpose of simplicity. (III.13) can be expressed as relative position error as the following equation.
\[ |\delta \mathbf{r}_u| = \left| \delta \mathbf{r}_{u,k+n} - \delta \mathbf{r}_{u,k} \right| = \begin{bmatrix} \delta p_{N,k+n}^y \\ \delta p_{E,k+n}^y \\ \delta p_{E,k+n}^z \end{bmatrix} \]

\[ \leq nT \left[ \frac{\delta v_{N,k}^y}{\delta v_{E,k}^y} \right] + \frac{1}{2} n^2 T^2 \left[ \frac{f_y}{f_x} \right] \left[ \delta v_{E,k}^y \right] + \frac{T^2}{2} \sum_{i=k}^{n+k-1} w_{f_{1,i}} + \frac{1}{6} T^3 \left[ \frac{f_x}{f_y} \right] \left[ \sum_{i=k}^{n+k-1} w_{f_{1,i}} \right] \]  

(III.14) shows that relative position error is composed of velocity and heading estimation error at previous GPS epoch, and IMU noise cumulation error. Also, it depends on GPS data rate. If error sources in (III.14) is substituted with the values in Table III–1, bounding value of relative position error can be obtained.

<table>
<thead>
<tr>
<th>Error factors</th>
<th>Velocity error: std ( \delta v_i ) = 0.01 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity, attitude error</td>
<td>std ( \frac{\delta v_{N,k}^y}{\delta v_{E,k}^y} ) = \sqrt{2} \text{ std } \delta v_i = 0.0141 m/s</td>
</tr>
<tr>
<td>heading error: std ( \delta \psi ) = 0.017 rad (1 deg)</td>
<td>[ \frac{f_x}{f_y} ] = 1 [m / s²]</td>
</tr>
<tr>
<td>Inertial sensor noise cumulation error</td>
<td>acc. noise cumulation: var ( \sum_{i=k}^{n+k-1} w_{f_{1,i}} ) = ( 2n \sigma_f ) = 2n × ( 0.02 \text{ [m / s²]} )² Sensor RMS spec.</td>
</tr>
<tr>
<td></td>
<td>gyro noise cumulation: var ( \sum_{i=k}^{n+k-1} w_{g_{1,i}} ) = ( n \sigma_g ) = ( n \times (1.31 \times 10^{-5} \text{ [rad / s]} )² Sensor RMS spec.</td>
</tr>
</tbody>
</table>

Table III–1 Relative position error sources

Using the bounding value and noise characteristic of carrier phase measurement, bounding value of the monitoring value residual is drawn in the following figure.
Figure III–6 shows that as GPS data rate gets faster, monitoring value residual gets smaller. Since 1 Hz GPS rate is the most commonly used one, this study implements cycle slip detection for 1 Hz GPS data rate. This implies that bias estimation and sensor calibration should be treated carefully, since it is assumed that the bias was assumed to be estimated and eliminated well when propagating the error equation.
IV. IMU calibration for enhanced performance

In this chapter, IMU calibration technique is introduced to improve cycle slip detection performance. The IMU calibration overcomes the differences caused by 2D navigation modeling from the actual 3D and the structural problems experienced by the smartphone IMU mounted on the WMR. In addition, the experimental results confirm that the cycle slip detection performance is improved through IMU calibration.

IV.1 IMU calibration

In II.3.2, IMU measurement model was introduced and error sources were examined. In this section, virtual measurement updates are introduced which can be used to reduce IMU errors. Plus, modeling and calibration of IMU errors is conducted, so that IMU errors can be effectively estimated by these virtual measurements.

IV.1.1 Virtual measurement update

For 2D WMR navigation, two kinds of virtual measurement can be used here. The first one is called Zero Velocity Update or simply
ZUPT [30]. ZUPT is applied for zero velocity movement which can be judged by the wheel command of WMR. It updates the north and east velocity to be zero. Measurement equation for ZUPT is expressed as follows:

\[
z_{v}\text{zv} = H_{v}\text{zv}x + v_{\text{zv}}
\]

\[
H_{\text{zv}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}
\]  

(IV.1)

where the subscript \(a\) stands for zero velocity, and \(v\) implies measurement error. Tuning the covariance or sigma value of \(v_{sv}\) determines how strong is the filter going to believe the zero-velocity virtual measurement.

The other virtual measurement is Zero Side Velocity Update. This one uses the nonholonomic constraints of WMR, i.e., that is cannot have velocity along its wheel axis direction. Unlike ZUPT, this update can be used for non-zero velocity movement as well. Measurement equation for Zero Side Velocity Update is expressed in the following equation.

\[
z_{sv} = H_{sv}x + v_{sv} = 0
\]

\[
H_{sv} = \begin{bmatrix} 0 & 0 & -\sin \psi & \cos \psi & 0 & 0 & 0 & 0 \end{bmatrix}
\]  

(IV.2)
where the subscript \( a_{sv} \) stands for side velocity, and \( v \) implies measurement error. Tuning the sigma value for \( v_{sv} \) determines how strong is the filter going to believe the side-velocity virtual measurement.

Unlike GPS measurement, these virtual measurements can be applied for every INS epoch. However, just implementing virtual measurement is not enough. To have best possible performance, IMU error should be modeled and calibrated appropriately.

### IV.1.2 Sensor noise and bias modeling/calibration

As explained in II.3.2, IMU measurement error model consists of sensor bias and sensor noise. Among them, the size of sensor noise is normally expressed in terms of its variance of RMS value. It is well known fact that vibration affects the IMU measurement, so usually the sigma value of sensor noise is set to be larger constant for dynamic case. However, it was confirmed that the characteristic of smartphone sensor is different due to the wheel structure of WMR. During several experiments, it was configured that the noise characteristic increases as the speed of WMR is increased. This is because vibration coming from the wheel is included in the IMU measurement. The following figure shows this for body x-axis
accelerometer.

From the data of Figure IV–1, it was confirmed that the standard deviation of IMU sensor increases linearly with respect to the wheel speed. Since sensor noise exists even when WMR stops, the sigma value of IMU sensor if modeled as 1\textsuperscript{st} order polynomial function of wheel speed. Least square fitting method is used for the modeling. The Speed dependent noise model is known in Figure IV–2.
Figure IV–2 Speed dependent noise model

Figure IV–2 only shows the noise model for accelerometer. Therefore, modeling coefficient is described in Table IV–1.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_f$</th>
<th>$\sigma_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>0.01 [m/s²]</td>
<td>0.08 [deg/s]</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.88 [m/s²/m/s]</td>
<td>4.65 [deg/s/m/s]</td>
</tr>
</tbody>
</table>

Table IV–1 Speed dependent noise model
In II.3.2, it was explained that IMU bias can be modeled with rate random walk to consider the bias drift or time varying bias. For 2D navigation, however, there’s much bigger influence to be considered. It is gravity influence. Even though 2D navigation is used, WMR experiences at least some amount of roll and pitch change. Even 1° change of roll or pitch can result in sensor bias change with almost 0.2m/s², which is very significant to low dynamics WMR.

![Gravity Influence Diagram]

**Figure IV–3 Gravity influence due to pitch change**

Therefore, rate random walk model should consider the gravity influence; the sigma value tuning should be done in a way so that filter can consider the gravity influence as bias drift. Since exact time varying bias reflecting the gravity influence cannot be known, frequency selective Low Pass Filter (LPF) is used to imitate the time varying bias as shown in Figure IV–4.
Figure IV–4 Time varying bias imitation through LPF

Numerical time derivative of imitated bias was conducted to check the noise characteristic of the bias rate.

Figure IV–5 Imitated IMU bias rate
Just as IMU noise was modeled, noise characteristic of IMU bias rate is also modeled linearly to the wheel speed.

![Speed Dependent Model](image)

\[ \sigma_{bf} = d_1 |v| \]

Figure IV–6 Speed dependent bias rate model

Note that for bias rate model, there is no constant term because there will be no roll or pitch change is WMR is stopped.

<table>
<thead>
<tr>
<th>(d_1)</th>
<th>(\sigma_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.7 , [m/s^3/m/s])</td>
<td></td>
</tr>
</tbody>
</table>

Table IV–2 Speed dependent bias rate model
Since gravity does not affect the gyroscope bias, it is modeled as constant.

IV.2 Experimental work and results

Experiments were conducted to verify the performance of cycle slip detection applying the suggested IMU calibration technique. After showing the result using low cost GPS receiver, the applicability of the proposed scheme to smartphone raw GPS measurement is treated.

IV.2.1 Experimental setup

Experimental environment is described in the following table and figure.

<table>
<thead>
<tr>
<th>Location</th>
<th>Seoul National University Building 312 Rooftop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date &amp; Time</td>
<td>Nov. 21\textsuperscript{th}, 2019 (5 min. stop, 2 min. square trajectory)</td>
</tr>
</tbody>
</table>
| Equipment            | GPS : Ublox 6T, Patch antenna – 1 Hz  
                      | IMU : Samsung Galaxy S8 – 100 Hz                          |
| True Reference       | Trimble NetR9 & TBC SW (1~2 cm level)                     |

Table IV–3 Experiment environment
As shown in Figure IV–7, 5 satellite were used with PRN24 to be the reference satellite. DOP is also shown in the figure.

Data of each sensor was collected by PC, and then it was post-processed through MATLAB program. Using high cost Trimble receiver and SW program, true reference position is obtained.
IV.2.2 Experiment results

It was confirmed using the reference position that there was no cycle slip during the experiment. Method applying proposed calibration that considers speed dependent vibration and attitude change is compared with conventional constant noise sigma model. Figure IV–9 shows the relative position error

![INS Predicted Position Error](image)

**Figure IV–9 Relative position error**

With conventional constant sigma noise model, relative position error exceeded cm–level. By applying the proposed method, relative position error was reduced by 35.2%. Since relative position error is closely related to the monitoring value for cycle slip detection, it can
be expected that monitoring value is also going to be reduced through the proposed method. The result is shown in the following figure.

![Monitoring Value](image)

**Figure IV–10 Monitoring value with no cycle slip.**

With conventional model, false alarm occurs for PRN10 and PRN21. If these false alarms are corrected, it will damage the positioning accuracy. Table IV–4 show the monitoring value sigma for each PRN. Every PRN experienced residual reduction, and the residual of PRN21 had largest reduction by 42%.
To verify whether 1 cycle slip is detectable, positive one cycle slip was inserted for PRN 10 and PRN 21. The result is shown in the following figure.

Table IV–4 Monitoring value result

<table>
<thead>
<tr>
<th>Sat.</th>
<th>PRN10</th>
<th>PRN15</th>
<th>PRN20</th>
<th>PRN21</th>
</tr>
</thead>
<tbody>
<tr>
<td>conventional</td>
<td>0.132</td>
<td>0.094</td>
<td>0.078</td>
<td>0.180</td>
</tr>
<tr>
<td>proposed</td>
<td>0.108</td>
<td>0.073</td>
<td>0.063</td>
<td>0.105</td>
</tr>
<tr>
<td>reduction</td>
<td>18.2%↓</td>
<td>22.9%↓</td>
<td>20.4%↓</td>
<td>42.0%↓</td>
</tr>
</tbody>
</table>

Figure IV–11 Monitoring value with cycle slip.

As it is shown in Figure IV–11, proposed method successfully
detects the inserted cycle slips. By taking round the monitoring value, proposed method determines both cycle slip inserted in PRN10 and PRN21 to be 1. Conventional method, on the other hand, still has a few false alarms. By taking round the monitoring value, conventional method determines the cycle slip inserted in PRN21 to be 2. If determined cycle slips are corrected, conventional method will experience several jumps in positioning because of false alarms and wrong determination of cycle slip.

Following figures shows the position result when proposed method is applied. Positioning of cycle slip correction is compared with that of uncorrected cycle slip.

*Figure IV–12 Horizontal position error norm*
It is shown that due to the inserted cycle slip, position jump occurs and WMR cannot maintain cm-level accuracy. However, through the cycle slip detection and compensation, WMR is possible to keep cm-level positioning consistently.

**IV.2.3 Smartphone CDGPS**

Previously presented experiment used low-cost GPS receiver for applying the presented scheme. In this section, the applicability of the proposed method on smartphone GPS system is treated by showing the positioning capability. It can be expected that if positioning capability is good enough, so will be IMU bias estimation quality for cycle slip detection.
Xiaomi Mi8, which is known to have no duty cycle issue, was used for user smartphone. For reference station, high cost Trimble receiver is used. With these equipment, zero-baseline experiment is conducted. A rebroadcaster was used to make zero-baseline possible for the high cost receiver and the smartphone.

GPS raw measurement was collected for 50 minutes. Total 7 satellite was used with PRN26 to be reference satellite. The following figure shows the sky plot for the conducted zero-baseline experiment.

![Sky Plot](image)

**Figure IV–14 Sky plot**

From (II.8), the double differenced distance term is zero, since the base line is zero. Therefore, the integer ambiguity can be
determined using (II.9). Figure IV–15 shows the double differenced carrier phase measurement residual, which can be obtained by subtracting determined ambiguity term from the double difference phase measurement.

![Trimble-Xiaomi Double Diff. Residual](image)

**Figure IV–15 Zero–baseline measurement residual**

Even though there are some unidentifiable jumps in the residual, the result resembles that of previous study [16]. Since the residual varies within about $\pm 0.1$ cycle, it can be expected that CDGPS solution will be good enough for IMU bias estimation, promising sufficient performance for cycle slip detection. The following figure shows the CDGPS solution calculated from the obtained measurement.
The 1σ value for horizontal position is \(8.8[\text{mm}]\), which is quite close to low-cost receiver positioning quality for static zero-baseline experiment. Despite the presence of unknown little jumps in the measurement shown in Figure IV–15, the positioning result showed the possibility of applying proposed method on smartphone GPS measurement.
V. Conclusion and future work

In this paper, smartphone IMU based cycle slip detection for WMR navigation is carried out. 2D GPS/INS integration for GPS cycle slip detection was conducted. The cycle slip monitoring value for CDGPS context was derived, and the relationship between the monitoring value and relative position error was analyzed. How the IMU error affects the monitoring value in a two-dimensional environment was mathematically verified.

Speed dependent IMU noise & bias instability modeling was carried out. This modeling was helpful especially with the use of additional virtual measurements. Proposed method is compared with conventional constant sigma model through experiment. As a result, monitoring value residual is reduced by approximately 42% at maximum. There was no false alarm for suggested method during the experiment. It was able to maintain cm-level positioning through suggested method. Finally, detection ability of 1 cycle slip by low-cost GPS equipment and smartphone IMU with simplified 2D navigation was achievable.

This study utilized low-cost GPS antenna and receiver for showing the experimental result of the proposed scheme. However, the applicability on smartphone GPS system was only treated for static zero-baseline environment. For further study, GPS/INS
integration purely using smartphone measurement is to be conducted. Integration of embedded IMU to Smartphone-based CDGPS for dynamic user should be tested with cycle slip detection capability.
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초 록

자율주행 기술이 급속도로 발전됨에 따라, 다양한 분야에서 정밀측위 및 항법에 대한 관심이 증가하고 있다. 이를 위해 다양한 센서들을 활용하고 통합하여 정밀한 위치를 계산하는 방법들이 제안되고 있다. 실외 환경에서 항법을 수행하는 데에는 위성항법 시스템이 가장 널리 활용되고 있으며, 특별히 cm급의 정밀항법을 위해서 반송파 측정치를 활용한 항법 기술이 많이 연구되고 활용되고 있다. 반송파 측정치를 활용한 항법에서는 반송과 측정치에 존재하는 미지정수에서 사이클 슬립에 대한 대처가 필수적인데, 이를 위해 많은 연구들이 진행되어왔으며 특별히 IMU의 결합을 통해 단일주파수 GPS 수신기만으로도 강건한 사이클 슬립을 검출하는 방법들이 제안되어 왔다. 하지만 그동안 제안되어 온 방법들은 다양한 분야에서의 적용 및 원가 절감을 위해 저가형 장비들을 활용하였지만 survey급 GPS 안테나를 사용하는 등 일부 장비를 고가형으로 사용하였다.

한편, 최근 10년간 널리 보급되어 온 스마트폰은 기술이 발전됨에 따라 여러가지 센서들을 포함하는 하나의 시스템이 되었다. 구글에서는 안드로이드 7.0 이상의 운영체제를 지원하는 스마트 기기에서 GPS의 raw measurement에 접근이 가능하도록 하였고, 스마트폰의 GPS 측정치를 활용하여 항법을 수행하는 연구들이 활발히 진행되고 있다. 정밀항법을 위해 스마트폰의 반송과 측정치를 활용하는 연구들이 진행되고는 있지만, 동적 사용자에 있어서 즉각적으로 사이클 슬립을 검출하고 보상하는 연구는 아직까지는 미비한 상황이다.
본 연구에서는 스마트폰 자체적으로 사이클 슬립 검출이 가능한 GPS/INS 통합항법을 구현하기 위해, 먼저 저가형 GPS patch 안테나와 수신기를 스마트폰의 IMU와 결합하는 연구를 수행하였다. cm급의 정밀 항법을 필요로 하는 작은 바퀴형 이동로봇에 2D GPS/INS 항법을 적용하였다. 기존 제안된 사이클 슬립 검출 기법을 2차원 상황에서 적용하였을 때 IMU 센서 오차와 사이클 슬립 검출 성능의 상관관계에 대한 분석을 진행하였고, 스마트폰 저가 IMU와 로봇의 특성으로 인해 생기는 한계점을 개선하기 위해 IMU 센서에러 모델링 및 calibration 기법을 제안하였다.

결과적으로 기존 모델링/calibration 기법을 사용했을 때 대비 최대 40% 정도 사이클 슬립 검출성능이 향상되었으며, GPS data rate이 높을 수록 유리한 사이클 슬립 검출에 있어서 저가 patch 안테나 및 수신기를 활용하고도 1Hz의 GPS data로 1 사이클 슬립 검출까지 가능하였다. 추가적으로 스마트폰 GPS 측정치에 대한 제안된 기법의 적용 가능성을 다루었다.

주요어: Cycle slip detection, Smartphone IMU, Smartphone raw GPS measurement, 2D GPS/INS integration, Extended Kalman Filter, INS error calibration
학 번: 2018-26372