Ph.D. DISSERTATION

Exploiting Velocity Information to Alleviate Association Uncertainty in Radar Dim Target Tracking

미세표적의 레이더 추적에서 연관 불확실성 완화를 위한 속도정보의 이용

BY

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지도교수 곽 노 준

이 논문을 공학박사 학위논문으로 제출함

2020년 1월

서울대학교 대학원

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이 규 정

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Abstract

The tracking is the problem to recursively estimate the state of the target of interest, based on measurements received from the sensor. This problem is normally formulated using the Bayesian filtering framework, specifically Bayes recursion, which consists of predict and update steps. Novel approaches to implement Bayes recursion have been studied, and these approaches can estimate the optimal state of the target given the noise corrupted target originated measurement. However, in the real-world environment, the challenge arises where the number of targets and measurements is different, so-called the 'association uncertainty.' Thus, two preprocessing stages, i.e., track initiation and data association, form the tracking system with filtering algorithms to distinguish target originated measurements. Most of the conventional approaches of preprocessing stages normally use spatial distance for the association uncertainty.

Recently, new threats of small aerial vehicles such as drones and missiles emerge in the radar tracking system. Such a target has a low SNR due to a small RCS and is called the dim target. The dim target induces more densely distributed measurements in an observation region due to the lowered detection threshold. In this circumstance, the target originated measurement cannot be recognized by using only spatial distance as in conventional approaches. This is because lots of measurements, including the target originated measurement, can be located at the adjacent distance. To address the limitation of conventional approaches for the dim target tracking, this dissertation proposes alternative approaches to exploit additional information for the preprocessing stage of the radar tracking system. The proposed approach formulates the novel algorithm to distinguish target originated measurements based on additional information that can be easily acquired from measurements. Then, the proposed algorithms are implemented to the preprocessing stage along with the conventional method as an ensemble manner to alleviate the measurement origin ambiguity. Velocity information between measure-

ments uses as additional information in this dissertation since it requires only a simple calculation and no extra data from other sensors.

The aim of this dissertation is not only investigating the causality between the number of measurements and the tracking performance but also providing viable methodologies to the association uncertainty in the radar dim tracking system. In support of these goals, the following studies are conducted. Firstly, the cause of the performance degradation of the tracking system in terms of the number of measurements is discussed with experimental demonstrations. The intuition toward exploiting velocity information is presented along with a comparison with the other tracking problem, i.e., visual tracking, that exploits a large amount of information. Secondly, the probabilistic track score is derivated, and the Non-Maximum Suppression(NMS) scheme, which is motivated by a standard approach of the clustering problem, is introduced for the track initiation stage. The weighted score is also proposed using signal intensity of the measurement. The simulation results show that the proposed algorithm can effectively suppress the false target initiation with maintaining the appropriate processing time. Lastly, additional criterions are designed for the data association stage, specifically the measurement validation, using velocity information from the estimated target trajectory. The trajectory based validation gate is set upon the conventional gate based on the newly designed criterion. The performance of the proposed algorithm is demonstrated in various scenarios and shown to outperform conventional approaches in a dense environment.

keywords: Dim Target, Radar, Tracking, Track Initiation, Data Association, Association Uncertainty, Measurement Validation

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Chapter 1

Introduction

1.1 Tracking System Overview

Tracking is the problem to find the current position of the *target of interest* for every sampling period based on the sensor's measurements. Depending on the environment, the target of interest and measurements would have various aspects. For example, the target of interest can be an aerial vehicle, ground vehicle, robot or communication signals and measurements can be acquired from various type of sensors such as radar, sonar, or wireless sensor network, etc[1].

The *filtering*, which is one of estimation problem, can effectively formulate the aforementioned problems. Especially, the *Bayesian filtering* aims to solve the filtering problem on the Bayesian point of view[2]. In the Bayesian filtering framework, the state and measurement are defined as a random vector or set, and the posterior density of the state is recursively estimated by following two steps: prediction and update. The current posterior density of the state is first predicted by using the *Chapman-Kolmogorov*(C-K) equation[3] (prediction step) and then filtered by using measurements under the rationale of the Bayes theorem (update step). This formulation also refers to the *Bayes recursion* which is compatible with the hidden Markov model(HMM) or state-space model(SSM)[4].

One of the representative implementation of the Bayesian filtering is the Kalman filter[5], which is a closed form solution of the Bayes recursion based on linear Gaussian assumption. It allows an accurate estimate to make in the presence of a noise corruption on measurements, even in the outer space. After a successful adaptation to the navigation system of the Nasa's Apollo spacecraft in the 1960s, Bayesian filtering have spanned various domains, disciplines, and applications such as surveillance[6, 7], reconnaissance[8, 9], navigation[10], biomedical engineering[11, 12], autonomous vehicles[13], wireless sensor networks(WSNs)[14], economy[15], mobile devices[16], and security[17].

In case of an ideal environment where the initial state of targets are given, and target originated measurement per each target are solely acquired, Bayesian filtering is able to provide an optimal estimation and applicable to the tracking problem directly. In contrast to the ideal environment, however, the real environment where tracking algorithms would be confronted only a series of measurements are given which are even noisy and indistinguishable. Therefore, generic Bayesian filtering cannot be directly applied to tracking problems for two reasons. Firstly, tracking cannot begin since the initial state of targets is not given. Secondly, even though the initial state of targets is given, a measurement to exploit by the update step is unable to specify due to the *measurement origin ambiguity*. The measurement origin ambiguity describes the circumstance where the target originated measurement is unrecognizable. It occurs when a set of measurements consist not only from the target but also from various sources(e.g., background, sensor's noise). Consequently, thus, the generic tracking problems focus on recursive estimate the posterior density of the state along with how to specify the target of interest and recognize the measurement originated from the target among a set of measurements[18].

In consideration of the measurement origin ambiguity, the generic tracking system consists of three sub-stages: track initiation, data association, and filtering, as shown in Fig.1.1. With regard to the limitations mentioned above, the former corresponding

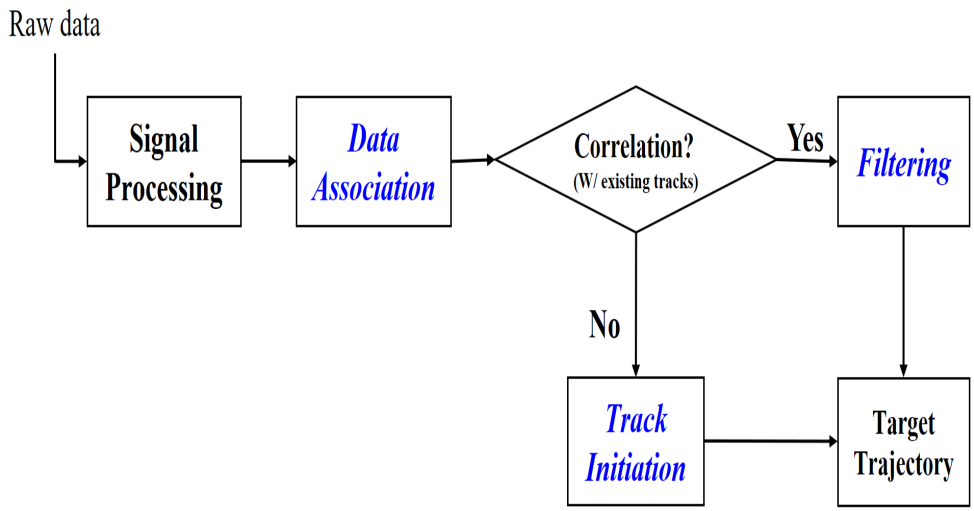


Figure 1.1: Overall procedure of the general tracking framework. In this dissertation, track initiation and data association stages are mainly considered.

to the track initiation stage and the latter corresponding to the data association stage. The brief description of each stage is as follows: First of all, the *track initiation* stage is to determine the target to track including initialization of the state of the target. This stage takes into account only the relationship between measurements regardless of the target being tracked. The measurement combination that satisfies the specific conditions determines as the track. The methods of the track initiation are divided into sequential method and batch method. The sequential method includes a logic-based approach and the batch method includes a Hough transform based approach and a scored based approach. The *data association* stage is to select measurements to use in the update step of Bayes recursion. The measurement is selected based on the likelihood function, which represents the relationship between measurements and the target being tracked. In order to get rid of unnecessary computation, *measurement validation*(or *gating*[1]), which is the process of selecting the validated measurement among

[1]In the literature, measurement validation and gating are used interchangeably, thus this dissertation follows the same usage for convenience purpose.

a set of measurement, is generally preceded. The most well-known algorithms for the data association stage are *probabilistic data association filter*(*PDAF*) and *multiple hypothesis tracker*(*MHT*). Recently, the random finite set(RFS) based filter is studied which to attempt to integrate sub-stages of track system into one. Although the track initiation and data association stage tackle different limitations, both stages share the objective that is finding a reliable association from given measurements. Besides, the *filtering* stage corresponds to the update step of the Bayes recursion and not directly relates to the association.

1.2 Motivation

The *association uncertainty* is the term that represents the measurement origin uncertainty in the association problem point of view[19]. Track initiation and data association stages relate to the association uncertainty in the tracking system. In the track initiation stage, it appears as the inability to determine the true track when multiple track candidates are generated. In the data association stage, the uncertainty represents the ambiguity in the target originated measurement, which is caused by the mismatch between the number of received measurement and the target being tracked. The degree of the association uncertainty relates the probability of selecting the false measurement among a set of received measurement. Intuitively, without prior information of the target, the degree of the association uncertainty can increase as the number of measurement grows. Thus, the significant association uncertainty may cause the initialization of track with false measurement or inaccurate update of the state's posterior density. Consequently, the degradation of tracking performance could be confronted.

The above uncertainty issues also present in the radar tracking system, which is a key sensor for military operations. *Radar*(radio detection and ranging) is a detection system developed during World War II. It acquires the information of the target such as range, azimuth, and velocity by processing the reflected signal from the target after

transmitting an electromagnetic wave. The probability of target detection is affected by the size of the radar cross section(RCS). Thus, targets with large RCS such as civil aircraft has been effectively detected and tracked using conventional tracking algorithms. However, small aerial vehicles such as drones and missiles, which is recent arises as new threats(Saudi Arabia case[20]), has small RCS then conventional targets. Since small RCS induces the low SNR, radar hardly detects compared to the average target. These targets are called the *dim target*[21] In order to detect the dim target, the radar needs to lower the detection threshold. As a result, the number of unexpected measurements(e.g., clutters or spurious measurement) grows, which leads to the emerging of significant association uncertainty. In other words, the parameter calibration to detect small aerial vehicles can cause inaccurate tracking. This phenomenon affects seriously in the military operation and must be addressed. Therefore, this dissertation focuses on performance degradation due to the association uncertainty in the dim target tracking in terms of the radar system.

The cause of performance degradation in a *dense environment*[2] can be explained by how conventional approaches evaluate the association. Since previous approaches solve the association uncertainty based solely on the distance between the target and measurement, the origin of measurement is indistinguishable. In order words, the likelihood of the measurement evaluates based on how close to other measurement or predicted target position. So the spurious measurement can get higher likelihood than target originated measurement. This can lead to update the target's state or initialize new target to track by false measurement. On the contrary, if the accurate discrimination of the target originated measurement is guaranteed, one can estimate the optimal state of the target using Bayes recursion. This point is where the central intuition underlying this dissertation is raised. That is, even though the overall tracking system proceeds under the Bayesian filtering framework, sub-stages related to association

[2]The environment where a massive number of measurement are concentrated due to low detection threshold.

problem focuses on the relationship inference between measurements and targets.

Various studies have been conducted to enhancing the discriminability of the target originated measurement in the dim target tracking. Most of the recent studies are based on multi-sensor fusion or the advanced radar system environments. In [22, 23, 24, 25, 26, 27], measurements from various domain sensors, such as radar, GPS, and vision, are integrated to provide meaningful information for the dim target tracking. In [28, 29, 30, 31], tracking algorithms for the dim target based on an advanced radar system with improved signal processing capability has been studied. Furthermore, in visual tracking, one of the active research domain in the tracking literature, the deep neural network is adopted to extract crucial information from the measurement directly[32, 33, 34]. However, the aforementioned approaches have limitations to apply to conventional radar tracking systems directly. The multi-sensor fusion or advanced radar system requires extra physical systems. Also, the measurement of the conventional radar system is not appropriate to extract deep features due to insufficient information. Nevertheless, exploiting extra information is the most intuitive and practical way to discriminate target originate measurement when significant uncertainty presents. Based on the above consideration, this dissertation proposes approaches to exploit additional information for association uncertainty based on the measurement of conventional radar systems. Thus, novel methods are proposed for track initiation and data association stage, respectively, and both of them are implemented within conventional methods in an ensemble way.

The measurement of the conventional radar system consists typically of position, signal, and Doppler information. Some researches exploit signal intensity and Doppler information for preprocessing stages to resolve the association uncertainty[35, 36, 37]. However, in the dim target tracking, one cannot guarantee that the signal intensity of the target is more prominent than clutters. Moreover, Doppler information can be vanished according to the direction of the target trajectory. The information to be considered in this dissertation is the *velocity information* that can be calculated from the

sequence of measurements, not from the measurement itself. The exploitation of velocity information is based on the deduction that the trajectory would have a specific velocity pattern since the tracking is the problem of estimating the state of the same target, recursively. It means that the proposed method acquires meaningful information of the target from the trajectory and exploits it to the subsequent tracking process. Note that the trajectory accumulates estimations of the state at each sampling time. Furthermore, since the velocity only requires simple calculation, the computational overhead can be minimized.

1.3 Problem

In this section, the problem that this dissertation mainly focuses on and their corresponding assumptions are briefly described. As mentioned in the previous section, this dissertation considers the problem of the association uncertainty in the radar dim target tracking. The association uncertainty relates to two sub-stages of the overall tracking system, i.e., track initiation and data association. Thus, the scope of this dissertation is to establish novel approaches for the "track initiation" and "data association" stages for the association uncertainty in the radar dim tracking system. Since the purpose and detailed process of the two preprocessing stages are different, proposed algorithms are designed separately to conform to the purpose of each stage. Experiments to verify the performance of the proposed algorithm are also conducted, respectively in its individual simulation environment.

The environment considered by this dissertation describes as follows: (1) radar system is required to detect small aerial vehicles('dim target') that have small RCS. (2) Many spurious measurements are received due to the low detection threshold. (3) The degree of the association uncertainty is raised by the number of measurements. The measurement acquired from the conventional radar system includes the following information: position(Cartesian coordinate), signal intensity, and Doppler frequency.

The problem of the track initiation stage is to identify the target originated measurement in the multi-target tracking where Y number of ground truth targets exist. Measurement sets for N consecutive sampling period is given as input data and no target being currently tracked is present. Output data is M measurements that determined to be target originated($N \geq M$). The performance of the track initiation method is evaluated by false track probability(P_{FA}), track detection probability(P_D), and average processing time, respectively. While P_{FA} measures how much the false target suppressed, P_D measures how correctly detect the ground truth target. Average processing time represents the computational overhead of the algorithm. The problem of the data association stage is also to identify the target originated measurement as analogous to the track initiation state. However, the task of this problem is the single-target tracking where a single ground truth target whose initial state is given exists. A set of measurements per each scan is provided as input data. Output data can be either one specific measurement or likelihood(weights) of all measurements according to the estimation strategy. The performance of the data association is evaluated root mean squared error(RMSE), and average processing time. RMSE measures the average distance between ground truth trajectory and estimated trajectory, and average processing time represents the same as the track initiation stage.

This dissertation aims to propose approaches using velocity information to mitigate performance degradation in a dense environment in terms of the association uncertainty. The proposed algorithm is designed in consideration of complement not only conventional methods in an ensemble way but also minimizes computational overhead. The underlying assumptions of the proposed algorithm are described as follows:

Target The target is always detected by the conventional pulse-Doppler radar[38], and generates a single measurement per each targets(point target assumption) except the RFS based filter simulation[3]. The target assumes to be traverse nearly constant velocity. Moreover, if the sampling period sufficiently short, the estimated velocity

[3]In the RFS based filter simulation, a target generates one measurement or not.

Table 1.1: The problem of this dissertation.

Problem	Track Initiation	Data Association
Target	Multi-target	Single-target
Input	Accumulated measurement sets for N consecutive scans	• A set of measurements. • Initial state of the target.
Output	M measurements	• A measurement. • Likelihood of all measurement.
Metric	• False track probability(P_F) • Track detection probability(P_D) • Average processing time	• Root mean squared error(RMSE) • Average processing time
Assumption	• The dynamic of the target is modeled using constant velocity model. • The target traverse is simulated for 100 scans in 2-D space. • Track initiation conducts using measurements during 4 scans($N = 4$). • The measurement[4]is acquired from a conventional pulse-Doppler radar. • The noise of measurement follows Gaussian distribution. • The number of measurements follows Poisson distribution. • The position of measurements follows Uniform distribution.	

of the target remains constant during the track initiation stage. The trajectory of the target simulates for 100 scans in 2-D space. The track initiation stage determines target originated measurement using the accumulated measurement for four scans($N = 4$) while the data association stage estimates the state of the target for 100 scans.

Measurement The measurement of the radar system consists of the following information: position(Cartesian coordinate, (x, y)), signal intensity, and Doppler information. Since the simulation environment omits the signal processing module of the radar system, the measurement of the dim target is modeled by the number of measurements, rather than the signal intensity. The modeling of the measurement needs to

[4]The measurement consists of position(Cartesian coordinate), signal intensity, and Doppler information.

consider three factors: noise, amount, and position. Each modeling factor establishes to follows the probability distribution. The noise is i.i.d random variable that follows Gaussian distribution. The number of measurements follows Poisson distribution and measurements are uniformly distributed in the observation region.

Table. 1.3 summarizes the problem of this dissertation. Note that, this dissertation proposes track initiation and data association stage algorithms, respectively, and experiments are conducted in separate environments.

The rest of this dissertation is organized as follows:

- Section 1.2 of this chapter describes overall tracking procedure and difference aspect of the association uncertainty for each stages. And section 1.3 of this chapter briefly states the contributions of this dissertation.
- Chapter 2 presents the problem environments of this dissertation and reviews conventional Bayesian filtering techniques including track initiation and data association techniques.
- Chapter 3 discusses a variety of the association uncertainty along with experimental demonstrations regarding the number of measurements. The cause and intuitions of the association uncertainty in terms of the number of measurements are suggested with a comparison of other studies. Moreover, recent studies using other information of measurement instead of the position information are introduced.
- Chapter 4 considers the association uncertainty in the track initiation stage. The novel track initiation approaches using the velocity based track score, which is derived in a probabilistic way, are proposed, including the scheme of limiting the number of initiated targets. In addition, the weighted track score is proposed using the signal intensity of the measurement. The proposed algorithm verifies through computer-based simulation.

- Chapter 5 considers the association uncertainty in the data association stage, especially for the measurement validation. The trajectory-based measurement validation approaches are proposed which include two types of criterions: gating and eNDS. The proposed approaches are first verified by the combination of standard Kalman filter and PDAF, then verified by more complex scenario such as a random number of targets(RFS based filter: Bernoulli filter).

- Chapter 6 provides a summarization of the central concept, methodologies, and experimental results then discusses what they stand for. Finally, future research directions are presented based on the limitations that were not addressed in this dissertation.

The remainder of this chapter proceeds as follows. In section 1.2, a outline of each sub-stage of the tracking system and the problem that arises when the number of measurement increases are discussed. In section 1.3 briefly presents the contributions of this dissertation.

1.4 Contributions

This dissertation exploits velocity information for the association uncertainty in the radar dim target tracking. The central thesis of this dissertation is that the exploitation of additional information for the association uncertainty in the radar dim target tracking is an experimentally reliable and practically viable methodology. In support of this thesis, the following fundamental contributions are established in the dissertation:

The first contribution of this dissertation is the experimental interpretation of the association uncertainty in the radar dim target tracking and presenting novel approaches to address using additional information. Specifically,

- Experimental demonstrations are provided to show the relationship between the tracking performance and the number of measurements in various applications

of the association: M/N logic based method, PDAF and Bernoulli Gaussian sum filter. The cause of the performance degradation is interpreted via the analysis of conventional approaches, and novel direction for this challenge is provided with examples from current researches. Further details are given in Chapter 3.

The second contribution is introducing the clustering approaches for resolving the association uncertainty in the track initiation stage. Conventional approaches have no logical scheme for limiting the number of initiated target, and the calculation of conventional track score is required Bayes recursion. Thus,

- the scheme of non-maximum suppression(NMS) is implemented to maintain the number of initiated targets. The priority of tentative tracks is determined by the newly derivated velocity-based track score based on simple conditions of conventional approaches. Furthermore, the weighted track score is proposed to redeem the fact that simple conditions are sensitive to measurement error. The simulation results show that the proposed algorithm can not only effectively suppress the number of false targets but also detect true targets accurately even if the measurement error present. Further details are given in Chapter 4 or the author's published papers[39, 40].

The third contribution is exploiting velocity information from the estimated target trajectory for enhancing the ability to distinguish in the measurement validation. Since conventional approaches evaluate the association between targets and measurements using a distance-oriented metric, the measurement origin ambiguity significantly increases if measurements are densely distributed. To address this issue,

- additional criterions are designed based on accumulated information from the estimated trajectory. The trajectory-based measurement validation is derivated using velocity information. This proposed validation gate is set upon the conventional validation gate as an ensemble manner. The performance of the proposed algorithm is verified in various tracking implementations and shows that

this novel approach not only outperforms conventional methods but also saving the processing time. Further details are given in Chapter 5 or the author's journal article[41].

Chapter 2

Backgrounds

Since this dissertation aimed to address the problem of the association uncertainty in the Bayesian tracking framework, it is important to examine not only details on the association related sub-stage of overall tracking procedure but also the fundamental of Bayesian filtering. Furthermore, it is also required to aware of the circumstances and causes of the problem. In this chapter, various backgrounds that necessary to understand the argument of this dissertation are covered as follows: Section 2.1 describes the environment that this dissertation focuses on. It includes an overview of the radar system and mathematical models for target and measurement. Additionally, the growth of the number of measurement due to the detection of a small target is discussed. Section 2.2 reviews the fundamental of Bayesian filtering and its variation such as Kalman filter, particle filter, and the RFS based filter. In section 2.3 and 2.4, details on conventional methods of the track initiation and the data association stage are presented, respectively.

Note that, the following sections describe with a mathematical formula, so it needs to clarify some notations. Bold upper case refers to a matrix and bold lower case refers to a vector. Others represent a scalar value. Subscripts refer to the index of the sampling period.

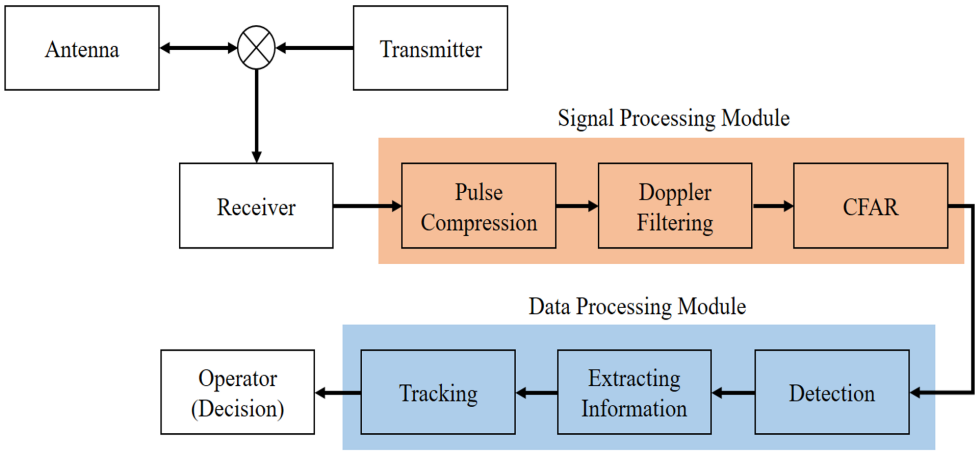


Figure 2.1: The electromagnetic wave received by the antenna is provided to the operator as the measurement after extracting by the signal processing and data processing module.

2.1 Environments

In this section, before the discussion on the tracking procedure, the environment that the tracking system operates is presented. The measurement acquiring process of the radar(sensor) is briefly described, as well as the modeling factors that need to be considered in designing the algorithm. Moreover, the causality between detecting small aerial vehicles and the number of measurement increments also describes.

2.1.1 Radar Systems

This dissertation considers conventional target tracking systems based on the standard radar system. The process of obtaining the measurement in the standard radar system is as follows. Once the transmitter emits the electromagnetic wave, echo signals which reflected from the target or the clutter are received by an antenna and then processed by a signal processing module(e.g., pulse compressing and constant false alarm rate(CFAR), etc.). The preprocessed signal is provided to the operator as the *measurement* after postprocessing by data processing modules(e.g., thresholding and

tracking, etc.) as shown in Fig. 2.1. The measurement represents the information about the source. This information consists of range, azimuth angle, elevation angle, signal intensity, and Doppler frequency shift in case of a 3-dimensional system. Range ρ_k is a distance between the target and the radar, regardless of direction. Azimuth angle θ_k is the antenna beam's angle on the local horizontal plane from a particular reference, such as true north. Elevation angle φ_k is the angle between the radar antenna beam's axis and the local horizontal plane. Signal intensity is the power that is reflected from the target. Doppler frequency is the frequency shift of the echo signal caused by a target's motion with respect to the radar[42, 43].

Assume that the target travels in 2-D space and corresponding clutters or false alarms also places in the same dimension. The position measurements, i.e., ρ_k and θ_k in 2-D polar coordinates are converted to Cartesian coordinates for the tracking system as follows[44]:

$$x_k = \rho_k \cos\theta_k \tag{2.1}$$

$$y_k = \rho_k \sin\theta_k. \tag{2.2}$$

Since this dissertation contains studies to address the intrinsic limitation of the radar system encountered in dealing with new threats, the description focuses on the radar system. Note that, compares the radar system to the sensor from other domains, there are only differences in the way that measurements are acquired, and the rest of the procedure(e.g., the mathematical model of target and measurement) is analogous. Thus, the proposed approaches that will describe in the rest of this dissertation could be applied to the other domain.

2.1.2 Targets

From the sensor point of view, the target can be modeled in two different ways: point target and extended target. The point target, which, if detected, produces only one measurement, focuses on the dynamic behaviors. In contrast, the extended target, which is

represented by a spatial feature, can cause more than one measurement. The extended target model can be formulated under the Bayesian filtering framework. However, it is more widely studied in vision-based trackings such as *radar high-resolution range profile*(HRRP)[45] radar or *visual object tracking*(VOT)[46, 47].

Likewise, target motions are generally classified into two classes: non-maneuver and maneuver[46]. A non-maneuvering target travels in straight and level motion and typically assumes that no thrust of target is engaged. The rest of the targets belongs to the maneuvering target. The motion of the target is represented by the following models: *constant-velocity*(CV), *constant-acceleration*(CA), and *constant-turn*(CT) model. CV model is the simplest model for a target motion and is the so-called white-noise acceleration model. The term implies that accelerations along each axis are modeled as small white noise. CA model is the second simplest model for a target motion and is the so-called Wiener-process acceleration model. It assumes that the acceleration is a process with independent increments. CT model is appropriate for the target in a maneuver. It assumes typically constant speed and turn rate. Note that, in order to verify the central intuition in terms of the number of measurements, this dissertation covers mainly the tracking performance of a simple target model, i.e., non-maneuvering point target with CV.

In the tracking literature, the *dim target* is a general term that represent target which induces weak signal[48, 49, 50, 51]. This term is used interchangeably in various areas such as radar, infra-red(IR) sensor, and sonar. In this dissertation, the dim target also represents the target with a low signal-to-noise ratio. Since no theoretical definition to discriminate against the dim target, however, the experimental environment for the dim target is indirectly set through the number of measurements instead of direct simulation of SNR.

2.1.3 Measurements

The measurement can be acquired from not only the target but also unwanted objects which present in the ground, sea, and atmosphere(rain, birds, and cloud). It also can be generated by internal interference of the sensor, such as thermal noise. In this regard, measurements classify into false measurement or clutter and are processed by different approaches in the signal processing module. However, this dissertation regards to data processing, where the source of measurement is not given. Therefore, in this dissertation, 'false measurement', 'clutter', and 'spurious measurement' are used interchangeably in the sense of the measurement whose origin is not the target for a convenience reason. [1]

The motion of the target is best described in Cartesian coordinates. However, measurements are acquired as in sensor coordinates. Thus, measurements are commonly converted from sensor to Cartesian coordinates, as explained in Sec. 2.1.1. For the experimental purpose, the measurement is generally modeled using the two following factors: First, the number of clutter points(N_c) in each scan k is assumed to have a Poisson distribution as shown in (2.3).

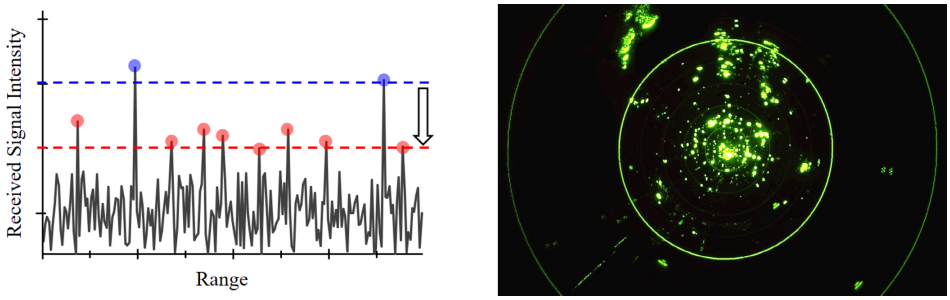
$$p_{N_c}(m) = \frac{\mu_c^m}{m!} \exp(-\lambda) \tag{2.3}$$

where μ_c is the expected number of clutter points. μ_c is determined by the total number of cells (N_{cells}) and the probability of false alarm P_{FA} as shown in (2.4)[52].

$$\mu_c = \lambda V = N_{\text{cells}} P_{\text{FA}} \tag{2.4}$$

where V is volume of the observation region and λ is a spatial density parameter. Second, the location of clutter points at scan k is assumed to have uniform distribution

[1] Likewise, in some studies, 'observation' refers to the echo signal from objects refer to instead of 'measurement'. This term emphasizes the fact that the other object except for the target is not actually measured, rather just received. Since, however, this dissertation does not focus on discrimination between measurement and observation, both terms will use interchangeably for a convenience reason.



(a) Low detection threshold

(b) Cluttered environment[2]

Figure 2.2: Illustration of the cluttered environment example and reason to establish. (a) If the detection threshold is lowered, low observable targets can be acquired but the unwanted measurement comes along. (b) So, measurements are densely distributed in an observation region.

in the observation region. The experiment of this dissertation also follows these factors when modeling the measurement condition.

For further details on the target and measurement modeling the reader is referred to [46, 53, 54]

2.1.4 The cluttered environment

The cluttered environment establishes due to the lower detection threshold, as shown in Fig. 2.2. However, to the best of the author's knowledge, no universal criterion for defining the cluttered environment in terms of the number of measures exists in the literature. Since the criterion is adjustable according to the sensor's processing capability. Instead, in [55], Edde describes three rules for *low observable targets* that can be formulated as follows.

- First, the target can be constructed so that its electo-magnetic impedance of 337 ohm will be absorb rather than reflect illumination energy.

[2]https://www.radartutorial.eu/

- Second, the target can be constructed so that reflected energy is concentrated into a few very narrow beams.
- Third, targets can be constructed to diffuse the energy reflected in the direction of the radar as much as possible*[55]*.

This dissertation aims to resolve the association uncertainty in the environment where the number of measurement grows due to the low detection threshold. The purpose of lowering the threshold is to detect low observable targets, i.e., small aerial vehicles. Thus, in the rest of this dissertation, assuming that the small aerial vehicle follows the rules given in the above quote. Based on this assumption, experiments regarding the association uncertainty in a dense environment are conducted through the performance evaluation according to the number of measurements increases.

2.2 Bayesian Filtering

The fundamental of the filtering theorem considers optimal estimation in the presence of a noisy measurement. That means the problem of the association uncertainty, which is caused by multiple measurements, is indirectly related to the filtering theorem. However, since the overall tracking procedure is established based upon the state-space model from Bayesian filtering framework, fundamental of filtering theorem cannot be overlooked. In other words, it is important to point out the filtering principle because of the track initiation or data association stages work in conjunction with the filtering algorithm. Therefore, in this section we briefly reviews the fundamental of Bayesian filtering and its implementation using Gaussian distribution(i.e. Kalman filter), sequential Monte-Carlo(SMC) methods(i.e. particle filter[56]), and random finite set(RFS)(i.e. Bernoulli filter[47]).

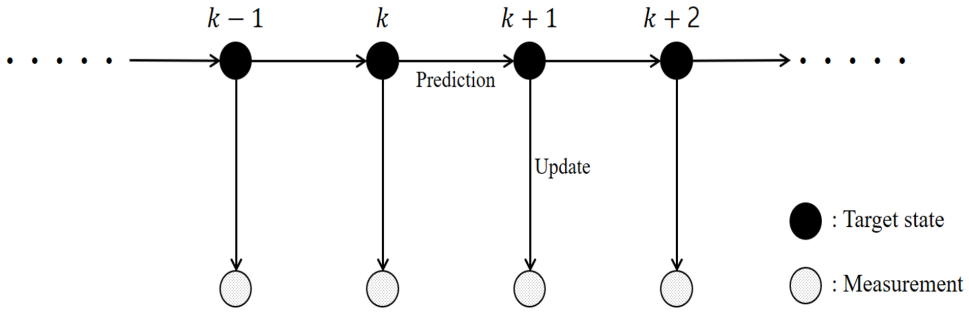


Figure 2.3: Illustration of the state space model. The aim of the filtering is to recursively estimate the information of the state based on the history of measurement up to current time. Prediction and update step is included.

2.2.1 Bayes Filter

In the tracking literature, filtering is a problem that extracts the information about the target of interest at time k by using measurements that acquired up to and including k. At each time k, information about the target of interest is represented by a state vector $\mathbf{x}_k$ in a state space $\chi \subseteq \mathbb{R}^{n_x}$ and measurements obtained from sensors are defined by a vector $\mathbf{z}_k$ with noise[57], where n_x is a dimension of the state space. Generic filtering problem is formulated as a dynamic state-space in the discrete-time domain as shown in Fig.2.3. The purpose of Bayes filter is recursively estimating the state vector $\mathbf{x}_k$ in the discrete-time state-space given the history of measurements $\mathbf{z}_{1:k}$ [2]

The Bayes filter consists of two following equation: state equation and measurement equation. The state equation describes the dynamics of target by using Markov transition.

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{v}_k) \tag{2.5}$$

where $\mathbf{v}_k$ denotes a dynamic noise that is described the uncertainty of the model. Markov transition can be represented by Markov transition density as in eq.2.6, in which case the current state x_k is simplified by eq.2.7 according to the Markov prop-

erty.

$$f_{k+1|k}(\mathbf{x}_{k+1} \mid \mathbf{x}, \mathbf{z}_{1:k}) \tag{2.6}$$

$$f_{k+1|k}(\mathbf{x}_{k+1} \mid \mathbf{x}_k) \tag{2.7}$$

The measurement equation in eq.2.8 describes measurement vector $\mathbf{z}_k$ by using the state vector $\mathbf{x}_k$ and the measurement noise ω_k.

$$\mathbf{z}_k = h(\mathbf{x}_k, \omega_k) \tag{2.8}$$

The relationship between the state and the measurement is represented in probability using the likelihood function $\mathrm{g}(\cdot \mid \cdot)$.

$$\mathrm{g}_k(\mathbf{z}_k \mid \mathbf{x}_k) \tag{2.9}$$

In the Bayesian point of view, the one which recursively estimates is the posterior density of the state $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ [58, 59, 60, 61, 62]. The state and the measurement equation are related to the prediction and update steps as follows, respectively.

Prediction step

In the prediction step, Chapman-Kolmogorov(C-K) equation is used to predict the probability density of the state $p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1})$, which acting as the prior density at time k, based on dynamic model.

$$p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}) = \int \pi_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1}) d\mathbf{x}_{k-1} \tag{2.10}$$

where $\pi_{k|k-1}(\cdot)$ is the transitional density.

Update step

In the update step, the posterior density of the state $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ is estimated according to the Bayes theorem using prior density from the prediction step, and likelihood.

$$p(\mathbf{x}_k \mid \mathbf{z}_{1:k}) = \frac{\mathrm{g}_k(\mathbf{z}_k \mid \mathbf{x}_k) p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1})}{\int \mathrm{g}_k(\mathbf{z}_k \mid \mathbf{x}_k) p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}) d\mathbf{x}_k} \tag{2.11}$$

where, denominator refers to evidence or marginal distribution. The recursive process between the prediction and update step is called the Bayes recursion.

Optimal filtering, that is, extracting optimal state vector from given estimated posterior density p_k at time k is related to a criterion that measures the optimality such as minimum mean-squared error(MMSE), maximum a posteriori(MAP) or maximum likelihood(ML), etc. Details of other criteria for measuring optimality are described in [2]. The Bayes risk of MMSE is the criterion of optimality used for Bayesian filtering. Due to the multiple integrations in the Bayes recursion, however, direct implementation usually intractable in practice. Hence in general, approximation approaches are used in practice, such as (1) sub-optimal estimator via Gaussian assumption(e.g., Kalman filter) and (2) optimal estimator exploiting sequential Monte-Carlo methods(e.g., particle filter). In the following subsections, the aforementioned practical implementation of the Bayes filter is presented. Furthermore, the RFS based filter is reviewed which is a recent study that assumes the number of targets as a random variable.

2.2.2 Kalman Filter

The standard Kalman filter is sub-optimal estimator which fixing the posterior density to take a priori form [63]. In this filter, the Bayes recursion formulates into the closed-form solution under the linear Gaussian assumption. That is, the state transition follows a linear dynamic model and the dynamic and measurement error follows Gaussian distribution. In the Kalman filter framework, one only needs to propagate the mean and covariance of the posterior density for every sampling steps. The dynamic and measurement model of the standard Kalman filter are shown below

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\mathbf{u}_k + \omega_{k-1} \tag{2.12}$$

$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + v_k \tag{2.13}$$

where $\mathbf{F}_{k-1}$ is transition matrix, $\mathbf{H}_k$ is measurement matrix, ω_{k-1} and v_k are process and measurement noises that follow i.i.d. white Gaussian distribution $\mathcal{N}(0, \mathbf{Q}_{k-1})$ and $\mathcal{N}(0, \mathbf{R}_k)$, respectively. Note that, $\mathbf{G}_{k-1}$ and $\mathbf{u}_k$ are terms related to the input signal and are included in the generic filtering problem. However, the input signal to the target usually not given, so that it is discarded in the tracking problem.

Since the Kalman filter operates on the assumption of linear Gaussian case, the transition density and likelihood function are described as follows:

$$f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}) = \mathcal{N}(\mathbf{x}_k; \mathbf{F}_{k-1}\mathbf{x}_{k-1}, \mathbf{Q}_{k-1}) \tag{2.14}$$

$$\mathrm{g}_k(\mathbf{z}_k \mid \mathbf{x}_k) = \mathcal{N}(\mathbf{z}_k; \mathbf{H}_k\mathbf{x}_k, \mathbf{R}_k) \tag{2.15}$$

The closed-form solution for the prediction step at time k are given by computing expectation and variance of the target's state as shown in eq.2.12

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1}^+ + \mathbf{G}_{k-1}\mathbf{u}_k \tag{2.16}$$

$$\mathbf{P}_k^- = \mathbf{F}_{k-1}\mathbf{P}_{k-1}^+\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} \tag{2.17}$$

where, $\mathbf{P}_k$ is error covariance matrix. Note that, superscript $-$ means predicted value of each term.

According to the Bayes theorem, formulations of the update step are given by

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^-) \tag{2.18}$$

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^- \tag{2.19}$$

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T\mathbf{S}_k^{-1} \tag{2.20}$$

$$\mathbf{S}_k^{-1} = (\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{2.21}$$

where $\mathbf{K}_k$ is Kalman gain, $\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^-$ is innovation and $\mathbf{S}_k$ is the innovation covariance. Note that, superscipt $+$ means updated value of each term. The equations for each process of the Kalman filter can be derivated based on the calculation of expectation and variance. See reference for details on derivation[64].

For every recursion, matrix inversion is required to obtain the Kalman gain $\mathbf{K}_k$ as shown in eq.2.20. Thus computational complexity exponentially increases as the dimension of state-space grows. α-β filter[65] exploits constant Kalman gain to avoid iterative calculation of matrix inversion. The standard Kalman filter is inefficient for estimating in the non-linear dynamic environment due to the linear Gaussian assumption. The extended and unscented Kalman filter are the variations of Kalman filter for corresponding the non-linear dynamic environment. Extended Kalman filter(EKF)[66] introduces local linearization approach by Taylor expansion of the transition matrix. Unscented Kalman filter(UKF)[67] applies sampling approach that using the sigma point generated by the unscented transform. Recently, the cubature Kalman filter[68] proposed the cubature rule implementation(e.g., squared root implementation) to improve the efficiency of multi-dimensional integral. Besides, Gaussian sum filter [69] is a method that parallelizes multiple standard Kalman filters in a multiple model manner and estimates states through the weighted sum of each result.

2.2.3 Particle Filter

While a family of the Kalman filter assumes linear Gaussian system where dynamics and measurement models are followed by Gaussian distribution, the particle filter is aimed to estimate true distribution of state's posterior by sequential Monte-Carlo sampling methods[4]. In other words, this approach does not make any explicit assumption about the posterior density form.

In order to estimate a probability distribution, one needs to guarantee a sufficient number of the population(sample) follows the distribution to estimate. However, the evidence of target's posterior density is intractable, so that samples cannot be drawn from the true distribution. Monte-Carlo sampling is the method to approximate the true distribution in this circumstance. Simple technique for implementing the Bayes recursion based on Monte-Carlo methods is sequential importance sampling(SIS) algorithm, also known as particle filter. The particle filter estimates the posterior distri-

bution of the state using a set of a random sample(i.e. particle) with associated weights. The information of the state is extracted through a weighted sum of particles. The importance sampling[70, 71] introduces proposal distribution $q(\cdot)$ that can easily draw samples and is also similar to $p(\cdot)$. True distribution $p(x)$ is approximated by the importance sampling as follows:

$$p(x) \approx \sum_{i=1}^{N_s} \bar{\omega}^i \delta(x - x^i) \tag{2.22}$$

where $\bar{\omega}^i$ is a normalized associated weights corresponding random variable x^i and N_s is the number of samples(or particles).

$$\bar{\omega}^i = \frac{\omega(x^i)}{\sum_{j=1}^{N_s} \omega(x^j)} \tag{2.23}$$

$$\omega(x^i) = \frac{p(x^i)}{q(x^i)} \tag{2.24}$$

The samples x^i are drawn from an importance density $q(\cdot)$ which is commonly uniform distribution or Gaussian distribution.

The posterior density is estimated given by

$$p(\mathbf{x}_k \mid \mathbf{z}_{1:k}) \approx \sum_{i=1}^{N_s} \omega_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \tag{2.25}$$

where ω_k^i is associated weights corresponding state $\mathbf{x}_k^i$ at time k. $\mathbf{x}_k^i$ is particles that drawn from importance density

$$\mathbf{x}_k^i \sim q(\mathbf{x}_k^i \mid \mathbf{x}_{k-1}^i, \mathbf{z_k}) \tag{2.26}$$

and associated weights(i.e. importance weight) are updated from the previous step.

$$\bar{\omega}_k^i = \frac{\omega_k^i}{\sum_{j=1}^{N_s} \omega_k^j} \tag{2.27}$$

$$\omega_k^i = \omega_{k-1}^i \frac{\mathrm{g}_k(\mathbf{z}_k \mid \mathbf{x}_k^i) f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1})}{q(\mathbf{x}_k^i \mid \mathbf{x}_{k-1}^i, \mathbf{z}_k)} \tag{2.28}$$

details of derivation of the weight update equation is referred to [56].

The standard SIS particle filter is suffered from its major drawback which is called as a degeneracy problem. Due to this problem, most particles will have a weight of almost zero except a few over time. This caused by the fact that the variance of the importance weights can only be increased over time[72]. The re-sampling method is known as effective approach to reducing the degeneracy problem[60]. This method is applied when the degeneracy of particle weight is significant, and the level of degeneracy is calculated by the effective sample size N_{eff} as follows.

$$N_{eff} = \frac{N_s}{1 + \text{Var}(\omega_k^{*i})} \tag{2.29}$$

where ω_k^{*i} is true weight that represents as

$$\omega_k^{*i} = \frac{p(\mathbf{x}_k \mid \mathbf{z}_{1:k})}{q(\mathbf{x}_k^i \mid \mathbf{x}_{k-1}^i, \mathbf{z}_k)}. \tag{2.30}$$

Since the true weight cannot be compute exactly, however, estimated effective sample size $\widehat{N_{eff}}$ are utilized in practice

$$\widehat{N_{eff}} = \frac{1}{\sum_{i=1}^{N_s} (\omega_k^i)^2} \tag{2.31}$$

where ω_k^i is the normalized weight which computed using eq.2.28. Particles with negligible weight are removed if $\widehat{N_{eff}}$ below the pre-defined threshold through the re-sampling procedure. Then, the weights of eliminated particles are accumulated to the particles that have large weight. The standard particle filter includes the re-sampling procedure and is called the sampling importance re-sampling(SIR) filter. The degeneracy problem also is addressed by auxiliary sampling importance re-sampling filter(ASIR)[73] and regularized particle filter(RPF)[74].

Moreover, various approaches are proposed such as Rao-blackwelliezed PF[75], jump markov system[76], and Gaussian sum particle filter[77]. Some studies have proposed methods that selection of importance density $q(\cdot)$ taking into account that $q(\cdot)$ has a significant impact to approximate the true distribution.

2.2.4 RFS based Filter

Compares to the previous filtering algorithms, the random finite set(RFS) based filter relaxes the assumption to reflect the real-world scenario. In order words, this approach considers the number of the target as a random variable instead of fixed the number of target as in previous approaches. The state of targets and measurements at time k is represented as a random finite set, and *finite set statistic*(FISST)[78] is introduced to formulate the Bayes recursion. The FISST denisty of RFS $\mathbf{X}$ is described given by

$$f(\{\mathbf{x}_1, ..., \mathbf{x}_n\}) = n! \cdot \rho(n) \cdot p_n(\mathbf{x}_1, ..., \mathbf{x}_n) \tag{2.32}$$

where $n \in \mathbb{N}_0$. Despite the FISST density is not probability density, it has been shown to be equivalent to probability density[79]. Thus the FISST density plays the same role as PDF in the Bayes recursion. The RFS includes Bernoulli RFS, Poisson RFS, and so forth.

As in the case of Kalman and particle filter, four simplifications of the RFS based filter have been studied: probability hypothesis density(PHD) filter[80, 81], cardinalized PHD filter[82, 83], Bernoulli filter[47], multi-Bernounlli filter[18].

As inspired by the strategy of constant-gain Kalman filter also called α-β filter the probability hypothesis density(PHD) filter is an approximation of multi-target recursive Bayes nonlinear filter that propagates a first-order multi-target statistical moment(i.e. intensity) called the PHD[84]. The cardinalized PHD filter jointly propagates cardinality distribution and intensity by relaxing the Poisson assumption for the number of targets[85, 86]. The Bernoulli filter is based on Bernoulli RFS, which allows joint detection of the target(i.e. track initiation) and tracking. The posterior density directly propagates instead of intensity and cardinality distribution[87, 88, 89, 90]. The Multi-Bernoulli filter is the extension of the Bernoulli filter for the multi-target problem and introduces the track label in consideration of compatibility to the tracking problem[91, 92, 93, 94, 95]. The common implementation of the RFS based filter is divide into two categories: Gaussian mixture model(GMM), sequential Monte

Carlo(SMC).

The other advantage of the RFS based filter is that external track initiation and data association stages are not required. Thus, in the RFS based filtering point of view, it insists that the explicit data association step can be avoided, so that the consideration of the association uncertainty is unnecessary[96]. This thought based on how the RFS based filter processing. The RFS based filter computes the weight of every combination between measurement and track, which is called hypothesis, every k time. Then, the new target initialized(if necessary), and the state of the RFS is updated after re-arrange the weights by the pruning and merging or the re-sampling. This process analogous to multiple hypothesis tracker(MHT)[97] that accounts for the target birth, death, and merge probability in the formulations. Although there is no explicit evaluation of the data association, the weight re-arrange step considers the association implicitly. Furthermore, the computational complexity of the calculation of weights is high due to the treatment of multi-target densities and multiple integrations. Thus, selecting the relevant measurements among the whole frame(i.e. the measurement validation) can be efficient to the RFS based filter. In this dissertation, the proposed approaches demonstrated on the Bernoulli filter in terms of the measurement validation.

Table 2.1: Summary of various filtering methods.

Filters	Approach	Variation
Kalman	Closed form solution with Gaussian assumption	EKF, UKF, cubature KF
Particle	Sequential Monte Carlo method to approximate true posterior distribution of the state	SIR, ASIR, RPF
RFS based	*FISST* based state estimation of the random number of targets.	PHD, CPHD, Bernoulli

2.3 Track Initiation

The main purpose of the track initiation stage is to determine the target to track from a series of measurement which consists of a measurement sets from N consecutive sampling period. Technically speaking, the track initiation stage operates the outside of the Bayesian tracking framework. However, since the Bayes recursion proceeds from the state information provided via this stage, the track initiation stage is an important contribution to tracking performance. The information on the target is usually not available, so conventional approaches of the track initiation stage mainly focus on the physical relationship between measurements such as velocity, acceleration or angle.

Previous researches are broadly divided into two categories: sequential methods and batch methods. In the following sub-sections, details on two categories including previous works are presented. Note that, general terms and their meanings for the track initiation stage are followed: the measurement combinations being evaluated is denoted as a "tentative track" and a tentative track that satisfies certain criteria is denoted as a "confirmed track", which is passed to the data association and tracking filter stages. A "deleted track" is a tentative track that does not satisfy the criteria, which is deleted from memory.

2.3.1 Sequential Methods

Sequential methods expand tentative tracks every sampling time using pre-defined condition. Then tentative tracks whose survive until N sampling time are assigned to confirmed track, which means initialized. This method is divided into two approaches: rule-based and logic-based.

The rule-based method is a simple heuristic approach defines limitation conditions using velocity, acceleration, and angle which are given below:

$$V_{\text{min}} \leq \left\| \frac{\mathbf{r}_{i+1} - \mathbf{r}_i}{t_s} \right\| \leq V_{\text{max}} \tag{2.33}$$

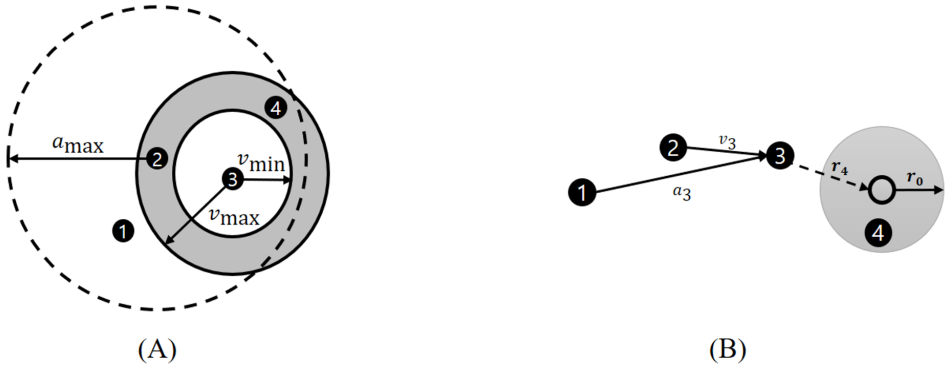


Figure 2.4: Concept of sequential methods of track initiation state. (A) rule based method (B) logic based method. The black circle indicates the measurement and the number inside the circle indicates the sampling index from which the measurement was received. Also, the shaded area represents the region that satisfies the condition. As the sequence progresses, the number of required conditions increases.

$$\left\| \frac{\mathbf{r}_{i+1} - \mathbf{r}_i}{t_s^2} - \frac{\mathbf{r}_i - \mathbf{r}_{i-1}}{t_s^2} \right\| \leq a_{\max} \tag{2.34}$$

$$\left| \cos^{-1} \left[\frac{(\mathbf{r}_{i+1} - \mathbf{r}_i)(\mathbf{r}_i - \mathbf{r}_{i-1})}{|\mathbf{r}_{i+1} - \mathbf{r}_i| \, |\mathbf{r}_i - \mathbf{r}_{i-1}|} \right] \right| \leq \phi_0 \tag{2.35}$$

where, $\mathbf{r}$ is position information and $0 < \phi_0 < \pi$. Note that, The application of each constraint depends on its availability.

The logic-based method is the analogous approach to the measurement validation of the data association. In this method, tentative tracks are expand using measurement which falls into the initiation gate. The initiation gate is set up around the predicted position of tentative tracks, based on velocity and acceleration as follows.

$$\|\mathbf{r}_k - \mathbf{r}^*\| \leq \mathbf{r}_0 \tag{2.36}$$

$$\mathbf{r}^* = \mathbf{r}_j + \mathbf{v} t_s \tag{2.37}$$

$$\mathbf{r}^* = \mathbf{r}_j + \mathbf{v} t_s + \frac{1}{2} \mathbf{a} t_s^2 \tag{2.38}$$

where, $\mathbf{r}_k$ is each validated measurements and $\mathbf{r}_j$ is tentative tracks. $\mathbf{r}$ applies eq. 2.37 and eq.2.38 depending on whether the acceleration is available.

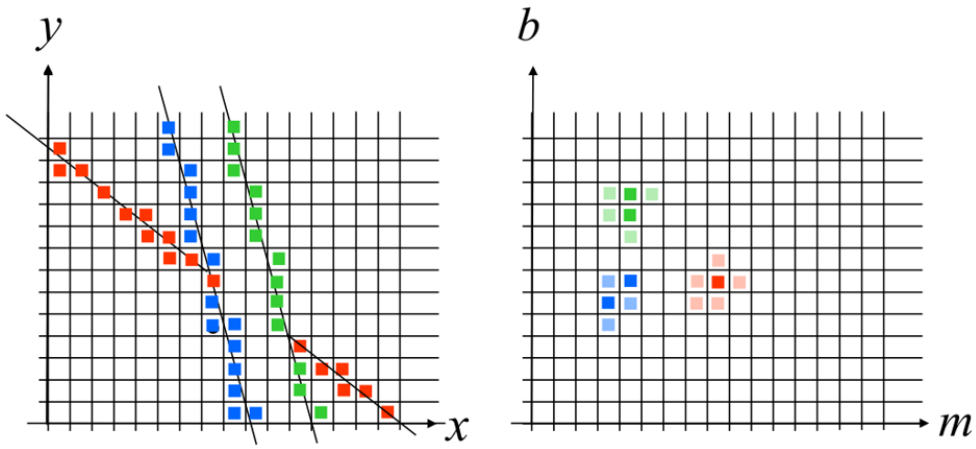


Figure 2.5: Illustration of standard Hough transform.

2.3.2 Batch Methods

The batch method stores a set of measurement from N consecutive sampling time and registers every possible combination of measurement as tentative tracks. Then tentative tracks which satisfy the certain condition are assigned as confirmed track. The condition establishes based on the notion of the Hough transform.

The Hough transform [98] is an algorithm used in the field of computer vision. It is mainly used to find the line or edge of an image. In this algorithm, data points in the image space represented by Cartesian coordinate (x, y) converted into parameter space ρ, θ then find result using a voting scheme. That is, the problem of finding a straight line in $x - y$ space is replaced by finding intersection point in $\rho - \theta$ space as shown in Fig.2.5. The relationship between (x, y) and (ρ, θ) is described as follows:

$$\rho = x \cos\theta + y \sin\theta \tag{2.39}$$

where, ρ is the distance between (x, y) and the origin, and θ is the angle to the normal of the x axis.

The Hough transform based method[99] utilizes the Hough transform that is effi-

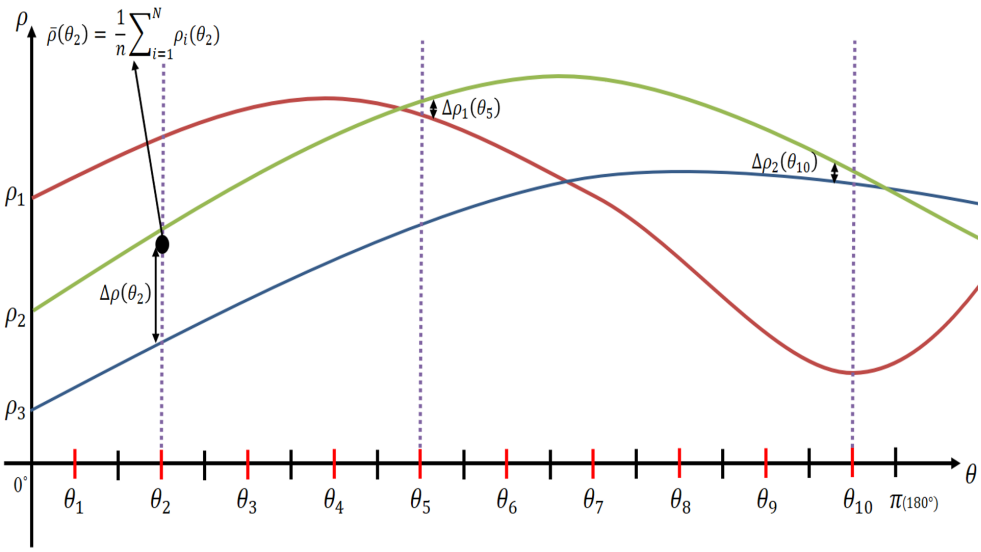


Figure 2.6: Concept of batch methods of the track initiation stage. The standard Hough based method uses ρ as shown on the right side, while the modified method uses θ as shown on the left side.

cient in finding a straight line[3]. This method determines confirmed track in terms of ρ as follows: Firstly, as the way of the batch method, every possible combination of a series of measurement during N scan stores as tentative tracks. The θ axis is then divided into N_θ cells, and the median of each cell is called θ_n.

$$\Delta\theta = \pi / N_\theta \tag{2.40}$$

$$\theta_n = (n - \frac{1}{2})\Delta_\theta, \quad n = 1, 2, \cdots, N_\theta \tag{2.41}$$

Typically, θ is limited to $180°$. Based on Hough transform, each (x_i, y_i) of the tentative track at scan i is converted to ρ_i using the median value of all cells, θ_n. The average of ρ_i at each of θ_n $\bar{\rho}(\theta_n)$, and the maximum deviation of ρ_i from their average $\Delta\rho(\theta_n)$ is evaluated

$$\rho_i(\theta_n) = x_i \cos\theta_n + y_i \sin\theta_n \tag{2.42}$$

[3]In this regard, the Hough transform-based method has advantages in finding the linear motion target.

$$\Delta\rho(\theta_n) = \max\{|\rho_i(\theta_n) - \bar{\rho}(\theta_n)|\} \quad i = 1, 2, \cdots, N \tag{2.43}$$

$$\Delta\rho = \min\Delta\rho(\theta_n), \quad n = 1, 2, \cdots, N \tag{2.44}$$

The tentative track that shows lower $\Delta\rho$ than pre-defined threshold g_0 is determined as confirmed track.

It is reported that compared to sequential methods Hough transform based methods can be effectively suppressed false track probability in a cluttered environment[100]. However, since not only it requires a series of measurement from a sufficient number of the scan but also inherent limitation of batch methods, it generally takes more processing time than the other[101]. Modified Hough transform based method have introduced to address the above limitation.

As similar to Hough transform based method, Modified Hough transform based method[102] also exploit the median value θ_n of each cell but determines confirmed track based on θ. Let each pair of measurements i and $i+1$, one can find $\theta^{(i)}$ which satisfies below relationship,

$$\Delta\rho_i\left[\theta^{(i)}\right] = \rho_{i+1}\left[\theta^{(i)}\right] - \rho_i\left[\theta^{(i)}\right] \tag{2.45}$$

$$= (x_{i+1} - x_i)\cos\theta^{(i)} + (y_{i+1} - y_i)\cos\sin^{(i)} \tag{2.46}$$

$$= 0. \tag{2.47}$$

Each $\theta^{(i)}$ is determined by the θ_n which minimizes $|\Delta\rho_i(\theta_n)|$, then the tentative track is assigned to the confirmed track if the variation of $\theta^{(i)}$ below than the pre-defined threshold ϕ_0.

$$\theta^{(i)} = \text{argmin}\{|\Delta\rho_i(\theta_n)|\} \tag{2.48}$$

$$\left|\Delta\theta^{(i)}\right| = \left|\theta^{(i+1)} - \theta^{(i)}\right| \leq \phi_0 \tag{2.49}$$

As such, by applying the sequential method approach that checks whether the tentative track meets the given condition every scan, the processing time can be saved as compared with the existing Hough transform based method.

In addition to the modified Hough transform based method, many studies have been proposed such as sequential Hough transform[103], partition Hough transform[104, 105], random Hough transform[106] to reduce the amount of computation and required memory capacity.

Table 2.2: Summary of various track initiation methods.

Categories	Methods	Summary
Sequential	Rule	Establishing track gate using $\mathbf{v}$, $\mathbf{a}$, θ
	Logic	Establishing track gate using $\mathbf{v}$, $\mathbf{a}$ using predicted position of measurements.
Batch	Hough transform(HT)	Determine confirmed track according to ρ
	Modified HT	Determine confirmed track according to θ

As the number of measurement increases, the computational requirement for the track initiation stage grows. Furthermore, tentative tracks can be assigned as confirmed tracks without quantity limitation, which leads to performance degeneracy of the tracking system. Details on the relationship between the track initiation stage and the number of measurement will discuss in Chap. 3.

2.4 Data Association

Tracking algorithms require the association of each measurement with the target being tracked to update the state[107]. The data association stage applies to fulfill the aforementioned requirement. The main purpose of the data association stage is providing information on the measurement to update the state's posterior density. The information stands for how relevant the measurement is to the target. This can be an index of a specific measurement or a parameter of each measurement, depends on the optimal estimation approach(refers to section 2.2). Regardless of the type of informa-

tion, the quantities are determined by likelihood function of the measurement given the predicted position of the target.

In the data association stage, the target being tracked exists and their predicted state is given by the Bayes recursion. To reduce the amount of computation, the measurement validation or gating is applied to select only part of the measurement based on the predicted state. The partial measurement selected by the measurement validation is referred to as validated measurements.

Two major approaches to adapt the data association stage into a standard Bayesian filtering framework exist. The probability data association filter (PDAF) is the MMSE approach that calculates the association probability of each validated measurements and updating the state based on this probability. The multiple hypothesis tracker(MHT) is the MAP approach that deferred the decision by maintaining every possible combination between measurements and targets as hypotheses.

In the following, details on conventional methods of the data association stage including measurement validation are discussed. Note that, performance of the data association in a cluttered environment will be discussed in Chap. 3.

2.4.1 Measurement Validation

The observation region means the spatial range that the sensor can detect the target. Theoretically, evaluating the association between the target and the measurement subject to all measurements from the observation region is required. As the number of measurement grows, however, more processing time demands on likelihood computing. The measurement validation is involved to address this circumstance.

In the Bayes recursion point of view, the uncertainty[4] of the state of the target being tracked can be estimated as error covariance. The possible region that the target originated measurement appears is described by the predicted position and error co-

[4]The uncertainty, which is stated here, represents the variance of the state and is different from the association uncertainty that this dissertation mainly considered.

variance of the target. In other words, one can reduce the amount of computation by omitting irrelevant measurement based on the information of the target's state. This approach, called measurement validation, is a methodology that determines the validity of measurements based on the statistical distance between the measurements and the predicted position of the target and is included in the data association process.

Let the predicted position of target is $\bar{\mathbf{z}}_k$ and a set of measurement is $\mathbf{z}$ at time k, function of measurement validation $\mathcal{V}_k(\cdot)$ is formulated as follows,

$$\mathcal{V}_k(\gamma) = \left\{\mathbf{z} : (\mathbf{z} - \bar{\mathbf{z}}_k)^T \mathbf{S}_k^{-1} (\mathbf{z} - \bar{\mathbf{z}}_k) \leq \gamma\right\} \tag{2.50}$$

where γ is the gate threshold corresponding to gate probability P_G which is selected from the chi-squared distribution. Under the assumption of Gaussian noise, the optimal value for the gate threshold γ is the quantile of the upper-tail of a chi-squared distribution with n_z degrees of freedom[108]. Since e.q.2.50 is equivalent to the squared Mahalanobis distance between $\mathbf{z}$ and $\bar{\mathbf{z}}_k$, it is called the normalised distance squared (NDS) criteria. The volume of gate region is declared by

$$\mathcal{V}_k = c_{n_z} \gamma^{\frac{n_z}{2}} \left|\mathbf{S}_k\right|^{\frac{1}{2}} \tag{2.51}$$

where c_{n_z} is denoted the volume of the n_z-dimensional unit hyper-sphere depends on the dimension n_z of the measurement(e.g. $c_1 = 2$, $c_2 = \pi$, and $c_3 = \frac{4\pi}{3}$). In the case of Linear Gaussian systems, ellipsoidal validation gates are known to be optimal and can be easily computed.

Various studies have conducted on the gate size estimation or adaptive gating in a cluttered environment. Details on the previous studies are discussed in Chap. 3.

2.4.2 Nearest Neighbor

Simple and heuristic approaches to solve association uncertainty are nearest neighbor(NN) or Strongest neighbor(SN) filter[109]. Note that, the term filter that is included in the data association algorithm represents the conjunction process of the data

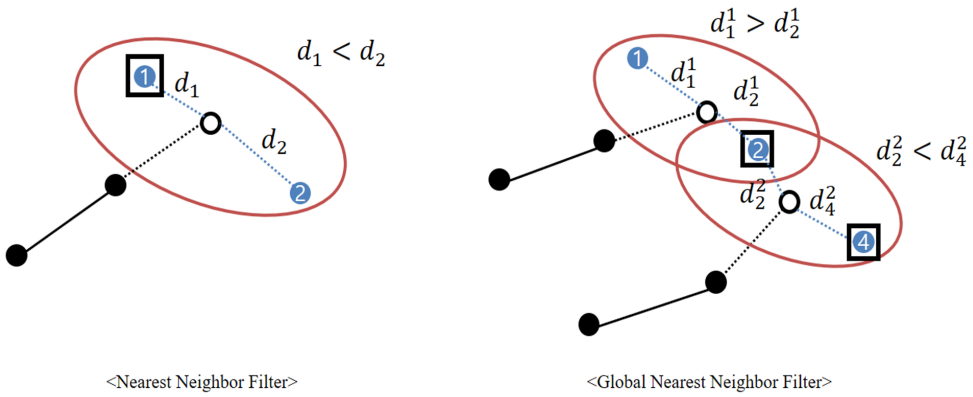


Figure 2.7: Comparison between NNF and GNNF. NNF selects the closest measurement, 1, according to the statistical distance. In contrast, GNNF selects measurements 2 and 4 based on the global constraint, respectively, even though the closest measurement of both targets is 2.

association algorithm with the Bayes filter. The NN filter selects the measurements which the closest distance to the predicted position of the target. The distance is measured by Euclidean distance or Mahalanobis distance as in eq. 2.50. The NN filter has the advantage of fast processing because it only considers distance, but is inefficient in a cluttered environment. While the NN filter is a greedy algorithm, Global NN(GNN) filter finds globally optimal in the multi-target tracking problem. The GNN filter solves the association problem subject to a constraint that allows measurements to be assigned to only one target[108]. Fig.2.7 illustrates the difference between the NN filter and GNN filter. Both tracks have the same nearest measurement 2, but 2 and 4 were selected by GNN's constraint, respectively.

The strongest neighbor(SN) filter have introduced to complement the limitations of the NN filter[110]. The SN filter exploits the signal intensity to either substitute or supplement of the distance of the NN filter. That is, the measurement with the strongest signal intensity is selected or the one with the closest distance whose signal intensity above the threshold. In addition, the probabilistic NN method has been

studied to formulate probabilistic metric without using deterministic distance or signal intensity[111, 112].

2.4.3 Probabilistic Data Association Filter

Probabilistic data association filter(PDAF) is an approach that updates the state of the target based on whole validated measurement instead of selecting one measurement[113, 114, 115]. Each measurement contributes to the update of the state with corresponding weights. The weight for each measurement which refers to *association probability* at time k are computed using the likelihood function $\mathfrak{L}$. The likelihood function evaluates the relationship between the target and measurement which accounts for the distance between the predicted position of the target, and measurement and the probability of false alarm $\mathrm{P_{PA}}$. Underlying assumptions for PDAF application are as follows.

- The number of target in an observation region is given, including the initial state of each target.
- Dynamic and measurement noise follows a zero-mean Gaussian distribution.
- The history of the target through time $k-1$ is summarized approximately by a sufficient statistic in the form of the posterior distribution.
- Only one of the validated measurements can be originated from the target at most, as in the point target assumption.
- The remaining measurements are either clutter or false alarms and are modeled according to the assumptions in section 2.1.

As follow the assumption, that each time $k = 0, 1, 2...$, only one target with the state x_k and the number of the corresponding validated measurement is m_k are present. The update equation of the Kalman filter is modified in consideration of multiple mea-

surements and corresponding association probability as follows

$$\hat{x}_k^+ = \hat{x}_k^- + K_k \sum_{i=1}^{m_k} \beta_i(k) \nu_k^i \tag{2.52}$$

where $\beta_i(k)$ is the association probability and $\nu_k^i = \mathbf{z}_k^i - \mathbf{H}_k \hat{\mathbf{x}}_k^-$ is innovation of ith measurement $\mathbf{z}_k^i$ at time k. Compared with the standard Kalman filter in eq.2.18, the innovation of each measurement is weighted by the corresponding $\beta_i(k)$. The association probability $\beta_i(k)$ is defined as:

$$\beta_i(k) = \begin{cases} \dfrac{\mathfrak{L}_i(k)}{1 - P_D P_G + \sum_{j=1}^{m_k} \mathfrak{L}_j}, & i = 1, \cdots, m_k \\ \\ \dfrac{1 - P_D P_G}{1 - P_D P_G + \sum_{j=1}^{m_k} \mathfrak{L}_j}, & i = 0 \end{cases} \tag{2.53}$$

where, and P_D and P_G denote the detection and gate probability, respectively. $\mathfrak{L}_i$ is the likelihood ratio of $\mathbf{z}_k^i$ originating from the target rather then from the clutter defined by

$$\mathfrak{L}_i \triangleq \frac{\mathcal{N}\left[\mathbf{z}_k^i; \bar{\mathbf{z}}_k, \mathbf{S}_k\right] P_D V_k}{m_k} \tag{2.54}$$

where V_k is the volume of the observation region. Note that, in order to compute the likelihood ratio, the probability of mass function(PMF) of the number of false measurement needs to consider. The PDAF has two versions for PMF: parametric and non-parametric[116]. The non-parametric PMF assumes diffuse prior, that is, information of the observation region regarding the spatial distribution of false measurements is vague. Thus, the spatial density is represented by in forms of $m(k)/V_k$ as shown in eq.2.54. On the other hand, the parametric PDAF assumes the spatial density λ is given, as shown in eq.2.3.

Meanwhile, the error covariance $\hat{P}_k^+$, which represent the uncertainty of the estimation of the target's state is modified as follows:

$$\hat{P}_k^+ = \beta_0(k) \hat{P}_k^- + \left[1 - \beta_0(k)\right] P_k^c + \tilde{P}_k \tag{2.55}$$

where, P_k^c is the covariance of the state updated with the correct measurement

$$P_k^c = \hat{P}_k^- - K_k S_k K_k^T \tag{2.56}$$

and $\tilde{P}_k$ is the spread of the innovation term as shown by,

$$\tilde{P}_k = K_k \left[\sum_{i=1}^{m_k} \beta_i(k)\nu_i(k)\nu_i(k)^T - \nu_i(k)\nu_i(k)^T \right] K_k^T. \tag{2.57}$$

Compare to eq.2.19, the innovation of the validated measurement affects error covariance update. The relationship between the number of validated measurement and error covariance in terms of tracking performance will be discussed later in this dissertation.

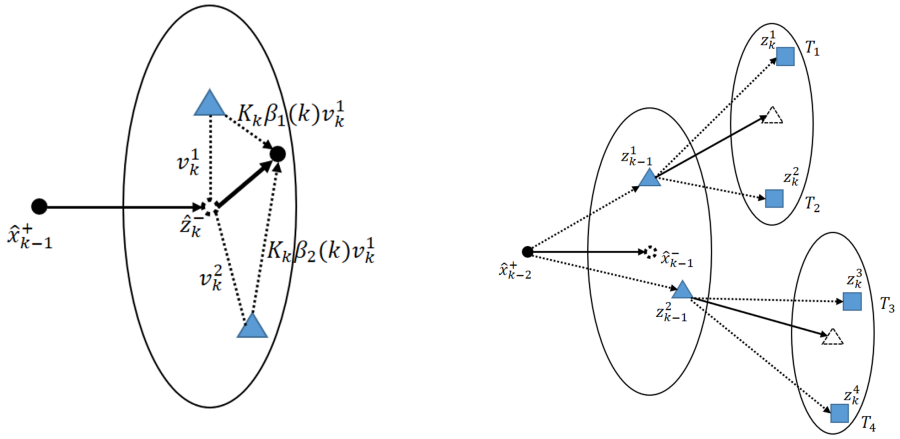
PDAF can also be implemented onto the particle filter. Compare to the KF implementation, in case of the particle filter with PDAF, the state of the target is represented by particles, which is recursively propagated. Thus, the independent weights for each particle compute according to association probabilities [117] as shown below

$$\omega_k^i = \sum_{j=0}^{m_k} \beta_j(k) p(\mathbf{z}_k^i \mid \mathbf{x}_k) \tag{2.58}$$

$$\beta_k^j = \sum_{\theta_k^j \in \theta} P(\theta \mid \mathbf{Z}_k) \tag{2.59}$$

where $p(\mathbf{z}_k^i \mid \mathbf{x}_k)$ is likelihood for ith measurement at time k. Analogous to modification of Kalman filter formulation, the weighted sum according to each association probability is added to the standard formulation in eq.2.25. Note that, as mentioned above, since the RFS based filter omits the explicit data association stage, it is not relevant to PDAF. Fig. 2.8(a) shows the concept of weight calculation and target status update in PDAF.

While PDAF considers the single target tracking problem, joint PDAF(JPDAF) solves the multi-target tracking problem[118]. Since JPDAF is extended from PDAF, it shares the underlying assumption and computes the association probability in a similar way as PDAF. While the association probability in PDAF only describes the relationship between measurement and the target of interest, in the case of JPDAF, it needs to



(a) PDAF

(b) MHT

Figure 2.8: Comparison between PDAF and MHT

compute in consideration of *persistent interference*. The *persistent interference*. represents the circumstance that measurement from one target falls in the validation region of an adjacent target. This leads the additional computation for the association probability. Thus, the number of the initialized target affects the computational complexity.

Variation of (J)PDAF has been widely studied. cheap JPDA[119], near-optimal JPDA[120], maximum-likelihood PDA[121], and sub-optimal JPDA[122] are proposed for reducing the computational burden. Integrated PDAF(IPDAF)[123, 124] integrates track initiation stage using the existence probability along with the association probability. JIPDAF[124] is an extension of IPDAF for multi-target tracking. Moreover, MCMCDA[125] approximates the true posterior distribution using Markov chain Monte Carlo method onto the form of (J)PDAF.

2.4.4 Multiple Hypothesis Tracker

Multiple hypothesis tracker (MHT) is a data association approach based on a probabilistic evaluation of all possible combinations of targets and measurements during a N scan[97]. It is also called as a deferred decision logic that assumes the associa-

tion ambiguity can be resolved through accumulated measurement. The log-likelihood ratio(LLR)[126] exploits as the probabilistic evaluation method.

The likelihood ratio(LR) is the ratio between likelihoods that the measurement originated from the (1) target or from (2) the clutter. The log-likelihood ratio(LLR) is a logarithm of the likelihood ratio. Let H_1 and H_0 are the hypothesis that the measurement originated from the target or the clutter, respectively. LR and LLR are described as follows:

$$\text{LR} = \frac{P(D \mid H_1)P_0(H_1)}{P(D \mid H_0)P_0(H_0)} \triangleq \frac{P_T}{P_F} \tag{2.60}$$

$$\text{LLR} = \ln\left[P_T \mid P_F\right] \tag{2.61}$$

where $P(D \mid H_i)$ is a probability density function that represents the probability that the measurement D will be received according to the hypothesis of H_i and $P_0(H_i)$ represents a priori probability of the hypothesis H_i, respectively.

LLR for each scan is accumulated in the form of track scores $L(k)$. $L(k)$ at time k is formulated in the recursive form as follows:

$$L(k) = L(k-1) + \Delta L(k) \tag{2.62}$$

$$\Delta L(k) = \begin{cases} \ln(1 - P_D) & \text{no update on scan } k \\ \Delta L_u(k) & \text{track update on scan } k \end{cases} \tag{2.63}$$

where P_D is the probability of detection and $L_u(k)$ is the gain in track score upon update.

MHT is divided into two categories: MOMHT and TOMHT. Measurement oriented MHT(MOMHT)[127] stores every measurement-to-measurement combinations during the N scan as hypotheses. Then, the hypothesis whose track score below the threshold is removed(pruning). Finally, the existing track is extended using the remaining hypotheses. On the other hand, track oriented MHT(TOMHT)[128] extends the tracks using every measurement-to-target combination at each scan. Then, the

track whose track score below the threshold is removed(pruning). MHT can effectively track the target but demands high computational requirement. The probabilistic multi-hypothesis tracking (PMHT) have been proposed to address this limitation[129].

Table 2.3: Summary of various data association methods

Methods	Approach	Variation
Measurement validation	Select the subset of measurements according to the statistical distance.	Validation gate
Heuristic	(MAP) Choose the closest measurement to the target according to metric	NNF, SNF
PDAF	(MMSE) Update the state based on whole validated measurement corresponding weights	JPDAF, (J)IPDAF
MHT	(MAP) Deferred decision until sufficient hypotheses accumulated to resolve ambiguity	PMHT

Chapter 3

Association Uncertainty in the Dim Target Tracking

In this chapter, the association uncertainty, which is the central problem of this dissertation is discussed along with an experimental demonstration. In particular, limitation of conventional methods is discussed in the radar dim target tracking point of view. Furthermore, the intuition that exploiting velocity information to the association uncertainty presents based on this discussion.

The association uncertainty occurs when the origin of received measurements ambiguous. This is because a set of measurement consists not only the echo signal from the target of interests but also sensor's noise and background signals[19]. As described in section 2.1, this phenomenon is even aggravated when tracking the target of interest whose radar cross section(RCS) is small such as drones, missiles due to the loose detection threshold. Consequently, the significant association uncertainty causes an increase in the number of initialized false targets and inaccurate updates of the state of the target.

The track initiation and data association stages play a role to discriminate the target originated measurement in the presence of the association uncertainty. However, conventional approaches cannot effectively mitigate the uncertainty in a dense environment due to the insufficient information. That is, since the association is evaluated using only the spatial distance, the target originated measurement cannot be distin-

guished when the number of measurement nearby the predicted position of the target increases. A common approach to address the aforementioned problem is introducing additional information. Recent studies are mostly based on advanced radar systems or multi sensors. These studies assume that proper hardware, which is expensive, implemented. In comparison, this dissertation aims to establish novel methods exploits only the given information of measurements from the conventional radar system. Specifically, velocity information is utilized to formulize proposed algorithms.

This chapter is organized as follows. In section 3.1, the overall tracking procedure, including two preprocessing stages, is described. In section 3.2, various aspect of the association uncertainty for each tracking procedure is described, including experimental demonstration and analysis. Then, in section 3.3, approaches that compensate the conventional method are discussed to maintain the tracking performance even in the presence of the significant association uncertainty by comparison with other tracking problems. Furthermore, recent studies addressing the problem of association uncertainty regarding the track initiation and data association stage using additional information introduces.

3.1 Tracking Procedure Overview

In the case of the radar system, raw signals that are reflected from various objects including target, are processed by signal processing modules. Then converted to the measurement presented by the Cartesian coordinate in the data processing modules. In the real environment where the state information of the target of interest is not given, the overall tracking procedure requires three sub-stages to estimate the state of the target. Fig.3.1 shows an illustration of the overall tracking procedure. A set of measurements is acquired for the entire observation region at each sampling period of the sensor. Afterwards, it is used to update(filtering) the state of the target according to the association with the target being tracked(data association) or for initializing a

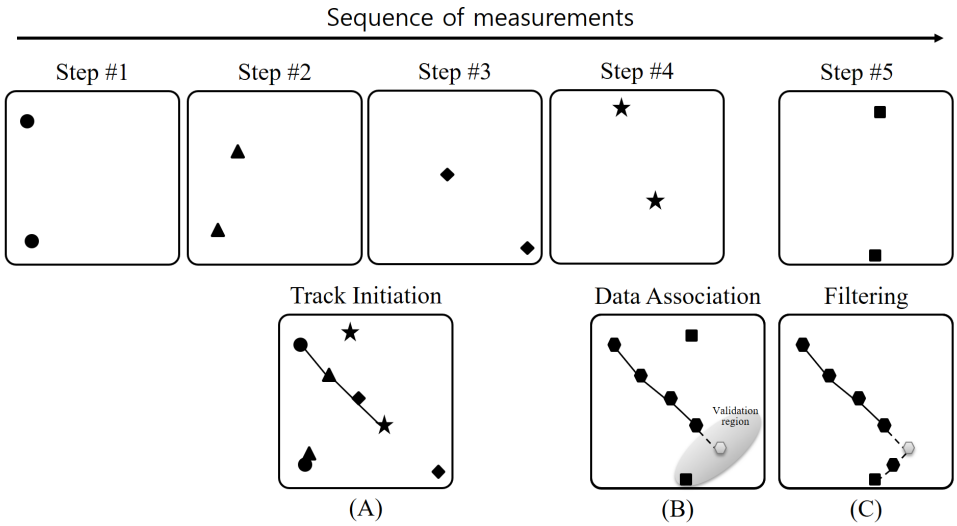


Figure 3.1: Illustration of concepts of operation for each sub-stage of tracking procedure. While no target is being tracked, only the track initiation stage operates until the initialization of the new target to track. (A) If the target to track is initialized through the track initiation stage at step 4, (B) the data association stage is performed for the measurement set of step 5, and finally (C) the state of the target is updated through the filtering stage using the selected measurement. This overall procedure is recursively repeated.

new target to track(track initiation). In the Bayesian filtering point of view, the data association stage involves in between the prediction and update steps, and the filtering step corresponds to the update step. Technically speaking, the track initiation stage operates onto the boundary of the Bayes recursion.

According to each stage of the tracking procedure, the association shows in different aspects. Therefore, the challenge that arises by the association uncertainty also reveals differently for each stages. In this section, outlines on each stage of the tracking procedure and corresponding relationship with the association uncertainty are described.

Track Initiation Since no target is being tracked, the track initiation stage only takes into account the relationship among measurements which is called the measurement-to-measurement association. In this stage, every possible combination of a series of measurements received for a certain period of time is assigned as hypothesis. This hypothesis is referred to as a 'tentative track'. Conventional approaches set up specific conditions based on velocity, angle, and so forth. Subsequently, tentative tracks that satisfy the condition are declared as targets to track. The state of each declared targets is recursively estimated by the data association and filtering stage without no exception until termination. The performance of the track initiation algorithm depends on how accurately the true target initialized and false track suppressed, simultaneously. Note that, the track initiation stage can be interpreted as the problem of finding a meaningful relationship among a set of measurements without any prior information, such as a clustering problem.

Data Association The data association stage considers the relationship between measurement and track(measurement-to-track association) to designate measurements for updating the state of the target. The index or contribution of the measurements is obtained based on the likelihood related to the predicted state of the target. Since the result of the data association stage directly uses to target's state update, accurate tracking cannot be guaranteed if the target originated measurement is not involved. Thus, the performance of the data association algorithm depends on how accurately discriminate between target originated measurement and others. Based on this consideration, this problem can be interpreted as a classification problem. Besides, apart from the accuracy of tracking, the growth of the number of measurements subject to likelihood calculations would increase the processing time.

Filtering In terms of association, the filtering stage plays a role to update the state of the target based on the initialize state from the track initiation stage and measurement information from the data association stage. Hence this stage is not directly related to the association uncertainty, and this dissertation mainly focuses on the track initiation

and data association stage.

3.2 Various Aspect of the Association Uncertainty.

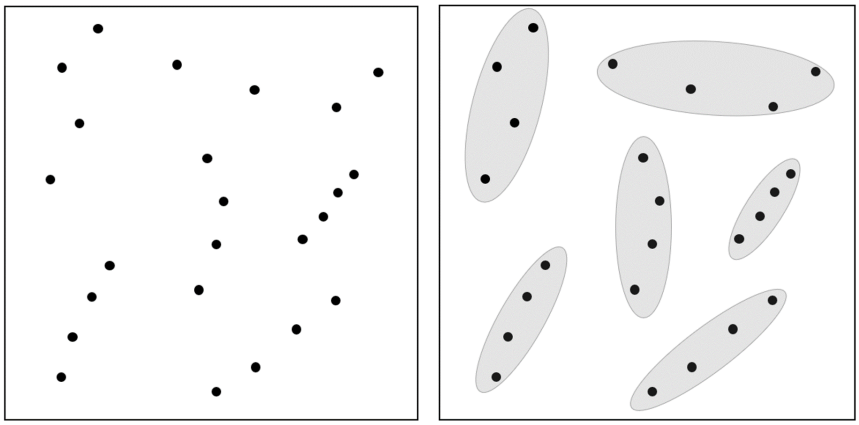
The aspect of the association for each stage of the tracking system is different and can be categorized into three classes as shown in Fig. 3.2[114]: measurement-to-measurement, measurement-to-track, and track-to-track association. While the track initiation stage considers the association among a series of measurement, the data association stage is based on the association between measurement and tracks. Thus, track initiation and data association stages correspond first and second class of association, respectively. The other class relates to the association between tracks which will not discuss in this dissertation.

In contrast to the filtering stage, the objective of the track initiation and data association stage is to find a reliable combination among the given measurements. However, as the aspect of the association is different, the performance degradation for these two stages reveals also differently. While the track initiation stage generates the excessive number of initiated targets, the data association stage fails to track the target. In this section, various aspect of the association is briefly described, and the cause of the performance degradation due to the association is discussed with experimental demonstrations.

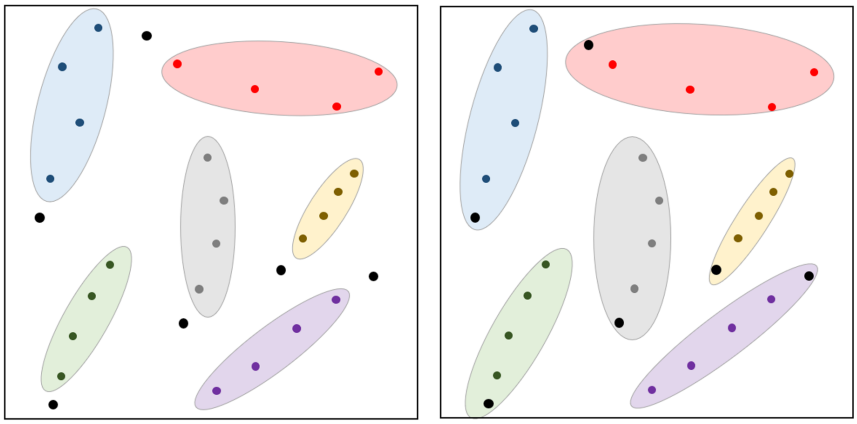
3.2.1 Measurement-to-Measurement

The track initiation stage is based on the concept of measurement-to-measurement association(MTMA). MTMA finds a subset of a series of measurements without prior information, as analogous to the clustering problem, see Fig.3.2(a). In this sense, velocity, acceleration, and angle of the tentative track play a role as a criterion.

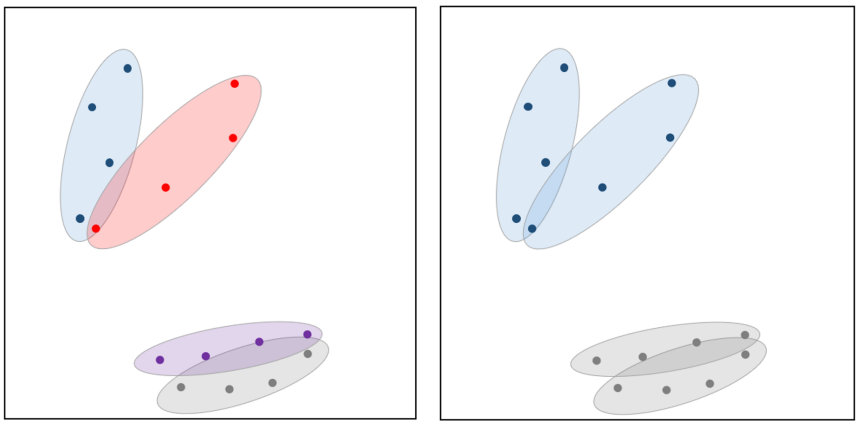
Strictly speaking, since not only no prior information on the number of ground truth target is given but also the number of the initialize target is not limited, the am-



(a) Measurement-to-measurement association

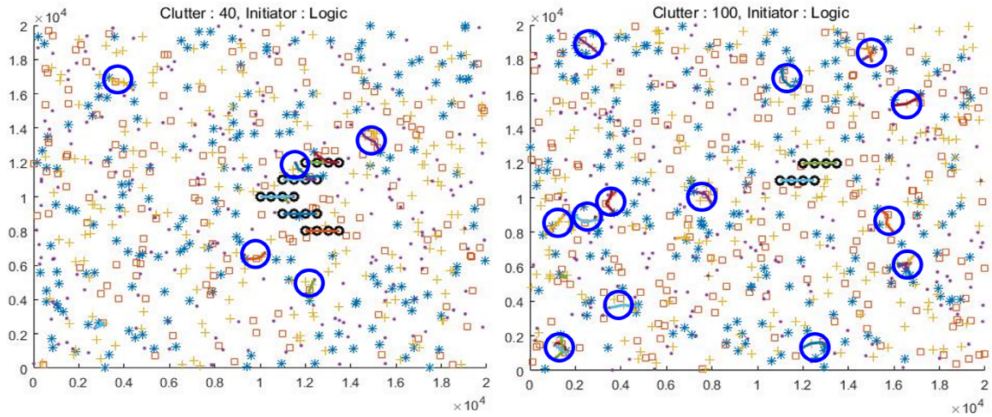


(b) Measurement-to-track association



(c) Track-to-track association

Figure 3.2: Various aspect of the association in the radar tracking system.



(a) The number of expected clutter, 40

(b) The number of expected clutter, 100

Figure 3.3: Comparison of the initialized track between different clutter distribution. Blue circles indicate the initiated tracks. As the number of clutter points increases the number of initiated targets grows.

biguity does not arise in the track initiation stage. In other words, the track initiation algorithms only need to find tentative tracks that satisfy the condition. However, the number of initiated target relates to the computational requirement of the Bayes recursion. The growth of the number of the target to track causes the raise not only processing time of the tracking system but also the probability of tracking false targets. Consequently, the track initiation stage can also lead to performance degradation in terms of the number of measurement. Fig. 3.3 illustrates the comparison in the number of the initialized target by M/N logic-based method according to the number of measurements. It consists not only the ground truth target but also false measurement. Nevertheless, every initialized targets equally contribute to the tracking system.

The conventional clustering algorithm, such as K-means and density-based spatial clustering of applications with noise(DBSCAN)[130], has a scheme that restricts the number of centroids. The number of centroids is determined by human preference based on prior information, or by the validation process. In a similar way, limiting the number of initialized target can be a reasonable approach to resolve the association

uncertainty regarding MTMA. The level of association for the initialized target needs to be defined, indeed.

3.2.2 Measurement-to-Track

Measurement-to-track association(MTTA), also called track maintenance or track updating, relates to the data association stage. MTTA infers the originate of measurements which can be the target or clutters.

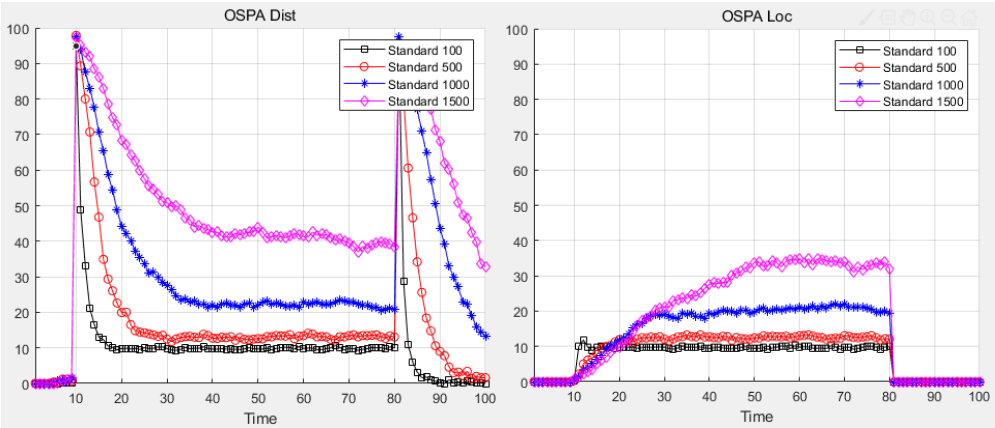
A similar approach can be found in a (binary) classification problem. Suppose that the target is labeled as the true class. MTTA can be seen as the problem to classify each measurement into true or false using a hard(MAP) or soft label(MMSE) based on the likelihood function. Also, since the trajectory consists of the history of estimation of the same target, a set of the estimated state can be interpreted as data of the true class, see Fig.3.2(b).

Recall likelihood function, that evaluates the level of association used commonly in conventional approaches as follow,

$$p(\nu_k^i) = \frac{1}{(2\pi)^{\frac{M}{2}} |\mathbf{S}_k|^{\frac{1}{2}}} \exp\left\{ \frac{-(d_i^k)^2}{2} \right\} \tag{3.1}$$

$$d_i^k = (\mathbf{z} - \bar{\mathbf{z}}_k^i)^T \mathbf{S}_k^{-1} (\mathbf{z} - \bar{\mathbf{z}}_k^i). \tag{3.2}$$

By assuming that the measurement is of dimension M, most of the conventional algorithm exploits the M-dimensional Gaussian association likelihood for the measurement based on the normalized innovation d_i^k. In other words, the association between the measurement and target is evaluated by spatial distance only. As the number of measurements increases, the clutter may present in the adjacent region of target originated measurement. In this situation, the source of measurement cannot be correctly classified depend only on spatial distance. Thus, one can be deduced that the uncertainty on MTTA relates to the insufficient use of information.

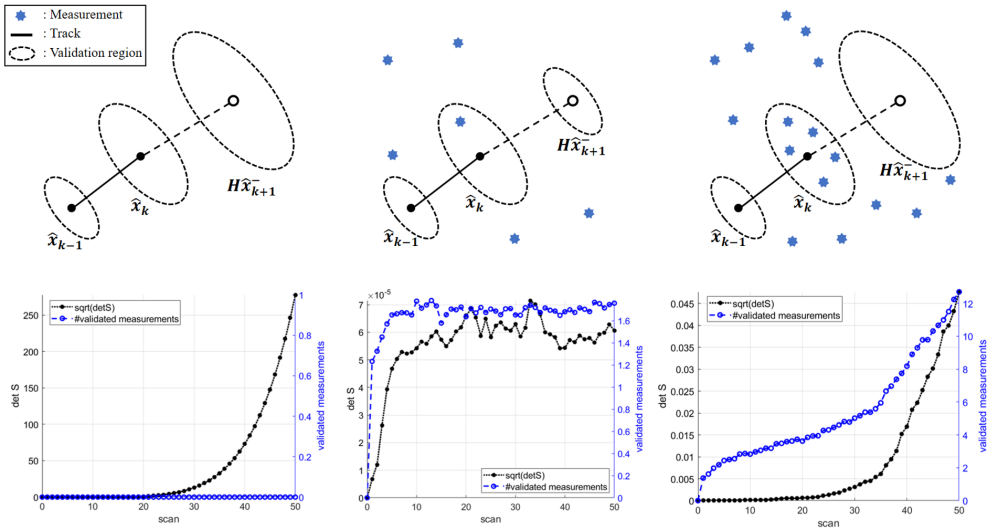


(a) OSPA Distribution (b) OSPA Location

Figure 3.4: Comparison of the performance of Bernoulli filter between different clutter distribution. Higher is worse. Details on the experimental setup follow the one with Chap. 5.

The grow of the number of measurement affects the tracking performance in two way. One is regarding performance degradation, and the other is related to the inaccurate measurement validation. Firstly, the performance of MTTA method is measured by the localization accuracy between the ground truth trajectory and the estimated trajectory. However, a large number of measurement leads to an inaccurate calculation of likelihood. In other words, high association probability can be assigned to the spurious measurement. Eventually, the target can be lost due to the incorrect update of the state. Fig.3.4 depicts the performance comparison of Bernoulli filter in a various expected number of the clutter. *OSPA* is the metric for measuring the accuracy of the RFS filter including position and cardinality. While OSPA location only considers the positional accuracy, OSPA distribution evaluates comprehensively. As the number of measurement increases, the accuracy of Bernoulli filter is degraded.

The other is related to measurement validation. This relationship can be clearly seen in PDAF formulations where the parametric error covariance is propagated. Recall the formulation of error covariance update $\hat{P}_k^+ = \beta_0(k)\hat{P}_k^- + [1-\beta_0(k)]\, P_k^c + \tilde{P}_k$,



(A) No measurement (B) $P_{FA} = 0.011$ (C) $P_{FA} = 0.044$

Figure 3.5: Relationship between P_{FA}, the number of validated measurements, and the volume of the validation region. (A) no measurement, (B) $P_{\text{FA}} = 0.011$ and (C) $P_{\text{FA}} = 0.044$. $|\mathbf{S}_k|$ and the number of validated measurements plotted in the bottom row are the average values of 1,000 trials. The volume of the validation region is increased in the absence of validated measurements and it will decrease when validated measurements are present. However, in a heavy-clutter environment, many unwanted measurements are included in validated measurements which is causing the volume of the validation region to increase.

especially $\tilde{P}_k$, the spread of the innovation term, of PDAF with standard Kalman filter,

$$\tilde{P}_k = K_k \left[\sum_{i=1}^{m_k} \beta_i(k)\nu_i(k)\nu_i(k)^T - \nu_i(k)\nu_i(k)^T \right] K_k^T. \tag{3.3}$$

The number of measurement is involved the update of the error covariance as in the middle term of the right side of eq. 3.3. Since it is positive semidefinite, thus, the measurement origin uncertainty causes an increase in the covariance of the update state of the target. This affects the subsequent process of measurement validation. The volume of validation region is proportional to the determinant of $\mathbf{S}_k$ where $\mathbf{S}_k \propto P_k$. Regardless of the accuracy of the tracking, consequently, the volume of validation region can be increased continuously. Fig.3.5 experimentally demonstrates the aforementioned relationship. When the tracking is steady, it is observed that the determinant of the innovation covariance maintains in a low cluttered environment. On the contrary, the number of validated measurement continuously increases as follows the determinant of $\mathbf{S}_k$ along with the volume of the validation region. In the case of the hypothesis based approaches such as MHT and the RFS based filter, the causality between the number of measurement and the volume of validation region is not explicitly revealed. However, one can be inferred the implicit relationship through eq. 3.4 as follows

$$I_k(\mathbf{z}) \approx \sum_{i=1}^{N+B} p_d(\mathbf{x}_{k|k-1}^i) g_k(\mathbf{z} \mid \mathbf{x}_{k|k-1}^i) \omega_{k|k-1}^i \tag{3.4}$$

where, $g(\cdot)$ is the conventional likelihood function of measurement $\mathbf{z}$ due to the target in state $\mathbf{x}$ and p_d is the probability of detecting the target in state $\mathbf{x}$. Eq. 3.4 is a part of the equation for calculating a target birth intensity I in the SMC implementation of Bernoulli filter. The estimation of the birth of a target is equivalent to the track initiation stage in the conventional target tracking system. For every recursion at k, the number of particles N is incremented by the birth particle B whose follows the number of received measurement. Thus, the number of particles, which represents the approximation of the true distribution of the state, can be insistently grows depending on the parameter setting for the shrinking step(e.g. pruning, merging, and re-sampling).

Consequently, one can deduce that the RFS based filter also requires gating strategies to guarantee computational tractability even though the explicit data association stage omitted

As described above, the uncertainty of MTTA is arisen due to the insufficient use of information, and it can be worse when tracking the dim target. One intuitive approach to address this limitation is enhancing the complexity of the evaluation(likelihood function) using additional information[131]. Furthermore, since the estimated trajectory is accumulated result for tracking the same target, novel criterions can be induced in consideration of the consistent information within the accumulated result.

3.2.3 Track-to-Track

Track-to-track association(TTTA) is also called track fusion or track correlation. In the multi-target tracking problem, if the multiple tracks are generated for the same target, those tracks are merged into single track based on TTTA, see Fig.3.2(c). It is mainly discussed in the multi-sensor environment and the performance improvement compared to the single sensor can be expected depending on the TTTA method[132, 133]. This dissertation, however, considers the environment established by changes in the target of interest, not in the sensor. Thus, the problem of TTTA is beyond the focus of this dissertation and will not be discussed.

3.3 Exploiting Velocity Information of the Measurement

Previous sections considered approaches that exploit additional information for the association uncertainty in the radar dim target tracking. In this section, information that is included in the measurement of the conventional radar system is described, and the proposed approach is discussed with a comparison of the visual tracking approach.

The aforementioned intuition, *exploiting additional information*, has been already introduced to the tracking literature. Especially in the visual tracking problem, it has

been extensively studied in the name of *track-by-detection*[32, 33]. This approach is based on the notion of neural network instead of the Bayesian tracking framework. That is, the variation of the target is encoded to the appearance model, which is a neural network during the training phase. Then the subsequent position of the target is inferred using the model in classification manner during the tracking phase. Surprisingly, it shows overwhelming tracking accuracy than conventional approaches. Likewise as the visual tracking, hence, interpreting the radar tracking problem as a classification or clustering problem is reliable. As discussed in the above sub-section, however, most of the conventional approaches evaluate how the measurement associate to the target based solely on the positional information. Thus, one can consider the direct implementation of a neural network into the radar tracking system, such as in the case of visual tracking.

Assume that the accumulated state estimation which belongs to the trajectory plays a role as a dataset in the radar tracking system. The available information for the feature extraction from conventional radar measurement is generally as follows[55, 134, 135]:

Position is measured by the time delay between the radar's transmission and the detection of the echo signal reflected by the target. It originally is described in range, azimuth angle, and elevation angle in the signal processor and then converted to Cartesian coordinate before the data processor.

Signal Intensity represents the echo power of the measurement, which propagates back to the radar and captured by the receiving antenna. According to the radar equation, it is proportional to the power transmitted, the target radar cross-section, and so forth.

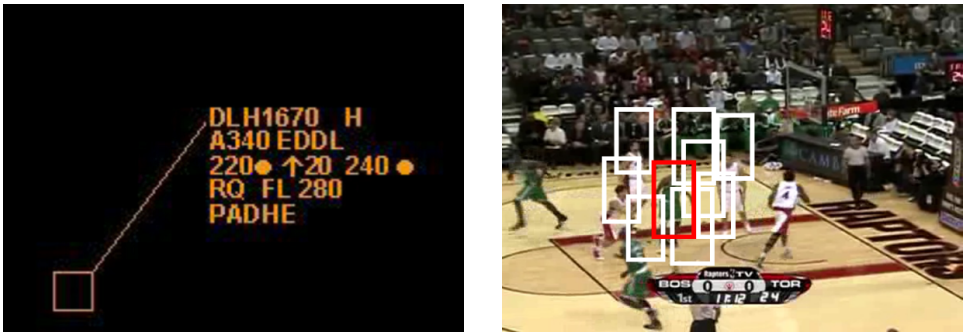
Doppler Information Doppler information regards to Doppler shift that is the frequency shift of the echo signal caused by the target's motion with respect to the radar. Target radial velocity is extracted from the Doppler frequency. It is effective in discriminating moving target from clutter.

In order to ensure the reliability of the neural network approach to the radar tracking system based on the above information, there are key factors that need to be considered in terms of the dataset as follows[136]:

- **Consistent Feature** Firstly, one needs to be able to design the feature which has a consistency in the same class from the dataset. In the conventional algorithms, the spatial distance between the measurement and the target plays a role as the feature. Since the origin of measurement is indistinguishable where measurements are densely distributed in an observation region by using either Euclidean or Mahalanobis, ambiguity occurs inevitably in a dense environment. This circumstance is caused by insufficient model complexity and is addressed by using additional features. Indeed, the radar measurement also has additional information except for the position. However, the signal intensity of the small target is not only unstable but also indistinguishable. Moreover, the Doppler information cannot be acquired depending on the direction of the target, occasionally. The alternative feature can be considered such as velocity or acceleration since the target is moving, namely the combination of existing features.

- **Quantity and Variation** The other factor relates to how much the dataset represents the various aspects of a class. This means that to ensure the robustness of the estimator about outlier or high leverage point, one requires not only sufficient population but also variation. Intuitively, it can be inferred that, as in MHT and the RFS based filter, the sufficient accumulation of the measurement can effectively reduce the association uncertainty. Since the radar tracking system needs to support the immediate decision, however, sufficient time for storing measurement cannot be conformed.

For the reasons described above, the direct implementation of the recently studied methods, such as a neural network, into the radar tracking system is infeasible. This also can be clearly verified by comparing the dimension of available information be-



(a) Radar measurement example(5) (b) Basketball sequence$(16 \times 47 \times 3)$

Figure 3.6: Comparison of dimension between radar measurement and visual tracking[140]. The numbers in the bracket indicate the dimension of data. Direct implementation of a sophisticated machine learning method, such as a neural network, is limited due to the insufficient information that the radar measurement contains.

tween radar tracking and visual tracking. Fig. 3.6 shows a comparison between the radar measurement[1] and a piece of data from the visual tracking problem. While the dimension of measurement of conventional pulse-Doppler radar is up to 5, the dimension of the patch of an image is at least $(16 \times 47 \times 3)$[137]. In order words, the information quantity of the radar measurement is insufficient to encode meaningful representation. Some types of radar(e.g. synthetic aperture radar(SAR)[138, 139]), which can provide visualization of the target, are studied, but they are not suitable for detecting remote targets.

Therefore, this dissertation exploits additional information to establish novel criterions, and apply it to conventional methods in an ensemble manner. Some studies tackle the association problem by using radar measurement information in a different way. These studies presented a methodology for using other information such as signal intensity and Doppler frequency, unlike the conventional approaches using only position information as follows. In the case of the track initiation stage, the signal intensity is converted to the track score and evaluate tentative tracks along with

[1]source: www.andreas-milde.de

conventional methods[141, 142]. This track score computed based on Bayes recursion. In [143, 144], the complementary methods based on Doppler information are proposed. In the case of the data association stage, the studies have focused mainly on the adaptive validation gate. Algorithms to estimate optimum gate size are proposed in [145, 146, 147, 148]. Algorithms including selection criteria been proposed in [149, 52]. In [149], they combined a conventional validation gate and a geometric distance measure which is called the Voronoi diagram. In [52], an approach to consider the balance between pure MMSE based DA and ML based DA algorithms is proposed as a methodology to maintain the robustness of the tracker, which is an important factor in high-clutter environments. Furthermore, the adaptive gating techniques for the RFS based filter incorporate with the elliptical gating technique have been proposed. In [150], [151], and [152], the gate size is enlarged adaptively in proportion to the likelihood of a single selected component. In [153], GM-CPHD is integrated with a square-root cubature Kalman filter and augmenting the threshold of the corresponding Mahalanobis distance by the weights. The spooky effect of GM-CPHD is tackled to improve computational efficiency in [154]. These previous approaches hint toward the use of additional information of the measurement to improve the performance of the tracking system.

Motivated by the above studies, methodologies for association uncertainty of the track initiation and data association stage are presented in the subsequent chapters. The velocity information is mainly exploited to design the additional criterion since it only requires the simple calculation and can be remained consistently in the trajectory if the proper assumption conformed. Moreover, appropriate schemes are also introduced for each stage's algorithm in consideration of the association aspect. The proposed algorithms for track initiation and data association are verified through individual simulation environments using proper metrics, respectively. The integrated experiment for the overall tracking procedure is not considered in this dissertation.

Chapter 4

Probabilistic Track Score for Multi-target Track Initiation

In previous chapters, the cause of the performance degradation due to the association uncertainty in the radar dim target tracking has been discussed and provided intuitions to exploit additional information. In later chapters, methodologies using velocity information will present for each stage of the radar target tracking system except the filtering stage[1].

The track initiation stage initializes the state of the target for Bayes recursion. It aims to find measurement combinations, which satisfies the predetermined condition, among the accumulated set of measurements for N consecutive sampling time. In other words, this problem is finding a subset of data that can be grouped into the same class. Thus, methods for the clustering problem can be effectively compatible. The association uncertainty in this stage mainly affects overall tracking performance in terms of the computational burden, see chapter 3. Because no explicit schemes to maintain the number of the confirmed tracks are implemented in the conventional approaches.

In this chapter, novel track initiation methods to effectively suppress the number of initialized false tracks while maintaining track detection probability are proposed.

[1]This chapter is reproduced based on the author's published papers[39, 40].

First, the probabilistic track score is defined for the comparison of the priority between tentative tracks using velocity information. Furthermore, to compensate for the measurement error sensitivity of velocity information, the weighted track score is also derivated using signal intensity. Second, *non-maximum suppression*(NMS), motivated by clustering problems, is introduced to maintain the number of confirmed targets based on the newly defined track score.

This chapter is organized as follows: Firstly, section 4.1 summarises the track initiation stage regarding the association uncertainty. Section 4.2 provides a formulation of the probabilistic track score using velocity information. Section 4.3 presents the weighted track score using signal intensity. Section 4.4 describes the non-maximum suppression scheme to maintain the number of confirmed targets based on the newly defined track score. Section 4.5 demonstrates experimental results with two scenarios. Finally, Section 4.6 discusses the contributions and limitations of the proposed approaches

4.1 Introduction

The track initiation is a very important stage in *multiple target tracking* (MTT) systems. Since a track initiation algorithm determines the initiation information for tracking targets, it directly influences the data association and tracking filter used[114, 115]. It aims to find the measurement trajectory that is closest to the actual target's maneuvering based on the received information from the radar without the state estimation process[155]. Generally, conventional track initiation algorithms use mainly the geometrical relationships among measurements that have to be combined (e.g. distance, velocity, angle). The measurement combination is a sequence of measurements obtained from consecutive radar scans, which can determine the initialization of tracking. If geometrical relationships of measurement combinations meet a predetermined condition (e.g. relationship score exceeding a threshold), they are regarded as targets

to track.

The key considerations that affect the performance of the track initiation algorithm can be summarized in two ways[44]. First, the track initiation algorithm requires to confirm the target as soon as it appears in the observation region. It focuses on guarantee effective operational decisions through immediate situational awareness. Second, the algorithm needs to suppress the false track probability. It aims to prevent unnecessary processing time to maintain false tracks generated during the track initiation stage. There is a trade-off between two considerations. If the sophisticated method is applied or measurements are accumulated for more scans to suppress the false track probability, the processing time increases. On the other hand, while naive algorithms can reduce the processing time, the number of false tracks can be exponentially grown in a dense environment. Thus, the algorithm needs to be designed, considering the balance between two considerations.

Conventional track initiation algorithms are categorized as sequential methods and batch methods[100]. The sequential method focuses on the fast processing time, the former consideration mentioned above, and produce the combination of measurements sequentially using the measurements that exceed a threshold for each radar scan. On the contrary, the batch method focuses on false track suppression, the latter consideration. In batch methods, every measurement from some period of radar scans is stored. Then, batch data of every possible measurement combinations are processed simultaneously. Details on each method refer to chapter 2. In a low clutter environment, sequential methods can effectively initiate targets. Since, however, every measurement that satisfies the condition of individual scans uses to expand tentative tracks, the false track probability increases in a dense environment[141]. Batch methods can accurately initiate targets in a high clutter environment because the statistics of the measurement combinations during several scan times are used to decide the initiation[100]. However, this approach works effectively only after many scans[156, 157], and requires sufficient memory for storing measurement combinations[101].

As mentioned above, the conventional track initiation algorithms operate effectively in normal clutter environments where the number of measurement combinations is rather small. However, since small aerial vehicles, which have recently increased threats such as drones and missiles, have a small radar cross section(RCS), the radar detection threshold needs to be lowered. [158, 159, 160]. Consequently, the number of measurement combinations increase exponentially due to a high cluttered environment[100, 161]. Conventional track initiation algorithms can be initiated a massive number of unnecessary targets, i.e., false track, in this environment. Furthermore, since false tracks require unnecessary computational power for data association and running tracking filters, the performances of MTT systems in heavy cluttered environments can be degraded eventually[44]. Therefore, studies on improving the track initiation stage are required regards to not only effectively suppress false targets but also false and accurate initiation[162, 163].

While conventional methods utilize measurement information deterministically, score-based algorithms such as Multiple hypothesis tracker(MHT) and Integrated Probabilistic Data Association Filter(IPDAF) use probabilistic approaches. Probabilistic approaches integrate the track initiation, data association, and filtering stages into one. Track scores are calculated based on error covariance matrix between measurement and prediction and the initiation of targets to track is determined according to these scores[164]. Since the probabilistic approach for targets as well as the estimation and prediction of the state are used, these algorithms are more accurate and suppress false track effectively[165, 156]. However, they require more computational resources because all stages of the MTT system are integrated and the number of measurement combinations is exponentially increased in a high clutter environment. Thus, it is preferred to use separate algorithms for each stage, especially in heavy cluttered environments[166].

As such, the conventional approach focuses only on finding subsets of measurements so that it is inefficient in the high clutter environment. Adding a clustering tech-

nique can improve the performance of existing algorithms. In other words, as analogous to limiting the number of centroids in clustering, maintaining the maximum number of confirmed tracks to remedy the computational burden of the tracking system. To keep the number of initiate target, one needs to remove redundant among tentative tracks in consideration of prioritizing. In this regard, the score based approach using the information of the measurement, such as velocity and signal intensity, can be exploited.

In following sections, a score-based track initiation algorithm combining probabilistic methods is proposed to address the limitations of conventional algorithms which is caused by the association uncertainty in the environment where the number of measurement increases. In contrast to conventional track scores using the covariance of the state, the proposed algorithm defines the track score based on the velocity information used in the traditional approaches to guarantee fast calculation even in a dense environment. The proposed algorithm can be understood as a kind of combination of a score-based algorithm and a conventional algorithm. Furthermore, the weighted track score is proposed to compensate for the velocity-based track score using signal intensity[167] and weighted Hough transform[168]. It considers the inaccurate velocity-based track score due to the measurement error. To limit the number of confirmed tracks ans determine the target to track, *the non-maximum suppression* (NMS)[169] method is introduced based on the track score. To verify the performance of the proposed algorithm, a track initiation simulator is designed for analyzing track detection probability (P_D), false track probability (P_F), and processing time. Calculation of P_D and P_F follows as in [100]. The result reveal that the proposed method yields considerable saving in processing requirements compared to the conventional methods without any significant degradation in performance.

4.2 Probabilistic Track Score

In this section, the derivation of the probabilistic track score, which represents the confidence of tentative tracks, is described. Since the track score calculation for conventional score based track initiation algorithms are based on error covariance, the Bayes recursion is essentially accompanied. However, it is inefficient to apply the Bayes recursion to every tentative track due to not only the matrix inversion but also the Monte Carlo sampling, especially in a dense environment. Based on this consideration, the probabilistic track score based on the conditions from conventional track initiation algorithms is proposed.

The newly defined track score requires to guarantee both accurate representations on the tentative track and fast calculation, simultaneously. In the following subsection, the track score E_v based on one of the conditions, the velocity, for track initiation is described along with proper assumptions. Note that, other conditions except the velocity, such as acceleration and angle, can also be exploited to formulate the track score with proper assumption.

4.2.1 Assumption

Assume that the target follows the CV(Constant velocity) model during N scan[46]. N is the number of reference scans for determining the confirmed track in the track initiation stage. The CV model assumes that the target's velocity remains almost constant during the tracking process, and $N = 4$ is generally applied in the conventional track initiation algorithm[100]. If the sampling period is short enough relative to the target's velocity, the change of the target's velocity would be negligible. Furthermore, the process of the track initiation stage requires only a few scans. So that the assumption can be valid in real-world problems. A constant target velocity implicates that the estimated velocity between measurements of the tentative track would also be nearly constant. Thus, as the previous assumption, if the variation of the target's velocity maintains

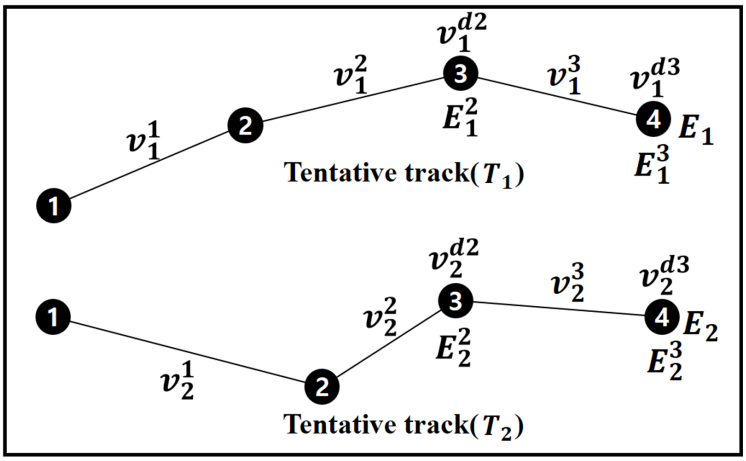


Figure 4.1: The concept of measurement combination. The black circle and the number indicated the measurement and the time when received, respectively.

constantly, the velocity change of the tentative track can be used as a condition(of feature) for track initiation. In order words, the velocity based track score is formulated that the tentative track with smaller velocity variation to have a more significant value. Both acceleration or angle can also be applied based on suitable assumptions as in the case of velocity.

4.2.2 Derivation of Track Score

Based on the assumption, the level of variation of estimated velocity can be used as a criterion of track initiation. In other words, the track score is defined to increase the probability that a tentative track with smaller velocity variation becomes a confirmed track. The process which converts the velocity variation of a tentative track into a track score is described below.

Every measurement combination which satisfies the gating criterion of velocity and angle is assigned as a tentative track. The track score of each tentative track is calculated based on variations of velocity information probabilistically.

Let k be a index of the tentative track and v_k^i is a velocity information of kth

tentative track at i^{th} scan. As shown in Fig. 4.1, the velocity variation, $v_k^{d_i}$, between adjacent scan is represented by e.q. (4.1).

$$v_k^{d_i} = \left| v_k^{i-1} - v_k^i \right| \tag{4.1}$$

In order to assign a high track score to the tentative track that shows small velocity variation, the relative velocity variation Δv_k^i is defined as follows:

$$\Delta v_k^i = \left| \max_k \left(v_k^{d_i} \right) - v_k^{d_i} \right| \tag{4.2}$$

And, the likelihood that the i-th measurement of the tentative track, T_k^i, originates from a true target(H_1) is defined as:

$$P(T_k^i \mid H_1) = \frac{1}{Z} \exp(\Delta v_k^i) \tag{4.3}$$

where, Z is the partition function that is calculated as the sum of $\exp(\Delta v_k^i)$ for every k, and H_1 is hypothesis that the tentative track is originated from a true target.

Finally, the track score of a tentative track, $E_v(T_k)$ is defined by the sum of log-likelihoods of each scan as the following:

$$E_v(T_k) = \sum_{i=2}^{N-1} \log P(T_k^i \mid H_1) \tag{4.4}$$

where the subscript v represents the velocity-based track score. Consequently, the tentative track with smaller velocity variation corresponds to having higher track score. Since track scores are calculated by summing log-likelihoods for several scans, confirmed tracks can be determined based on accumulated information.

4.3 Weighted Score based Approach

$E_v(T_k)$ described in the previous section is a simple approach that calculated using only velocity information. The velocity variation of T_k is calculated through a simple equation based on the positional information of the measurements. Thus, one cannot

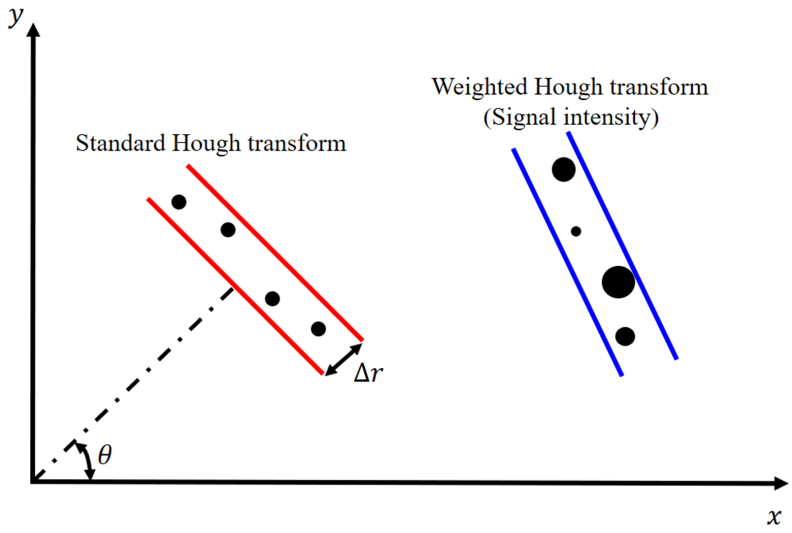


Figure 4.2: While measurements between the red line indicate standard Hough transform, those between the blue line represent weighted Hough transform. In the weighted Hough transform, the size of the points is determined according to the signal intensity.

guarantee the reliability of the condition for track initiation when significant measurement error in the positional information occurs. It can also be interpreted as an insufficient representation power of the estimator for the dataset in the machine learning perspective. The track score sensitive to error can lead to ambiguity in target originated track decisions. Consequently, the accuracy of the track initiation stage is degraded.

To address this limitation, in this section, additional methods are proposed to complement the uncertainty in $E_v(T_k)$ due to the measurement error based on the weighted Hough transform[168]. The proposed methods introduce additional information about the measurement, signal intensity, to support $E_v(T_k)$. It divides into two sub-methods depending on how to integrate with $E_v(T_k)$: binary weight and weighted sum. The integrated track score refers to the weighted track score.

4.3.1 Weighted Hough Transform

Weighted Hough transform is first introduced in [168] for improving the accuracy of the standard Hough transform by weighting the contribution of each measurement according to the quantity of its information, as expressed in eq.4.5.

$$F(\rho, \theta, x, y) = \begin{cases} kK(x, y) & \rho = x\cos\theta + y\sin\theta \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

$$F'(\rho, \theta, x, y) = \begin{cases} 1 & \rho = x\cos\theta + y\sin\theta \\ 0 & \text{otherwise} \end{cases} \tag{4.6}$$

where, $K(\cdot)$ is denoted as a kernel function for which computing the weight of each measurement and k is a normalizing constant. When mapping the measurement located in (x, y) to ρ-θ space, each measurement is given a weight to make a difference between the measurements. This is different from the standard Hough transform $F(\rho, \theta, x, y)$ mapping all measurements to the same weight of 1. Fig.4.2 illustrates the comparison between the standard and weighted Hough transform.

4.3.2 Derivation of Weighted Track Score

Two types of methods are proposed based on the weighted Hough transform approach. Both of them additionally exploit the signal intensity of measurements to compensate for inaccurate track initiation due to the insufficient use of information. The proposed methods refer to the binary weight and weighted sum, respectively. Fig. 4.3 depicts the concept of each proposed methods. The size and color of each point represent the level of signal intensity, and scan index in which the measurement is received, respectively. The line between the points shows how tentative(dot line) or confirmed tracks(solid line) are formed. Moreover, shaded points or lines represent removed measurements or tentative tracks during the process of proposed methods. The detailed procedure of each proposed method is described below.

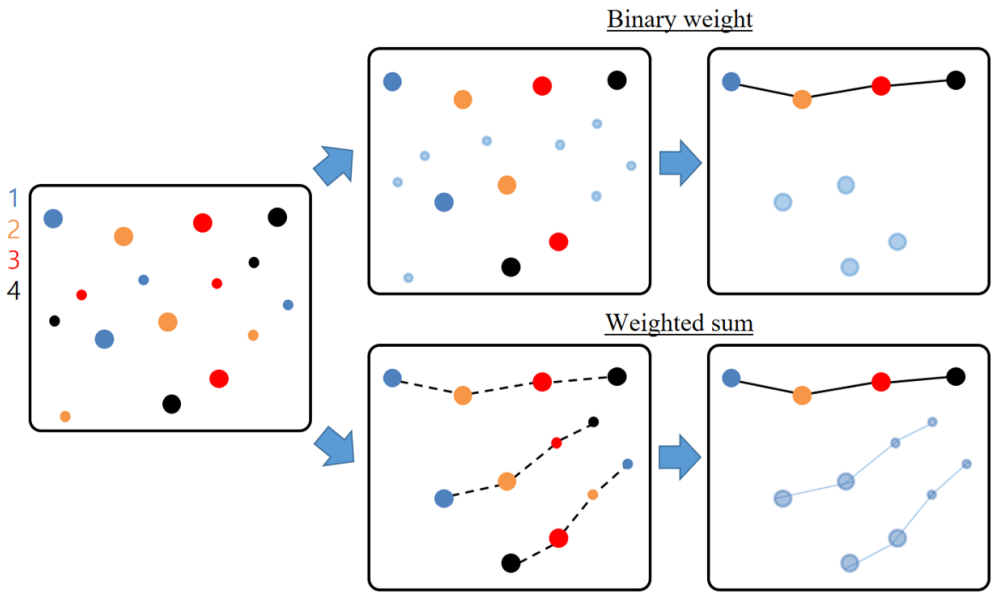


Figure 4.3: Concept of weighted Hough transform based track initiation approaches. Each point represents the accumulated measurement during $N = 4$ scans. The size and color of point mean the signal intensity level and scan time in which the measurement is acquired, respectively. The dot and solid line show how tentative and confirmed tracks are formed. The shaded dot and line represent the removed measurement or tentative track during the track initiation process. While the binary weight removes insignificant measurement before the track initiation stage according to the weight, the weighted sum uses the weight with a conventional approach simultaneously.

Binary Weight

The binary weight is the simple heuristic approach using only a subset of measurement whose signal intensity above a certain threshold at each scan. In other words, this approach tries to resolve the association uncertainty in the track initiation stage by reducing the number of tentative tracks. Suppose that $\mathbf{z}_k$ is a set of measurement acquired at scan k and A_k^i is the signal intensity of ith measurement from $\mathbf{z}_k$. The binary weight chooses measurements to form T_k among $\mathbf{z}_k$ prior to the track initiation stage as follows

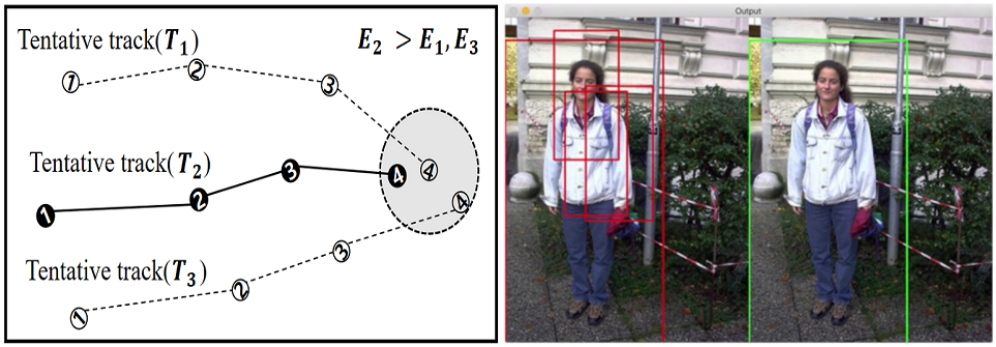
$$\mathcal{W}_k(\eta) = \left\{\mathbf{z} : A_k^i \geq \eta\right\} \tag{4.7}$$

where, η is pre-determined threshold. As shown in the top of Fig. 4.3, among the accumulated measurements during $N = 4$ scan, measurements with insignificant A_k^i, i.e., negligible weight, are removed before the track initiation stage proceeded. This method can be universally applicable to other track initiation algorithms since it only operates to limit the number of measurements regardless of track initiation schemes.

Weighted Sum

In the weighted sum, an additional track score is defined using the signal intensity in an analogous manner as $E_v(T_k)$. Due to the small RCS, one cannot clearly discriminate between small aerial vehicles and clutters by only A_k^i, as mentioned previously. However, assuming that A_k^i of the target maintains a constant level during the track initiation stage, the accumulated A_k^i for T_k can be used as a criterion for distinguishing the target from the clutter. Moreover, one can ensure the consistent criterion even in the presence of high signal intensity clutters, using $E_v(T_k)$ in conjunction. In this way, the association uncertainty from $E_v(T_k)$ can be resolved. In other words, this approach tries to resolve the association uncertainty in the track initiation stage by enhancing the discriminability.

Let $E_A(T_k)$ is the signal intensity based track score which is calculated based on



(a) Track initiation stage (b) Object detection in computer vision domain

Figure 4.4: Comparison of NMS concept between track initiation stage and object detection in computer vision domain.

the signal intensity as follows:

$$E_A(T_k) = \sum_{i=1}^{N} \log \frac{A_k^i}{\sum_k A_k^i} \tag{4.8}$$

As shown in the bottom of Fig. 4.3, regardless of the signal intensity, T_k is firstly assigned among the series of measurement sets received during $N = 4$ scans. Then, confirmed tracks are determined based on the integrated track score $E(T_k)$ as in Eq. 4.9.

$$E(T_k) = E_v(T_k) + \lambda E_A(T_k) \tag{4.9}$$

$E(T_k)$ is defined as the sum of $E_v(T_k)$ and $E_A(T_k)$ weighted by λ, which refers to the weighted track score.

4.4 Non-Maximum Suppression(NMS)

The proposed algorithm is based on track scores, so we have to consider how to determine confirmed tracks among tentative tracks. The scheme of non-maximum suppression(NMS) can effectively be implemented to determine confirmed tracks and reducing false track probability. The NMS is widely used for object detection tasks (es-

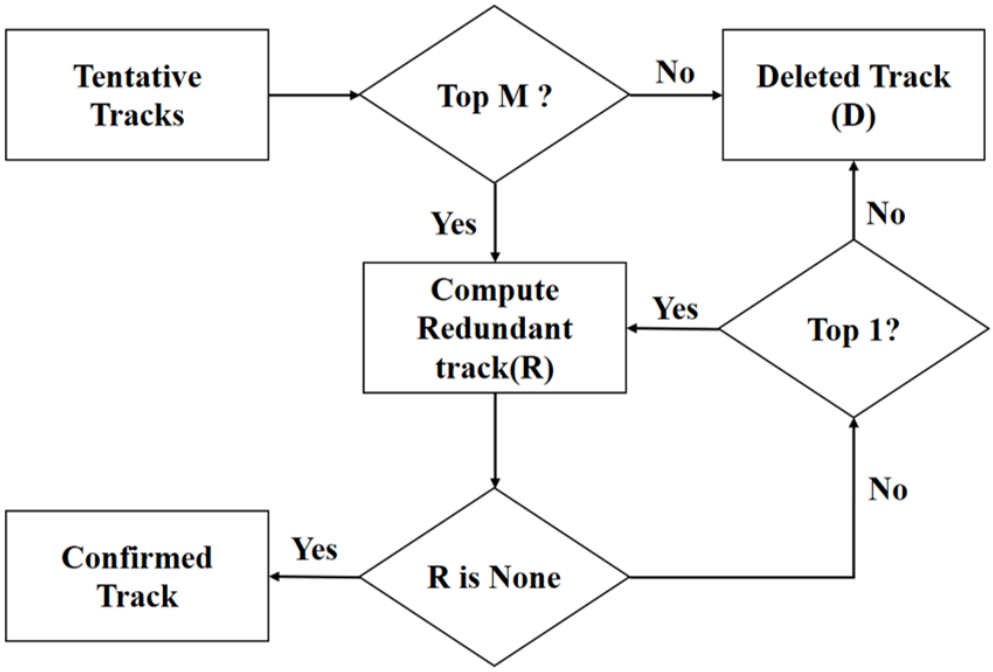


Figure 4.5: NMS block diagram.

pecially edge detection techniques) in the computer vision domain. The conventional process of NMS in computer vision is as follows: First, it sorts all detection boxes by their scores. Then, the detection boxes with a significant overlap are suppressed except the one with the maximum score. Inspired by this process, the analogous process, as used in object detection tasks, is proposed. The process of NMS in the proposed track initiation algorithm is as follows. First, to reduce processing time, TOP-M T_k based on the track scores are selected as the NMS input, where M is the average number of clutter points per scan. Next, adjacent T_k in distance satisfying (4.10) are assigned as a ”redundant track”, R_k. The distance between T_k is calculated based on the N_{th} measurement of each tracks where N is the final scan of the initiation process.

$$\left\|T_k^N - T_m^N\right\| \leq \delta_0, \qquad m = 1, 2, \cdots, K \tag{4.10}$$

where, K is the number of T_k and δ_0 is the NMS threshold.

As shown in (4.11), R_k are assigned as deleted track, D_k, except the one with the highest track score. The NMS process is iterated until there is no more R_k present. After the NMS process is done, remaining T_k are assigned as confirmed tracks.

$$T_k = \begin{cases} T_k & \text{if } T_k = \underset{T_k \in R_k}{\text{argmax}} E(T_k) \\ D_k & \text{otherwise} \end{cases} \tag{4.11}$$

Fig.4.4(a) shows an example of the NMS process. Suppose there are three T_k in an observation region. The number in circle indicates the radar scan time of measurement. For T_1, gray circle indicates a threshold for NMS process, so its R_1 are T_1, T_2 and T_3. Thus, only T_2 with the maximum track score survives and T_1 and T_3 are assigned as deleted tracks.

The block diagram of the proposed algorithm is shown in Fig.4.5. Through the NMS process, T_k that are assigned to the same measurement or approximately at the same position can be removed according to the threshold. However, finding the optimal threshold with respect to the environment is an open question, so this will be investigated in subsequent studies.

4.5 Experimental Results

The studies presented in this section verify the performance of the proposed algorithms, and show that it outperforms conventional approaches in certain circumstances.

Simulation environments In order to verify the performance of the proposed algorithm, the simulator for the track initiation($N = 4$) stage is built based on Matlab 2019b and conducts 1000 Monte Carlo simulations. Fig. 4.6 illustrates the simulation environment of the simple scenario. The ground truth target and clutter during the track initiation stage are presented in an $2D$ observation region, simultaneously. The black line and other symbols represent the trajectory of the target and clutter, respectively. 5 ground-truth targets with linear motion are present on the different starting points but the same velocity and direction. Each ground truth targets are simulated to have

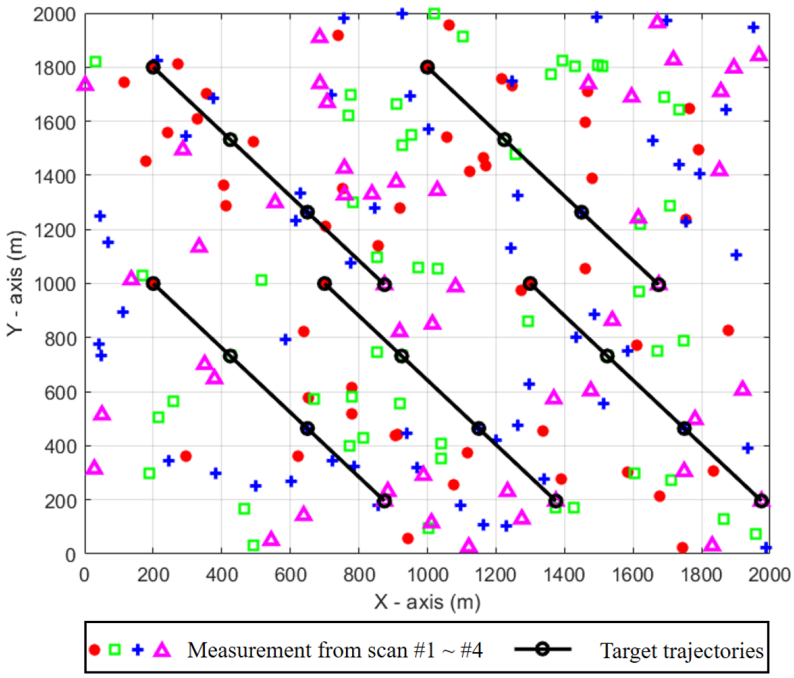


Figure 4.6: Simulation scenarios for track initiation algorithms. The black line and other symbols represent the trajectory of the target and clutter, respectively. 5 ground-truth targets with linear motion are present on the different starting points but the same velocity and direction.

random measurement error in range and azimuth. At each scan, the number of clutter points is determined randomly by Poisson distribution with an expected number of clutter points and uniformly distributed in the observation region. The measurement error in range and azimuth follows Gaussian distribution. The expected number of clutter points and the standard deviation of measurement errors are set differently according to the purpose of experiments. The signal intensity of the target and clutters follows Rayleigh distribution[42] as in Eq.4.12 and Eq.4.13, respectively.

$$P_1(s) = \frac{s}{1+d} \exp\left[\frac{-s^2}{2(1+d)}\right], \quad s \geq 0 \tag{4.12}$$

Table 4.1: Simulation parameters for track initiation experimental environment.

Parameters	Value(default)
Sampling time (t_s)	$1s$
Observation region	4km^2
Target velocity(v)	350m/s
Target angle(θ)	$-50°$
Total scan of track initiation process(N)	4
Number of scans for track confirmation(M)	3
Minimum velocity threshold(v_{min})	40m/s
Maximum velocity threshold(v_{max})	700m/s
Acceptance gate threshold(r_0)	80m
Angular threshold(ϕ_0)	$15°$
NMS threhsold(δ_0)	20m
Expected signal intensity(d)	10dB

$$P_0(s) = s\exp(\frac{-s^2}{2}), \quad s \geq 0 \tag{4.13}$$

where d is the expected signal intensity of the target and the RCS model for the target assumes to be Swerling 1. Other parameters for the simulation environment follows in Table.4.5.

Metric Two metrics are generally used for evaluating the performance of the track initiation algorithm: false track probability and track detection probability[100]. The false track probability(P_F) stands for the ratio between the expected number of clutter points and the number of the initiated false track at each scan. The track detection probability(P_D) is computed by the ratio between the number of true targets that exist and the number of true targets confirmed. In addition, the processing time is also measured to verify the effectiveness of the proposed algorithm. The processing time is the

time from the first scan to the track confirmation.

$$P_D = \frac{\text{Number of correct track formed}}{\text{Number of true track present}} \tag{4.14}$$

$$P_F = \frac{\text{Number of false track formed}}{\text{Number of average clutter point}} \tag{4.15}$$

The proposed algorithm compares to baselines such as the M/N logic based and modified Hough transform based track initiation algorithms. In order to verify the effect of the weighted Hough transform based methods, experiments are conducted using the velocity based track scores, binary weights, and weighted sums, separately.

Comparison of P_F for the expected number of clutter points. Fig. 4.7 represents the P_F change of each track initiation algorithm according to the number of clutter points. The black and green line indicates conventional algorithms, which are the M/N logic based and the modified Hough transform based algorithm, respectively. The rest of the lines are indicated the variation of the proposed algorithm, respectively: E_v only(the blue line), E_v with the binary weight(the red line), and E_v with the weighted sum(the magenta line). In order to conduct experiments while maintaining the proper P_D for every algorithm, the range error sets $\sigma_r = 10$m. The clutter density is increased up to $1.25 \times 10^{-5}\text{m}^{-2}$ for experimental purposes.

As shown in Fig. 4.7, the proposed algorithm shows up to 40% lower P_F than conventional algorithms. It means that the proposed algorithm can effectively suppress the initiation of false tracks in a dense environment. Specifically, while P_F of conventional algorithms increases proportionally to N_c, P_F of proposed algorithms can be maintained at a certain level. This result can be understood as the effect of the NMS scheme motivated by the clustering approach. As the probabilistic track score point of view, it is verified that a simple condition of conventional track initiation algorithms can be utilized as the track score if the proper converting process is given. Furthermore, weighted score based methods have lower P_F than when using E_v only. It implies that the association uncertainty in the track initiation stage can effectively be resolved by using additional information.

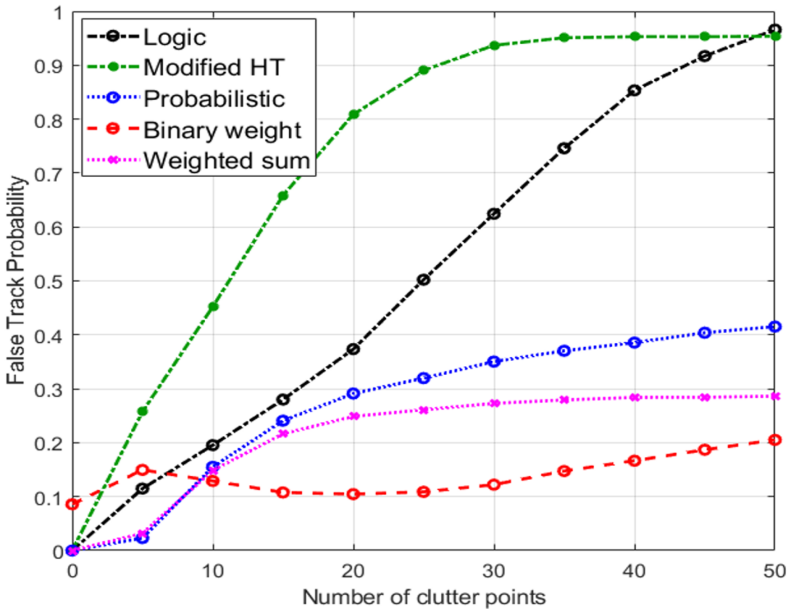


Figure 4.7: Comparison of P_F for the expected number of clutter points N_c. The track initiation algorithms for each line indicates in the legend.

Table 4.2 presents the average processing time for each algorithm at various N_c. For every N_c, the proposed algorithm requires more processing time than the M/N logic based track initiation algorithm, which is a sequential method, but similar to the modified Hough transform based track initiation algorithm, which is a batch method. Even though E_v based algorithm uses the velocity as the same as the M/N logic based algorithm, it requires times to not only compute track scores but also remove redundant track through the NMS scheme. Likewise, the weighted sum method also needs a similar processing time as E_v based algorithm. On the contrary, since the number of measurements diminishes before the track initiation stage, average processing times of the binary weight method close to the M/N logic based algorithm.

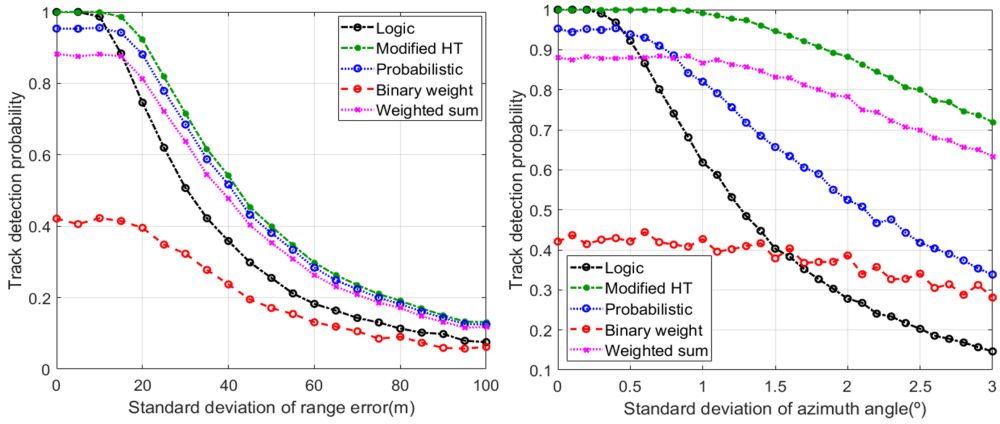
Comparison of P_D for measurement errors. Fig. 4.8 represents the P_D change of each track initiation algorithm according to the measurement error: range error(σ_r) and azimuth angle error(σ_ϕ). The notation of lines and symbols is the same as Fig. 4.7.

Table 4.2: Comparison of average processing time for various N_c.

Number of expected clutter points(N_c)	30	40	50
M/N logic based	0.16s	0.22s	0.26s
Modified Hough transform	0.25s	0.64s	0.90s
Probabilistic	0.29s	0.72s	0.99s
Probabilistic + Binary weight	0.13s	0.27s	0.38s
Probabilistic + Weighted sum	0.29s	0.71s	0.99s

In order to conduct experiments while maintaining the proper P_F for every algorithm, the expected number of clutter points sets $N_c = 30$. To clearly verify the effect of measurement error, σ_r is set 0 when conducting experiment for σ_ϕ, and vice versa. σ_r and σ_ϕ are increases up to 100m and 3° for experimental purposes, respectively.

As shown in Fig. 4.8, P_D decreases as the measurement error increases for every algorithm, including the proposed algorithm. Specifically, P_D is similar to or lower than that of conventional algorithms if E_v is solely used. Thus, one can verify that the track initiation algorithm based on the positional information, i.e., velocity and angle, is sensitive to measurement error. On the other hand, the weighted track score based algorithm, especially the weighted sum method, show a favorable P_D compared to conventional algorithms as well as E_v based algorithm. This can be interpreted as the result of resolving the ambiguity of E_v due to the measurement error, by using the additional information of the measurement, which is signal intensity, through the weighted sum method. On the contrary, the binary weight method is not suitable for small RCS target since the result is sensitive to the threshold. Consequently, one can verify that the weighted sum based algorithm is able to accurately initiate the target for track not only in a dense environment but also in the presence of measurement errors. Note that, the reason why the proposed algorithm cannot achieve 100% track initiation at $\sigma_r = 0$ and $\sigma_\phi = 0$ is that the ground truth target can be deleted unexpectedly during NMS processing. In other words, the proposed algorithm results in an efficiently



(a) error in range σ_r

(b) error in azimuth angle σ_ϕ

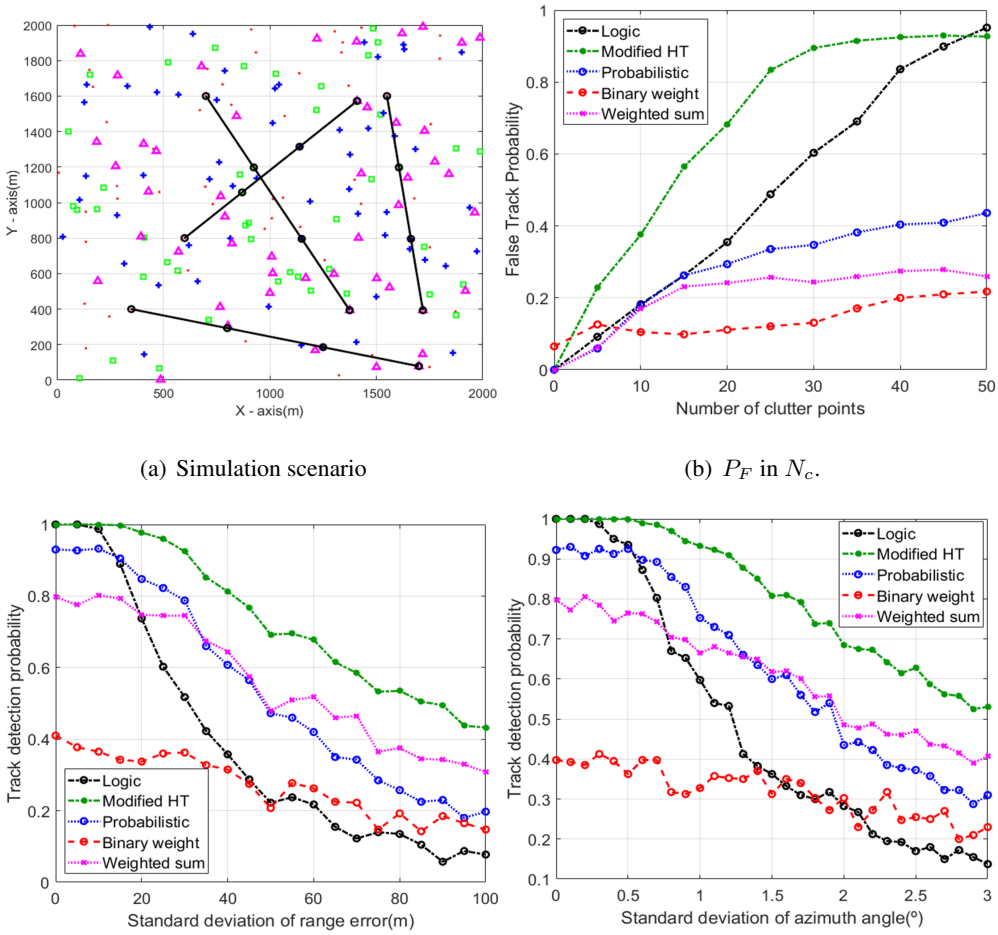
Figure 4.8: Comparison of P_D for measurement errors. The legend of lines denotes the same as Fig.4.7.

suppressing in the number of false targets with a slight compromise in accuracy, so that further studies are required.

Meanwhile, the results in Fig. 4.8 can imply that the performance of the proposed algorithm tends to sensitive for σ_r. However, this result is not due to the methodology of the proposed algorithm but to environmental factors. That is, since targets move in the direction perpendicular to the origin as shown in Fig. 4.6, so σ_r affects the target position more than the σ_ϕ.

Fig.4.9 present the performance of track initiation algorithms in a more complex scenario((a): target trajectories in an observation region, (b): P_F in N_c, (c): P_D in σ_r, and (d): P_D in σ_ϕ). 4 ground-truth targets with linear motion are present on the different starting points, and velocities and directions also vary. The other parameters and notations are the same as the previous scenario. Analogous results as the experiments in the previous scenarios indicate that the proposed algorithm can effectively track initiation even in complex scenarios. However, to guarantee the robustness of the proposed algorithm, the NMS threshold δ_0 needs to be considered as follows.

Comparison of performances for NMS thresholds. As δ_0 increases, more redundant



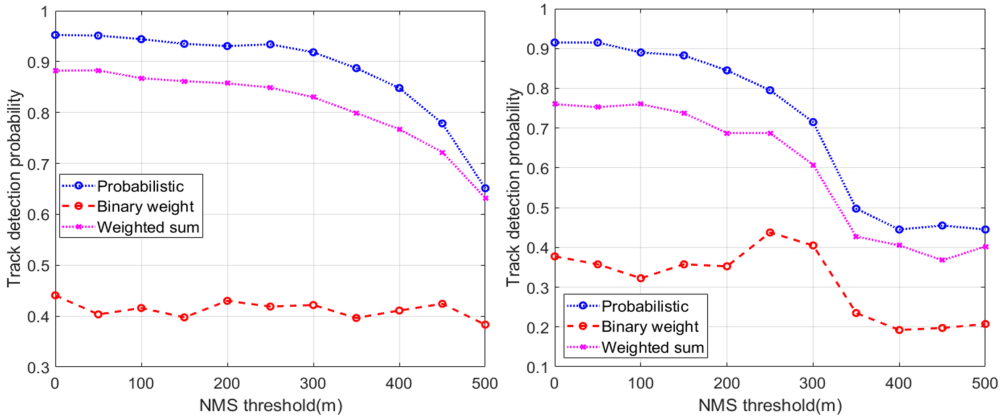
(a) Simulation scenario

(b) P_F in N_c.

(c) P_D in σ_r

(d) P_D in σ_ϕ

Figure 4.9: More complex scenario for the track initiation and results.



(a) Simple Scenario

(b) Complex Scenario

Figure 4.10: Comparison of P_D for NMS threshold. Ground truth targets can be discarded according to the spatial distribution of targets in an observation region.

tracks can be grouped for a single tentative track. By definition, only one tentative track among redundant tracks survives. Thus, to suppress the P_F efficiently, a large value of δ_0 can be suggested. However, according to how targets are distributed in an observation region, the result may not be as expected, that is, the degradation of P_D. Fig. 4.10 depicts the the comparison of P_D in δ_0((a) simple scenario, and (b) complex scenario). As shown in (b), the targets do not travel in parallel so that tentative tracks can be adjacent at $N = 4$. Hence, it can be seen that P_D of (b) is decreased in the small δ_0 compares to (a). Note that, since a plurality of tentative tracks can be assigned using the same measurement at $N = 4$, R_k can exist even in $\delta_0 = 0$. In this circumstance, the ground truth target can be deleted according to $E(T_k)$. So, P_D of the proposed algorithm can be below 1 in $\delta_0 = 0$.

4.6 Discussion

Conventional track initiation algorithms suffer from the association uncertainty in a cluttered environment. It leads to not only increasing P_F of the track initiation stage

but also performance degradation of the overall tracking system. Since conventional approaches have no limitation of the number of initiated target, significant performance degradation emerges in a dense environment. Thus, introducing the scheme of limiting the number of initiated targets can mitigate the above problem. Obviously, the proper method to prioritize tentative tracks need to be defined, Indeed.

In this chapter, novel approaches are proposed for association uncertainty due to the dim target tracking in the multi-target track initiation. In the proposed algorithms, the track score is defined probabilistically based on the plural conditions of conventional algorithms without the Bayes recursion. In addition, the scheme of NMS is applied to maintain the number of initiated target. Specifically, the track scores are defined (1) based solely on the velocity information, then to verify the effectiveness of additional information in terms of representation power, (2) based on the signal intensity using weighted Hough transform, respectively.

In order to verify the performance of the proposed algorithm, experiments are conducted in the simulator for the track initiation stage. The result of the simulation reveals that the proposed algorithm has up to 40% performance improvement on P_F compared to conventional algorithms without any significant requiring in processing time in heavy clutter environments. Thus, one can deduce that the use of the NMS scheme provides the ability to keep low false track probability even though the number of clutter points increases. Furthermore, through the experimental result on the weighted score, it can be verified that supplementation through additional information contributes to performance improvement. Meanwhile, the proposed algorithm has a slightly lower P_D compared to conventional algorithms on the measurement error. It means that there is a trade-off between P_D and P_F, which can be addressed by introducing an additional representation of the measurement.

However, to enhance the robustness of the proposed algorithm, some issues must be discussed as follows. First, as shown in experimental results on the binary weight method in measurement errors, P_D and P_F are sensitive to η. Since the binary weight

method has an advantage in terms of processing time, more studies on selecting η while maintaining an appropriate P_D need to be conducted. Second, the adaptive δ_0 or adaptive NMS process needs to be studied regarding the environmental factor, because the track initiation performance can be degraded due to δ_0 according to the distribution of targets in an observation region. Lastly, more studies on the representation of the additional measurement information need to be conducted. More representation of the measurement can resolve the trade-off between P_D and P_F due to the association uncertainty.

Chapter 5

Trajectory based Measurement Validation for Single-target Data Association

In this chapter, the association uncertainty in the data association stage is discussed[1]. The data association stage evaluates the relationship(or association) between the target and measurement to determine the measurement to use for the update step of the Bayes recursion. It aims to find target originated measurement among a set of measurements at time k given the target's predicted state $\hat{\mathbf{x}}_k^-$. Before the determination of the measurement, only a subset of measurement is select to evaluation in order for computational efficiency. This procedure is so-called the measurement validation, see Chap. 2.

Theoretically, it assumes that the estimation history of the target is represented by current conditional probability density of $\hat{\mathbf{x}}_k^-$. Conventional approaches use the conditional probability density of $\hat{\mathbf{x}}_k^-$ to reduce the number of measurements in the measurement validation process. Specifically, the validation gate sets up around the predicted position of the target based on Mahalanobis distance according to the covariance of the probability density. In other words, the source of the measurement is inferred based solely on the spatial distance between the target and measurement. This

[1]This chapter is reproduced based on the author's published journal article[41].

is the reason for the performance degradation of conventional approaches in a dense environment, see chap 3. However, the history of state estimation is also accumulated in the trajectory, and it can be used to establish explicit criterions. That is, consistent information can be extracted within the accumulated data, and use to infer the origin of the measurement.

Based on the above consideration, the novel *trajectory-based measurement validation* is proposed using additional information. The proposed algorithm mainly focuses on the measurement validation process. The additional information of the measurement is introduced to supplement the ambiguity of the existing criterion in a dense environment as an ensemble manner. Velocity information is used as additional information since it requires only simple calculations and can remain nearly constant in the trajectory within proper assumptions. Two criteria are derived using velocity information: gating and empirical normalized distance squared(eNDS). The proposed algorithm firstly verify the effectiveness in the simple single tracking scenario and extend to a random number of target and multi-target scenario.

The remainder of this chapter is organized as follows: section 5.1 summarises the data association stage regarding the association uncertainty. Section 5.2 provides formulations of the target, measurement and clutter model, and derivation of trajectory based measurement validation methods, including the extension. Section 5.3 demonstrates the performance of the proposed algorithm in various cases. Finally, section 5.4 discusses the contributions and limitations of the proposed methods.

5.1 Introduction

In a target tracking system based on radar, *data association*(DA) is a problem that requires matching measurements received from the radar to other measurements or tracks in order to update the state of the target being tracked [116, 108]. One of the critical factors to deal with the DA problem is discrimination of the source of mea-

surements, which is figuring out whether the measurement originates from the target, from the clutter, or other targets[44]. This is because, if measurements that originated from the clutter or noise are used to update the target state, the tracking error may be increased or the target may be lost[170]. However, it is hard to figure out the origin of measurements without additional information such as secondary radar signals[2]. Therefore, the updated target states are estimated based on all measurements[171, 121] or hypotheses that include all measurement-to-track combinations[97].

Meanwhile, it is computationally inefficient to apply every measurement in the observation region to the DA algorithm. Thus, the process of *measurement validation* is applied to reduce the number of measurements that need to be processed in the DA algorithm. In the measurement validation, the validation gate, which is the possible range of the target-originated measurements is set up around the predicted position of the target according to the error covariance. Measurements within this validation gate are used to update the target state, and these measurements are called "the validated measurements." There are various types of validation gates such as annular, ellipsoidal, rectangular, and specific sector[44]. It has been reported that measurement validation can effectively reduce the number of measurements [116, 171].

In the tracking literature, various approaches have been studied to solve the DA problem along with the measurement validation. These approaches, such as NNF, SNF, PDAF, and MHT, can track the target accurately in a low-clutter environment, see chap. 2 for further information. However, potential threats of dim targets such as Unmanned Aerial Vehicles (UAVs) and, missiles have arisen[172] lately. The detection threshold adjustment increases the number of measurements in an observation region, and this environment is the so-called dense environment[173, 174]. The growth of the number of measurement necessarily causes an increase in the number of validated measurements. As such, in an environment where the association uncertainty is significantly

[2]Identification Friend or Foe(IFF) is one of the secondary radar. The target that responds inappropriately to the IFF's transponder is recognized as an enemy.

raised, not only the accuracy of conventional algorithms for DA can be degraded but also more processing time required[171, 175]. In order to alleviate this problem, many studies have been proposed with an emphasis on measurement validation, as described in chapter 3. Especially in [149, 52], the selection criteria have been introduced to the measurement validation. These previous approaches hint toward the use of the selected validated measurement to improve the performance of the tracking system.

Conventional approaches infer the source of measurements based solely on the spatial distance. Thus, the performance degradation may emerge in a dense environment since the boundary between the target and the clutter gets unclear. In other words, the information(i.e., spatial distance) used in conventional approaches is *insufficient* to infer the target and others, especially in a dense environment. Based on the this consideration, one can expect that increasing the complexity of the boundary between the target and clutter by using the additional information of the measurement can mitigate performance degradation.

In the following section, novel algorithms for DA in terms of the measurement validation are proposed. The proposed algorithm chooses only a part of the measurement from validated measurements as similar to the ensemble approach and uses it to update the state of the target. Different from conventional selection criteria[149, 52], the proposed algorithm exploits the accumulated estimation history in the target trajectory rather than the *likelihoods*. In other words, the accumulated estimation history of the target's state in the target trajectory plays roles as a dataset for a classification problem and the *feature* is designed based on this dataset. In this regard, the proposed algorithm is called as the *trajectory-based measurement validation*(TbMV)[3]. The information of the trajectory to design the new criteria can be velocity, acceleration, Doppler, and signal intensity. In this dissertation, velocity information is used to design the criterion and demonstrate the proposed algorithm. Based on the newly designed criteria,

[3]However, the proposed algorithm does not train the model as in the generic classification problem due to the intrinsic limitation of radar measurements. See chap 3.

an additional validation gate is set up within the conventional validation gate in an ensemble manner. To effectively select validated measurements based on the velocity information, the use of the gating criteria and empirical normalized distance squared (eNDS) criteria are proposed. The gating criteria provide a range of velocities for the validated measurements, and the eNDS defines a velocity oriented statistical distance. Furthermore, the extension of the proposed algorithm to a random number of the target and multi-target scenarios is also introduced to guarantee the robustness in a variety of tracking algorithms.

To verify the performance of the proposed algorithm, the data association simulator is designed and conducts experiments in various control factors. The performance of the algorithm is determined using the root mean squared (RMS) position error except for the scenario whose number of the target is random. For the RFS based filter, optimal subpattern assignment(OSPA) metric[176] is used.

5.2 Trajectory based Measurement Validation

The association uncertainty in the data association stage can be resolved if the origin of measurements identifies unambiguously. Also, the processing time also can be saved if the discrimination is achieved in the measurement validation. The objective of the proposed algorithm, which presents in this section, is to guarantee the robustness of the tracking filter, by reducing the number of validated measurements using trajectory information of the target that is being tracked. The performance degradation caused by the excessively increasing of the association uncertainty is avoided by combining the conventional validation gate and the proposed algorithm.

If the tracking system is estimating a target position accurately, the target trajectory can contain information reflecting the true dynamics of the target. In other words, the accumulated information of the target trajectory can be interpreted as a dataset for the ‘target class’ and ‘feature’ can be designed based on the accumulated information.

Since, however, the radar measurement is not only insufficient information but also hard to acquire massive data for training, it is limited to apply the neural network to train the estimation model as in the generic classification problem. From this point of view, the purpose of the proposed algorithm is to introduce additional criterions in an ensemble manner to complements the feature of conventional measurement validation approaches in a dense environment. To demonstrate the proposed algorithm, the estimated velocity is used as the additional feature, because estimating the velocity requires little computational power, and also we can use the assumption about the positional measurement such as the Gaussian noise.

The proposed algorithm is divided into two types according to the selecting criteria based on velocity information. The gating criterion uses the Euclidean distance, while the eNDS uses Mahalanobis distance. Since eNDS requires the variance calculation, it stores a history of velocity information. Details of each criterion are described in the following subsections. Measurement validation conducts independently of the Bayes recursion, so the prediction and update steps use conventional Bayesian tracking formulations.

In the following, the system model and derivation of the proposed algorithm are presented in the PDAF framework with the standard Kalman filter. Then, the extension of the proposed algorithm to the RFS based model is described.

5.2.1 System Model

Let $\mathbf{r}_k = [x_k \quad y_k]^T$ is the positional information of the measurement in Cartesian coordinate, additional information such as estimated velocity $\mathbf{v}_k$ and acceleration $\mathbf{a}_k$ of a target can be obtained using $\mathbf{r}_k$ and the sampling rate t_s of radar at scan k.

$$\mathbf{v}_k = (\mathbf{r}_{k+1} - \mathbf{r}_k)/t_s \tag{5.1}$$

$$\mathbf{a}_k = \left[(\mathbf{r}_{k+1} - \mathbf{r}_k)/t_s - (\mathbf{r}_k - \mathbf{r}_{k-1})/t_s\right]/t_s \tag{5.2}$$

Note that, since estimated information can be obtained not only between track-to-measurement but also between measurement-to-measurement pairs, it used to in conventional track initiation algorithm [177]. As mentioned above sections, $\mathbf{v}_k$ uses as an additional feature in the proposed algorithm.

Let the target state vector at scan k is $\mathbf{x}_k = [x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k]^T$. x_k, $\dot{x}_k$, y_k, $\dot{y}_k$ denotes position information of each dimension and corresponding velocities in Cartesian coordinates. The measurement vector is $\mathbf{z}_k$ and the target and measurement state space model at time k given by

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \boldsymbol{\omega}_k, \tag{5.3}$$

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \boldsymbol{v}_k \tag{5.4}$$

where $\boldsymbol{\omega}_t$, $\boldsymbol{v}_k$ are the zero-mean Gaussian noise with covariance $\mathbf{Q}$ and $\mathbf{R}$, respectively. $\mathbf{F}$ is the state transition matrix, $\mathbf{H}$ is the measurement function. Target state, and dynamics model are mutually independent. Matrices for each model are followed CV model as follows:

$$\mathbf{F} = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{5.5}$$

$$\mathbf{Q} = \sigma \begin{pmatrix} \frac{T^4}{4} & \frac{T^3}{2} \\ \frac{T^3}{2} & T^2 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} T^2 & 0 \\ 0 & T^2 \end{pmatrix} \tag{5.6}$$

where, σ is noise variance and T is the sampling time.

The number of clutter points(N_c) in each scan k is assumed to have a Poisson distribution as shown in e.q. 2.3. μ_c is determined by the total number of cells (N_{cells}) and the probability of false alarm P_{FA} as shown in e.q. 2.4.

$$\mu_c = \lambda V = N_{\text{cells}} P_{\text{FA}}$$

where V is volume of the observation region and λ is a spatial density parameter. The location of clutter points at scan k is assumed to have uniform distribution in the observation region.

In the prediction stage, predicted target state $\hat{\mathbf{x}}_k^-$ and error covariance $\hat{\mathbf{P}}_k^-$ are computed by the mean and covariance of the target's state space model, respectively, as shown by:

$$\hat{\mathbf{x}}_k^- = \mathbf{F}\hat{\mathbf{x}}_{k-1}, \tag{5.7}$$

$$\mathbf{P}_k^- = \mathbf{F}\mathbf{P}_{k-1}\mathbf{F}^T + \mathbf{Q} \tag{5.8}$$

Predictions related to the measurement are defined by the measurement function H as follows:

$$\bar{\mathbf{z}}_k = \mathbf{H}\hat{\mathbf{x}}_k^-, \tag{5.9}$$

$$\mathbf{S}_k = \mathbf{H}\mathbf{P}_k^-\mathbf{H}^T + \mathbf{R} \tag{5.10}$$

where, $\bar{\mathbf{z}}_k$ and $\mathbf{S}_k$ are the target's predicted measurement and covariance of the innovation, respectively.

5.2.2 Gating Criterion

Before describing each criterion, one needs to present how the additional feature designed. At each scan k, the estimated velocity $\mathbf{v}_k^i$ of ith validated measurement, as the additional feature, from the estimated target position $\hat{\mathbf{x}}_{k-1}$ is obtained by

$$\mathbf{v}_k^i = \frac{(\mathbf{z}_k^i - \mathbf{A}\hat{\mathbf{x}}_{k-1})}{t_s} \tag{5.11}$$

where $\mathbf{A}$ is $I_{2\times 2}$, see Fig. 5.1. Likewise, the estimated velocity of the target $\mathbf{v}_k^{\text{target}}$ at each scan k is obtained from the target trajectory as follows,

$$\mathbf{v}_k^{\text{target}} = \frac{(\hat{\mathbf{x}}_{j+1} - \hat{\mathbf{x}}_j)}{t_s}. \tag{5.12}$$

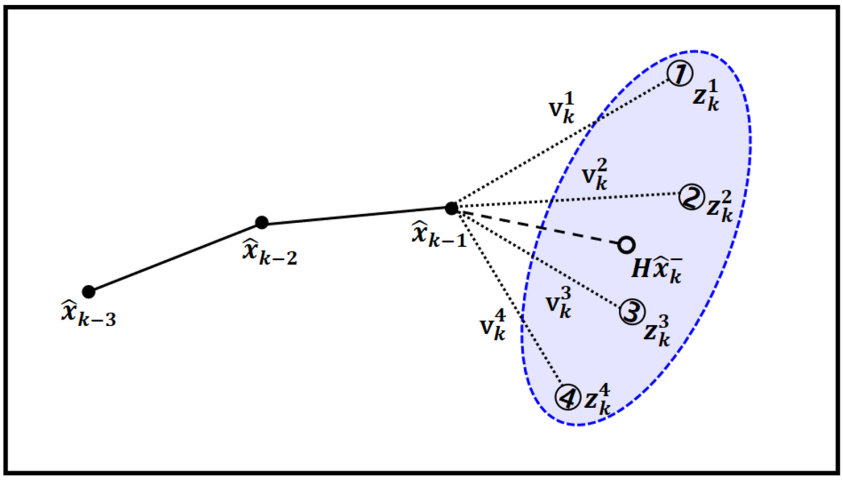


Figure 5.1: Estimated velocity $\mathbf{v}_k^i$ related to the measurement $\mathbf{z}_k^i$ is calculated between filtered target positions at $k-1$ and the current predicted target position.

Since the feature of the target needs to contain previous information of the target trajectory, the weighted average of $\mathbf{v}_k^{\text{target}}$ up to $(k-1)$ scans uses for establishing the criterion given by the following equation:

$$\mathbf{v}_{\text{target}} = \frac{\sum_{j=1}^{k-1} \zeta^{(k-j)} \mathbf{v}_k^{\text{target}}}{\sum_{j=1}^{k-1} \zeta^{(k-j)}}, \quad k \geq 2 \tag{5.13}$$

where, ζ is a discount factor which determines the importance of past information. If the discount factor is close to 0, early $\mathbf{v}_k^{\text{target}}$ almost ignored and $\mathbf{v}_{\text{target}}$ only contains late $\mathbf{v}_k^{\text{target}}$.

The gating criterion is a simple heuristic approach based on the Euclidean distance between $\mathbf{v}_k^{\text{target}}$ and $\mathbf{v}_k^i$. Validated measurements that satisfy the following condition are used in the data association step.

$$R_k(\eta_g) = \left\{ \mathbf{z}_k : \left\| \mathbf{v}_{\text{target}} - \mathbf{v}_k^i \right\| \leq \eta_g \right\} \tag{5.14}$$

where, η_g is predefined velocity threshold for the gating criterion.

The gating criterion is a method of estimating the possible velocity range of the target using the weighted average velocity of the target trajectory as reference velocity.

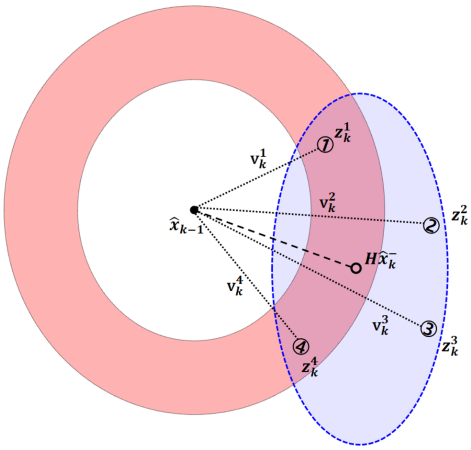


Figure 5.2: The concept of gating criterion. Additional validation gate based on estimated velocities of the trajectory (red) is added onto conventional validation gate (shaded blue).

This newly defined gate is added to the conventional validation gate to select appropriate measurements. The concept of the gating criterion is shown in Fig. 5.2. Only $\mathbf{z}_k^1$, $\mathbf{z}_k^4$ of the four validated measurements that satisfy additional validation gate based on $\mathbf{v}_{\text{target}}$ are used in the data association step.

5.2.3 Empirical NDS(eNDS)

As described in Chap. 2, the PDAF assumes that the process and measurement noise follows a Gaussian distribution, and this is the reason that the validation gate has an elliptical shape. The volume of the validation gate implicitly represents the uncertainty of the estimation, and it is calculated by innovation error covariance $\mathbf{S}_k$. During the tracking process, $\mathbf{S}_k$ is recursively updated by the process noise covariance $\mathbf{Q}$, measurement noise covariance $\mathbf{R}$, and corresponding Kalman gain $\mathbf{K}_k$. In the case of the PDAF, the level of association uncertainty is also used to update $\mathbf{S}_k$ [171]. However, Since $\mathbf{Q}$ and $\mathbf{R}$ are pre-selected fixed constant values, they limit the representation of the true uncertainty of the estimation, and this can be a major restriction on the

tracking system performance. To address this limitation, the empirical NDS(eNDS) which is a gating criterion with the Gaussian distribution assumption is proposed as described below. Note that, the term eNDS implies that the formulation is similar to the NDS criteria(see. sec. 2.4.1) except for the normalizing covariance which is empirically acquired.

Using (5.11), the set of estimated velocities of validated measurements at each scan k can acquired by

$$\mathbf{v}_k = \begin{bmatrix} \mathbf{v}_k^1 & \mathbf{v}_k^2 & \mathbf{v}_k^3 & \cdots & \mathbf{v}_k^{m_k} \end{bmatrix}. \tag{5.15}$$

It can be assumed that $\mathbf{v}_k$ is sampled from the Gaussian distribution, since the measurements follow the Gaussian distribution. Then, an unbiased sample mean (or empirical mean) and sample covariance of the estimated velocity can be estimated using $\mathbf{v}_k$ as the sample instead of a fixed value of noise covariances.

$$\bar{\mathbf{v}}_k = \frac{1}{m_k} \sum_{i=1}^{m_k} \mathbf{v}_k^i \tag{5.16}$$

$$\sigma_{\mathbf{v}_k} = \frac{1}{m_k - 1} \sum_{i=1}^{m_k} (\mathbf{v}_k^i - \bar{\mathbf{v}}_k)^2. \tag{5.17}$$

Let $\mathbf{S}_{\mathbf{v}_k}$ be the empirical error covariance of the estimated velocity, and assume that the measurement validation step conduct independently across time. At each scan, $\mathbf{S}_{\mathbf{v}_k}$ is iteratively updated by using a newly obtained validated measurement as follows:

$$\mathbf{S}_{\mathbf{v}_k} = \frac{\mathbf{S}_{\mathbf{v}_k}^{\text{cov}} + \mathbf{S}_{\mathbf{v}_k}^{\text{mean}}}{\mathrm{M}_{k-1} + m_k - 1} \tag{5.18}$$

where, $\mathrm{M}_{k-1} = \sum_{j=1}^{k-1} m_j$ is total number of validated measurements up to scan $k-1$ and the variance term of $S_{\mathbf{v}_k}$ is given as follow:

$$\mathbf{S}_{\mathbf{v}_k}^{\text{cov}} = (\mathrm{M}_{k-1} - 1)\mathbf{S}_{\mathbf{v}_{k-1}} + (m_k - 1)\sigma_{\mathbf{v}_k} \tag{5.19}$$

and the mean term of $S_{\mathbf{v}_k}$ is obtained by

$$\mathbf{S}_{\mathbf{v}_k}^{\text{mean}} = \mathrm{M}_{k-1}(\mathrm{E}_{\mathbf{v}_{k-1}} - \mathrm{E}_{\mathbf{v}_k})^2 + m_k(\bar{\mathbf{v}}_k - \mathrm{E}_{\mathbf{v}_k})^2. \tag{5.20}$$

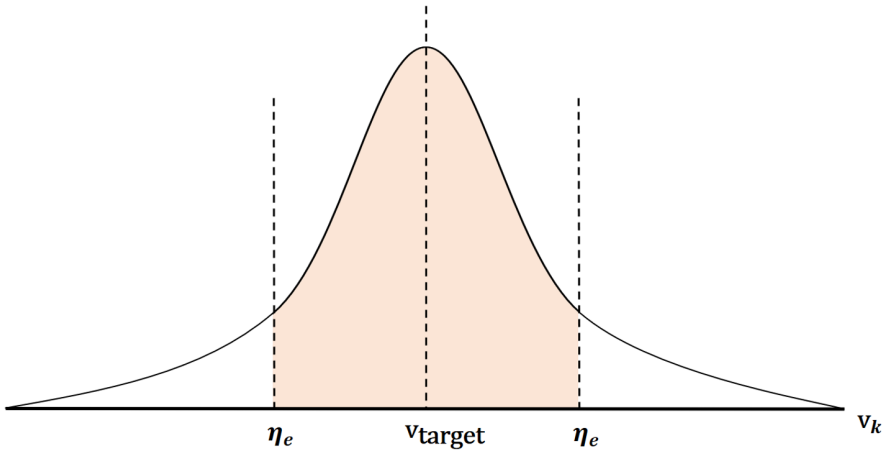


Figure 5.3: The concept of eNDS. Validated measurements are confirmed by the statistical distance from $\mathbf{v}_{\text{target}}$.

where, $\mathrm{E}_{\mathbf{v}_k}$ is

$$\mathrm{E}_{\mathbf{v}_k} = \frac{\mathrm{M}_{k-1}\mathrm{E}_{\mathbf{v}_{k-1}} + m_k\bar{\mathbf{v}}_k}{\mathrm{M}_{k-1} + m_k}. \tag{5.21}$$

Finally, validated measurements which satisfy the eNDS condition are used in the data association step as follows:

$$\mathcal{V}_{\mathbf{v}_k}(\eta_e) = \left\{\mathbf{z}_k : (\mathbf{v}_k^i - \mathbf{v}_{\text{target}})^T \mathbf{S}_{\mathbf{v}_{k-1}}^{-1} (\mathbf{v}_k^i - \mathbf{v}_{\text{target}}) \leq \eta_e\right\} \tag{5.22}$$

where, η_e is the threshold of the eNDS. Likewise the gating criterion, the eNDS at scan k uses $\mathbf{v}_{\text{target}}$ accumulated up to $k-1$ scan.

Note that the purpose of introducing the eNDS is solely to complement $\mathbf{Q}$ or $\mathbf{R}$ in terms of forming the validation gate. So that, as the same as the gating criterion, $\mathbf{Q}$ and $\mathbf{R}$ are still utilized to calculate the predicted error covariance P_k^- and the Kalman gain K_k.

Fig. 5.3 depicts the concept of the eNDS. At each scan k, $\mathbf{v}_{\text{target}}$ is used as a reference point. Compare to the gating criterion, the eNDS uses the square root of Mahalanobis distance as the condition instead of the Euclidean distance. Furthermore, in

contrast to how it is done in the conventional validation gate method, an empirical error covariance obtained from the target being tracked is used. Therefore, a measurement that is more probable to be a target can be selected for the data association step. Note that, the empirical validation gate may not be set up around the predicted measurement as opposed to the conventional validation gate because it does not use prediction.

5.2.4 Measurement *Re*-Validation

The proposed algorithm conducts an additional confirmation process on the validated measurement of the conventional method using accumulated information of the target trajectory. This confirmation process is called as *measurement re-validation*. Fig. 5.4 shows the block diagram of the PDAF including the proposed measurement re-validation step. The measurement re-validation step is performed when a specific condition is satisfied during tracking. This condition is described below.

As mentioned before, if the tracker is tracking the target correctly, information about the target can be contained in the target trajectory and measurements that are used to update the target state. As target tracking progresses, information about the target is accumulated. Thus, by the law of large numbers, the accumulated information can eventually represent the distribution of the target dynamics. However, if the number of samples i.e., information about the target is limited such as in the early stage of tracking or due to inaccuracies in the tracking, then the accumulated information can lead the measurement re-validation in the wrong direction. In other words, the accumulated information can be biased if there is insufficient or inaccurate data.

Taking this consideration into account, the measurement re-validation step is applied according to the $\mathbf{S}_k$ as described following:

$$|\mathbf{S}_k| \geq \alpha \, |\mathbf{S}_{k-1}| \tag{5.23}$$

where, α is pre-determined ratio between error covariance at step k and $k-1$. e.q.5.23 is designed to avoid (a) a measurement re-validation step in the early stages of tracking or (b) a rapid increase in the volume of the validation region.

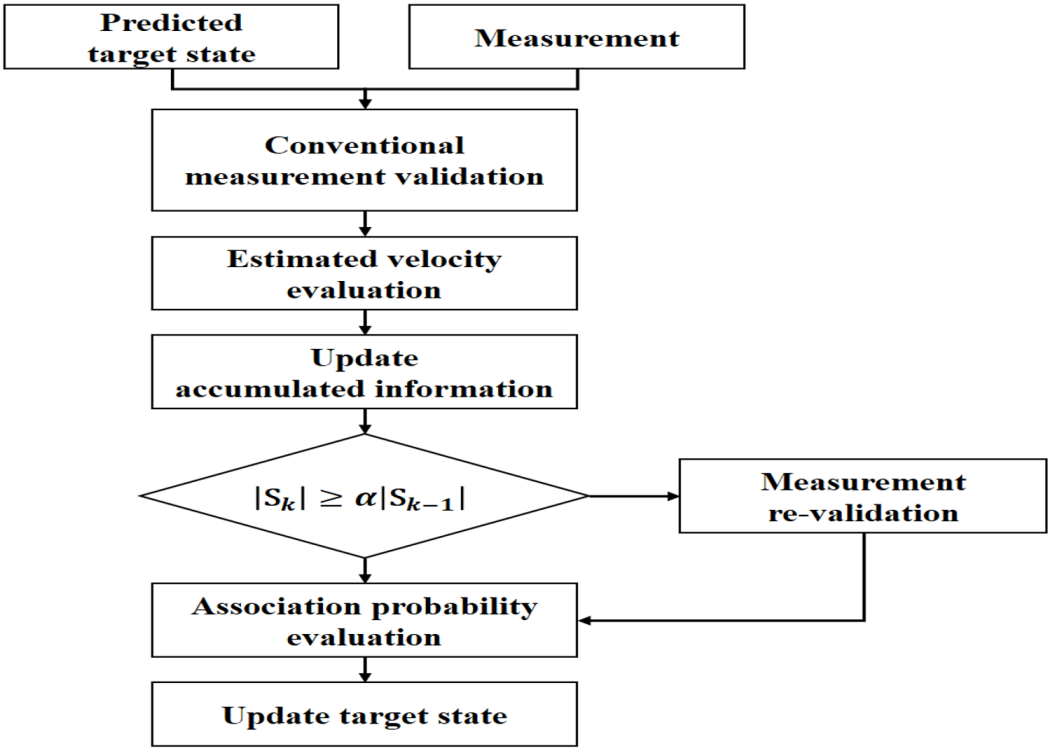


Figure 5.4: Block diagram of the PDAF including the proposed algorithm at scan k.

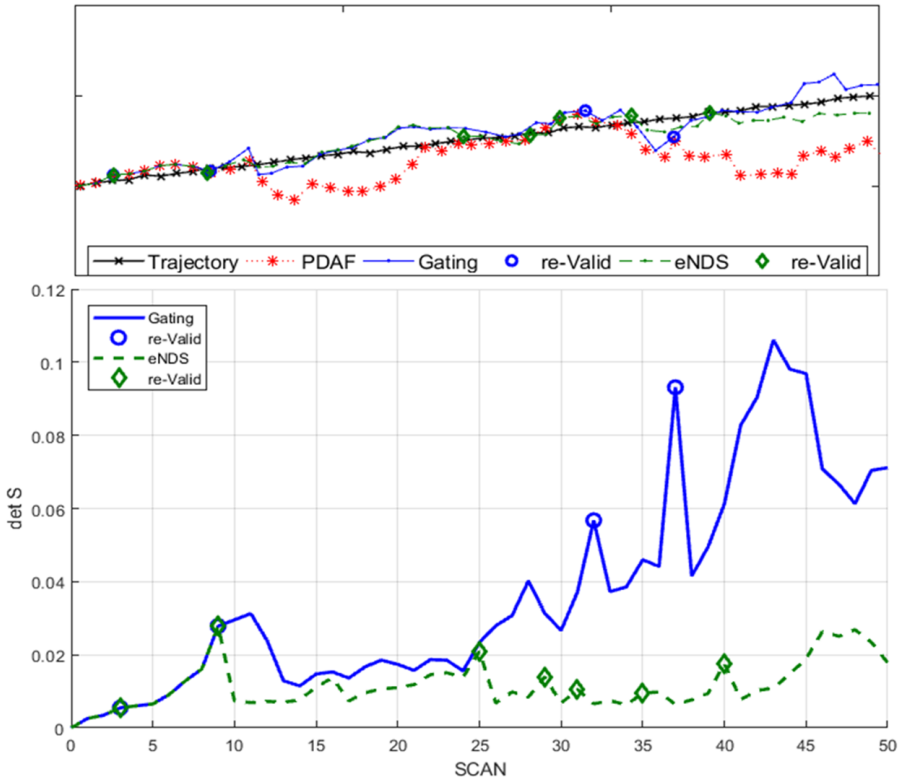


Figure 5.5: Example of resulting estimated target trajectory using measurement re-validation(top) and $|\mathbf{S}_k|$ at each scan(bottom). $P_{FA} = 0.044$, $\eta_g = 0.4$, $\eta_e = 1.8$ and $\alpha = 1.8$ is applied. The measurement re-validation is activated(circle) when $|\mathbf{S}_k|$ increases sharply.

Fig. 5.5 depicts an example of the resulting estimated target trajectory using measurement re-validation and $|\mathbf{S}_k|$ at each scan. The circles in the figure indicate that the measurement reconfirmation has been applied. The measurement re-validation is applied when $|\mathbf{S}_k|$ increases sharply. Also, due to the effect of measurement re-validation step, one can see that the target is tracked correctly compared to the conventional PDAF.

5.2.5 Extension to RFS based model

As described earlier, the proposed algorithm is designed in consideration of an universally application regardless the Bayes recursion implementation. To guarantee the robustness into a variety of tracking algorithms, an extension of the proposed algorithm to a single target RFS based model, Bernoulli filter, is discussed.

The RFS based filter derives a mathematically rigorous Bayes recursion for tracking a target which generates a random number of measurements. Two implementations are proposed to solve the Bayes recursion: Gaussian sum filter and sequential Monte Carlo Filter. Both implementations have in common that they approximate true distributions using a weighted sum of sub-components.

Suppose that $s_{\cdot|\cdot}(\mathbf{x})$ is the spatial PDF and $q_{\cdot|\cdot}$ represents the cardinality distribution of RFS $\mathbf{x}$. Since this dissertation focuses on the association uncertainty according to the number of measurement growth, Gaussian sum filter implementation of Bernoulli filter(which is so-called Bernoulli GSF), whose underlying assumption is linear Gaussian, is mainly described. The prediction equations of Gaussian sum filter implementation are formulated below:

$$\begin{aligned} s_{k|k-1}(\mathbf{x}) =& \frac{P_B(1-q_{k-1|k-1})}{q_{k|k-1}} \sum_{i=1}^{N_{b,k}} \omega_{b,k}^i \mathcal{N}(\mathbf{x}; \mathbf{m}_{b,k}^i, \mathbf{Q}_{b,k}^i) + \\ & \frac{P_S q_{k-1|k-1}}{q_{k|k-1}} \sum_{i=1}^{N_{k-1}} \omega_{k-1}^i \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k-1}^i, \mathbf{P}_{k|k-1}^i), \end{aligned} \tag{5.24}$$

$$q_{k|k-1} = P_B(1-q_{k-1|k-1}) + P_S q_{k-1|k-1}, \tag{5.25}$$

where P_B and P_S are the birth and survival probability which account for the target existence. ω^i is the weight of Gaussian component i. The update equations are then:

$$\begin{aligned} s_{k|k}(\mathbf{x}) =& \frac{(1-P_D)}{1-\Delta_k} s_{k|k-1}(\mathbf{x}) + \\ & \frac{P_D}{1-\Delta_k} \sum_{\mathbf{z}\in\mathbf{Z}_k} \sum_{i=1}^{N_{k|k-1}} \frac{\omega_{k|k-1}^i q_k^i(\mathbf{z})}{\lambda_c} \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k-1}^i, \mathbf{P}_{k|k-1}^i) \end{aligned} \tag{5.26}$$

$$q_{k|k} = \frac{1-\Delta_k}{1-q_{k|k-1}\Delta_k} q_{k|k-1}, \tag{5.27}$$

$$\Delta_k = P_D \left[1 - \sum_{\mathbf{z}\in\mathbf{Z}_k} \sum_{i=1}^{N_{k-1}} \frac{\omega_{k-1}^i q_k^i(\mathbf{z})}{\lambda_c} \right], \tag{5.28}$$

where $q_k^i(\mathbf{z})$ accounts for the estimated measurement, which follows the Gaussian distribution with mean $\mathbf{H}_k\mathbf{m}_{k|k-1}^i$ and covariance $\mathbf{S}_{k|k-1}^i$.

Since the existence and birth of the target should also be estimated, every possible combination between measurements received at time k and the estimated state of each component at $k-1$ are generated as hypotheses. Then, the number of hypotheses is reduced by pruning, merging, or re-sampling after the Bayes recursion for all hypothesis. This methodology is analogous to the MHT approach and the equivalent between MHT and the RFS based filter(GMCPHD[4]) has been proved in [178]. Therefore, an explicit association probability evaluation process such as in PDAF is omitted, but the measurement validation still available to reduce the number of hypotheses by selecting an appropriate measurement.

Compare to PDAF, since the Gaussian component of Bernoulli GSF is estimated by standard KF formulation, the association uncertainty not involves in the error covariance calculation. See e.q.2.19 and 2.57. Thus, validation gates need to set up separately based on the error covariance of each Gaussian component. The proposed algorithm stores the accumulated information of each estimated trajectory until the Gaussian component merged or pruned. Furthermore, validated measurements are excluded from the accumulation if no measurement from targets are present according to the cardinality estimation.

Note that the study that applies the gating scheme into the Bernoulli filter has proposed in the labeled Bernoulli filter[94] for multi-target tracking. In this algorithm, the state of targets is updated in parallel based on the spatial grouping of targets or measurements. This results in a drastic reduction in computation times with a slight

[4]Gaussian mixture cardinalized Probabilistic Hypothesis Density filter

compromise in accuracy. Likewise, since the implementation of the Bernoulli filter is the sampling-based approximation of true FISST of the state, accuracy could degrade if the number of validated measurements reduced. In other words, the gating approach in the Bernoulli filter requires the balance between accuracy and the number of measurements for saving processing time. This trade-off also can be found in the Bernoulli GSF formulation. As shown in eq. 5.27 and 5.28, the number of $\mathbf{Z}_k$ and the Gaussian components are involved in the estimation of $q_{k|k}$. Since cardinality estimation is one of the essential measures when evaluating Bernoulli filter performance, the trade-off between the accuracy and the number of measurements is one of the major challenges for the extension of the proposed algorithm. The proposed algorithm can provide a balance based accumulated information of the target.

5.3 Experimental results

In this section, the performance of the proposed algorithm demonstrates in three tracking frameworks: standard Kalman filter with PDAF, and Bernoulli GSF. Both two frameworks track a single target. The targets for all scenarios assume to follow the linear Gaussian assumption. The simulators for the proposed algorithm is designed in Matlab R2019b and 1,000 Monte-Carlo trials were carried out to evaluate the performance, respectively. The simulation environment and experimental result for each scenario present follow.

5.3.1 Kalman filter with PDAF

Simulation environments Fig.5.6 depicts the simulation scenario for the standard Kalman filter with PDAF. The trajectory of the ground truth target is generated with the known initial state $\mathbf{x}_0 = [0.1 \quad 0.3 \quad 3.0 \quad 0.01]^T$ in xy-plane along with clutters, simultaneously. The black line and blue dots represent the trajectory of the target and clutter, respectively. The ground truth target traverses linearly in the observation re-

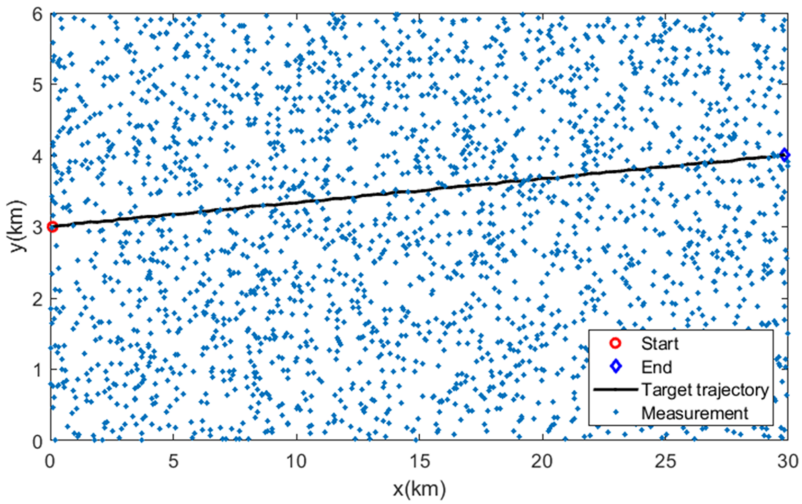


Figure 5.6: Simulation scenario for Kalman filter with PDAF. Target trajectory(the black line) and measurements(the blue dot) with $P_{\text{FA}} = 0.044$

gion and is always detected as a measurement with noise. The number of clutter points and their corresponding distribution is simulated as described in Sec. 5.2.1. The other parameters for the simulation are listed in Table 5.3.1. The hyper parameters such as η_g, η_e, ζ, and α were chosen experimentally. The PDAF with a conventional measurement validation scheme is used as a baseline to verify the performance of the proposed algorithm.

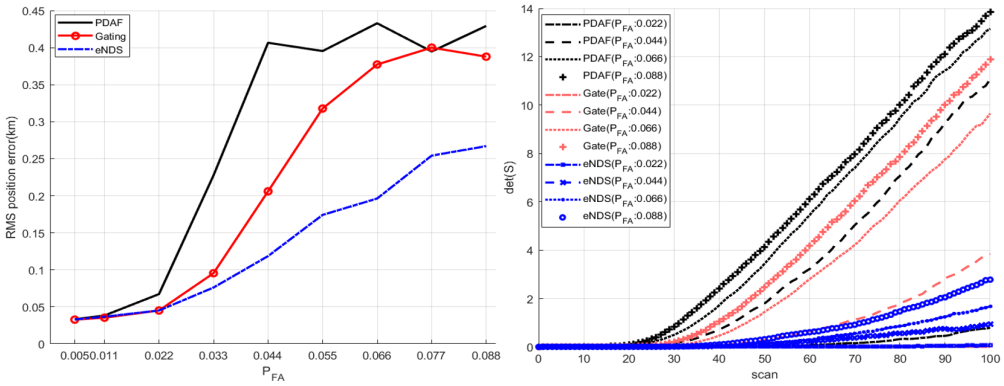
Metric The performance of each algorithm measure using RMS position error, $|\mathbf{S}_k|$, and average processing time. Note that, in order to compare the performance between algorithms accurately, the performance of the proposed algorithm analyses based on the average of all trials except two cases as follows: (1) case where estimated target trajectories have RMS position errors exceeding 1.5km (single scan), and (2) case where no measurements were received in 5 consecutive scans. These cases are called as an excepted track and present the number of excepted tracks for each simulation. The reason for defining excepted tracks is to prevent inaccurate interpretation of simulation results due to excessive errors, not for discussing the determination of the track loss.

Table 5.1: Simulation parameters for TbMV in the standard Kalman filter with the PDAF scenario.

Parameters	Value
Total number of scan	100
Sampling rate	$1s$
Observation region	$180 \times 10^6 \text{m}^2$
Number of radar cells	$54,000$
Probability of detection	0.95
Probability of gate	0.99
Probability of FA (e^{-3})	1.1 2.2 3.3 4.4 5.5 6.6 7.7 8.8
Measurement error	50m
Discount factor	1
Threshold η_g, η_e	0.1 0.3 0.5 0.7 0.9 1.3 1.6 1.9
Ratio α	1.0 1.25 1.5 1.75 2.0 2.25 2.5 2.75

The problem of determining the lost track in a heavy-clutter environment is beyond the scope of this paper.

Comparison of RMS position errors for the probability of false alarm. Fig. 5.7 shows the RMS position error and $|\mathbf{S}_k|$ of each algorithm against various P_{FA}s. The gating threshold and ratio is set to be $\eta_g = 0.4$, $\eta_e = 1.8$ and $\alpha = 1.8$, respectively. As mentioned earlier in this chapter, in a low-clutter environment such as $P_{\text{FA}} = 0.005$ and $P_{\text{FA}} = 0.011$, not only the target can be tracked accurately, but also the number of validated measurements can be maintained. Therefore, there is no difference between the performance of conventional measurement validation and the proposed algorithm. However, in a dense environment, both gating criterion and the eNDS show a lower RMS position error than the conventional measurement validation algorithm. Furthermore, $|\mathbf{S}_k|$ of the eNDS is not increased significantly despite of an increase in the P_{FA}.



(a) RMS position error

(b) $|\mathbf{S}_k|$

Figure 5.7: Comparison of RMS position error(A) and $|\mathbf{S}_k|$(B). eight different P_{FA} were used.($\eta_g = 0.4$, $\eta_e = 1.8$ and $\alpha = 1.8$)

Table 5.2: Average processing Time(s) for one cycle of tracking for the standard Kalman filter with the PDAF scenario.

P_{FA}	0.011	0.022	0.044	0.088
Conventional PDAF	0.0205	0.0574	1.4654	3.3428
Gating	0.0212	0.0502	1.0917	1.9479
eNDS	0.0283	0.0498	0.9435	1.8673

This is because the proposed algorithm carries out the measurement re-validation to reduce the effect of the measurement origin uncertainty. Thus, the robustness of the selected measurement by the measurement re-validation using the accumulated information is verified.

Table 5.2 presents the average processing time for one cycle of tracking. The proposed algorithm prevents an increase in the volume of validation region by suppressing the origin of measurement uncertainty. With this effect, the number of validated measurements is reduced compared to the conventional method, which can also reduce the processing time.

Comparison of RMS position errors for the threshold The results are shown in

Fig. 5.8(a) representing the effect of η in the proposed algorithm. $P_{FA} = 0.044$ and $\alpha = 1.8$ are used, respectively. As η increases, the effect of measurement re-validation decreases, which can be confirmed by an increase in RMS position error. Also, one can see there is a difference in η, which has the best performance due to the difference in the method of calculating the distance between the gating criterion and the eNDS. Note that, at $\eta = 0.1$, the RMS position error has increased. This is the case where the tracking performance is degraded due to the strict threshold. In other words, a strict threshold can lead to measurement missing and it degrades the tracking performance. The performance of the proposed algorithm in terms of the ratio α is shown in Fig.5.8(b). The hyperparameter is set as $P_{FA} = 0.044$, $\eta_g = 0.4$ and $\eta_e = 1.8$, respectively. one can see that the effect of the bias is due to the insufficiency of data. Referring to [177], if 'α' is close to 1, the measurement re-validation process can be applied in the early stage of tracking because it is independent of the increase of $|\mathbf{S}_{k-1}|$. This means that the accumulated information is insufficient to represent the target, RMS position error increases near $\alpha = 1$, as shown in the figure. On the other hand, since the gating criterion accounts for accumulated data not as much as the eNDS does, it is less affected by data insufficiency than the eNDS.

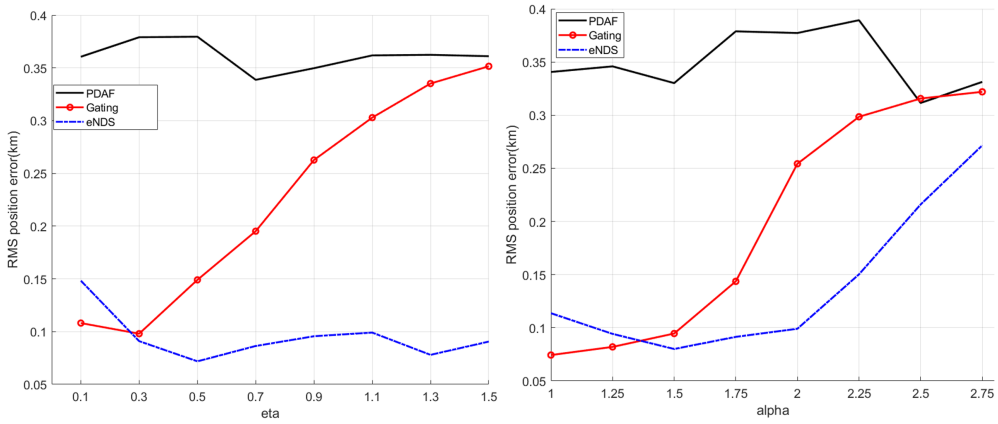
Table 5.3 represents the probability of excepted tracks for each experiments. The probability was calculated from the number of times an excepted track occurred in $1,000$ trials. The probability of excepted tracks is similar to the RMS position error of each experiments. However, the eNDS shows a higher probability of excepted tracks in a low-clutter environment than conventional PDAF. This result implies that the insufficiently accumulated information can lead the biased estimation.

5.3.2 Bernoulli GSF

Simulation environments In order to verify the performance of the proposed algorithm in the Bernoulli GSF, experimental environments set up the same as in [47], and the gating criterion and eNDS are implemented upon this environment. The simulator

Table 5.3: In the experiment, track results with an excessive error are excepted for calculating RMSE to avoid inaccurate interpretation. The numbers in this table represent the ratio of excepted tracks during Monte Carlo trials for each algorithm and higher is worse. The proposed algorithms have less number of excepted tracks than conventional algorithms in a dense environment.

P_{FA}	0.011	0.022	0.033	0.044	0.055	0.066	0.077	0.088
Conventional PDAF	0.031	0.071	0.237	0.368	0.443	0.453	0.433	0.425
Gating	0.032	0.064	0.166	0.285	0.364	0.405	0.385	0.399
eNDS	0.054	0.162	0.278	0.286	0.326	0.358	0.376	0.387
η	0.1	0.3	0.5	0.7	0.9	1.3	1.6	1.9
Conventional PDAF	0.390	0.390	0.390	0.390	0.390	0.390	0.390	0.390
Gating	0.440	0.280	0.284	0.308	0.380	0.390	0.367	0.385
eNDS	0.452	0.428	0.411	0.414	0.427	0.347	0.333	0.311
α	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75
Conventional PDAF	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388
Gating	0.260	0.258	0.282	0.298	0.308	0.370	0.316	0.338
eNDS	0.308	0.286	0.276	0.269	0.290	0.316	0.336	0.340



(a) different η (b) different α

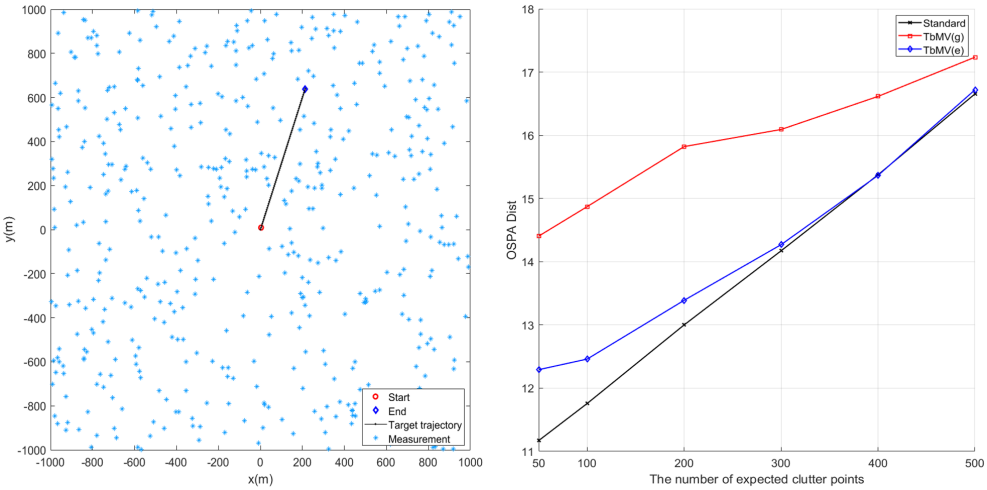
Figure 5.8: Comparison of RMS position error for (a) different η ($P_{FA} = 0.044$ and $\alpha = 1.8$) and (b) different α ($P_{FA} = 0.044$, $\eta_g = 0.4$, and $\eta_e = 1.8$).

is modified based on the published code on the project page[5] of [47]. Fig. 5.9(a) depicts the experimental scenario for Bernoulli GSF. Measurements are acquired during $k = 100$ scans in the observation region. A single ground truth target in linear motion is only detected between $k = 20$ and $k = 80$. No initial state of the target is given. The experiment is conducted on the three cases using various numbers of the expected number of measurements. Bernoulli GSF with a standard validation gate is used as the baseline. The model parameters are set as in the published code and thresholds for TbMV are set as follows: $\eta_g = 0.5$, $\eta_e = 0.8$, and $\alpha = 1.8$

Metric Since the Bernoulli filter includes cardinality estimation, accurate performance evaluation is limited with RMSE alone. Thus, OSPA[176] distance $\bar{d}_p^c$ is used as a metric. Let $X = \{x_1, ..., x_m\}$ and $Y = \{y_1, ..., y_m\}$ are arbitrary point patterns and Π_k is the set of permutation on $\{1, 2, ..., k\}$. OSPA distance between two point pattern X and Y defines as follows:

$$\bar{d}_p^c := \left[\frac{1}{n} \left(\min_{\pi \in \Pi_n} \sum_{i=1}^{m} d^c(x_i, y_{\pi(i)})^p + c^p(n-m) \right) \right]^{\frac{1}{p}} \tag{5.29}$$

[5] http://ba-tuong.vo-au.com/codes.html



(a) Scenario (b) OSPA dist for various cases

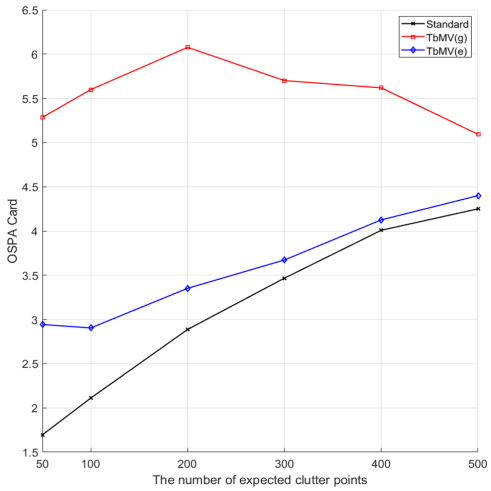
(c) Average processing time for various cases

Clutter points	100	200	300	400	500
Standard	0.0704	0.1139	0.1702	0.2503	0.3593
TbMV(Gating)	0.0765	0.1149	0.1634	0.2301	0.3207
TbMV(eNDS)	0.0780	0.1178	0.1695	0.2470	0.3373

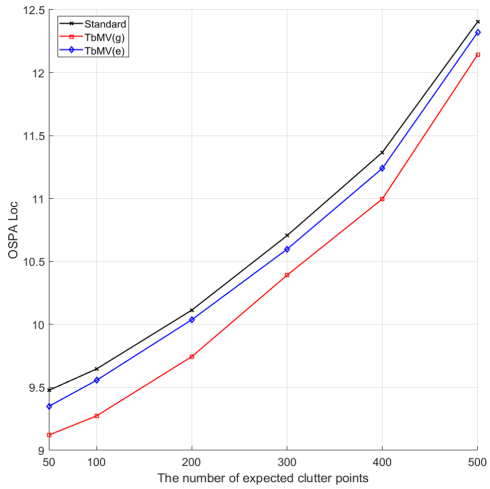
Figure 5.9: Experimental scenario and result for the Bernoulli GSF. Higher is worse.($\eta_g = 0.5$, $\eta_e = 0.8$, and $\alpha = 1.8$).

where, p is order and c is cut-off of the OSPA metric. $d^c(x_i, y_{\pi(i)})^p$ is the pth order Wasserstein metric with cut-off c. The OSPA metic is divided into two components, each accounts for localization(first term in eq. 5.29) and cardinality(second term in eq. 5.29), respectively. For further details on the OSPA metic the reader is referred to [176, 179]

Comparison of performances for the expected number of clutter points Fig. 5.9(b) and (c) shows the OSPA dist and the average processing time for the various numbers of expected clutter points, respectively. The experimental results show that the proposed algorithm cannot effectively validate the redundant measurement in a low cluttered environment and even requires more processing time. However, in a dense



(a) OSPA card



(b) OSPA loc

(c) The ratio of excepted tracks($1,000$ trials)

Clutter points	100	200	300	400	500
Standard	0.012	0.051	0.083	0.183	0.267
TbMV(Gating)	0.054	0.150	0.238	0.360	0.441
TbMV(eNDS)	0.027	0.081	0.119	0.225	0.305

Figure 5.10: OSPA results and the ratio of excepted tracks for Bernoulli GSF scenario. Higher is worse. ($\eta_g = 0.5$, $\eta_e = 0.8$, and $\alpha = 1.8$).

environment, the proposed algorithm shows similar accuracy while slightly saving processing time. Results for each part of OSPA dist provide detailed explanations. In particular, inaccurate estimation in a low cluttered environment is caused by inaccurate estimation of existence probability due to the insufficient number of components as shown in Fig. 5.10(a). This experimental results can support the deduction for the relationship between the performance degradation and existence probability. Compare to the standard measurement validation process, the proposed algorithm increases for OSPA card while maintains similar results for OSPA loc as shown in Fig. 5.10(b). On the other hand, performance results for a large number of measurements verify that the proposed algorithm can provide a balance between the accuracy and the number

of measurement in a dense environment. Nevertheless, one needs to be addressed that the number of excepted tracks of the proposed algorithm maintains a higher value than the standard method as shown in Table. 5.10(c). Thus, to guarantee the robustness of the proposed algorithm, the study to select the appropriate measurement while maintaining accuracy needs to conduct.

5.4 Discussion

Conventional data association algorithms suffer from ambiguity of the origin of measurements in a dense environment. This association uncertainty leads to an inaccurate estimation of the target's state, and even the target can be lost occasionally. Most of the conventional approaches evaluate the association between target and measurements based solely on spatial distance. Thus, one can deduce that the insufficient use of information causes the association uncertainty in this stage.

In this chapter, the trajectory-based measurement validation(TbMV) is proposed for the association uncertainty in the data association stage. The proposed algorithm is a refinement procedure for preventing an excess increase in the number of validated measurements in a heavy-clutter environment. To select relevant measurements to the target, the additional *criteria* is newly defined based on the accumulated information contained in the target trajectory. In particular, the estimated velocity of the target and measurement is used to design the criteria for this dissertation. Selecting criteria for TbMV are established by using the (1) Euclidean distance(gating), and (2) Mahalanobis distance(eNDS), respectively.

The performance of the proposed algorithm is verified in the various tracking frameworks as follows: (1) standard KF with PDAF, (2) Bernoulli GSF. The simulation result for the simplest case, the standard KF with PDAF, shows that the proposed algorithm exhibits up to 50% lower RMS position error than conventional measurement validation in a dense environment. Thus, it can be confirmed that the selected validated

measurement using the accumulated information is valid for updating the target state. Furthermore, due to the effect of limiting the number of the validated measurement, processing time is also decreased. Compared to the gating criterion, the eNDS can track the target more effectively in a dense environment. Since both algorithms use the same accumulated information, one can analyze the difference as the effect of normalization by the empirical covariance. The effectiveness of the proposed algorithm can also find in other experiments. The simulation results for more complex environments show that the proposed algorithm can also marginally resolve the association uncertainty in the data association stage regardless of the tracking framework.

However, to guarantee the robustness of the proposed algorithm, some issues must be cleared as follows: First, referring to the experimental result on the α, the insufficiently accumulated information can induce the bias to the estimation. This phenomenon can be severely aggravated when tracking a maneuvering target whose dynamic changes widely. Thus, more studies on the feature that adequately represent the maneuvering target need to be conducted. Second, η_g and η_e need to be selected in consideration of the unexpected tracking failure, especially in case of sampling-based implementations such as Bernoulli GSF. Since the RFS based filter estimates the state of the target along with its existence probability, tracking can fail due to incorrect estimation of the existence probability even if the state estimates accurately. For this reason, this issue also requires further studies.

Chapter 6

Conclusion

This chapter presents the conclusion of this dissertation. A summary of the problem, central intuition, and contributions are described. Then, experimental results for each proposed algorithms are discussed, and directions for future studies are presented.

6.1 Summary

This dissertation consider the association uncertainty in the radar tracking system based on the Bayesian filtering framework. In order to implement the Bayesian filtering framework in the radar tracking problem, two preprocessing stages essentially are required to account for the measurement origin uncertainty: track initiation and data association stage. Various approaches have been proposed for preprocessing stages, such as logic based method, Hough transform based method, (J)PDAF, MHT, and the RFS based filter, and verified that the target could be tracked accurately even if the association uncertainty present. However, the threat of small aerial vehicles, such as drones and missiles, is increasing as in the drone attack against Saudi oil facilities. These target with small SNR is called the dim target, and the radar's detection threshold needs to be lowered to detect the dim target. In this circumstance, the number of measurements in an observation region is expanded, the association uncertainty of the

radar tracking system tends to worsen, consequently. The aim of this dissertation is to address the performance degradation of conventional algorithms caused by significant association uncertainty.

The Bayesian filtering framework, specifically Bayes recursion, recursively estimates the posterior distribution of the state given a noisy measurement. Theoretically, Bayes recursion considers only one measurement whose source is the target and provides optimal estimation for the state. Thus, it can be deduced that the optimal solution estimation is guaranteed if the target originated measurement identified within two stages whose take responsibility for resolving the measurement origin ambiguity. However, from a simple and heuristic algorithm such as NNF to a sophisticated algorithm such as Bernoulli filter, conventional algorithms calculate the association between the target and measurement depend solely on spatial distance. Therefore, the environment where the measurement is densely distributed in an observation region, the origin of measurement cannot be distinguished, and optimal estimation by Bayes recursion cannot be expected as a result.

Based on this intuition, this dissertation aims to provide alternative approaches for the association uncertainty in the radar dim target tracking. The proposed approach uses additional information of measurement to supplement conventional methods of the track initiation and data association stages as in an ensemble manner. Different from recent studies that apply the advanced radar or additional sensors, the proposed method exploits velocity information as additional information to minimize the computational overhead due to the proposed method. This dissertation demonstrates the applicability of this alternative approach to the radar tracking system.

The key contributions of this dissertation is summarized as follows:

- First, the association uncertainty in the radar tracking system is experimentally described, and the novel interpretation and corresponding approaches for the association uncertainty in the radar dim target tracking are provided.

- Second, novel track scores are proposed using velocity information about the

measurement for the track initiation stage. Furthermore, motived by the clustering problem, the scheme of non-maximum suppression(NMS) is proposed to maintain the number of initiated targets. The experimental result has verified that the association uncertainty is effectively resolved by suppressing the number of false targets based on priority.

- Last, the novel trajectory-based measurement validation(TbMV) method is proposed for the data association stage. The proposed algorithm, motived by the (binary) classification problem, define the new feature based on the accumulated information from the target trajectory. Furthermore, selecting criteria are established based on the statistical distance between features. The result of experiments shows that the proposed algorithm can effectively resolve the association uncertainty on various tracking frameworks.

6.2 Discussions

Various simulations have conducted in support of what this dissertation insists on and the superiority of the proposed algorithm. In the following, experimental results are discussed with a brief description of the proposed algorithms.

The relationship between the number of measurements and tracking performance has experimentally demonstrated to account for the association uncertainty. The experiment conducted on logic-based track initiation method, PDAF, and Bernoulli filter using a standard set up except for the number of measurements. The results confirm that the performance of the tracking algorithm can be degraded by solely on the growth of the number of measurements.

The design of the proposed track initiation algorithm mainly concentrated on how to maintain a constant level of the number of initiated targets by applying a clustering problem approach. For this purpose, the track score defined to determine the priority between tentative tracks. The scheme of non-maximum suppression, which is com-

monly used in the object detection task in the computer vision domain, is introduced to limit the number of confirmed tracks. Only one tentative track remained among redundant tracks, a group of adjacent tentative tracks, according to the track score. A method that converts simple conditions of conventional approaches to probabilistic track scores is proposed based on the fact that the conventional track score suffers from processing time in a dense environment due to the Bayes recursion. Furthermore, two weighted score based approaches are proposed using additional information of the measurement: binary weight and weighted sum. The weighted score complements the inconsistency of conventional conditions in the presence of measurement error. Experiments are conducted on a computer-based simulator. The performance of the proposed algorithm is verified in both simple and complex scenarios with linear motion multi-target. The result confirmed that the proposed algorithm can effectively suppress the initiation of false targets while maintains favorable processing time in a dense environment compared to M/N logic based and modified Hough transform based method. It can be interpreted that the track score indicates the priority of the tentative track as intended and the redundancy is removed through the NMS process. In the case of measurement error, while the probabilistic track score based method suffers from initiation failure like conventional algorithms, weighted score based methods can find ground truth target even in the significant measurement error. This result shows that the appropriate additional feature can effectively resolve the association uncertainty in the track initiation stage. However, since measurements are excessively removed, the binary weight method tends to lose the ground truth target and the ground truth target was removed in certain scenarios according to the NMS threshold. This phenomenon is related to thresholds setting and need to subsequent studies.

The proposed data association stage mainly focuses on how to reduce irrelevant measurements as much as possible in a dense environment by exploiting additional information. In this regard, the novel trajectory-based measurement validation(TbMV) algorithm is designed and extracts meaningful information from the estimated trajec-

tory. Two approaches are proposed using additional information of the measurement based on the fact that conventional measurement validation approaches suffer from measurement origin ambiguity in a dense environment due to lack of representation power. TbMV has not substituted the conventional method, instead, it selects validated measurements along with the conventional method in an ensemble manner. The proposed algorithm is based on the newly defined information. The estimated velocity is used as an additional information. In each scan, the velocity of ground truth target calculated using the target's estimated positions, which accumulated in trajectory. Also, the velocity of measurements calculated using the previous position of the target and current position of each measurement. The difference between the two velocities becomes a new criterio to infer the association. According to the way of defining the difference, the proposed validation gate is set around the predicted position of the target in two approaches: gating criterion and eNDS. The gating criterion and eNDS use the Euclidean and Mahalanobis distance, respectively. In particular, since eNDS requires the variance calculation, it stores a history of measurement features. Furthermore, measurement re-validation is a method for preventing the unwanted exception of proper measurements caused by biased features due to insufficiently accumulated data. Extension to the RFS based filter is introduced to verify the robustness in various scenarios. Experiments are also conducted on a computer-based simulator. From a fixed number of the target to a random number of the target, the performance of the proposed algorithm is verified with various scenarios and tracking methods. The simulation result confirmed that the proposed algorithm not only performs efficiently in a dense environment but also saving processing time compared to the conventional method. This can be interpreted that the additional feature effectively mitigates the ambiguity of the measurement origin. Furthermore, the average processing time saved by selecting a subset of measurements. The performance results on Bernoulli filter showed the proposed algorithm is applicable to the real-world scenario. However, some issues must be considered to guarantee robustness. First, since the existence probability esti-

mation of the Bernoulli filter relates to the number of measurements, it needs to design the measurement validation that takes into account the existence probability. Second, regarding the underlying assumption that the target travels in a linear trajectory with constant velocity, it is essential to design features that are suitable for the maneuvering target, which dynamically changes in motion.

6.3 Direction for Future Studies

The challenges that are required more studies still remain to integrate machine learning approaches into the radar tracking system.

Adaptive thresholds Regardless of the detailed schemes of each proposed algorithms, empirical thresholds are used in all methods to establish criterions. These thresholds are determined by the validation process and tend to depend on environmental factors. As a result, inappropriate thresholds may lead to inaccurate tracking results. Experiments on NMS, gating, and eNDS thresholds confirm the above issues. Thus, studies on the threshold whose value *adaptively* change without human intervention depending on the environment need to be conducted.

Features that invariant to maneuvering target This dissertation investigates the analogies between the radar tracking system and machine learning problem to address the association uncertainty in terms of the number of measurements. Experimental results confirm that proposed algorithms can successfully track the linear motion target in a dense environment. Subsequent studies need to extense to account for the non-linear motion target, so-called maneuvering target. To clearly classify that the origin of measurements is whether the target or clutter, the feature is required to represent the various aspect of the target. The maneuvering target moves in a dynamic direction, speed, and acceleration. This means that the variance of the estimated trajectory is larger than the linear motion target, and more data is required to sufficient representation of the target. However, since adequate time is needed to accumulate the estimated

trajectory, it may lead to an inaccurate inference of measurement origin by the proposed algorithm. Therefore, future studies need to focus on the feature which invariant to complex dynamic change.

Toward deep learning At the beginning of the study on this subject, the author, stimulated by the recent development of deep learning algorithms, aimed at the direct implementation of a deep neural network to the radar tracking system. In particular, the successful integration of deep features in visual object tracking[34] seemed to promise the applicability to the radar tracking system. The method to infer the association probability or confirmed track by training the model has been studied but encountered intrinsic limitations that the radar tracking system has, eventually. Firstly, the dimension of data is insufficient for training a neural network. The dimension of the pulse-Doppler radar is up to five and three of them represent positional information which is independent of the characteristic of the target. This becomes a major obstacle in the direct implementation of deep learning to the radar tracking system. Note that, the dimension of a well-known dataset, MNIST, which consists of a tiny size digit image is 28×28. Lastly, the massive number of a dataset is hardly acquired. This is because the difference between target and clutter cannot be distinguished due to the aforementioned reason, even though many trajectories can be obtained.

However, this dissertation has provided approaches to enhance the representation power using the combination of features in the radar tracking system. Based on these approaches, studies can be conducted toward the direct integration of deep learning methods into the radar tracking system. Recent studies have been conducted on this approach. [180, 181] regards the data association problem as classification in a multi-source environment. In [182], the image frame of the operator's console inputs to the neural network directly as an image. [183, 184, 185] trains a neural network with a set of simulation trajectories and infers the target's state. None of the studies tackle the association uncertainty in a single source.

Based on approaches that are proposed in this dissertation and recent studies using

the deep neural network, one can consider an efficient deep learning method for the radar tracking system. Besides, fusing raw signals from the range-Doppler map[186] to measurement, or using the recurrent model[187] as in visual object tracking can be alternative approaches.

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초 록

추적은 센서에서 수신 한 측정값을 기반으로 관심 대상의 상태를 지속적으로 추정하는 문제이다. 이 문제는 일반적으로 베이지안 필터링 프레임 워크, 특히 예측 및 업데이트 단계로 구성된 Bayes 재귀를 사용하여 공식화된다. Bayes 재귀를 구현하는 다양한 접근법이 연구되었으며 이 접근법들은 대상의 최적 상태를 제공하는 것으로 알려져 있다. 그러나 실제 환경에서는 목표와 측정 수가 일치하지 않는 소위 '연관 불확실성'의 문제가 발생한다. 이를 해결하기 위해 추적 시스템에 2개의 전처리 단계를 두고 있다. 이 단계의 기존 기법은 주로 측정치의 위치를 기반으로 한다.

최근, 레이더 추적 시스템에서 드론 및 미사일과 같은 소형 항공기의 새로운 위협이 대두되고 있다. 희미한 표적이라고 불리는 이러한 표적의 탐지는 낮은 검출 임계값을 필요로 하며 관측 영역에서 더 조밀하게 분포 된 측정을 유도한다. 이로 인해 연관 불확실성이 증가되고 그에 따라 종래의 접근법의 비효율적인 추적을 초래한다. 이러한 기존 접근법의 한계를 해결하기 위해 추가정보를 활용한 접근법을 제안한다. 제안 접근법은 앞서 언급한 추적 시스템의 2개의 전처리 단계에 초점을 맞추며 측정치로부터 쉽게 얻을 수 있는 추가정보를 기반으로 한다. 이를 바탕으로 제안된 기법은 기존 기법과 앙상블 방식으로 적용되며 이때 연산량의 오버헤드를 최소화 하기위해 간단한 계산으로 얻을 수 있는 속도정보를 추가정보로 활용하다.

이 논문의 목적은 측정 횟수와 추적 성능 사이의 인과성 조사를 통해 레이더 추적 시스템에서 연관 불확실성을 해결하기 위한 실행 가능한 방법론을 제공하는 것이다. 이러한 목적을 달성하기 위해 다음과 같은 연구를 수행되었다. 먼저, 추적 시스템의

성능저하 원인이 측정치 수의 증가 측면에서 실험 데모를 통해 논의된다. 이를 바탕으로 레이더 추적문제의 연관 불확실성을 위해 속도정보를 해결하는 직관이 또다른 추적문제의 분야인 영상추적과의 비교를 통해 제시된다. 두 번째로, 속도기반의 확률적 트랙 스코어가 제안되고, 클러스터링 문제의 일반적인 접근법으로부터 착안한 비최대치 억제방식이 추적 초기화 단계에 도입된다. 시뮬레이션 결과는 제안 된 알고리즘이 적절한 처리 시간을 유지하면서 거짓 표적의 초기화를 효과적으로 억제할 수 있음을 보여준다. 마지막으로, 추정된 표적궤적에 누적된 레이더 측정 정보를 사용하여 데이터 연관 단계 중 측정 검증을 위한 속도기반의 새로운 기준이 설계된다. 궤적 기반 검증 게이트는 새롭게 설계된 기준에 기초하여 종래의 게이트와 함께 설정된다. 다양한 시나리오에서 제안 된 알고리즘의 성능을 확인하였으며 측정치가 많이 분포된 환경에서 기존 방식보다 성능이 우수함을 검증하였다.

주요어: 미세표적, 레이더, 추적, 추적 초기화, 데이터 연관, 연관 불확실성, 측정 검증

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2015년 10월, 공군 장교의 신분을 잠시 내려놓고 박사과정 진학이라는 새로운 모험을 시작한 지 4년의 시간이 지났습니다. 연구주제를 잡지못해 고민하였던 시간, 주제를 잡고 난 뒤 논문을 게재 하기위해 마음을 조렸던 시간들을 지나 박사학위 취득이라는 결과를 얻을 수 있었습니다. 이러한 결과는 많은 분들의 도움 없이는 이룰 수 없었을 것입니다.

먼저, 학문적 소양이 부족한 저를 연구실의 일원으로 받아주시고 학위취득까지 이끌어주신 지도교수 곽노준 교수님께 가장 먼저 감사의 인사를 남깁니다. 또한 학위논문 심사를 통해 많은 조언을 해주신 이교구, 이원종, 전동석, 이교구 교수님께 이 글을 빌어 감사의 인사를 전합니다.

4년이라는 연구실 생활은 함께 동거동락했던 연구실 동료들이 없다면 끝까지 해내지 못했을 것입니다. 2016년 2월 처음 연구실 문을 두드렸을 때 연구실 적응에 많은 도움을 주신 연구실 선배님 오지용, 이지은, 박성헌, 김대식 박사님, 원혜민, 이혁진, 황영규, 조영수, 이용수, 황지혜, 이상호, 유재영, 손성준님 감사합니다. 연구실 동기 양한열, 박효진, 박규태, 이경민님, 서울대 야구리그에 입문하게 해준 이명기, 이상국님, 그리고 연구실 생활에 많은 도움을 받았던 최준영, 김장호, 김선훈, 조원민, 박샘, 장시명, 최재석, 박성욱, 이호준, 현민성, 정인섭, 정서형, 유지윤, 유자연, 서건석, 송예지님께 감사의 인사를 드립니다.

또한 같은 방에서 많은 이야기를 함께 나누었던 김지수, 정지수님, 연구 외적인 부분까지 많은 고민을 공유할 수 있었던 공군후배 박동석님, 3년간 과제를 함께 수행하며 연구방향을 잡고 박사학위까지 취득하는데까지 많은 도움을 받았던 권지훈

박사님, 이승의님께 감사의 인사를 전합니다.

끝으로 박사과정 기간동안 물심양면으로 지원해준 부모님 이덕기, 황춘자, 김시영, 임성자님 나의 심장과 같은 두 딸 이규민, 이채민, 그리고 아내 김민주님께 진심을 담아 감사의 마음을 전합니다.

이제 연구실을 떠나 다시 공군으로 돌아갑니다. 몸은 연구실을 떠나지만 지난 4년의 시간이 헛되지 않도록 새로운 목표를 향해 계속 정진하겠습니다. 감사합니다.