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Two Essays on Moral Hazard and Risk Selection when Insurable Asset and Income are Separable

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Abstract

Two Essays on Moral Hazard and Risk Selection when Insurable Asset and Income are Separable

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This thesis investigates the optimal insurance contract with asymmetric information when an insurable asset and income are separable. In the first chapter, I justify a need for a two-argument expected utility model by considering the irreplaceability of income and the asset.

In the second chapter, I develop a model of moral hazard and analyze the optimal insurance choice when an insurer cannot observe an individual's choice of self-protection. Without moral hazard, optimal insurance can vary from no insurance to over insurance depending on the individual's preferences towards income and the asset. Under moral hazard, on the contrary, optimal insurance is partial insurance in which coverage is less than that without moral hazard. The optimal effort level under moral hazard is also less than that without moral hazard. On the other hand, those who have greater marginal utility of income than that of the asset will suffer a lot of disutility from premium payments, and therefore will not buy any insurance at all regardless of the presence or absence of moral hazard. This result stands in contrast to the conventional conclusion that partial insurance is the only equilibrium under moral hazard. Moreover, I compare the equilibrium result in the separation case with that in the non-separation case. This study also demonstrates that the relative importance of moral hazard may differ according to the separability of income and the asset as well as the interaction between income and the asset.

In the third chapter, I develop an endogenous selection model under asymmetric information, in which risk types are endogenously determined by individuals. By assuming heterogeneity in the asset sensitivity that is inherent in a two-argument utility function, we find that in equilibrium, an asset sensitive type of individual may invest in self-protection and become a low-risk, whereas an insensitive type never chooses to expend effort. Unlike the standard model of Rothschild and Stiglitz (1976), this study demonstrates that the sensitive type (low-risk) demands more insurance than the insensitive type (high-risk) under advantageous selection. We also find other types of equilibrium such as adverse selection, separating equilibrium for a single premium rate, partial pooling equilibrium, and pooling equilibrium. In contrast to all previous papers, the equilibrium results obtained in this study reflect the reality that individuals make trade-offs between an income and an insurable asset.

Keywords: Two-argument utility function; Income; Insurable asset; Self-protection effort; Moral Hazard; Advantageous selection; Asset sensitivity

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Chapter 1

Introduction

Insurance is a means to transfer risk and decrease the financial burden for an insured. Specifically, an insurance contract is one whereby the insured can recover the unexpected loss in an insurable asset by receiving the indemnity from the insurer in exchange for the premium payments.

Previous studies emphasize the role of insurance as a mechanism for consumption smoothing for risk-averse individuals. The logic underlying this argument rests on the assumption that income and the insurable asset can quickly and easily be converted to each other. This assumption corresponds to assuming that the insurable asset is as liquid as income. Under this assumption, income and the insurable asset become indistinguishable, thus combined into wealth or simply income, which leads to the conventional one-argument expected utility models. Most of the standard results in insurance economics are derived from these one-argument utility models.

The idea of the convertibility of income and the asset is closely related to the presumption that an insurable asset is a liquid asset and behaves in the same way as a financial asset. In this case, income can be easily converted into other financial assets, and vice versa.

However, this presumption is not applicable to many insurable assets. Assets that can be insured are typically real, not financial. Insurable assets include real assets such as homes, vehicles, and health. The distinction between financial and real assets is especially important in terms of illiquidity and irreplaceability. For instance, trading of a house is subject to high transaction costs and liquidity risks. Thus, a house is not easily converted to income. Moreover, the benefit as a shelter cannot be readily substituted by that of a financial asset. As a result, it is not natural to assume that a house is fully convertible with income or financial assets.

Another example is health which is not directly replaceable by cash and cannot be assessed by market price as individuals have their own valuation of health (Cook and Graham, 1977). Health is also one of the most illiquid assets. Income and health cannot be quickly and easily converted with each other.

Based on this observation, this thesis aims to investigate the optimal insurance contract under a two-argument expected utility model in which income and (the benefit from) an insurable asset separately affect the individual's utility, following Lee (2007). We treat income and the asset as if they are two different goods. Individuals will make insurance decisions, considering their relative preferences on income and the asset.

In this thesis, we assume that one insurable (real) asset is separated from income. Following Lee (2007), we utilize a two-argument utility function $\mathrm{U(C, A)}$, where C and A represent income and the asset, respectively. Then, we take account of the interaction between the two goods, $\mathrm{U_{CA}}$, the cross-second derivative of utility.[1] Whereas Lee (2007) only verified the condition for the optimality of full-insurance, we extend his model to include the insured's private information regarding the care activity to reduce the loss probability, which is often referred to as "self-protection".[2]

[1] Interaction between income and asset can be expressed as follows: $\mathrm{U_{CA}(C, A)} = \frac{\partial^2 \mathrm{U(C,A)}}{\partial \mathrm{C}\, \partial \mathrm{A}}$.

[2] According to Ehrlich and Becker (1972), the effort to reduce the risk falls into two categories: loss prevention (self-protection) and loss reduction. Self-protection effort is a care activity that lowers the probability of an accident. By contrast, loss reduction is an activity that decreases the amount of loss.

In chapter two, we investigate the optimal choice of insurance coverage and self-protection effort under moral hazard. First, if moral hazard is not present, optimal insurance can vary from no insurance to over insurance depending on an individual's preferences. Second, if moral hazard is present, optimal insurance is partial insurance of which coverage is less than that without moral hazard. The optimal effort level is also less than that without moral hazard. By contrast, an individual whose disutility from the premium payment is too high will not purchase any insurance at all regardless of the presence or absence of moral hazard. This result stands in contrast to the conventional conclusion that partial insurance is the only equilibrium under moral hazard. This study also demonstrates that the relative extent of moral hazard may differ according to the separability of income and the asset as well as the interaction between income and the asset.

In chapter three, we assume the heterogeneity in individuals' preferences which is represented by asset sensitivity – sensitivity to the change in an asset value – and provide the equilibrium results under risk selection. We find that in equilibrium, an asset sensitive type of individual may invest in self-protection and become a low-risk, whereas an insensitive type never chooses to expend effort. Unlike the standard model of Rothschild and Stiglitz (1976), the present study demonstrates that the sensitive type (low-risk) demands more insurance than the insensitive type (high-risk) under advantageous selection. We also find other types of equilibrium such as adverse selection, separating equilibrium for a single premium rate, partial pooling equilibrium, and pooling equilibrium. In contrast to all previous papers, the equilibrium results obtained in this study reflect the reality that individuals make trade-offs between income and the asset.

This paper proceeds as follows. Chapter 2 presents insurance and moral hazard when the insurable asset and income are separable. Chapter 3 describes insurance and risk selection in the separation case.

Chapter 2

Insurance and Moral Hazard when Insurable Asset and Income are Separable

2.1 Introduction

Economic analysis of insurance markets with asymmetric information has focused on phenomena such as selection and moral hazard. In this study, we develop a model of moral hazard, in which the adverse selection problem is ignored. As Marshall (1976) pointed out, moral hazard is defined as any misallocation of resources as a result of insurance protection. An individual with insurance will only be partially responsible for the consequences of his or her action. Specifically, the action that we are concerned with is the self-protection effort to reduce loss probability. In this context, the problem raised by moral hazard is that the more coverage there is against loss, the less incentive there is to exert effort in self-protection.

When moral hazard is present, the standard model predicts that partial insurance is optimal (Pauly, 1974; Shavell, 1979). Apart from the variety of insurance demands in reality, including no insurance, the concern is that the analytical framework of the conventional model is an oversimplification. Because the utility function depends only on wealth, they obtain the optimality of insurance regardless of the individual's preferences towards income and the insurable asset.

In fact, many important decisions in economics often involve the allocation of various resources with some of them being risky. Given this, a two commodity model is commonly used to capture the trade-offs between two goods in decision-making (Dardanoni, 1988;

Picone, Uribe, and Wilson, 1998; Eeckhoudt, Rey, and Schlesinger, 2007; Liu and Menegatti, 2019). However, much of the work in insurance literature typically adopts an oversimplified model with utility depending only on wealth. Although a one-argument utility framework has the advantage in mathematical tractability, the solutions obtained by this framework may neglect some important aspects of the real world, especially when dealing with the economic problems involving more than one good. For instance, Doherty and Schlesinger (1983) examined the optimal choice of insurance in the economy with multidimensional risks, by using a one-argument utility function.

Rey (2003), by contrast, adopted a two-argument utility function to consider two sources of uncertainty, an insurable risk and an uninsurable risk. The optimality of insurance thereby depends not only on the correlation between two risks, but also on the variation of the marginal utility of the insurable asset with respect to the uninsurable asset.

However, as Lee (2007) pointed out, this literature still does not take into account an important feature of an insurance contract that practically, the premium is paid from one's income and indemnity is made against an insured asset. In this respect, even in the simplest model with a single insurable risk in the asset, it is plausible to assume that utility depends on both income and the asset. This key assumption is particularly important in that an individual gets different utilities from income (or the consumption of composite good) and the consumption of the benefit generated by the asset. This is because insurable assets such as houses, property, and health, are irreplaceable goods. For example, the benefit of a house as a shelter cannot be readily substituted by that of composite goods. Moreover, health is essentially unique and cannot be easily replaced by income. Based on this observation, we developed a model in which income and the asset separately affect the individual's utility.

Conversely, a strand of literature examined the demand for insurance against a risk in an irreplaceable good (Cook and Graham, 1977; Dionne, 1982; Schlesinger, 1984; Huang and Tzeng, 2006; Hong and Seog, 2020). This literature emphasized the fact that the optimal insurance is affected by the individual's own valuation of the irreplaceable good, which may not be equivalent to the market price. In this sense, not only a monetary compensation but also a sentimental compensation might be needed to fully recover the utility level in the loss state compared with that in the no loss state.[3] However, none of this literature separated income from the asset, and therefore, lacked consideration of the important characteristics of an insurance contract as described above. Although Huang and Tzeng (2006) utilized a two-argument utility function that depends on monetary wealth and sentimental value, their model does not capture the trade-offs between income and the asset in making insurance decisions.

In contrast to previous studies, we decompose wealth into income and the asset, and therefore, a decision about insurance coverage depends on the individual's preferences towards each good. This is because additional coverage has opposite effects on income and the asset. In other words, additional coverage increases the expected value of the asset but reduces income by increasing the premium. Under this assumption, Lee (2007) demonstrated that the optimal insurance depends largely on income and preferences, rather than on the magnitude of the loading factor.

This study contributes to this body of literature by further exploring the demand for insurance with asymmetric information. Lee (2007) only reexamined the condition for the optimality of full-insurance, but his model can be extended to include a moral hazard problem. Moral hazard is pervasive throughout the economy with imperfect information about an individual's behavior (Cummins and Tennyson, 1996; Wang et al., 2008; Koç, 2011; Rowell et al., 2017). If it is either too expensive or impossible for the insurer to

[3] Consideration of an irreplaceable good can be reflected in a state-dependent utility model, in which the utility function changes when a loss occurs.

observe the individual's care activity, the optimality of insurance without considering moral hazard is often doubtful. Indeed, we find that a moral hazard problem still exists under a two-argument utility framework. Therefore, we consider two cases with and without moral hazard, depending on the observability of the individual's behavior, and compare the results of each case.

Our focus is on the choice of the optimal insurance coverage and of the optimal amount of self-protection effort when income is separated from an insurable asset. Following Lee (2007), we denote a two-argument utility function by $U(C, A)$, where C and A respectively indicate the composite good (income) and the benefit generated by an asset. Then, an individual's preferences towards income and the asset can be represented by the marginal rate of substitution (MRS) between two goods. According to these preferences, our model predicts diverse coverage rates to be optimal, from no insurance to full insurance, if moral hazard is not present. However, if an insurer cannot observe an individual's choice of effort, the individual has an incentive to underinvest in self-protection. To induce the insured to exert effort, less insurance coverage (partial insurance) is provided under moral hazard than that without moral hazard. The optimal effort level is also less than that without moral hazard. By contrast, an individual whose disutility from the premium payment is too high will not purchase any insurance at all regardless of the presence or absence of moral hazard. Moral hazard is relevant only for those who purchase insurance.

In fact, the conventional model employing a one-argument utility function, which is referred to as the non-separation case, is a special case of the separation case in our model utilizing a two-argument utility function. Because income and the asset are separated in our model, we can consider the individuals' preferences and the interaction between income and the asset of any sign, which draws a variety of equilibrium results depending on the shape of the utility.

In particular, we compare the equilibrium result in a separation case with that in a non-separation case. If the MRS and marginal utility with respect to the asset in the separation case is large (small) enough, the optimal coverage and effort level in the separation case can be simultaneously greater (smaller) than those in the non-separation case.

In addition, we adopt Dionne's (1982) concept of the relative importance of moral hazard, and analyze the effect of the separability of income and the asset on the relative importance of moral hazard.[4] This study shows that there is a trade-off between the demand for insurance and the willingness to exert effort in determining the relative importance of moral hazard. We show that when the optimal coverage is large enough, the moral hazard problem is more likely to be severe in the separation case if the marginal utility with respect to the asset in the separation case is larger than that in the non-separation case.

Because we separate income from an asset, the present model can also take account of the interaction between two goods, that is, U_{CA}, the cross-second derivative of utility. As an individual receives different utilities from the consumption of composite goods and the benefit generated by an asset, we can easily imagine a case in which income and the asset are neutral to each other. Whether U_{CA} is positive or negative can be affected by behavioral factors. As for a house, first note that we consider the house a use good that provides benefits such as a shelter, warmth, comfort, and private family space (Doling and Ronald, 2010). Now consider a situation in which the house is destroyed, or the roof is leaking. Then, for some individuals, the consumption of the same food in this low-quality

[4] For this analysis, we define the utility function as the weighted average of a two-argument utility function and a one-argument utility function. Then, the separability of income and the asset is measured by the weight on the two-argument utility function.

house will provide them with less marginal utility, because they are no longer able to enjoy their food in a warm and comfortable space. That is, U_{CA} is positive. However, for others with a low-quality house, the consumption of composite goods may still give them greater marginal utility if they are grateful for what they can still consume in their lives. Then, U_{CA} may be negative.

Second, let us consider health as the asset. In this case, U_{CA} can be positive, because generally food loses its relish when one is ill. However, the effects of loss of health can vary according to the type of illness. For example, an individual with mental health problems, such as lethargy and anxiety, would get less marginal utility from the consumption of composite goods, and therefore U_{CA} is positive. Conversely, some illnesses that involve mobility-related physical and functional losses, such as a fracture or Parkinson's disease, can lead to (long-term) immobility and increase the marginal utility of the consumption of consumer goods such as public transport and taxis. That is, U_{CA} is negative.

Note that some types of composite goods are substitutes and others are complements to an asset. Then, the net effect of loss in the asset depends on which effect dominates. Because of different preferences and socioeconomic status, we can expect heterogeneity across individuals whether substitution or complementarity dominates. In summary, U_{CA} may be positive or zero or negative in reality, and to reflect this realism in the model, we need to separate income from the asset.

According to the variation of U_{CA}, we investigate the relative importance of moral hazard. Our model demonstrates that, under some conditions, moral hazard can be more or less severe with a marginal increase in the degree of U_{CA}. This result implies that the concern about moral hazard may be excessive or should be higher depending on U_{CA}.

The remainder of this paper is organized as follows. Section 2 outlines the model.

Section 3 and 4 describe the equilibrium in a non-separation and a separation case, respectively. Section 5 discusses the relative importance of moral hazard. Section 6 provides numerical examples and Section 7 concludes.

2.2 Model

In a two-argument utility model, income and an insurable asset separately affect the individual's utility. We refer to this case as the "separation case". Let $U(C, A)$ denote the two-argument utility function that depends on C and A, where C and A respectively indicate the composite good (income) and the benefit generated by the asset. We assume that $U(\cdot,\cdot)$ is continuous, increasing, and strictly concave in both arguments. That is, the utility function has positive marginal utilities with respect to income and the asset, and its Hessian is negative definite. Note that we have $U_C(C, A) > 0$, $U_A(C, A) > 0$, $U_{CC}(C, A) < 0$, and $U_{AA}(C, A) < 0$, where subscripts in the utility function denote the partial derivatives such as $U_C(C, A) = \partial U(C, A)/\partial C$ and $U_{CC}(C, A) = \partial^2 U(C, A)/\partial C^2$. Moreover, we do not impose any restriction on the cross-second derivative of utility, that is, $U_{CA} = \partial^2 U(C, A)/\partial C\,\partial A$.

In a one-argument utility model, by contrast, income and the asset are combined into monetary wealth, which is denoted by $M = C + A$. Contrary to the separation case, we refer to this case as the "non-separation case." Because income and the asset are not distinguishable, the first derivatives with respect to income and the asset are the same, and represented equally by U_M. That is,

$$U_M(C, A) \equiv U_C(C, A) = U_A(C, A) \tag{1}$$

for all (C, A) in our support. Moreover, the second derivatives of utility are also the same, and are represented by U_{MM}. That is,

$$U_{MM} \equiv U_{CC} = U_{CA} = U_{AA} < 0 \tag{2}$$

for all (C, A) in our support. The second derivatives of utility in the non-separation case are always negative, whereas U_{CA} can have any sign in the separation case.

Notice that the one-argument utility function in the non-separation case is a special case of the two-argument utility function in the separation case, which satisfies equations (1) and (2) for all (C, A) in our support. In this study, we utilize the two-argument utility function $U(C, A)$, but we distinguish the separation and the non-separation case by whether equations (1) and (2) are satisfied or not.

We consider a simple model with two states of the world, in one of which the loss of the asset will occur with probability $p(\varepsilon)$. It is assumed that there is no uncertainty about income in both states. By exerting self-protection effort, $\varepsilon \geq 0$, an individual can reduce the probability of loss. We assume that the loss probability function is decreasing and strictly convex in ε. That is, $p_\varepsilon < 0$ and $p_{\varepsilon\varepsilon} > 0$, where the subscript in the loss probability function denotes the derivative with respect to ε. The cost of effort $c(\varepsilon)$ is measured in utility units, where $c_\varepsilon > 0$, $c_{\varepsilon\varepsilon} > 0$, and $c(0) = 0$.

Initial income and the asset are given by y and w, respectively.[5] An individual faces

[5] If the initial resource allocation is endogenously determined, we can solve the following maximization problem with budget constraint:

$$\mathrm{Max}_{\{\varepsilon,I\}} p(\varepsilon)U(y - Q(\varepsilon, I), w - D + I) + \big(1 - p(\varepsilon)\big)U(y - Q(\varepsilon, I), w) - c(\varepsilon)$$
$$\text{subject to } \ y + w \leq M$$
$$Q(\varepsilon, I) = p(\varepsilon)I,$$

a potential loss of $D < w$, the whole or part of which can be covered by insurance. Let us denote insurance premium and indemnity by Q and I, respectively, where $0 \leq I \leq D$. Note that the insurance premium is set to be actuarially fair and linear in indemnity, that is $Q(\varepsilon, I) = p(\varepsilon)I$. We assume that all individuals in the model are identical, and thus, the adverse selection problem does not occur. We also assume that the insurance market is competitive, so each insurer makes zero profit.

An individual's problem is to choose the optimal insurance coverage and effort level to maximize his or her expected utility:

$$EU(I, \varepsilon) = p(\varepsilon)U(y - Q, w - D + I) + \big(1 - p(\varepsilon)\big)U(y - Q, w) - c(\varepsilon). \qquad (3)$$

First, in the separation case, an individual will allocate his/her limited resources according to his or her preferences by means of insurance. This is because the insurance premium and indemnity each have an effect on different arguments of the utility function. In other words, the insurance premium decreases the income and indemnity is made against the asset if a loss occurs. Note that the individual's preferences towards income and the asset can be represented by the MRS between two goods:

$$MRS(Q, I; \varepsilon) \equiv \frac{p(\varepsilon)U_A(y - Q, w - D + I)}{(1 - p(\varepsilon))U_C(y - Q, w) + p(\varepsilon)U_C(y - Q, w - D + I)},$$

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where M denotes the available budget or total monetary wealth. The first set of constraints describes the budget constraint. If the asset is illiquid, we can consider the transaction costs or liquidity risks in the budget allocation. For simplicity, it is assumed that the asset is perfectly liquid. If a moral hazard problem exists, we must consider an additional incentive constraint (see Equation (6)). Because the aim of this study is the optimal insurance contract under a two-argument utility model, we simplify our model with given y and w. Note that whatever the initial allocation of resources, the decision about insurance coverage depends on the individual's own preferences.

where (Q, I) and ε are a chosen insurance contract and self-protection effort, respectively.[6] Here, the MRS is expressed in expectation form, because there are two possible states in the model. Therefore, in the separation case, MRS implies the rate at which an individual can give up some amount of the expected consumption of composite goods to premium payments in exchange for the expected indemnification against loss of the asset, while maintaining the same level of expected utility.

Second, in the non-separation case, an individual does not have any preferences towards income and the asset, because these two goods are not distinguishable. Therefore, insurance protection only affects the total monetary wealth in each state of the world, and plays the role of consumption smoothing for risk-averse individuals. Note that in the non-separation case, the MRS reflects the degree of concavity of utility, not the preference towards income and the asset.

2.3 Equilibrium in a non-separation case

We consider an individual who maximizes his or her expected utility as in equation (3). Recall that in the non-separation case, the utility function satisfies equations (1) and (2) for all (C, A) in our support.

2.3.1 Equilibrium without moral hazard in a non-separation case

If an insurer can monitor the individual's choice of effort at a cost that can be ignored, the choice of optimal insurance coverage and self-protection is obtained by maximizing the

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6 Originally in economics, the MRS between x and y is denoted by $MRS(x, y) = U_x/U_y$, where $U_x = \frac{\partial U(x,y)}{\partial x}$ and $U_y = \frac{\partial U(x,y)}{\partial y}$.

expected utility:

$$\mathrm{Max}_{\{\varepsilon,I\}} p(\varepsilon)U(y - Q(\varepsilon, I), w - D + I) + \big(1 - p(\varepsilon)\big)U(y - Q(\varepsilon, I), w) - c(\varepsilon)$$

$$\text{Subject to } \; Q(\varepsilon, I) = p(\varepsilon)I.$$

Now, the Lagrangian can be expressed as follows:

$$L = p(\varepsilon)U(y - p(\varepsilon)I, w - D + I) + \big(1 - p(\varepsilon)\big)U(y - p(\varepsilon)I, w) - c(\varepsilon). \tag{4}$$

For simplicity, we denote w_0 and w_1 to be the asset in the no loss and loss states, respectively: $w_0 = w$ and $w_1 = w - D + I$. Let us also denote y_1 as the income remaining after paying premiums, that is, $y_1 = y - Q$. We assume the validation of the first-order approach. Then the first order conditions are:

$$\begin{aligned} L_\varepsilon = p_\varepsilon[U(y_1, w_1) - U(y_1 - w_0)] + p(\varepsilon)U_C(y_1, w_1)(-p_\varepsilon I) \\ + \big(1 - p(\varepsilon)\big)U_C(y_1, w_0)(-p_\varepsilon I) - c_\varepsilon = 0, \end{aligned} \tag{5.1}$$

$$\begin{aligned} L_I = p(\varepsilon)\big[U_C(y_1, w_1)\big(-p(\varepsilon)\big) + U_A(y_1, w_1)\big] \\ + \big(1 - p(\varepsilon)\big)\big[U_C(y_1, w_0)\big(-p(\varepsilon)\big)\big] = 0. \end{aligned} \tag{5.2}$$

Solving equation (5.2), we have the following results:

Lemma 1 [Non-separation case without moral hazard]

In a non-separation case, full insurance is optimal.

Proof. See the Appendix. //

In the non-separation case without moral hazard, full insurance is always optimal if it is actuarially fair, except in the case of the state-dependent utility function.[7] Because income and the asset are not distinguishable, consumption smoothing is the only concern. Therefore, a perfect hedge against the risk in the asset is the only equilibrium.

The optimal effort level is given by equation (5.1). This implies that the optimal effort is determined such that the expected marginal benefit of expending effort equals its marginal cost. Note that the marginal benefit of exerting effort is represented by the reduction in both loss probability and the insurance premium. If $L_\varepsilon = 0$ does not hold, then $\varepsilon(Q, I) = 0$ and $p_\varepsilon[U(y_1, w_1) - U(y_1 - w_0)] + p(0)U_C(y_1, w_1)(-p_\varepsilon I) + \big(1 - p(0)\big)U_C(y_1, w_0)(-p_\varepsilon I) < c_\varepsilon$.

2.3.2 Equilibrium with moral hazard in a non-separation case

If an insurer cannot observe the individual's behavior, the insured with a given insurance contract will choose the amount of effort which does not depend on the contract. Then, the optimal effort for the given insurance contract (Q, I) is obtained by maximizing the expected utility over ε:

$$\mathrm{Max}_\varepsilon p(\varepsilon)U(y - Q, w - D + I) + \big(1 - p(\varepsilon)\big)U(y - Q, w) - c(\varepsilon).$$

Assuming an interior solution, the first order condition can be expressed as:

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[7] Under a state-dependent utility function, full insurance is optimal if the marginal utility is equal between the two states, or if the marginal utility in the loss state is greater than that in the no loss state. Conversely, partial insurance is optimal if the marginal utility in the loss state is less than that in the no loss state. For further information about the state-dependent utility function, see Dionne (1982) and Hong and Seog (2020).

$$p_\varepsilon[U(y-Q,w-D+I)-U(y-Q,w)]-c_\varepsilon=0. \qquad (6)$$

Equation (6) represents incentive compatibility under moral hazard. Compared with equation (5.1), equation (6) indicates that the marginal benefit of expending effort is reduced to the marginal benefit of reducing the loss probability, and it is null when full insurance is provided. Moreover, we assume that a chosen ε is positive at $I=0$ in order that moral hazard is possible.

If an individual purchases some insurance at the optimum, equation (6) should hold. Then, the individual solves the following maximization program:

$$\mathrm{Max}_{\{\varepsilon,I\}} p(\varepsilon)U(y-Q(\varepsilon,I),w-D+I)+\big(1-p(\varepsilon)\big)U(y-Q(\varepsilon,I),w)-c(\varepsilon)$$

$$\text{subject to } Q(\varepsilon,I)=p(\varepsilon)I,$$

$$p_\varepsilon[U(y-Q,w-D+I)-U(y-Q,w)]-c_\varepsilon=0.$$

To solve this program, let us denote λ for the Lagrange multiplier attached to the last constraint. Now the Lagrangian can be expressed as follows:

$$\hat{L}=p(\varepsilon)U(y-p(\varepsilon)I,w-D+I)+\big(1-p(\varepsilon)\big)U(y-p(\varepsilon)I,w)-c(\varepsilon)$$
$$+\lambda[p_\varepsilon\{U(y-p(\varepsilon)I,w-D+I)-U(y-p(\varepsilon)I,w)\}-c_\varepsilon]. \qquad (7)$$

The first-order conditions are:

$$\begin{aligned}\hat{L}_{\varepsilon} = p_{\varepsilon}[U(y_1, w_1) - U(y_1 - w_0)] + p(\varepsilon)U_C(y_1, w_1)(-p_{\varepsilon}I) \\ + \big(1 - p(\varepsilon)\big)U_C(y_1, w_0)(-p_{\varepsilon}I) - c_{\varepsilon} \\ + \lambda[p_{\varepsilon\varepsilon}\{U(y_1, w_1) - U(y_1, w_0)\} \\ + p_{\varepsilon}\{U_C(y_1, w_1) - U_C(y_1, w_0)\}(-p_{\varepsilon}I) - c_{\varepsilon\varepsilon}] = 0,\end{aligned} \tag{8.1}$$

$$\begin{aligned}\hat{L}_{I} = p(\varepsilon)\big[U_C(y_1, w_1)\big(-p(\varepsilon)\big) + U_A(y_1, w_1)\big] \\ + \big(1 - p(\varepsilon)\big)U_C(y_1, w_0)\big(-p(\varepsilon)\big) \\ + \lambda p_{\varepsilon}[\{U_C(y_1, w_1) - U_C(y_1, w_0)\}\big(-p(\varepsilon)\big) + U_A(y_1, w_1)] \\ = 0,\end{aligned} \tag{8.2}$$

$$\hat{L}_{\lambda} = p_{\varepsilon}[U(y_1, w_1) - U(y_1, w_0)] - c_{\varepsilon} = 0. \tag{8.3}$$

It is well known that $\lambda > 0$ for an interior optimum in the conventional model.[8] A positive λ implies that the insurer would like to induce the individual to expend more effort at the optimum. Now, we obtain the following Lemma by solving the first-order conditions of the Lagrangian.

Lemma 2 [Non-separation case under moral hazard]

In a non-separation case, partial insurance is optimal.

Proof. See the Appendix. //

[8] Notice that the separation case is a general version of the non-separation case, and the sign of λ in the non-separation case is the same as that in the separation case. See Lemma 4 in the next section for more details.

If moral hazard is present, the optimal insurance policy always involves partial coverage in the standard model where income and the asset are not separable (Pauly, 1974; Shavell, 1979).

2.4 Equilibrium in a separation case

2.4.1. Consumption Risk Sharing for ASEAN and Its Subgroups

In this section, we consider the equilibrium without moral hazard in a separation case. The Lagrangian in the separation case can be expressed similarly to that in equation (4), but the utility function does not need to satisfy either (1) or (2). Then the first order conditions are expressed the same as in equation (5.1) and (5.2). Solving (5.2), we obtain lemma 3.

Lemma 3 [Separation case without moral hazard]

(A) Full or over insurance is optimal if and only if $\mathrm{p}(\varepsilon)\mathrm{U_C}(\mathrm{y}-\mathrm{p}(\varepsilon)\mathrm{I},\mathrm{w}-\mathrm{D}+\mathrm{I}) + \big(1-\mathrm{p}(\varepsilon)\big)\mathrm{U_C}(\mathrm{y}-\mathrm{p}(\varepsilon)\mathrm{I},\mathrm{w}) = \mathrm{U_A}(\mathrm{y}-\mathrm{p}(\varepsilon)\mathrm{I},\mathrm{w}-\mathrm{D}+\mathrm{I})$, where $\mathrm{I} \geq \mathrm{D}$.

(B) Partial insurance is optimal if and only if $\mathrm{p}(\varepsilon)\mathrm{U_C}(\mathrm{y},\mathrm{w}-\mathrm{D}) + \big(1-\mathrm{p}(\varepsilon)\big)\mathrm{U_C}(\mathrm{y},\mathrm{w}) < \mathrm{U_A}(\mathrm{y},\mathrm{w}-\mathrm{D})$ and $\mathrm{U_C}(\mathrm{y}-\mathrm{p}(\varepsilon)\mathrm{D},\mathrm{w}) > \mathrm{U_A}(\mathrm{y}-\mathrm{p}(\varepsilon)\mathrm{D},\mathrm{w})$.

(C) No insurance is optimal if and only if $\mathrm{p}(\varepsilon)\mathrm{U_C}(\mathrm{y},\mathrm{w}-\mathrm{D}) + \big(1-\mathrm{p}(\varepsilon)\big)\mathrm{U_C}(\mathrm{y},\mathrm{w}) \geq \mathrm{U_A}(\mathrm{y},\mathrm{w}-\mathrm{D})$.

Proof. See the Appendix. //

Similar to the results of Lee (2007), Lemma 3 shows that an optimal insurance can vary from no insurance to over insurance depending on the individual's preferences. For an interior solution, the optimal coverage is determined such that the expected marginal disutility from premium payment equals the marginal utility from the insurance protection in the loss state. In other words, the MRS with the chosen insurance contract and effort is equal to the unit price of insurance at the equilibrium. That is, $\mathrm{MRS}(\mathrm{p}(\varepsilon)\mathrm{I}, \mathrm{I}; \varepsilon) = \mathrm{p}(\varepsilon)$. Therefore, the optimal coverage is determined such that the value of additional coverage equals the marginal cost of it. Note that full insurance is optimal even if $\mathrm{MRS}(\mathrm{p}(\varepsilon)\mathrm{D}, \mathrm{D}; \varepsilon) > \mathrm{p}(\varepsilon)$, if over insurance is not allowed. Now consider an individual of the opposite extreme with respect to his or her preferences towards two goods. In regard to an additional unit of insurance at the point of no insurance, the increase in marginal utility due to the increase in coverage in the loss state does not account for the increase in expected marginal disutility due to the increase in premium, if $\mathrm{MRS}(0, 0; \varepsilon) \leq \mathrm{p}(\varepsilon)$. For this type of individual, disutility from the decrease in income is too high after paying the insurance premium, so he or she will not purchase any insurance at all. The results of Lemma 3 imply that the individual's preferences towards income and the asset are critical in determining the optimal insurance coverage, because additional coverage has the opposite effect on each argument.

Contrary to the non-separation case, the individual's preferences towards income and the asset play a vital role in making the insurance decision in the separation case. From Lemmas 1 and 3, we can conclude that without moral hazard, the equilibrium result in the non-separation case is only a part of the possible equilibrium results in the separation case. Similar to the separation case, the optimal effort level in the non-separation case is given by equation (5.1).

2.4.2. Equilibrium with moral hazard in a separation case

Now, we consider the equilibrium with moral hazard in a separation case. Note that the Lagrangian can be expressed the same as in equation (7), but the utility function does not need to satisfy neither (1) nor (2). The first order conditions are also expressed the same as in equation (8.1), (8.2), and (8.3). Now, we obtain Lemma 4.

Lemma 4

$\lambda > 0$ for an interior optimum.

Proof. See the Appendix. //

Lemma 4 is equivalent to Proposition 1 in Hölmstrom (1979). Similar to the non-separation case, $\lambda > 0$ in the separation case. Again, a positive λ implies that the insurer would like to induce the individual to expend more effort at the optimum. Now, we can verify the sign of the variation of effort with respect to coverage by differentiating equation (6):

$$\{p_{\varepsilon\varepsilon}[U(y_1, w_1) - U(y_1, w_0)] - c_{\varepsilon\varepsilon}\}\frac{d\varepsilon}{dI} + p_{\varepsilon}\big[\{U_C(y_1, w_1) - U_C(y_1, w_0)\}\big(-p(\varepsilon)\big) + U_A(y_1, w_1)\big] = 0$$

Rearranging the above equality, we obtain:

$$\varepsilon_I = -\frac{p_{\varepsilon}\big[\{U_C(y_1, w_1) - U_C(y_1, w_0)\}\big(-p(\varepsilon)\big) + U_A(y_1, w_1)\big]}{p_{\varepsilon\varepsilon}[U(y_1, w_1) - U(y_1, w_0)] - c_{\varepsilon\varepsilon}}. \tag{9}$$

We can show that the expression in parentheses in the numerator of equation (9) is positive. From equation (8.2), we have the following relation:

$$(p(\varepsilon) + \lambda p_{\varepsilon})\big[\{U_C(y_1, w_1) - U_C(y_1, w_0)\}\big(-p(\varepsilon)\big) + U_A(y_1, w_1)\big] > \hat{L}_I = 0.$$

Because $\lambda > 0$, we have $p(\varepsilon) + \lambda p_{\varepsilon} > 0$ (see equation $(A.3)$ in the appendix). Therefore, the expression in parentheses in the numerator is positive. By contrast, the denominator equals the second order condition of the optimality of effort, and it is negative at the optimum. Therefore, $\varepsilon_I < 0$, because $p_{\varepsilon} < 0$. Note that negative ε_I implies that an increase in coverage reduces the effort level because the insurer cannot observe the individual's choice of effort. Moreover, for a given level of indemnity, we can compare the effort levels with and without moral hazard.

Lemma 5

Given identical insurance contracts, spending less effort is optimal under moral hazard rather than without moral hazard.

Proof. See the Appendix. //

Given that the effort level is chosen such that equation (6) is satisfied for any level of indemnity, the insurer will set an insurance premium that considers the change in expected reimbursement resulting from a reduction in the effort level under moral hazard. Note that the solutions of Lagrangian (7) satisfy these constraints.

Let us examine the optimal coverage and optimal level of effort under moral hazard. We denote the equilibrium without moral hazard as (I^*, ε^*) and that with moral hazard as $(\hat{I}^*, \hat{\varepsilon}^*)$. Because the insurer cannot observe the insured's choice of effort, an increase in insurance coverage reduces the incentive to invest in self-protection. Given that $\lambda > 0$ and $\varepsilon_I < 0$, it is more likely that the insurer induces the insured to invest in self-protection by offering less insurance coverage under moral hazard.

Proposition 1 [Separation case under moral hazard]

(A) Those who purchase full insurance without moral hazard are provided with less insurance (partial insurance) under moral hazard.

(B) Those who purchase partial insurance without moral hazard are provided with less insurance (partial insurance) under moral hazard.

(C) Those who do not purchase insurance without moral hazard, also do not purchase insurance under moral hazard.

Proof. See the Appendix. //

As we assume an interior solution to the choice of the optimal effort level, full coverage cannot be optimal under moral hazard. This is because the incentive to expend effort is reduced after purchasing an insurance contract. In Proposition 1, those who purchase positive insurance in the absence of moral hazard are provided with positive but lower insurance under moral hazard than that without moral hazard. Similar to Shavell (1979) argument, moral hazard alone cannot eliminate the possibility of insurance when the insurable asset and income are separable. By providing less indemnity, the insurer induces the insured to invest in self-protection under moral hazard.

However, an individual, who does not buy insurance in the absence of moral hazard, will also not buy any insurance under moral hazard. This is because disutility from the premium payment is too high for this type of individual. In this case, private information about the choice of effort does not play any role in decision-making for the individual. Moral hazard is relevant only for those who purchase insurance.

From Proposition 1, we can conclude that, under moral hazard, the insurer induces the insured to expend more effort by providing less coverage, but the selected effort level is less than that without moral hazard. That is, $\hat{\varepsilon}^* < \varepsilon^*$ and $\hat{I}^* < I^*$. Conversely, the relative size of the insurance premium is not decisive, that is, $\widehat{Q} = p(\hat{\varepsilon}^*)\hat{I}^* \gtreqless p(\varepsilon^*)I^* = Q$, where $\widehat{Q}$ denotes the premium at the equilibrium under moral hazard.

Contrary to the non-separation case, not only partial insurance but also no insurance can be optimal in the separation case under moral hazard. Note that our model is the general version of the conventional model, and therefore, not surprisingly, the equilibrium results under moral hazard in the non-separation case are included in the possible equilibrium results of our model in the separation case.

Now, in the following proposition, we attempt to compare the equilibrium results of the non-separation case and the separation case under moral hazard. Because the utilities in each case are different, we define the following utility function:

$$\mu(C, A; t) = (1 - t)U^n(C, A) + tU^s(C, A) \tag{10}$$

where $0 \leq t \leq 1$, and the superscripts in the utility function denote the separability of income and the asset, that is, "n" denotes the non-separation case and "s" denotes the separation case. Note that $t \in [0, 1]$ denotes the degree of separability. We may interpret t as the degree of irreplaceability of income and the asset. If $t = 0$, the asset can be directly replaceable by income. Then, $\mu(C, A; 0)$ represents the non-separable utility function such that $\mu_C = \mu_A$ and $\mu_{CC} = \mu_{CA} = \mu_{AA} < 0$ for all (C, A) in our support. By contrast, if $t > 0$, the asset cannot be perfectly substituted by income, and $\mu(C, A; t)$ represents the separable utility function. Then, we will compare the equilibrium results of some cases in which $t = 0$ and $t > 0$.

From equation (8.2), the optimal coverage depends largely on the individual's preferences or the MRS. On the other hand, the optimal effort level is determined by equation (8.3). Note that the choice of effort is induced by the choice of insurance coverage at the equilibrium. In general, greater coverage leads to lower investment of effort, and vice versa.

However, the optimal effort level is also affected by the difference between the utilities in the loss state and no loss state (henceforth DU), that is, $U(y_1, w_1) - U(y_1, w_0)$. This is because, for a given risk reduction technology, the greater the DU, the greater the care activity. Especially in the separation case, if the marginal utility with respect to the asset is large enough, the DU can be large even with a relatively high insurance coverage. Now, we obtain the following proposition.

Proposition 2 [Moral hazard case]

The relative sizes of the optimal coverage and effort level in a separation case and a non-separation case are ambiguously determined. If the MRS and marginal utility with respect to the asset in the separation case are large (small) enough, the optimal coverage and effort level can be simultaneously greater (smaller) in the separation case than those in the non-separation case.

Proof. See the Appendix. //

Proposition 2 emphasizes that the mechanism of the equilibrium of insurance coverage and effort decisions differs according to the separability of income and the asset. From Proposition 1, it is clear that the optimal coverage depends largely on an individual's MRS. This is because additional coverage is relatively more valuable for an individual with a greater MRS. Moreover, insurance protection reduces the care activity. However, the MRS is not the only factor that affects the optimal choice of effort in the separation case. The optimal level of effort is affected by MRS as well as the marginal utility with respect to the asset.

From the above observation, Proposition 2 provides the simultaneous existence of higher (lower) coverage and higher (lower) effort level in the separation case. If the MRS in the separation case is greater enough than that in the non-separation case, the optimal coverage under moral hazard in the separation case can be greater than that in the non-separation case. Even with greater insurance coverage, the individual may also choose a higher effort level, if his or her utility decreases a lot when a loss occurs. Therefore, the individual in the separation case, whose marginal utility with respect to the asset is large enough, and whose MRS is greater enough than those of the non-separation case, will demand more insurance and exert more effort at the same time.

Proposition 2 compares the equilibrium results of the separation case and the non-separation case under moral hazard. This comparison provides insight into the determinants of insurance coverage and self-protection effort when income and the asset are separated. However, the relative sizes of the optimal coverage and effort level do not directly provide information about the relative severity of the moral hazard problem. In the following section, we adopt Dionne's (1982) concept of the relative importance of moral hazard, and attempt to understand the extent of moral hazard in the present model.

2.5 The relative importance of moral hazard

2.5.1 The relative importance of moral hazard in the non-separation and separation case

Because, for some reason, the insurer cannot observe the individual's choice of effort, an increase in insurance coverage reduces the optimal effort level under moral hazard. In other words, moral hazard is still important in a two-argument utility model because $\varepsilon_I <$

0. From equation (9), the absolute degree of ε_I depends on the shape of the utility function.

Dionne (1982) measured the importance of moral hazard by the sensitivity change of moral hazard with the variation of a factor of interest, say "a." Note that the sensitivity of moral hazard is defined by ε_I, and the relative importance of moral hazard is defined by $\frac{d\varepsilon_I}{da}$. Because ε_I is negative, $\frac{d\varepsilon_I}{da} < 0$ implies that moral hazard is more important as "a" increases, and vice versa.

In this section, we verify whether the degree of the separation affects the importance of moral hazard. We consider the utility function in equation (10):

$$\mu(C, A; t) = (1 - t)U^n(C, A) + tU^s(C, A)$$

where $0 \leq t \leq 1$. The shape of $\mu(C, A; t)$ depends on the assumptions about $U^n(C, A)$ and $U^s(C, A)$. Let us check the derivatives of the utility with respect to income and the asset, respectively.

$$\mu_C(t) = (1 - t)U^n_M(C, A) + tU^s_C(C, A).$$

$$\mu_A(t) = (1 - t)U^n_M(C, A) + tU^s_A(C, A).$$

Then, the weighted average of ε_I in the separation and non-separation cases is

$$\begin{aligned}\varepsilon_I(t) = -\frac{p_\varepsilon}{D_1}\big[&(1 - t)[\{U^n_M(y_1, w_1) - U^n_M(y_1, w_0)\}(-p) \\ &+ U^n_M(y_1, w_1)] \\ &+ t[\{U^s_C(y_1, w_1) - U^s_C(y_1, w_0)\}(-p) \\ &+ U^s_A(y_1, w_1)]\big],\end{aligned} \tag{11}$$

where p represents $p(\varepsilon)$, and the denominator of equation (11) is defined as $D_1 \equiv p_{\varepsilon\varepsilon}\big[(1-t)[U^n(y_1,w_1)-U^n(y_1,w_0)]+t[U^s(y_1,w_1)-U^s(y_1,w_0)]\big]-c_{\varepsilon\varepsilon}$.[9]

Now, we differentiate equation (11) with respect to t:

$$
\begin{aligned}
\frac{d\varepsilon_I}{dt} = \frac{p_\varepsilon}{\{D_1\}^2}\Big[&\big[[\{U_M^n(y_1,w_1)-U_M^n(y_1,w_0)\}(-p)+U_M^n(y_1,w_1)] \\
&- [\{U_C^s(y_1,w_1)-U_C^s(y_1,w_0)\}(-p)+U_A^s(y_1,w_1)]\big] \\
&\cdot \{p_{\varepsilon\varepsilon}[U^n(y_1,w_1)-U^n(y_1,w_0)]-c_{\varepsilon\varepsilon}\} \\
&+ [\{U_M^n(y_1,w_1)-U_M^n(y_1,w_0)\}(-p)+U_M^n(y_1,w_1)] \\
&\cdot p_{\varepsilon\varepsilon}[-\{U^n(y_1,w_1)-U^n(y_1,w_0)\} \\
&+ \{U^s(y_1,w_1)-U^s(y_1,w_0)\}]\Big].
\end{aligned} \tag{12}
$$

We assume that the chosen coverage and effort level do not change with a marginal increase in t. Then, $\frac{d\varepsilon_I}{dt}$ represents the change in $\varepsilon_I(t)$ with respect to the weight t. From equation (12), two factors in this expression reflect, respectively, the following changes in ε_I with a slight increase in t:

$$
\begin{aligned}
F_1 = p_\varepsilon\big[&[\{U_M^n(y_1,w_1)-U_M^n(y_1,w_0)\}(-p)+U_M^n(y_1,w_1)] \\
&- [\{U_C^s(y_1,w_1)-U_C^s(y_1,w_0)\}(-p)+U_A^s(y_1,w_1)]\big]\cdot\{p_{\varepsilon\varepsilon}[U^n(y_1,w_1) \\
&- U^n(y_1,w_0)]-c_{\varepsilon\varepsilon}\} \\
F_2 = p_\varepsilon&[\{U_M^n(y_1,w_1)-U_M^n(y_1,w_0)\}(-p)+U_M^n(y_1,w_1)] \\
&\cdot p_{\varepsilon\varepsilon}[-\{U^n(y_1,w_1)-U^n(y_1,w_0)\}+\{U^s(y_1,w_1)-U^s(y_1,w_0)\}]
\end{aligned}
$$

[9] The sensitivity of moral hazard, $\varepsilon_I(t)$, is evaluated at the optimum.

The sign of $\frac{d\varepsilon_I}{dt}$ is identical to the sign of $F_1 + F_2$. Note that the sign of F_1 depends on the relative size of $[\{U_M^n(y_1, w_1) - U_M^n(y_1, w_0)\}(-p) + U_M^n(y_1, w_1)]$ and $[\{U_C^s(y_1, w_1) - U_C^s(y_1, w_0)\}(-p) + U_A^s(y_1, w_1)]$, and the sign of F_2 depends on the relative size of $\{U^n(y_1, w_1) - U^n(y_1, w_0)\}$ and $\{U^s(y_1, w_1) - U^s(y_1, w_0)\}$. Therefore, F_1 is associated with the relative size of the marginal utility with respect to the asset, and F_2 is related to the DU. Notice that if the optimal coverage is big enough, F_2 is close to zero. For comparison, we can consider the case in which the optimal coverage is big enough, so that the effect of F_2 on $\frac{d\varepsilon_I}{dt}$ is negligible to a certain extent. Now, we have the following result.

Proposition 3

Moral hazard is more important in a separation case if the sum effect of F_1 and F_2 is negative. When the MRS is large enough, if marginal utility with respect to the asset in the separation case is large (small) enough, the sum effect of F_1 and F_2 may be negative (positive).

Proof. See the text below. //

When determining the relative importance of moral hazard, there is a trade-off between the demand for insurance and the willingness to exert effort. Note that F_1 and F_2, respectively, are associated with the demand for insurance and the incentive to expend effort. If the marginal utility with respect to the asset in the separation case is large enough, then F_1 is likely to be negative, but DU in the separation case in F_2 can be large. The sign of $\frac{d\varepsilon_I}{dt}$ is ambiguously determined and affected by the shape of the utility functions, U^n and U^s. Therefore, for comparison, we consider the case where the optimal coverage is big enough so that F_2 is negligible to some extent. Then, Proposition 3 indicates that the moral hazard problem can be more (less) severe in the separation case if the marginal utility with respect to the asset in the separation case is large (small) enough. This result implies that the separation of income and the asset is particularly important in analyzing the moral hazard problem.

2.5.2 The relative importance of moral hazard according to the interaction between income and the asset

Unlike a one-argument utility model, our model can take the interaction between income and the asset into consideration. This section aims to study the relative importance of moral hazard according to the variation of U_{CA}. Following Dionne (1982), we represent the relative importance of moral hazard by the sensitivity change of ε_I with the variation of U_{CA}.

Now, we will focus on the interaction between income and the asset. However, the variation of U_{CA} also effects U_C and U_A. Given that, we define $U(C, A)$ as follows:

$$V(C, A) = K_1U^1(C) + K_2U^2(A) + K_3U^3(C, A) \tag{13}$$

where $V(C, A)$ is increasing and concave in C and A. Note that the last term in equation (13), that is, $K_3U^3(C, A)$, picks up the interaction between C and A. Without loss of generality, we assume that $U^3(C, A)$ has positive first derivatives and negative second derivatives. That is, $U_C^3(C, A) > 0$, $U_A^3(C, A) > 0$, $U_{CC}^3(C, A) < 0$, $U_{AA}^3(C, A) < 0$, and $U_{CA}^3(C, A) < 0$. Then, the sign of $V_{CA}(C, A)$ is determined by the sign of K_3. If $K_3 \gtreqqless 0$, we have $V_{CA}(C, A) \lesseqqgtr 0$. Notice that if $K_3 < 0$, the magnitude of K_3 should be limited so that $V(C,A)$ is increasing and concave. Then, K_3 is the parameter of interest, which indicates the degree of $V_{CA}(C,A)$. Using equation (13), we rewrite equation (9) as follows:

$$\varepsilon_I(K_3) = -\frac{p_\varepsilon\left[-pK_3\left(U_C^3(y_1,w_1)-U_C^3(y_1,w_0)\right)+K_2U_A^2(w_1)+K_3U_A^3(y_1,w_1)\right]}{p_{\varepsilon\varepsilon}\left[K_2[U^2(w_1)-U^2(w_0)]+K_3[U^3(y_1,w_1)-U^3(y_1,w_0)]\right]-c_{\varepsilon\varepsilon}}, \tag{14}$$

where p represents $p(\varepsilon)$. Let us denote D_2 as the denominator of equation (14).

Now, we differentiate (14) with respect to K_3:

$$\begin{aligned}\frac{d\varepsilon_I}{dK_3} = \frac{p_\varepsilon}{\{D_2\}^2}\Big[\Big[p\left(U_C^3(y_1,w_1)-U_C^3(y_1,w_0)\right) \\ - U_A^3(y_1,w_1)\Big]\left[p_{\varepsilon\varepsilon}K_2[U^2(w_1)-U^2(w_0)]-c_{\varepsilon\varepsilon}\right] \\ + K_2U_A^2(w_1)p_{\varepsilon\varepsilon}[U^3(y_1,w_1)-U^3(y_1,w_0)]\Big].\end{aligned} \tag{15}$$

Note that $\frac{d\varepsilon_I}{dK_3}$ represents the change in $\varepsilon_I(K_3)$ with a marginal increase in K_3. From equation (15), three factors in this expression reflect, respectively, the following changes in ε_I with a small increase in the degree of V_{CA}:

$$G_1 = p_\varepsilon p\left(U_C^3(y_1,w_1)-U_C^3(y_1,w_0)\right)\left[p_{\varepsilon\varepsilon}K_2[U^2(w_1)-U^2(w_0)]-c_{\varepsilon\varepsilon}\right] > 0$$

$$G_2 = -p_\varepsilon U_A^3(y_1,w_1)\left[p_{\varepsilon\varepsilon}K_2[U^2(w_1)-U^2(w_0)]-c_{\varepsilon\varepsilon}\right] < 0$$

$$G_3 = p_\varepsilon K_2U_A^2(w_1)p_{\varepsilon\varepsilon}[U^3(y_1,w_1)-U^3(y_1,w_0)] > 0$$

The sign of $\frac{d\varepsilon_I}{dK_3}$ is identical to the sign of $G_1 + G_2 + G_3$. Note that G_2 has a negative effect on $\frac{d\varepsilon_I}{dK_3}$, whereas G_1 and G_3 have a positive effect on $\frac{d\varepsilon_I}{dK_3}$. For a given K_3, if the optimal coverage is big enough, G_1 and G_3 are close to zero, but G_2 is still negative. In this case, if the cost function is convex enough so that $c_{\varepsilon\varepsilon}$ is big enough, then the sum of G_1, G_2, and G_3 can be negative. If MRS is not that big for a given K_3, then, on the contrary, $G_1 + G_2 + G_3$ can be positive.

If $K_3 = 0$, income and the asset are independent to each other. In this case, it is evident that the relative importance of moral hazard is not affected. By contrast, if $K_3 < 0$ and $\frac{d\varepsilon_I}{dK_3} > 0$ $\left(\frac{d\varepsilon_I}{dK_3} < 0\right)$, we interpret that moral hazard is less (more) important as the degree of V_{CA} decreases with a marginal increase in K_3. On the contrary, if $K_3 > 0$, then $\frac{d\varepsilon_I}{dK_3} > 0$ $\left(\frac{d\varepsilon_I}{dK_3} < 0\right)$ implies that moral hazard is less (more) important as the degree of V_{CA} increases with a marginal increase in K_3. Now, we obtain the following proposition.

Proposition 4

(A) When $V_{CA} = 0$, the importance of moral hazard is not affected.

(B) When $V_{CA} > 0$, moral hazard is less (more) important with a marginal increase in the degree of V_{CA}, if the sum of the positive effect of G_1 and G_3 is less (greater) than the negative effect of G_2.

(C) When $V_{CA} < 0$, moral hazard is less (more) important with a marginal increase in the degree of V_{CA}, if the sum of the positive effect of G_1 and G_3 is greater (smaller) than the negative effect of G_2.

Proof. See the Appendix. //

Proposition 4 indicates that under a two-argument utility framework, the relative importance of moral hazard may differ according to V_{CA}. The sign of $\frac{d\varepsilon_I}{dK_3}$ is determined by the sum effect of G_1, G_2, and G_3, and the relative magnitude of each factor is affected by the shape of the utility function. Notice that if the optimal coverage is large, $\frac{d\varepsilon_I}{dK_3}$ is more likely to be negative. Because the MRS is affected by V_{CA}, the optimal coverage and the sensitivity change in moral hazard are also affected by K_3. Therefore, under some conditions, moral hazard can be more or less important with a marginal increase in the degree of V_{CA}. The concern about moral hazard may be excessive or should be higher depending on V_{CA}.

2.6 Numerical example

In this section, we provide numerical examples to illustrate Propositions 1, 2, 3, and 4. We consider the power utility functions that are increasing and concave in C and A as follows:

$$U^n(C, A) = \frac{K}{1-\gamma}(C + A)^{1-\gamma}, \tag{16}$$

$$U^s(C, A) = \frac{K_1}{1-\gamma_1}C^{1-\gamma_1} + \frac{K_2}{1-\gamma_2}A^{1-\gamma_2} + \frac{K_3}{1-\gamma_3}(C \cdot A)^{1-\gamma_3}, \tag{17}$$

where $0 < \gamma, \gamma_1, \gamma_2, \gamma_3 < 1$, $K, K_1, K_2 > 0$, and K_3 can have any sign. The sign of $U^s_{CA}(C, A)$ is determined by the sign of K_3. If $K_3 \gtreqless 0$, we have $U^s_{CA}(C, A) \gtreqless 0$. Notice that if $K_3 < 0$, the magnitude of K_3 should be limited so that $U^s(C, A)$ is increasing and concave. Further, we assume that $p(\varepsilon) = \frac{1}{1.5+\varepsilon}$ and $c(\varepsilon) = 0.5\varepsilon^2$ for $\varepsilon \geq 0$. Let us set $y = 5$, $w = 4$, $D = 3$, and $\gamma = \gamma_1 = \gamma_2 = \gamma_3 = 0.5$.

2.6.1 Numerical examples for Proposition 1

Let us consider numerical examples to illustrate Proposition 1. In the following examples, we do not allow over insurance. We consider the separable utility function $U^s(C, A)$ in equation (17). In Proposition 1, we compared the equilibrium result with moral hazard and that without moral hazard. In the following, we consider 3 cases in which optimal coverage without moral hazard is full insurance, partial insurance, and no insurance.

First, consider the case in which full insurance is optimal without moral hazard. We choose the parameters K_1, K_2, and K_3 appropriately so that the MRS between income and the asset is large enough to obtain the optimality of full insurance. We set $K_1 = 2$ and $K_2 = 3$, and consider 3 cases where $K_3 = -0.5$, 0, and 0.5, respectively. Notice that if $K_3 = -0.5$, then, $U^s_{CA}(C, A) > 0$, and $U^s(C, A)$ is increasing and concave for $C, A > 0$. Note that the optimal coverage without moral hazard is full insurance, that is, $I = D = 3$. However, with moral hazard, partial insurance is optimal, as shown in table 2. The optimal effort level with moral hazard is also less than that without moral hazard. Table 2's results are consistent with Proposition 1 (A).

Second, we illustrate Proposition 1 (B) with the following example in which partial insurance is optimal without moral hazard. We set $K_1 = 2$ and $K_2 = 2$, and consider 3 cases where $K_3 = -0.5$, 0, and 0.5, respectively. In all the cases, the optimal coverage and effort level with moral hazard are both less than those without moral hazard, as shown in table 3.

Lastly, we consider the case in which no insurance is optimal without moral hazard. We set $K_1 = 3$ and $K_2 = 1.2$, and consider 3 cases where $K_3 = -0.5$, 0, and 0.5, respectively. Notice that the MRS between income and the asset is small enough that the disutility from the premium payment is too high. In this case, an individual will choose not to buy any insurance regardless of the presence or absence of moral hazard. As shown in table 4, the optimal coverage is zero, and the optimal effort level is the same with and without moral hazard. These results correspond with Proposition 1 (C).

2.6.2 Numerical examples for Proposition 2

Now, let us consider the numerical examples to illustrate Proposition 2. We consider the utility function in equation (10):

$$\mu(C, A; t) = (1 - t)U^n(C, A) + tU^s(C, A),$$

where $U^n(C, A)$ and $U^s(C, A)$ are the power utility functions as in equations (16) and (17). We consider the positive consumption of composite goods and the benefit generated by the asset, that is, $C, A > 0$. Let us check the derivatives of the utility with respect to income and the asset, respectively. We also check the cross-second derivative of the utility.

$$\mu_C(C, A; t) = (1 - t)(C + A)^{-\gamma} + t[K_1C^{-\gamma_1} + K_3(C \cdot A)^{-\gamma_3}]$$
$$\mu_A(C, A; t) = (1 - t)K(C + A)^{-\gamma} + t[K_2A^{-\gamma_2} + K_3(C \cdot A)^{-\gamma_3}]$$
$$\mu_{CA}(C, A; t) = -t\gamma_3K_3(C \cdot A)^{-(1+\gamma_3)}$$

Note that $t \in [0, 1]$ denotes the degree of separability. If $t = 0$, $\mu(C, A; t)$ represents the non-separable utility function. Conversely, if $t > 0$, then $\mu(C, A; t)$ represents the separable utility function. In the following examples, we compare the relative sizes of the optimal coverage and the optimal effort level with moral hazard for different choices of t.

In the first example, we set $K = 2$, $K_1 = 3$, $K_2 = 3$, and $K_3 = 0$. As shown in table 5, when $t = 0$, $I = 0.9091$ and $\varepsilon = 0.4171$. However, when $t = 0.2$, the optimal coverage is greater than 0.9091 and the optimal effort level is smaller than 0.4171. In other words, in the separation case, the relative size of optimal coverage can be greater and the optimal effort level can be smaller than those in the non-separation case ($t = 0$). Recall that in this section, we consider the equilibrium results with moral hazard. From equation (8.3), the decision about the effort level is induced by the choice of insurance coverage, and therefore, higher coverage leads to a lower effort level, in general.

By contrast, the optimal coverage and effort can be simultaneously greater in the separation case than those in the non-separation case, for example, when $t = 0.4$. Note that for $t > 0$, the marginal utility with respect to the asset, that is, $\mu_A(C, A)$, is large enough, and the marginal utility with respect to income, that is, $\mu_C(C, A)$, is not that big, so that the optimal coverage and effort level are likely to be greater for a higher t.

In the second example, we set $K = 2$, $K_1 = 4$, $K_2 = 2$, and $K_3 = 0$. Compared with the first example, K_1 becomes larger and K_2 becomes smaller, so that for $t > 0$, the MRS between income and the asset is relatively small. When $t = 0.2$, the optimal coverage is less than that in the non-separation case ($t = 0$), as shown in table 6. Conversely, the optimal effort level is greater than that in the non-separation case.

Lastly, in the third example, we set $K = 2$, $K_1 = 1.5$, $K_2 = 1$, and $K_3 = 0$. Note that for $t > 0$, the marginal utility with respect to the asset, that is, $\mu_A(C, A)$, is small enough so that the optimal choice of effort is likely to be smaller. As shown in table 7, the optimal coverage and effort level in the separation case can be simultaneously less than those in the non-separation case, for example, when $t = 0.2$.

In summary, the relative sizes of the optimal coverage and effort level in the separation case ($t > 0$) and the non-separation case ($t = 0$) are ambiguously determined. The choices of the coverage and effort level are largely affected by the shape of the utility functions. In this section, we provide various examples to illustrate all possible cases described in Proposition 2.

2.6.3 Numerical examples for Proposition 3

Let us consider the numerical examples to illustrate Proposition 3. We consider the utility function in equation (10) as in the numerical example for proposition 2. Then, t is the parameter of interest, which represents the degree of separability. We interpret that the degree of separability increases if t increases.

From equation (11), the sensitivity of moral hazard is defined by ε_I as a function of t:

$$\varepsilon_I(t) = -\frac{p_\varepsilon\left[\{\mu_C(y_1, w_1) - \mu_C(y_1, w_0)\}\left(-p(\varepsilon)\right) + \mu_A(y_1, w_1)\right]}{p_{\varepsilon\varepsilon}[\mu(y_1, w_1) - \mu(y_1, w_0)] - c_{\varepsilon\varepsilon}} = -\frac{N_1}{D_1},$$

where $y_1 = y - p(\varepsilon)I$, $w_1 = w - D + I$, and $0 < I < D$.

By differentiating $\varepsilon_I(t)$ with respect to t, we obtain the sensitivity change in moral hazard, $\frac{d\varepsilon_I}{dt}$, which is evaluated at the optimum. For a given t, $\varepsilon_I(t)$ represents the sensitivity of moral hazard at the optimum, and $\frac{d\varepsilon_I}{dt}$ shows the sensitivity change in moral hazard with a marginal increase in t.

From equation (12), we have the followings factors, and the sign of $\frac{d\varepsilon_I}{dt}$ is identical to the sign of the sum of the factors.

$$
\begin{aligned}
F_1 = p_\varepsilon\Big[&[\{U^n_{\mathcal{W}}(y_1,w_1) - U^n_{\mathcal{W}}(y_1,w_0)\}(-p) + U^n_{\mathcal{W}}(y_1,w_1)] \\
&- [\{U^s_C(y_1,w_1) - U^s_C(y_1,w_0)\}(-p) + U^s_A(y_1,w_1)]\Big] \\
&\cdot \{p_{\varepsilon\varepsilon}[U^n(y_1,w_1) - U^n(y_1,w_0)] - c_{\varepsilon\varepsilon}\},
\end{aligned}
$$

$$
\begin{aligned}
F_2 = p_\varepsilon[&\{U^n_{\mathcal{W}}(y_1,w_1) - U^n_{\mathcal{W}}(y_1,w_0)\}(-p) + U^n_{\mathcal{W}}(y_1,w_1)] \\
&\cdot p_{\varepsilon\varepsilon}[-\{U^n(y_1,w_1) - U^n(y_1,w_0)\} + \{U^s(y_1,w_1) - U^s(y_1,w_0)\}].
\end{aligned}
$$

The sign of $(F_1 + F_2)$ depends on the shape of the utility functions $U^n(C,A)$ and $U^s(C,A)$ as well as the degree of separability, t, because t affects the equilibrium result (I,ε), and $\frac{d\varepsilon_I}{dt}$ is evaluated at the equilibrium. For comparison, let us set the parameters appropriately so that the MRS is large, and therefore, the optimal coverage is large and F_2 becomes small. In the following examples, we set $K_1 = 1$, $K_2 = 2$, and $K_3 = 0$. Then, the sign of $\frac{d\varepsilon_I}{dt}$ is largely affected by the sign and the magnitude of F_1.

As a first example, we set $K = 2$, and show the results of I, ε, $\varepsilon(t)$, $\varepsilon_I(t)$, and $\frac{d\varepsilon_I}{dt}$ in table 8. Note that the relationship between $\frac{d\varepsilon_I}{dt}$ and t is not necessarily monotonic. Because the optimal coverage for each t is large and the marginal utility with respect to the asset in the separation case is small enough, F_1 is negative, and we obtain negative $\frac{d\varepsilon_I}{dt}$. Therefore, in this case, moral hazard is more severe with a marginal increase in the degree of separability. The values of $\frac{d\varepsilon_I}{dt}$ with respect to t are illustrated in Figure 1.

In the second example, we set $K = 4$. As shown in table 9, the sign of $\frac{d\varepsilon_I}{dt}$ changes from negative to positive when t is approximately 0.279. We can show that for $t \in [0, 0.2785]$, $\frac{d\varepsilon_I}{dt} < 0$, and for $t \in [0.279, 1]$, $\frac{d\varepsilon_I}{dt} > 0$. Therefore, for t below approximately 0.2785, moral hazard becomes more important with a marginal increase in the degree of separability. On the contrary, for t above approximately 0.279, moral hazard is less important with a marginal increase in the degree of separability. The values of $\frac{d\varepsilon_I}{dt}$ with respect to t are illustrated in Figure 2.

2.6.4 Numerical examples for Proposition 4

We consider the power utility function as in equation (17), which is increasing and concave in C and A. In this example, K_3 is the parameter of interest, which indicates the degree of $U^s_{CA}(C, A)$. The interpretation is that a higher K_3 implies higher (lower) degree of U^s_{CA} if K_3 is positive (negative). In other words, if $K_3 > 0$, the degree of U^s_{CA} increases as K_3 increases above 0. If $K_3 < 0$, the degree of U^s_{CA} decreases as K_3 increases up to 0. By contrast, if $K_3 = 0$, income and the asset are independent of each other, and the degree of $U^s_{CA}(C, A)$ does not matter.

We consider the positive consumption of composite goods and the benefit generated by the asset, that is, $C, A > 0$. From equation (14), the sensitivity of moral hazard is defined by ε_I as a function of K_3:

$$\begin{aligned}\varepsilon_I(K_3) &= -\frac{p_\varepsilon\left[-pK_3\left(U^3_C(y_1, w_1) - U^3_C(y_1, w_0)\right) + K_2U^2_A(w_1) + K_3U^3_A(y_1, w_1)\right]}{p_{\varepsilon\varepsilon}\left[K_2[U^2(w_1) - U^2(w_0)] + K_3[U^3(y_1, w_1) - U^3(y_1, w_0)]\right] - c_{\varepsilon\varepsilon}} \\ &= -\frac{N_2}{D_2},\end{aligned}$$

where $y_1 = y - p(\varepsilon)I$, $w_0 = w$, $w_1 = w - D + I$, and $0 < I < D$. Note that

$$N_2 = -\frac{1}{(1.5+\varepsilon)^2}\left[-\frac{1}{1.5+\varepsilon}K_3\left\{\left(\left(y - \frac{1}{1.5+\varepsilon}I\right)(w - D + I)\right)^{-\gamma_3} - \left(\left(y - \frac{1}{1.5+\varepsilon}I\right)\cdot w\right)^{-\gamma_3}\right\} + \right.$$

$K_2(w - D + I)^{-\gamma_2} + K_3\left(\left(y - \frac{1}{1.5+\varepsilon}I\right)(w - D + I)\right)^{-\gamma_3}\Bigg]$, and

$D_2 = \frac{2}{(1.5+\varepsilon)^3}\left[\frac{K_2}{1-\gamma_2}\{(w - D + I)^{1-\gamma_2} - w^{1-\gamma_2}\} + \frac{K_3}{1-\gamma_3}\left\{\left(\left(y - \frac{1}{1.5+\varepsilon}I\right)(w - D + I)\right)^{1-\gamma_3} - \left(\left(y - \frac{1}{1.5+\varepsilon}I\right)\cdot w\right)^{1-\gamma_3}\right\}\right] - 1.$

By differentiating $\varepsilon_I(K_3)$ with respect to K_3, we obtain the sensitivity change in moral hazard, $\frac{d\varepsilon_I}{dK_3}$, which is evaluated at the optimum. For a given K_3, $\varepsilon_I(K_3)$ represents the sensitivity of moral hazard at the optimum, and $\frac{d\varepsilon_I}{dK_3}$ shows the sensitivity change in moral hazard with a marginal increase in K_3. The relationship between $\frac{d\varepsilon_I}{dK_3}$ and K_3 is not necessarily monotonic.

From equation (15), we have the followings factors, and the sign of $\frac{d\varepsilon_I}{dK_3}$ is identical to the sign of the sum of the factors.

$$G_1 = \left(-\frac{1}{(1.5+\varepsilon)^2}\right)\left(\frac{1}{1.5+\varepsilon}\right) \cdot \left[\left(\left(y - \frac{1}{1.5+\varepsilon}I\right)\cdot(w - D + I)\right)^{-\gamma_3} - \left(\left(y - \frac{1}{1.5+\varepsilon}I\right)\cdot w\right)^{-\gamma_3}\right] \cdot \left[\frac{2}{(1.5+\varepsilon)^3}\frac{K_2}{1-\gamma_2}\{(w - D + I)^{1-\gamma_2} - w^{1-\gamma_2}\} - 1\right] > 0$$

$$G_2 = \frac{1}{(1.5+\varepsilon)^2}\left(\left(y - \frac{1}{1.5+\varepsilon}I\right)\cdot(w - D + I)\right)^{-\gamma_3} \cdot \left[\frac{2}{(1.5+\varepsilon)^3}\frac{K_2}{1-\gamma_2}\{(w - D + I)^{1-\gamma_2} - w^{1-\gamma_2}\} - 1\right] < 0$$

$$G_3 = -\frac{1}{(1.5+\varepsilon)^2} \cdot K_2 (w - D + I)^{-\gamma_2} \cdot \frac{2}{(1.5+\varepsilon)^3} \cdot \frac{1}{1-\gamma_3} \left[\left(\left(y - \frac{1}{1.5+\varepsilon} I \right) (w - D + I) \right)^{1-\gamma_3} - \left(\left(y - \frac{1}{1.5+\varepsilon} I \right) \cdot w \right)^{1-\gamma_3} \right] > 0$$

Because $\varepsilon_I < 0$, $\frac{d\varepsilon_I}{dK_3} > 0$ signifies that moral hazard is less important with a marginal increase in K_3. Therefore, if $U_{CA}^s > 0$, that is, $K_3 < 0$, $\frac{d\varepsilon_I}{dK_3} > 0$ implies that moral hazard is less important as the degree of U_{CA}^s decreases, that is, K_3 marginally increases below 0. On the contrary, if $U_{CA}^s < 0$, that is, $K_3 > 0$, $\frac{d\varepsilon_I}{dK_3} > 0$ implies that moral hazard is less important as the degree of U_{CA}^s increases, that is, K_3 marginally increases above 0.

Let us set $K_1 = 1.5$ and $K_2 = 1$. Moreover, we constrain the range of the parameter K_3 to $-0.3 < K_3 < \infty$, so that the utility function is increasing and concave, and we obtain an interior solution.[10] In table 10, we show the results of I, ε, $\varepsilon_I(K_3)$, and $\frac{d\varepsilon_I}{dK_3}$, where I and ε are the optimal coverage and the optimal effort level, respectively, under moral hazard.

The results in table 10 show that $\frac{d\varepsilon_I}{dK_3}$ is more likely to be negative if the optimal coverage is big enough. In this example, the sign of $\frac{d\varepsilon_I}{dK_3}$ changes from positive to negative

[10] For $K_3 \in [-0.3, \infty)$, $V(C, A)$ has positive marginal utilities, and the Hessian of the utility function is negative definite. Moreover, the optimal insurance is partial insurance, that is, $0 < I < D$.

when K_3 is approximately 0.561. For $K_3 \in [-0.3, 0.561]$, $\frac{d\varepsilon_I}{dK_3} > 0$, and for $K_3 > 0.562$, we have $\frac{d\varepsilon_I}{dK_3} < 0$. First, for $K_3 \in [-0.3, 0)$, $\frac{d\varepsilon_I}{dK_3} > 0$ implies that moral hazard is more important with a marginal increase in the degree of U_{CA}^s. Secondly, when $U_{CA}^s < 0$, for K_3 below approximately 0.561, moral hazard is less important with a marginal increase in the degree of U_{CA}^s. For K_3 above approximately 0.562, by contrast, moral hazard is more important with a marginal increase in the degree of U_{CA}^s. The values $\frac{d\varepsilon_I}{dK_3}$ with respect to K_3 are illustrated in Figure 3.

Notice that the sign of $\frac{d\varepsilon_I}{dK_3}$ is likely to be negative when the optimal coverage is large. The degree of U_{CA}^s affects the MRS, and therefore, affects on the choice of insurance coverage. Moreover, the relative magnitudes of the factors G_1, G_2, and G_3 are affected by the shape of the utility function. For a given K_3, if the optimal coverage is large, the sign of $\frac{d\varepsilon_I}{dK_3}$ can change sensitively with K_3.

2.7 Conclusion

This study develops a model of moral hazard and demonstrates the optimal insurance choice when an insurer cannot observe the individual's self-protection choice. The moral hazard problem in insurance has been widely studied, but much of the work is based on the standard utility function and ignores the fact that the premium is paid from income, rather than from the asset. This study attempts to fill this gap by using a two-argument utility function that depends on income and the asset. We suggest that the equilibrium results of our model reflect the trade-offs that individuals make between two goods.

In the absence of moral hazard, our model predicts diverse optimal coverage rates, from no insurance to full insurance, according to the individual's preferences. If moral hazard is present, however, less insurance coverage (partial insurance) and a lower effort level are optimal, compared with the equilibrium result without moral hazard. By contrast, if the marginal utility of income is much greater than that of the asset, the disutility from premium payments is too high and therefore, no insurance is optimal regardless of the presence or absence of moral hazard. In summary, moral hazard affects the equilibrium only for those who purchase positive insurance. This result stands in contrast to the conventional conclusion that partial insurance is the only equilibrium under moral hazard.

In fact, the non-separation case in the conventional model is a special case of the separation case in our model. As income and the asset are separated in our model, we can consider the individuals' preferences and the interaction between income and the asset of any sign, which draws a variety of equilibrium results depending on the shape of the utility. In particular, we show the conditions for both optimal coverage and effort level under moral hazard in the separation case to be simultaneously greater (less) than those in the non-separation case.

This study also demonstrates the effect of the separability of income and the asset on the relative importance of moral hazard. When the optimal coverage is large enough, the moral hazard problem is more likely to be severe in the separation case if the marginal utility with respect to the asset in the separation case is a large enough than that in the non-separation case. This result implies that the separation of income and the asset is particularly important in analyzing the moral hazard problem.

In addition, the relative importance of moral hazard is also affected by the interaction between income and that asset, that is, U_{CA}. We show that under some conditions, moral hazard can be more or less severe as the degree of U_{CA} increases. Therefore, the concern about moral hazard may be excessive or insufficient depending on U_{CA}.

There remains an issue about the degree of the observability or the monitoring cost of an insurer. Similar to Shavell's (1979) results, if an insurer can imperfectly observe the individual's action without cost, then observations will improve the individual's welfare. Moreover, our results provide a policy implication that the penalty for moral hazard should be adjusted according to the shape of the utility. Given the realism of the model and the importance of the topic, we hope that this model will be broadly applied to other areas and future empirical studies of moral hazard.

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Table 1. Notations

$U(C, A)$	A two-argument utility function, which is twice-differentiable, increasing and concave in both arguments, income (C) and asset (A).
$y > 0$	Initial income.
$w > 0$	Initial insurable asset.
$D > 0$	Loss size, where $D < w$.
$\varepsilon \geq 0$	Self-protection effort.
$p(\varepsilon) > 0$	Probability of loss, where $p_\varepsilon < 0$ and $p_{\varepsilon\varepsilon} > 0$.
$c(\varepsilon) \geq 0$	Cost of effort, where $c_\varepsilon > 0$, $c_{\varepsilon\varepsilon} > 0$ and $c(0) = 0$.
(Q, I)	Insurance contract with premium Q and indemnity I, where $0 \leq I \leq D$.
$MRS(Q, I; \varepsilon)$	Marginal rate of substitution between income and asset, where (Q, I) and ε respectively denote a chosen insurance contract and effort.

Table 2. Equilibrium results with and without moral hazard, when full insurance is optimal without moral hazard

	$K_1 = 2,\ K_2 = 3,$ $K_3 = -0.5$		$K_1 = 2,\ K_2 = 3,$ $K_3 = 0$		$K_1 = 2,\ K_2 = 3,$ $K_3 = 0.5$	
	I	ε	I	ε	I	ε
No MH	3	0.6181	3	0.6700	3	0.7183
MH	2.9394	0.0277	2.4545	0.2706	2.2121	0.4398

Table 3. Equilibrium results with and without moral hazard, when partial insurance is optimal without moral hazard

	$K_1 = 2,\ K_2 = 2,$ $K_3 = -0.5$		$K_1 = 2,\ K_2 = 2,$ $K_3 = 0$		$K_1 = 2,\ K_2 = 2,$ $K_3 = 0.5$	
	I	ε	I	ε	I	ε
No MH	2.3030	0.5638	2.7273	0.6625	2.6970	0.7243
MH	1.5455	0.2579	1.7273	0.3902	1.7576	0.5078

Table 4. Equilibrium results with and without moral hazard, when no insurance is optimal without moral hazard

	$K_1 = 3$, $K_2 = 1.2$, $K_3 = -0.5$		$K_1 = 3$, $K_2 = 1.2$, $K_3 = 0$		$K_1 = 3$, $K_2 = 0.5$, $K_3 = 0.5$	
	I	ε	I	ε	I	ε
No MH	0	0.0668	0	0.5636	0	0.6806
MH	0	0.0668	0	0.5636	0	0.6806

Table 5. Equilibrium results in relation to t, when $K = 2$, $K_1 = 3$, $K_2 = 3$, $K_3 = 0$

t	I	ε
0	0.9091	0.4171
0.2	1.4242	0.4023
0.4	1.5758	0.4249
...	...	...
1	1.6970	0.5240

Table 6. Equilibrium results in relation to t, when $K = 2$, $K_1 = 4$, $K_2 = 2$, $K_3 = 0$

t	I	ε
0	0.9091	0.4171
0.2	0.3939	0.5299
0.4	0.2727	0.5925
...	...	...
1	0.1818	0.7326

Table 7. Equilibrium results in relation to t, when $K = 2$, $K_1 = 1.5$, $K_2 = 1$, $K_3 = 0$

t	I	ε
0	0.9091	0.4171
0.2	0.8485	0.4143
0.4	0.8485	0.4029
...	...	...
1	0.7879	0.3765

Table 8. Sensitivity of moral hazard in relation to the degree of separability, when $K = 2$, $K_1 = 1$, $K_2 = 2$, and $K_3 = 0$

t	I	ε	$\varepsilon_I(t)$	$\frac{d\varepsilon_I}{dt}$
0	0.9091	0.4171	-0.1196	-0.1326
0.0101	1	0.4060	-0.126041	-0.1283
…	…	…	…	…
0.9899	2.9091	0.0385	-0.4059	-0.1016
1	2.9091	0.0386	-0.4069	-0.1015

Table 9. Sensitivity of moral hazard in relation to the degree of separability, when $K = 4$, $K_1 = 1$, $K_2 = 2$, and $K_3 = 0$

t	I	ε	$\varepsilon_I(t)$	$\frac{d\varepsilon_I}{dt}$
0	0.8485	0.6685	-0.1665	-0.0387
0.0101	0.9091	0.6559	-0.1702	-0.0359
…	…	…	…	…
0.2727	1.6667	0.4692	-0.2278	-3.7266×10^{-4}
0.2828	1.6970	0.4609	-0.2278	0.0013
…	…	…	…	…
0.9899	2.9091	0.0388	-0.4086	0.1842
1	2.9091	0.0386	-0.4069	0.1844

Table 10. Sensitivity of moral hazard in relation to the degree of U_{CA}^{S}, when $K_1 = 1.5$, $K_2 = 1$, and $K_3 \in [-0.3, \infty)$

K_3	I	ε	$\varepsilon_I(K_3)$	$\frac{d\varepsilon_I}{dK_3}$
-0.3	0.9697	0.3685	-0.1711	0.0166
-0.2768	1.0606	0.3619	-0.1721	0.0148
…	…	…	…	…
0.5596	1.6667	0.3975	-0.1746	7.9844×10^{-6}
0.5828	1.6667	0.4006	-0.1743	-7.4529×10^{-5}
…	…	…	…	…
1.9768	1.7576	0.5353	-0.1681	-0.00329
2	1.7576	0.5376	-0.1680	-0.00331

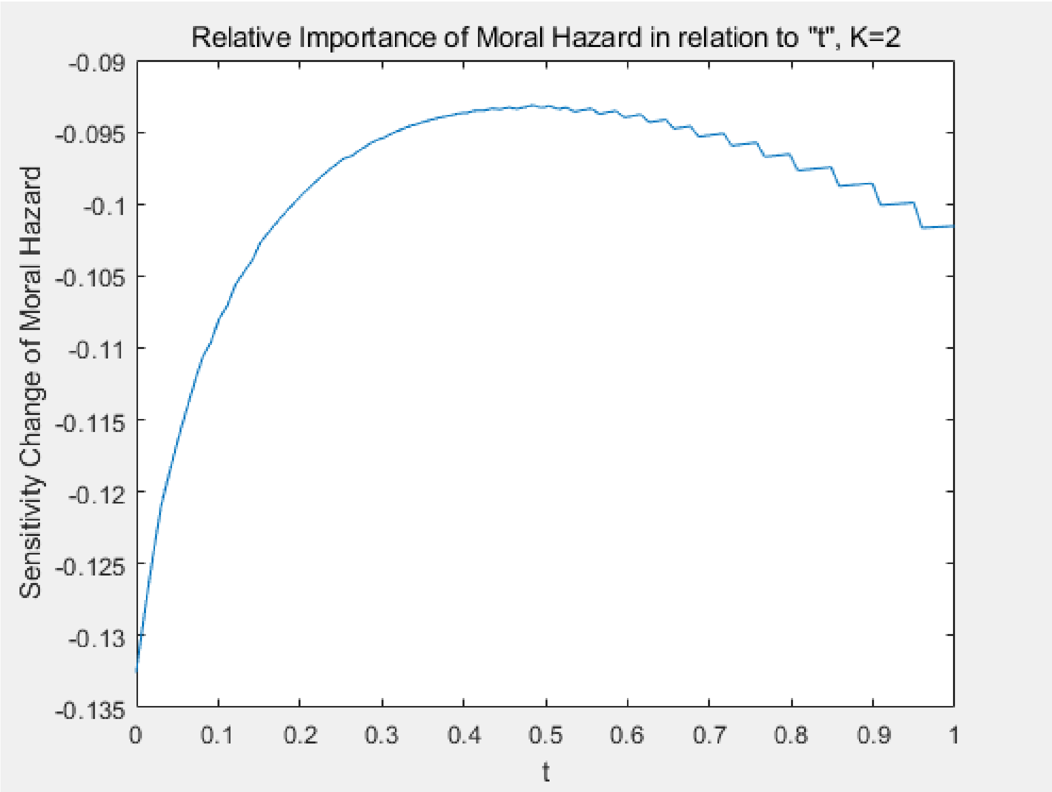

Figure 1. Sensitivity change in moral hazard in relation to the degree of separability, when K = 2

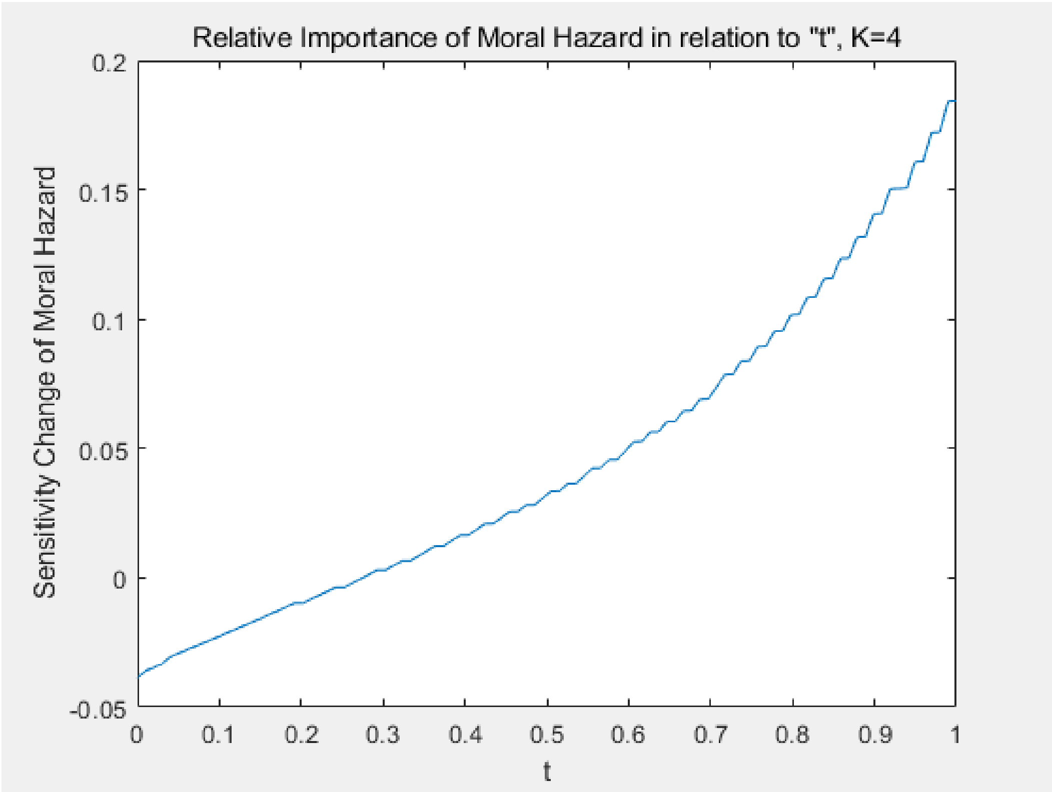

Figure 2. Sensitivity change in moral hazard in relation to the degree of separability, when K = 4

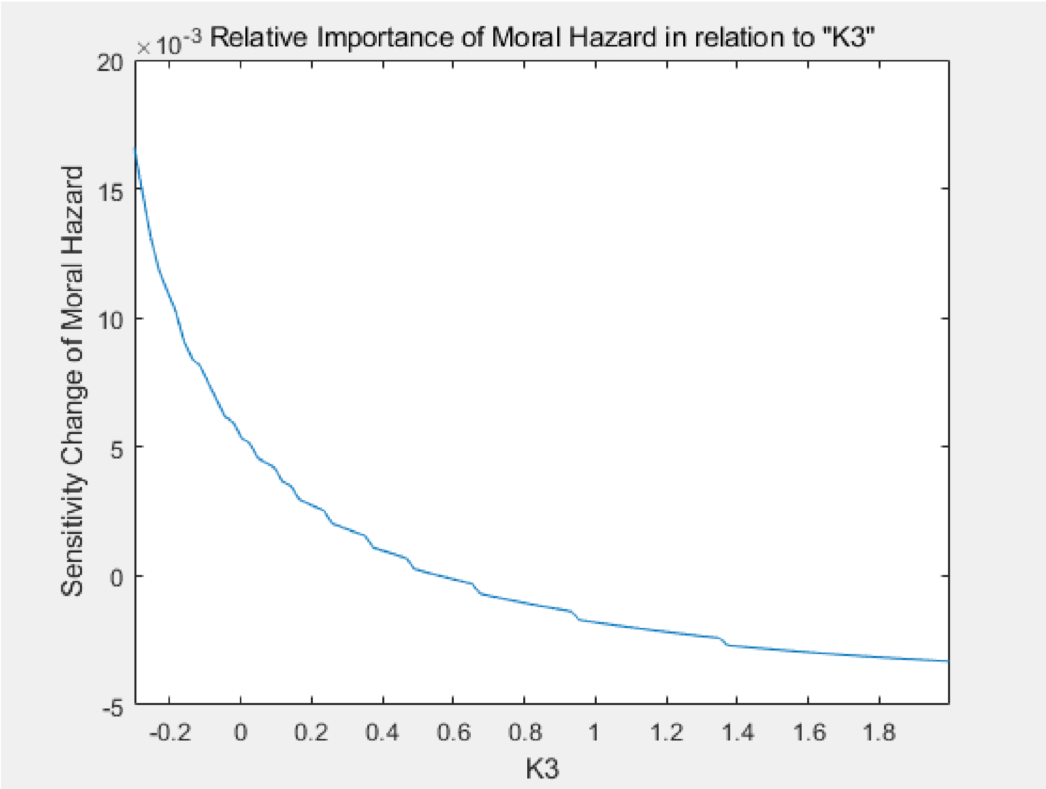

Figure 3. Relative importance of moral hazard in relation to the degree of U_{CA}^{S}

Appendix

1. Proof of Lemma 1

From (5.2), the optimal coverage is given by $p(\varepsilon)U_C(y - p(\varepsilon)I, w - D + I) + \big(1 - p(\varepsilon)\big)U_C(y - p(\varepsilon)I, w) = U_A(y - p(\varepsilon)I, w - D + I)$. Since $U_C(y_1, w_1) = U_A(y_1, w_1) \equiv U_M(y_1,\ w_1)$, we have $U_M(y_1, w_0) = U_M(y_1, w_1)$. Therefore, $I^* = D$. //

2. Proof of Lemma 2

From (8.2), and since $U_C(y_1, w_1) = U_A(y_1, w_1) \equiv U_M(y_1,\ w_1)$,

$\hat{L}_I = p(\varepsilon)\big(1 - p(\varepsilon)\big)\{U_M(y_1,\ w_1) - U_M(y_1,\ w_0)\} + \lambda p_\varepsilon[\big(1 - p(\varepsilon)\big)U_M(y_1,\ w_1) + p(\varepsilon)U_M(y_1,\ w_0)] = 0.$

Note that $\hat{L}_I$ has additional λ-term compared to L_I. Since $\lambda > 0$ and $p_\varepsilon < 0$, λ-term is negative. Therefore, $0 < \hat{I}^* < I^* = D$ under moral hazard, because it cannot be equal either to zero or to D. That is, $\hat{L}_I\big|_{I=D} = \lambda p_\varepsilon U_M(y - p(\varepsilon)D,\ w) < 0$ and $\hat{L}_I\big|_{I=0} = L_I|_{I=0} > 0$. //

3. Proof of Lemma 3

(A) From (5.2), full insurance is optimal if $L_I|_{I=D} = 0$. If $L_I|_{I>D} = 0$, over insurance is optimal. If over insurance is not allowed, full insurance is optimal. //

(C) From (5.2), no insurance is optimal if $L_I|_{I=0} \leq 0$. //

(B) From the proof of (A) and (C), we can easily prove Lemma 1 (B). //

4. Proof of Lemma 4

Suppose on the contrary that $\lambda \leq 0$. From (8.2), we have the following equation:

$$\begin{aligned}\big(1-p(\varepsilon)\big)U_C(y_1,w_0)+p(\varepsilon)U_C(y_1,w_1)& \\ +\lambda p_\varepsilon\{U_C(y_1,w_1)-U_C(y_1,w_0)\}& \\ =U_A(y_1,w_1)\left[1+\lambda\frac{p_\varepsilon}{p}\right]&.\end{aligned} \tag{A.1}$$

From (8.1), we obtain the following equation:

$$\begin{aligned}\big[\big(1-p(\varepsilon)\big)U_C(y_1,w_0)+p(\varepsilon)U_C(y_1,w_1)& \\ +\lambda p_\varepsilon\{U_C(y_1,w_1)-U_C(y_1,w_0)\}\big](p_\varepsilon I)& \\ =\lambda[p_{\varepsilon\varepsilon}\{U(y_1,w_1)-U(y_1,w_0)\}-c_{\varepsilon\varepsilon}]&.\end{aligned} \tag{A.2}$$

From $(A.1)$, we can rewrite $(A.2)$ as follows:

$$U_A(y_1,w_1)\left[1+\lambda\frac{p_\varepsilon}{p}\right](p_\varepsilon I)=\lambda[p_{\varepsilon\varepsilon}\{U(y_1,w_1)-U(y_1,w_0)\}-c_{\varepsilon\varepsilon}]. \tag{A.3}$$

Firstly, if $\lambda < 0$, the sign of the left-hand side of $(A.3)$ is negative, since $p_\varepsilon < 0$. On the other hand, the sign of the right-hand side of $(A.3)$ is positive because the second order condition of the choice of the optimal effort is negative at the optimum. Secondly, if $\lambda = 0$, $\hat{L}_\varepsilon = \{\big(1-p(\varepsilon)\big)U_C(y_1,w_0)+p(\varepsilon)U_C(y_1,w_1)\}(-p_\varepsilon I) > 0$, which is a contradiction. Therefore, $\lambda > 0$. //

5. Proof of Lemma 5

Given insurance coverage I', let us assume that $\hat{\varepsilon}'$ is the optimal level of effort under moral hazard, that satisfies equation (6). Now consider $L_\varepsilon|_{I'}$ which is the first order condition with respect to ε, without moral hazard. Denote ε' to be the optimal level of effort given I', without moral hazard, that is, $\mathrm{L}_{\varepsilon}|_{(\varepsilon',\mathrm{I}')} = 0$. Let us show that ε' is higher than $\hat{\varepsilon}'$. Suppose on the contrary that $\varepsilon' \leq \hat{\varepsilon}'$. Then, we have $\mathrm{L}_{\varepsilon}|_{(\hat{\varepsilon}',\mathrm{I}')} > 0$, which is a contradiction. There, $\varepsilon' > \hat{\varepsilon}'$. //

6. Proof of Proposition 1

(A) From (8.3), full insurance implies that $\hat{\mathrm{L}}_{\lambda} < 0$, which contradicts the first order condition. Therefore, $\hat{\mathrm{I}}^* < \mathrm{D}$. On the other hand, if $\hat{\mathrm{I}}^* = 0$, we can ignore (8.3) as moral hazard does not exist. Then, no insurance implies that $\hat{\mathrm{L}}_{\mathrm{I}}\big|_{\hat{\mathrm{I}}=0} = \mathrm{L}_{\mathrm{I}}|_{\mathrm{I}=0} > 0$, which is a contradiction. Therefore, $0 < \hat{\mathrm{I}}^* < \mathrm{I}^* = \mathrm{D}$. //

(B) Without moral hazard, we can reach the global optimum at $(\mathrm{I}^*, \varepsilon^*)$.[11] However, we only can obtain the constrained optimum at $(\hat{\mathrm{I}}^*, \hat{\varepsilon}^*)$ satisfying $\hat{\mathrm{L}}_{\lambda} = 0$, under moral hazard. This is a second-best solution, which trades off some of the risk-sharing benefits for investment in self-protection. Note that our objective function is given by equation (1).

Firstly, by Lemma 5, it is obvious that optimal coverage and effort level cannot be simultaneously greater than those without moral hazard. We want to show that the only possible equilibrium is where both coverage and effort level is less than those without moral hazard. That is, the equilibrium with moral hazard is $(\hat{\mathrm{I}}^*, \hat{\varepsilon}^*)$, where $\hat{\mathrm{I}}^* < \mathrm{I}^*$ and $\hat{\varepsilon}^* < \varepsilon^*$. Recall that $(\hat{\mathrm{I}}^*, \hat{\varepsilon}^*)$ should satisfy the constraint $\hat{\mathrm{L}}_{\lambda} = 0$.

[11] We can achieve the first-best solution by applying the calculus of variations and solving the first-order conditions. To validate this approach, we assume that a single global optimum exists and is differentiable.

(i) Suppose on the contrary that $\hat{I}^* > I^*$ and $\hat{\varepsilon}^* < \varepsilon^*$. Then, if we move from $(\hat{I}^*, \hat{\varepsilon}^*)$ to $(\hat{I}^* + dI, \hat{\varepsilon}^* + d\varepsilon)$, where $dI < 0$ and $d\varepsilon > 0$, we can reach closer to the global optimum. Note that both $(\hat{I}^*, \hat{\varepsilon}^*)$ and $(\hat{I}^* + dI, \hat{\varepsilon}^* + d\varepsilon)$ are located in the solution space with moral hazard, satisfying $\hat{L}_\lambda = 0$. However, since $(\hat{I}^*, \hat{\varepsilon}^*)$ is the constrained optimum, $EU(\hat{I}^*, \hat{\varepsilon}^*) > EU(\hat{I}^* + dI, \hat{\varepsilon}^* + d\varepsilon)$. This contradicts the fact that $EU(I^*, \varepsilon^*)$ is the single global optimum.

(ii) Suppose that $\hat{I}^* < I^*$ and $\hat{\varepsilon}^* > \varepsilon^*$. Then, if we move from $(\hat{I}^*, \hat{\varepsilon}^*)$ to $(\hat{I}^* + dI, \hat{\varepsilon}^* + d\varepsilon)$, where $dI > 0$ and $de < 0$, we can reach closer to the global optimum. However, we can make a similar argument for case (ii) as in case (i), and obtain the following contradiction: $EU(\hat{I}^*, \hat{\varepsilon}^*) > EU(\hat{I}^* + dI, \hat{\varepsilon}^* + d\varepsilon)$ as $(\hat{I}^*, \hat{\varepsilon}^*)$ is the constrained optimum. //

(C) Suppose that $\hat{I}^* > 0$ and $\hat{\varepsilon}^* < \varepsilon^*$. Since (I^*, ε^*) is the first-best solution, $EU(I^*, \varepsilon^*) > EU(\hat{I}^*, \hat{\varepsilon}^*)$. However, we can reach the global optimum by choosing no insurance. In no insurance case, the incentive compatibility is ignorable and moral hazard is irrelevant. That is, $\hat{L}_I\big|_{\hat{I}=0} = L_I|_{I=0} \leq 0$, as desired. //

7. Proof of Proposition 2

For $t > 0$, if MRS of U^s is greater (smaller) enough than U^n, the optimal coverage under moral hazard is greater (less) than that in the non-separation case $(t = 0)$. Then, greater (less) coverage generally leads to lower (higher) effort level, which follows by (8.3).

If U_A^s is big enough and U_C^s is not that big, we can have large enough MRS and greater DU for $t > 0$. Then, for very small but positive t, the optimal coverage is greater but the optimal effort level is lower than the non-separation case, by (8.3). However, for higher value of $t \leq 1$, DU can be greater even with greater coverage. In this case, the optimal coverage and effort level can be simultaneously greater than that in the non-separation case.

Similarly, if U_A^s is small enough, we can have small enough MRS and smaller DU for $t > 0$. Then, for very small but positive t, the optimal coverage is smaller but the optimal effort level is greater than the non-separation case, by (8.3). However, for higher value of t, DU can be smaller even with less coverage. In this case, the optimal coverage and effort level can be simultaneously less than that in the non-separation case. //

8. Proof of Proposition 4

(A) If $V_{CA} = 0$, the importance of moral hazard is not affected by K_3. //

(B) The sign of $\frac{d\varepsilon_I}{dK_3}$ is determined by the sum effect of G_1, G_2, and G_3, and the relative magnitude of each factor is affected by the shape of the utility function. Moreover, as $\frac{d\varepsilon_I}{dK_3}$ is evaluated at the optimum, its sign can change sensitively with K_3. Therefore, we determined the sign of $\frac{d\varepsilon_I}{dK_3}$ in a limited sense. When the optimal coverage is big enough, G_1 and G_3 are close to zero, while G_2 is still negative. In this case, if the cost function is convex enough, $\frac{d\varepsilon_I}{dK_3}$ can be negative. If the optimal coverage is not that big, the opposite is also possible. //

(C) Similar logic as in (B) can be applied. //

Chapter 3

Insurance and Risk Selection

when Insurable Asset and Income are Separable

3.1 Introduction

The relationship between risk-taking behavior and insurance purchase with asymmetric information has been widely studied from two perspectives: selection and moral hazard. Adverse selection and moral hazard promote inefficient risk sharing in insurance markets, whereas advantageous selection brings a favorable result. In this paper, we study the endogenous selection in which risk types are endogenously determined by individuals. Under asymmetric information, the insured is assumed to have private information about his/her preferences that are both relevant to the choice of insurance contract and the choice of action which is unobservable to the insurer. In regard to the hidden action, we are specifically concerned with the self-protection effort to reduce the loss probability, which depends on the heterogeneity of asset sensitivity. Asset sensitivity that inheres in utility function indicates how much an individual suffers disutility from a loss in an insurable asset.

The traditional model of insurance with asymmetric information predicts a positive correlation between risk and insurance coverage in equilibrium, whereas empirical tests have exhibited mixed results. The classic equilibrium models developed by Rothschild and Stiglitz (1976) indicate that insurers provide a menu of contracts to screen the insured so that high-risks choose full coverage and low-risks choose partial coverage under adverse selection. However, in some markets, such as automobile insurance (Richaudeau, 1999;

Chiappori and Salanie, 2000), commercial fire insurance (Wang et al., 2009), health insurance (Cardon and Hendel, 2001; Cutler, Finkelstein, and McGarry, 2008), annuity (Finkelstein and Poterba, 2004), life insurance (Cawley and Philipson, 1999), long-term care insurance (Finkelstein and McGarry, 2006), and Medigap insurance (Fang et al., 2008), there was even negative correlation or insignificant evidence of positive correlation between insurance coverage and ex post loss.

De Meza and Webb (2001) tried to explain these inconsistent results by heterogeneous risk tolerance. They demonstrated that risk-averse individuals purchase insurance and make an effort to reduce the loss probability, whereas risk-neutral individuals neither purchase insurance nor expend effort. Moreover, Huang et al. (2010) proposed the heterogeneity in risk perception, and showed that a rational individual takes precautions to reduce the loss probability and purchase higher insurance coverage compared to an overconfident individual who will not make any effort. Similarly, Spinnewijn (2013) explained the negative correlation between risk and insurance coverage by heterogeneous beliefs about the risk. On the other hand, Seog (2009) decomposed the risk into a general risk and a specific risk, and showed that the relationship between risk and insurance coverage is not in one direction when insurers and insureds both have superior information about the risk. In this paper, however, we provide a theoretical basis for the existence of advantageous selection and other types of equilibrium by asset sensitivity that reflects an insured's preferences towards an income and an asset. The rationale for this approach is based on a two-argument utility function.

In fact, many theoretically interesting economic problems that reflect reality are developed by a two commodity model, capturing the trade-offs that individuals make between two different goods in decision making (Dardanoni, 1988; Picone, Uribe, and Wilson, 1998; Eeckhoudt, Rey, and Schlesinger, 2007; Liu and Menegatti, 2019). In insurance literature, by contrast, economic models are typically oversimplified with a one-argument utility function. Although this framework has the advantage in mathematical tractability, it may fail to solve the economic problems with more than one good. For example, even dealing with two sources of uncertainty, insurable risk and uninsurable background risk, Doherty and Schlesinger (1983) used a one-argument utility function.

Rey (2003), meanwhile, introduced a two-argument utility function and further considered the variation of the marginal utility of the insurable asset with respect to the uninsurable asset. However, the analysis still ignores an important feature of an insurance contract. Lee (2007) is the first to point out this feature that insurance premium is in practice paid out of one's income, and indemnity is made against an insured asset. In this respect, even when considering a single insurable asset using the simplest model, it is plausible to assume that utility depends on income and an insurable asset. Lee (2007) demonstrated that the demand for insurance under this assumption depends largely on an income and preferences regardless of the magnitude of the loading factor. However, he only considered the optimality of full insurance in a market that is free from asymmetric information. Adding to Lee (2007), we are concerned with insurance demand and an incentive to spend on self-protection when an insured holds private information about his/her preferences towards an income and an asset.

Following Lee (2007), we adopt a two-argument utility function $U(C, A)$, where C and A respectively indicate the composite good (income) and an insurable asset or, technically, the benefit generated by an asset. Note that income and the asset are two different goods, of which consumption is not of the same dimension. This is because insurable assets such as a house, property, and health, are irreplaceable goods. In addition, income and the asset are not easily exchangeable in dollars. For example, a house is not easily and quickly converted into income, and the benefit as a shelter cannot be readily substituted by that of composite goods. Moreover, health is neither easily convertible into, nor directly replaceable by income. This realism justifies the need for a two-argument utility function.

As we assume that utility depends on an income and an asset, the decision to purchase an additional insurance coverage depends on the preferences towards these two goods. This is because the increase in insurance premium reduces an income rather than an asset, and the additional indemnity increases the consumption of the benefit generated by the asset if a loss occurs. This trade-off between income and asset can be represented by the marginal rate of substitution between an insurance premium and indemnity.

Moreover, decomposing wealth into income and the asset is especially important in the sense that individuals are heterogeneous in asset sensitivity – sensitivity to the change in the asset value – as they have their own valuation on the asset. For instance, if one places higher value on health, he or she will be more sensitive to the change in health status. In this paper, we define an asset sensitive type of individual as one who has relatively greater marginal utility with respect to the asset. Furthermore, we assume that individuals can be divided into two types according to their asset sensitivity: the sensitive type (s) and the insensitive type (t). Then, if a loss occurs, the sensitive type suffers greater disutility than the insensitive type. Therefore, individuals with the same initial income and asset but different asset sensitivity will make different choices of self-protection and insurance coverage. Our model indicates that the sensitive type is more likely to invest in self-protection and may become a low-risk in equilibrium, whereas the insensitive type never chooses to expend effort. On the other hand, the choice of insurance coverage largely depends on the preferences towards income and asset. If the sensitive type expends effort and at the same time has more incentive to sacrifice an income to recover a loss in an asset by means of insurance, an equilibrium with advantageous selection may occur in the market.

Moreover, the existence of equilibrium depends on the proportion of the sensitive type in the market. According to this proportion, separating equilibrium or pooling equilibrium can occur.

By assuming heterogeneity in asset sensitivity, we find five Nash equilibrium configurations in the insurance market. Contrary to the results reported by Rothschild and Stiglitz (1976), our model indicates that advantageous selection and even pooling equilibrium can occur. In a separating equilibrium with advantageous (adverse) selection, the sensitive type of insured invests in self-protection and demands more (less) insurance. In a partial pooling equilibrium, the insensitive type mixes between two contracts, and sensitive type chooses the contract with higher coverage out of the two contracts. In a pooling equilibrium, on the other hand, both types of insureds choose the same contract. Furthermore, the possible equilibrium includes the case, in which both types of insured do not invest in self-protection, but the sensitive type purchases higher coverage. Unlike Huang et al. (2010) and Spinnewijn (2013), who introduce irrational (optimistic) individuals into their models, we provide such results under full rationality. Most importantly, in contrast to all previous studies, we propose a model reflecting the reality, which captures the trade-offs that individuals make between an income and an asset in decision making. Moreover, this study demonstrates that, even under the equilibrium concept of Rothschild and Stiglitz's (1976), a complete risk pooling can occur in the competitive insurance market where two different types of insured exist.

The remainder of this paper is organized as follows. Section 2 outlines the model, and Section 3 describes the market equilibrium. Section 4 discusses the differentiation of our results from the literature, and Section 5 concludes the paper. Some proofs and figures are provided in the appendix.

3.2 Model

Each individual is an expected utility maximizer with an endowment income of y and an insurable asset of w. Note that an income and an asset are different kinds of goods that are not easily exchangeable in dollars. An individual faces a fixed loss of $D < w$, and there is no uncertainty about income. A loss occurs with probability $p(\varepsilon)$, where $p'(\varepsilon) < 0$ with self-protection effort $\varepsilon \geq 0$. An individual can affect the probability of loss by spending ε at the expense of $c(\varepsilon)$ in utility units, where $c'(\varepsilon) > 0$ and $c(0) = 0$. Without loss of generality, we assume that individuals face a binary choice about whether to make an effort, i.e., $\varepsilon \in \{0, e\}$, where $e > 0$. Let us denote p_e and p_0 to be the loss probabilities with and without effort, i.e., $p(e) = p_e$ and $p(0) = p_0$, where $p_e < p_0$. Since the investment of self-protection cannot be observed by insurers, the insurers provide a menu of insurance contracts (Q, I) to screen the individuals, where Q and I denote an insurance premium and indemnity, respectively, where $0 \leq I \leq D$.[12] We consider a competitive insurance market, in which no insurance company can make a positive profit.

Following Lee (2007), we adopt a two-argument utility function $U(C, A)$, where C and A indicate, respectively, the consumption of composite good and the benefit generated by the asset. $U(\cdot,\cdot)$ is assumed to be continuous, increasing, and concave in both arguments. Decomposing the wealth into an income and an asset is especially important in the sense that individuals are heterogeneous in asset sensitivity – sensitivity to the change in the asset value. Asset sensitivity is difficult to be expressed using a one-argument utility function because asset preference is not distinguishable from income preference. It is thus

[12] After purchasing an insurance contract (Q, I), the individual chooses whether or not to expend effort e. Since an insurer cannot observe the individual's action, he will never provide full or over insurance at the insurance premium rate of p_e. In this study, we do not allow over insurance, regardless of the unit price of insurance, as commonly assumed in the literature.

important to consider a two-argument utility function. In this paper, we define the degree of asset sensitivity as the marginal utility with respect to the asset, i.e., $U_A(C, A)$, where the subscript in the utility function denotes partial derivative such as $U_A(C, A) = \partial U(C, A)/\partial A$. Individuals with relatively greater $U_A(C, A)$ will suffer ceteris paribus a larger disutility when a loss occurs. Thus, we regard an individual as more asset sensitive if he/she has greater $U_A(C, A)$. Now, we assume that individuals can be divided into two types according to their asset sensitivity: the sensitive type (s) and the insensitive type (t). Then, $U_A^s(C, A) > U_A^t(C, A)$, for all (C, A) in our support, where superscripts in the utility function denote the types of individuals.[13] We do not impose any restriction on U_{CA}, the cross second derivative of utility. That is, U_{CA} can have any sign, but we assume that the sign of U_{CA} is the same across the types of individuals.

In a two-argument utility framework with the univariate insurable risk in asset, we can define the Arrow-Pratt concept of risk aversion with respect to the asset.[14] Then, type s with greater $U_A(C, A)$ can be less risk averse in terms of asset. Note that asset sensitive types are not necessarily less risk averse with respect to asset, because, by definition, we need $U_A^s(C, A) > U_A^t(C, A)$ to hold only for each (C, A) in our support. However, we can generally demonstrate the cases, in which those who are more asset sensitive are less risk averse with respect to asset. Even though the form of risk aversion in the present model is similar to that in the standard model, the implication of risk aversion can be quite different.

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[13] Our support is the set of (C, A), where $y - \overline{Q} \leq C \leq y$ and $w - D \leq A \leq w$. Note that $\overline{Q}$ denotes the maximum premium, i.e., $\overline{Q} = p_o D$.

[14] Note that the implication of risk aversion in a two-argument utility framework with bivariate risks can be more complicated, considering the correlation between the risks (see Courbage, 2001). In the present model, however, there exists uncertainty only in the asset, and we simply consider Arrow-Pratt concepts of relative and absolute risk aversion with respect to asset, which are represented by $-A\frac{U_{AA}}{U_A}$ and $-\frac{U_{AA}}{U_A}$, respectively.

We describe this in Section 4.

Now, we consider the individual's preferences towards income and asset. As we adopt a two-argument utility function, individuals will allocate their limited resources according to their preferences by means of insurance. This is because the insurance premium and indemnity each have an effect on different arguments of the utility function. In other words, the insurance premium decreases the income and indemnity is made against an insured asset if a loss occurs. Then, given insurance contract (Q, I), the individual of type i's preferences towards income and asset can be represented by the marginal rate of substitution between the insurance premium and indemnity, denoted by $\mathrm{MRS}^i(Q, I; p) \equiv \frac{pU_A^i(y-Q,w-D+I)}{(1-p)U_C^i(y-Q,w)+pU_C^i(y-Q,w-D+I)}$, where p is the probability of loss. $\mathrm{MRS}^i(Q, I; p)$ is a kind of MRS, which represents the preferences for an additional insurance coverage, because it captures the trade-offs between the benefit of indemnity and the cost of insurance premium.[15] As described later in this section, $\mathrm{MRS}^i(Q, I; p(\varepsilon))$ corresponds to the slope of an indifference curve of type i in the (Q, I) plane, when the insured expends effort of ε, for a given insurance contract (Q, I). However, as we do not put restrictions on U_{CA}, and U_C can differ across the types, the relationship between $\mathrm{MRS}^s(Q, I; p)$ and $\mathrm{MRS}^t(Q, I; p)$ is not clearly decisive. Thus, we further assume that $\frac{pU_A^s(y-Q,w-D+I)}{(1-p)U_C^s(y-Q,w)+pU_C^s(y-Q,w-D+I)} > \frac{pU_A^t(y-Q,w-D+I)}{(1-p)U_C^t(y-Q,w)+pU_C^t(y-Q,w-D+I)}$ for all (C, A) in our

[15] MRS refers to the marginal rate of substitution between two goods, x and y. Originally in economics, $\mathrm{MRS}(x, y) = U_x/U_y$, where $U_x = \frac{\partial U(x,y)}{\partial x}$ and $U_y = \frac{\partial U(x,y)}{\partial y}$. Similarly to the original MRS, $\mathrm{MRS}^i(Q, I; p)$ is the rate at which an individual can give up some amount of expected consumption of composite good by premium payments in exchange for the expected loss recovery in asset by receiving the indemnity, while maintaining the same level of expected utility. When full insurance is provided, $\mathrm{MRS}^i(Q, D; p) = p\frac{U_A^i(y-Q,w)}{U_C^i(y-Q,w)} = p \cdot \mathrm{MRS}(y - Q, w)$.

support.[16] What this assumption means is that, given the same insurance contract, type s is asset sensitive enough to value an additional insurance coverage more than type t, when each type chooses the same level of effort.

Then, let us describe the optimal level of effort for an individual of type i. For a given insurance contract (Q, I), if an individual expends effort $\varepsilon \in \{0, e\}$, the expected utility of type i can be represented by a two-argument utility function as follows:

$$EU^i(Q, I; \varepsilon) = p(\varepsilon)U^i(y - Q, w - D + I) + \big(1 - p(\varepsilon)\big)U^i(y - Q, w) - c(\varepsilon). \quad (1)$$

Let us assume that an individual's outside option provides no insurance. Moreover, no firms will provide the contracts with an inordinately favorable premium that generates an overall negative profit. Therefore, we only consider the acceptable insurance contracts from the perspective of both insurers and the insured. Because the demand for insurance depends largely on the preferences towards income and asset, we suppose that the insurers have sufficient contract space for the voluntary participation of the insured.[17] After purchasing the acceptable insurance contract (Q, I), the individual will decide whether or not to make an effort e to maximize her expected utility. The increase in expected utility of type i from the investment in self-protection is as follows:

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[16] Note that MRS is invariant to affine transformation. Given that $MRS^s(Q, I; p) > MRS^t(Q, I; p)$, we rule out the cases where utility function of type s and that of type t represent the same preference.

[17] For this to hold, we assume that an individual has small enough $U_C(C, A)$ relative to $U_A(C, A)$. This is because, if disutility from the decrease in income is too large after paying the insurance premium, the individual will not purchase the insurance contract. Since the optimal insurance coverage depends on MRS, full insurance is not always optimal even if the premium is actuarially fair. See the appendix for more details.

$$\begin{aligned}\Delta^{i}(Q,I) &= EU^{i}(Q,I;e) - EU^{i}(Q,I;0) \\ &= (p_e - p_0)[U^{i}(y-Q,w-D+I) - U^{i}(y-Q,w)] - c(e),\end{aligned} \tag{2}$$

An individual of type i will invest in self-protection if and only if $\Delta^{i}(Q,I) \geq 0$. We intend to demonstrate that there may be an asset sensitivity threshold where no investment occurs. Here we exclude full coverage because an individual with full insurance has no incentive to invest in self-protection. For each acceptable partial insurance contract (Q,I), it is obvious that $\Delta^{s}(Q,I) > \Delta^{t}(Q,I)$:

$$\begin{aligned}&U^{s}(y-Q,w) - U^{s}(y-Q,w-D+I) \\ &\qquad > U^{t}(y-Q,w) - U^{t}(y-Q,w-D+I).\end{aligned} \tag{3}$$

Equation (3) follows because $U_A^{s}(C,A) > U_A^{t}(C,A)$, for all (C,A). Therefore, when we consider appropriate risk reduction technology, there exists an asset sensitivity threshold below which investment in self-protection does not occur. For simplicity, let us assume that $\Delta^{t}(Q,I) < 0$ for all acceptable insurance contracts (Q,I), so that type t will never choose to invest in self-protection.[18]

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[18] As an extreme case, we will check the condition for $\Delta^{t}(Q,I) < 0$ to hold for all (Q,I). From Equation (2), an individual is more likely to invest in self-protection when he/she is provided with less indemnity. By contrast, the premium that increases the incentive to invest in self-protection depends on the sign of U_{CA}. To consider the insurance contract that gives the individual incentives to expend effort, one can refer to the next paragraph in the main body of our text, along with the footnote 8, for the details on the locus of (Q,I) such that $\Delta^{t}(Q,I) = 0$. Now consider the contract $(Q',0)$ that maximizes the incentive to invest in self-protection. If $U_{CA} \geq 0$, $Q' = 0$, and if $U_{CA} < 0$, $Q' = p_0 D$. Then the type t will never expend effort no matter what contract is given, if $\Delta^{t}(Q',0) < 0$. That is, $\Delta^{t}(Q',0) = (p_e - p_0)[U^{t}(y-Q',w-D) - U^{t}(y-Q',w)] - c(e) < 0$. The above inequality holds if U_A^{t} is small enough and risk reduction technology is less efficient. Then, $\Delta^{t}(Q,I) < 0$ for all (Q,I).

In the following section, we present the possible scenarios diagrammatically in the (Q, I) plane. In all of the figures, the x-axis represents the indemnity, whereas the y-axis signifies the insurance premium. Let us denote curve Z^i to be the locus of (Q, I) such that $\Delta^i(Q, I) = 0$. That is, $Z^i = \{(Q, I) | \Delta^i(Q, I) = 0\}$. However, we do not consider Z^t, because we assume that $\Delta^t(Q, I) < 0$. Now let us consider the shape of Z^s. First, if $U^s_{CA} > 0$, curve Z^s is a downward sloping locus that partitions the space into the lower region where type s invests in self-protection, and the upper region where he/she does not. Since we assume that $U^s_{CA} > 0$, it follows that $\frac{dQ}{dI} < 0$.[19] Secondly, if $U^s_{CA} = 0$, Z^s becomes the vertical straight line partitioning the space into the left region where type s expends effort, and the right region where he/she does not. Lastly, in the case of $U^s_{CA} < 0$, curve Z^s is an upward sloping locus in the (Q, I) plane. Type s invests in self-protection in the upper region of Z^s. Contrarily, he/she does not expend effort in the lower region of Z^s.

Indifference curves are drawn in (Q, I) space assuming that individuals choose optimal level of effort between 0 and e. Let us denote the indifference curves of type i as J^i, where $i = \{s, t\}$. The slope of J^i in the (Q, I) plane is given by

$$\frac{dQ}{dI} = \frac{p(\varepsilon)U^i_A(y_1, w_1)}{p(\varepsilon)U^i_C(y_1, w_1) + \big(1 - p(\varepsilon)\big)U^i_C(y_1, w_0)} = \frac{p(\varepsilon)U^i_A(y_1, w_1)}{E\big[U^i_C\big]} > 0, \qquad (4)$$

where $y_1 = y - Q$, $w_0 = w$ and $w_1 = w - D + I$. Thus the indifference curves are increasing, and we assume that they are concave in the (Q, I) space.[20] Notice that

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[19] The slope of Z in the (Q, I) space is given by: $\frac{dQ}{dI} = \frac{U^i_A(y-Q,w-D+I)}{U^i_C(y-Q,w-D+I)-U^i_C(y-Q,w)}$.

[20] It is easy to prove that the second derivative, i.e., $\frac{d^2Q}{dI^2}$, is negative if $U_{CA} \geq 0$. Thus, the

Equation (4) coincides with $\mathrm{MRS}^{\mathrm{i}}(\mathrm{Q},\mathrm{I};\mathrm{p}(\varepsilon))$. The probability of loss for type t is always p_0, because we assume that $\Delta^{\mathrm{t}}(\mathrm{Q},\mathrm{I})<0$ for each acceptable insurance contract (Q,I). For type s, however, if $\Delta^{\mathrm{s}}(\mathrm{Q},\mathrm{I})\geq 0$, he/she will expend effort, and J^{s} becomes flatter.[21] Thus, the indifference curves of type s are kinked where they cross Z^{s}. If $\mathrm{U}^{\mathrm{s}}_{\mathrm{CA}}>0$, the probability of loss is raised in the region above Z^{s}, so J^{s} is steeper. Likewise, if $\mathrm{U}^{\mathrm{s}}_{\mathrm{CA}}=0$, the indifference curves of type s are steeper in the right region of Z^{s}. Lastly, if $\mathrm{U}^{\mathrm{s}}_{\mathrm{CA}}<0$, J^{s} is steeper below Z^{s}.

Now we examine the condition for the single crossing property (SCP). In the region where type s invests in self-protection, the slope of J^{s} is:

$$\left.\frac{\mathrm{dQ}}{\mathrm{dI}}\right|_{\mathrm{J}^{\mathrm{s}},\Delta^{\mathrm{s}}(\mathrm{Q},\mathrm{I})>0}=\frac{\mathrm{p_e}\mathrm{U}^{\mathrm{s}}_{\mathrm{A}}(\mathrm{y}-\mathrm{Q},\mathrm{w}-\mathrm{D}+\mathrm{I})}{\mathrm{p_e}\mathrm{U}^{\mathrm{s}}_{\mathrm{C}}(\mathrm{y}-\mathrm{Q},\mathrm{w}-\mathrm{D}+\mathrm{I})+(1-\mathrm{p_e})\mathrm{U}^{\mathrm{s}}_{\mathrm{C}}(\mathrm{y}-\mathrm{Q},\mathrm{w})}.$$

Consider the case where J^{s} is steeper than J^{t} in the relevant region where $\Delta^{\mathrm{s}}(\mathrm{Q},\mathrm{I})\geq 0$:[22]

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indifference curves are increasing and concave if $\mathrm{U}_{\mathrm{CA}}\geq 0$. In the case of $\mathrm{U}_{\mathrm{CA}}<0$, the sign of $\frac{\mathrm{d}^2\mathrm{Q}}{\mathrm{dI}^2}$ is not determined. However, if the utility function is concave enough, or if the income and the asset are almost independent of each other, $\frac{\mathrm{d}^2\mathrm{Q}}{\mathrm{dI}^2}$ is negative. For simplicity, we assume that $\frac{\mathrm{d}^2\mathrm{Q}}{\mathrm{dI}^2}<0$ when $\mathrm{U}_{\mathrm{CA}}<0$, so that the indifference curves are increasing and concave, regardless of U_{CA}.

[21] It is easy to prove that $\frac{\mathrm{d}}{\mathrm{dp}}(\frac{\mathrm{dQ}}{\mathrm{dI}})>0$, regardless of the interaction between income and asset.

[22] The area of interest in the (Q,I) plane is the neighborhood of the zone where indifference curves with equilibrium contracts exist. Throughout this paper, we refer to this area as the relevant region.

$$\frac{p_e U_A^s(y-Q, w-D+I)}{p_e U_C^s(y-Q, w-D+I)+(1-p_e)U_C^s(y-Q, w)} > \frac{p_0 U_A^t(y-Q, w-D+I)}{p_0 U_C^t(y-Q, w-D+I)+(1-p_0)U_C^t(y-Q, w)}. \tag{5}$$

Equation (5) implies that even after type s expends effort and J^s becomes flat, he/she is asset sensitive enough to have much greater U_A relative to U_C than that of type t in the relevant region.[23] By contrast, in the region where $\Delta^s(Q, I) < 0$, the slope of J^s becomes steeper as the probability of loss increases; besides this, we assume that $MRS^s(Q, I; p_0) > MRS^t(Q, I; p_0)$. Therefore, if Equation (5) is satisfied, SCP will hold. Now, we consider the case in which J^s is flatter than J^t in the relevant region where $\Delta^s(Q, I) \geq 0$:

$$\frac{p_e U_A^s(y-Q, w-D+I)}{p_e U_C^s(y-Q, w-D+I)+(1-p_e)U_C^s(y-Q, w)} < \frac{p_0 U_A^t(y-Q, w-D+I)}{p_0 U_C^t(y-Q, w-D+I)+(1-p_0)U_C^t(y-Q, w)}.$$

Recall that $MRS^s(Q, I; p_0) > MRS^t(Q, I; p_0)$ in the region where $\Delta^s(Q, I) < 0$. Then, double crossing of indifference curves is obtained by combining the two parts of the region where $\Delta^s(Q, I) \geq 0$ and $\Delta^s(Q, I) < 0$. In summary, the double crossing property (DCP) holds if $MRS^s(Q, I; p_e) < MRS^t(Q, I; p_0)$ in the relevant region where $\Delta^s(Q, I) \geq 0$.

[23] Note that this condition is a weaker condition of SCP, as equation (5) should be satisfied in the relevant region, i.e., the indifference curves of type *s* and *t* cross only once, or double crossing does not apply only in the relevant region.

3.3 Market equilibrium

We consider the Nash equilibrium in the insurance market in which insurers are perfectly competitive, so that the equilibrium contracts break even. In this study, we adopt the equilibrium concept of Rothschild and Stiglitz (1976): (a) Each individual chooses at most one insurance contract from the provided menu that maximizes his/her expected utility; (b) each equilibrium contract makes nonnegative profit to an insurer; and (c) there is no other insurance contract that will make a nonnegative profit. An insurer assumes that other insurers do not change contracts after new contracts are offered. In a separating equilibrium, each type prefers his/her equilibrium contract to the contracts chosen by other types (incentive constraint). In a pooling equilibrium, by contrast, only the pooling contract is offered, and everybody chooses it. In a partial pooling equilibrium, at least some of each type of the insured purchase the pooling contract, and the rest of the insured purchase other contracts. In equilibrium, an individual purchases an insurance contract that is at least as good as no insurance (participation constraint). Moreover, an individual will invest in self-protection if the investment yields nonnegative expected utility (effort incentive). In this section, we will demonstrate that five Nash equilibrium configurations can occur in this market:

(i) a separating equilibrium with advantageous selection,
(ii) a separating equilibrium with adverse selection,
(iii) a separating equilibrium for a single premium rate,
(iv) a pooling equilibrium, and
(v) a partial pooling equilibrium.

In all the figures, the zero profit offer curves corresponding to the probability of loss

p_0 and p_e are depicted as P_0 and P_e, respectively. Moreover, $\overline{P}$ is the zero profit offer curve under the pooling contract with the probability of loss $\bar{p} = \theta p_e + (1-\theta)p_0$, where θ is the proportion of type s who invest in self-protection.

Now, let us consider the following contracts. First, consider the contract denoted by $C_A = (Q^A, I^A)$ that is located at the tangency of J^t and P_0. Then, C_A maximizes the type t's expected utility under the line P_0. Similarly, consider the contract denoted by $C_B = (Q^B, I^B)$ that maximizes the expected utility of type s under the line P_e.[24] Moreover, let us denote $C_D = (Q^D, I^D)$ and $C_{D'} = (Q^{D'}, I^{D'})$ as contracts that are located at the intersection of J^t passing through C_A and the line P_e, where $I^D > I^{D'}$. Then, the type t is indifferent about purchasing insurance contracts C_A, C_D, and C'_D. That is, $EU^t(Q^A, I^A; 0) = EU^t(Q^D, I^D; 0) = EU^t(Q^{D'}, I^{D'}; 0)$. Next, let contract $C_F = (Q^F, I^F)$ denote the intersection of Z^s and the indifference curve of type t that is tangent to the line P_0. Then, $EU^t(Q^A, I^A; 0) = EU^t(Q^F, I^F; 0)$ and $EU^s(Q^F, I^F; e) = EU^s(Q^F, I^F; 0)$. Moreover, let us denote $C_G = (Q^G, I^G)$ as the contract located at the intersection of $\overline{P}$ and Z^s. Then, $EU^s(Q^G, I^G; e) = EU^s(Q^G, I^G; 0)$. We also consider the indifference curve of type s that is tangent to the line P_0. Let us denote $C_H = (Q^H, I^H)$ as a contract that is located at the tangency of the indifference curve of type s and P_0.[25]

In addition, we denote C^{i*} as an equilibrium contract for type i. For example, if type

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[24] If $EU^s(Q^B, I^B; e) < EU^s(Q^B, I^B; 0)$, we consider the most preferable indifference curve of type s who invests in self-protection. Let the contract $(p_e I^{thresh}, I^{thresh})$ denote the intersection of Z^s and P_e, where $I^{thresh} < D$, and replace the contract C_B by $(p_e I^{thresh}, I^{thresh})$.

[25] Recall that we do not allow over insurance in this model. Therefore, if $I^H > D$, we consider the most preferable indifference curve of type s that cuts $p_0 I$ at point $(p_0 D, D)$, and replace the contract C_H by $(p_0 D, D)$. Note that $I^A < I^H$. This is because type s has more incentive to sacrifice income to recover a loss in an asset by means of insurance than that of type t, as we assume that $MRS^s(Q, I; p_0) > MRS^t(Q, I; p_0)$.

t chooses C_A in equilibrium, we denote the equilibrium contract for type t as C_A^{t*}. On the other hand, if both types choose the same contract C_G in equilibrium, we denote this contract by C_G^{p*}, where the superscript p denotes the pooling contract.

In the first place, we suppose that the SCP holds. In a separating equilibrium, if it exists, the following Proposition holds.

Proposition 1 [Separating equilibrium under SCP]

(A) If $EU^s(Q^D, I^D; e) \geq EU^s(Q^D, I^D; 0)$ and $EU^t(Q^D, I^D; 0) \geq EU^t(Q^B, I^B; 0)$, then there exists separating equilibrium with advantageous selection in which type t chooses C_A^{t*} and type s chooses C_B^{s*}. In this case, both types obtain their first-best contracts.

(B) Suppose that $EU^s(Q^D, I^D; e) > EU^s(Q, I; e)$, where the contract (Q, I) is on the pooling price line $\overline{P}$. If $EU^s(Q^D, I^D; e) \geq EU^s(Q^D, I^D; 0)$ and $EU^t(Q^D, I^D; 0) < EU^t(Q^B, I^B; 0)$, then, there exists separating equilibrium with advantageous selection in which type t chooses C_A^{t*} and type s chooses C_D^{s*}. In this case, type t obtains the first-best contract, whereas type s obtains the best fair contract such that type t has no incentive to buy this contract.

(C) Suppose that $EU^s(Q^H, I^H; 0) > EU^s(Q^{D'}, I^{D'}; e)$. Further, suppose that $EU^s(Q^H, I^H; 0) > EU^s(Q, I; e)$, where the contract (Q, I) is on the pooling price line $\overline{P}$. Then, there exists separating equilibrium for a single premium rate in which type t chooses C_A^{t*} and type s chooses C_H^{s*}. In this case, both types do not invest in self-protection in equilibrium.

Proof: See the Appendix. //

Unlike the standard model of Rothschild and Stiglitz (1976), Propositions 1 (A) and

(B) indicate that the sensitive type (low-risk) demands more insurance than the insensitive type (high-risk) under advantageous selection. In the case of Proposition 1 (A), both types obtain their first-best contracts that are the optimal contracts under full information. In this case, there is no inefficiency in the market, even though an insurer cannot observe the individual's action. In the case of Proposition 1 (B), by contrast, type t obtains her first-best contract under P_0, whereas type s can only obtain the best fair contract that is not her first-best. This is because insurers should offer higher coverage (C_D) to type s than her optimal level (C_B) to screen the individuals.

The key that causes a difference between Propositions 1 (A) and (B) is $MRS^s(Q, I; p_e)$ which represents the type s's preference towards income and asset, after investing in self-protection. From Equation (2), type s is asset sensitive enough so that he/she invests in self-protection. In other words, the optimal choice of effort for type i largely depends on U^i_A. However, the optimal choice of insurance contract for type i depends on $MRS^i(Q, I; p(\varepsilon))$. That is, it depends not only on U^i_A but also on U^i_C (see the appendix). In the case of Proposition 1 (A), type s has high enough $MRS^s\left(Q^B, I^B; p_e\right)$ so that he/she expends effort while demanding much higher coverage than type t. On the other hand, in the case of Proposition 1 (B), type s is asset sensitive enough to invest in self-protection, but he/she does not have sufficiently high $MRS^s\left(Q^B, I^B; p_e\right)$ to be separated from type t.

In the case of Proposition 1 (C), as type s cannot be separated from type t while expending effort, he/she refuses to invest in self-protection and maximizes his/her expected utility by choosing C^{s*}_H. In this case, both types do not invest in self-protection in equilibrium, but type s purchases higher coverage than type t. It is clear because we assume that $MRS^s(Q, I; p_0) > MRS^t(Q, I; p_0)$. Therefore, the equilibrium contracts consist of two insurance contracts with the same premium rate, p_0, but different insurance coverage. Notice that the separating equilibrium for a single premium rate indicates the separation of the type of asset-sensitivity in equilibrium, not the separation of risk type. This separating equilibrium is consistent with Proposition 2 of Huang et al. (2010).

Now, we consider the case of pooling equilibrium. The results are summarized in Proposition 2.

Proposition 2 [Pooling equilibrium under SCP]

(A) Suppose that the slope of J^s at C_G is steeper than the unit price of pooling contract, $\bar{p}$. If $EU^s(Q^G, I^G; e) > EU^s(Q^H, I^H; 0)$, $EU^s(Q^G, I^G; e) > EU^s(Q^{D'}, I^{D'}; e)$ and $EU^t(Q^G, I^G; 0) > EU^t(Q^A, I^A; 0)$, there exists pooling equilibrium in which both types choose the pooling contract C_G^{p*}.

(B) Suppose that $EU^s(Q^F, I^F; e) > EU^s(Q, I; e)$, where the contract (Q, I) lies on the pooling price line $\bar{P}$. If $EU^s(Q^F, I^F; e) > EU^s(Q^{D'}, I^{D'}; e)$ and $EU^s(Q^F, I^F; e) > EU^s(Q^H, I^H; 0)$, then there exists partial pooling equilibrium in which type t chooses C_A^{t*} and type s chooses C_F^{s*}.

Proof: See the Appendix. //

Contrary to the seminal work of Rothschild and Stiglitz (1976), Propositions 2 (A) and (B) indicate that pooling equilibrium can exist. In these cases, an actuarially unfair insurance contract is provided to type s. In a pooling equilibrium demonstrated in Proposition 2 (A), all risk types choose the same insurance contract C_G^{p*}. Since risk type is endogenously determined by individuals, the area of interest for existence of a pooling contract is where $\Delta^s(Q, I) \geq 0$. As illustrated in Figure 4, the only possible pooling contract is a particular point (C_G^{p*}), i.e., the point of intersection between the pooling price line $\bar{P}$ and Z^s. Therefore, unlike the results of the conventional model, a pooling contract can remain stable under some conditions described in Proposition 2 (A). In a pooling equilibrium, the sensitive type of the insured (low-risk) subsidizes the insensitive type of the insured (high risk).

In a partial pooling equilibrium described in Proposition 2 (B), there is a potential for a positive profit in the competitive market. This is because type t is indifferent between contracts C_A^{t*} and C_F^{s*}. As De Meza and Webb (2001) pointed out, a chance of a positive profit is an artifact of the discontinuous choice of effort in the model. Suppose that only some of type t choose to purchase C_F^{s*} and the others purchase C_A^{t*}, so that C_F^{s*} generates zero profit. Then, a partial pooling equilibrium can be maintained in the competitive market in which no firms can earn a positive profit. In this case, the sensitive type of the insured (low-risk) provides a subsidy only to the insensitive type of the insured (high-risk) who chooses C_F^{s*}. This partial pooling equilibrium is consistent with the results of a partial pooling equilibrium of De Meza and Webb (2001) and Huang et al. (2010).

In the second place, we suppose that double crossing property (DCP) holds, i.e., $\frac{p_e U_A^s(y-Q,w-D+I)}{p_e U_C^s(y-Q,w-D+I)+(1-p_e)U_C^s(y-Q,w)} < \frac{p_0 U_A^t(y-Q,w-D+I)}{p_0 U_C^t(y-Q,w-D+I)+(1-p_0)U_C^t(y-Q,w)}$ in the relevant region of the contract space. The following proposition indicates the possible cases in which a separating equilibrium occurs in the market.

Proposition 3 [Separating equilibrium under DCP]

(A) Suppose that $EU^s\left(Q^{D'}, I^{D'}; e\right) > EU^s(Q^H, I^H; 0)$. Further, suppose that $EU^s(Q^{D'}, I^{D'}; e) > EU^s(Q, I; e)$, where the contract (Q, I) lies on the pooling price line $\overline{P}$. Then, there exists separating equilibrium with adverse selection in which type t chooses C_A^{t*} and type s chooses $C_{D'}^{s*}$.

(B) Suppose that $EU^s(Q^H, I^H; 0) > EU^s(Q^{D'}, I^{D'}; e)$. Further, suppose that $EU^s(Q^H, I^H; 0) > EU^s(Q, I; e)$, where the contract (Q, I) lies on the pooling price line $\overline{P}$. Then, there exists separating equilibrium for a single premium rate in which type t chooses contract C_A^{t*} and type s chooses C_H^{s*}. In this case, both types do not invest in self-protection in equilibrium.

Proof: See the Appendix. //

If DCP holds in the relevant region, adverse selection may occur in the market, as depicted in Figure 6. In this case, risk and coverage are positively correlated. Another possible equilibrium is the separating equilibrium for a single premium rate, as demonstrated in Proposition 3 (B). The equilibrium result is similar to Proposition 1 (C). That is, both types do not invest in self-protection, but type s purchases higher coverage than type t in the equilibrium. The only difference between Proposition 1 (C) and Proposition 3 (B) is whether the SCP holds or not. In Proposition 3 (B), the degree of asset sensitivity for type s is not that high, but he/she has large enough $MRS^s(Q, I; p_0)$, so that he/she obtains higher expected utility by choosing C_H^{s*}, rather than any other contracts located in the region where $\Delta^s(Q, I) \geq 0$. Again, the type of asset sensitivity, not the risk type, is revealed in the separating equilibrium for a single premium rate.

Now, we will demonstrate that a pooling equilibrium is ruled out when DCP holds, as described in Proposition 4.

Proposition 4 [No pooling equilibrium under DCP]

If double crossing property holds, pooling equilibrium cannot exist.

Proof: See the Appendix. //

In this paper, the individuals' preferences towards income and asset are the main key determining the type of equilibrium. Additionally, the proportion of the sensitive type in the market (θ) is another factor that affects the existence of equilibrium. If θ is not sufficiently low, the separating equilibrium may not exist, as we have discussed in Propositions 1 and 3. According to this proportion, there may also exist a pooling equilibrium. In particular, we have the interval of value θ satisfying the following inequality for a complete risk pooling to exist:

$$\frac{p_e U_A^s(y - Q^G, w - D + I^G)}{p_e U_C^s(y - Q^G, w - D + I^G) + (1 - p_e) U_C^s(y - Q^G, w)} > \bar{p} = \theta p_e + (1 - \theta) p_0.$$

In conclusion, when we consider some appropriate utility functions of type i and the proper risk reduction technology, the proportion of the sensitive type in the market plays an important role in determining the existence of equilibrium.

3.4 Discussion

In a one-argument utility framework, full insurance is optimal if it is actuarially fair (Mossin, 1968; Smith, 1968). However, in a two-argument utility framework, a fair premium is neither necessary nor sufficient for optimality of full insurance (Lee, 2007). In this paper, optimal level of insurance coverage depends on the individual's preference towards income and asset, which is represented by MRS between insurance premium and indemnity. Therefore, the degree of asset sensitivity also affects the optimality of insurance contracts. Note that the equilibrium contract for type s obtained in Proposition 1 (A) is partial coverage insurance such that the marginal expected benefit of an additional coverage equals its marginal expected cost.

Unlike the standard model of Rothschild and Stiglitz (1976), when individuals differ with respect to both loss probability and degree of risk aversion, it is well recognized that more risk-averse individuals may purchase more insurance in the market with asymmetric information.[26] For example, Smart (2000) and Wambach (2000) demonstrated that at least for one risk group, more risk-averse individuals purchase more insurance than less risk-averse individuals in the same risk group. The key to these results is that high-risk types with higher risk aversion are less distracted by a partial insurance contract that is offered to the low-risk types than high-risk types with lower risk aversion. In the model of de Meza and Webb (2001), by contrast, the risk type is not exogenously given but is determined by the degree of risk aversion. They demonstrated that those with lower risk aversion buy less insurance and take fewer precautions than those with higher risk aversion. The mechanism of this equilibrium is that less risk-averse individuals can tolerate higher uncertainty than more risk-averse individuals.

From a different perspective, as we utilize a two-argument utility function, we can define risk aversion with respect to asset. As mentioned in Section 2, type s can be less risk averse in terms of asset. Lower risk aversion does not necessarily imply that this type of individual can tolerate higher uncertainty. Rather, type s with lower risk aversion with respect to asset is more likely to invest in self-protection. In addition, we can easily demonstrate that in separating equilibrium with advantageous selection, type s with lower risk aversion invests in self-protection and purchases more insurance than type t with higher risk aversion who never expends effort (see, for example, Kim and Seog, 2019). This result is perhaps surprising given that it is opposite to that of the standard model. Note that the

[26] In a one-argument utility framework, the utility function depends only on wealth. Then, the coefficient of absolute risk aversion is defined by $\mathrm{A(w)} = -\frac{\mathrm{u''(w)}}{\mathrm{u'(w)}}$, where $\mathrm{u'(w)}$ and $\mathrm{u''(w)}$ denote the first and second derivatives with respect to w of $\mathrm{u(w)}$.

key to this result is not the risk aversion but the degree of asset sensitivity. However, the degree of asset sensitivity and risk aversion with respect to asset are both relevant to the shape of a two-argument utility function associated with an asset. Moreover, we can generally demonstrate the cases, in which those who are more asset sensitive are less risk averse with respect to asset. Our finding emphasizes the fact that less risk averse individuals can expend more efforts and purchase more insurance than those with higher risk aversion; this implies that the present model considers the characteristics of utility, which cannot be captured by the standard model.

Other explanations about advantageous selection explored in the literature are cognitive biases. Huang et al. (2010) and Spinnewijn (2013) modeled the individual's decision making as if the individual were trying to maximize a perceived expected utility function that incorporates risk perception bias, when the true expected utility function is that of an unbiased individual. By contrast, the first-best decision is the optimal level of effort and insurance coverage that maximize the true expected utility. Therefore, in equilibrium, welfare loss arises not only because the insurer cannot observe the individual's action but also because the optimistic individual believes that his subjective loss probability is lower than his objective loss probability. In other words, a full information contract does not maximize the true expected utility of an optimistic individual. Huang et al. (2010) argued that in the first case of Proposition 1, an optimistic individual does not impose any negative externality on an unbiased individual. However, this outcome is clearly not first-best. By contrast, in Proposition 1 (A), we show that both types obtain their first-best contracts and that there is no welfare loss even though an insurer cannot observe the individual's action. The present model is thus differentiated from those of Huang et al. (2010) and Spinnewijn (2013) in that we provide our results under full rationality.

In the seminal work of Rothschild and Stiglitz (1976), pooling equilibrium cannot exist. In practice, however, pooling contracts are prevalent, notably in health insurance. Moreover, in group life insurance, a single contract covers an entire group of people who differ in their risks. Under the alternative equilibrium concept of Wilson (1977), however, pooling equilibrium can occur in a competitive insurance market.

Even under the Rothschild-Stiglitz conjecture, however, we proved that pooling equilibrium can exist, as demonstrated in Proposition 2 (A). Wambach (2000) also demonstrated that as a very special case, a complete risk pooling can occur. However, he introduced four types of individuals and defined a complete risk pooling as a situation in which more than one risk type chooses one specific contract. That is, not all of the individuals in the economy, but three types of individuals out of four types, purchase a particular contract. In this study, by contrast, there are only two types of individuals, and in a pooling equilibrium, all individuals in the economy choose the same contract. Even under Rothschild-Stiglitz conjecture, this is a new finding that demonstrates the existence of a complete risk pooling in the competitive insurance market, in which two different types of the insured exist.

3.5 Conclusion

According to the asset sensitivity, people have different preferences towards income and asset. This paper develops an endogenous selection model under asymmetric information and investigates how insurers screen individuals who differ in asset sensitivity. The analysis indicated that in equilibrium, the asset sensitive type may invest in self-protection and become a low-risk, whereas the insensitive type never chooses to expend effort. Unlike the standard model of Rothschild and Stiglitz (1976), we demonstrated that sensitive type (low-risk) purchases higher insurance coverage than insensitive type (high-risk) under advantageous selection. We also find other types of equilibrium including adverse selection, single premium rate, and pooling equilibrium.

These results are partially analogous to those of De Meza and Webb (2001), Huang et al. (2010), and Spinnewijn (2013). However, in contrast to all previous papers, we propose a model reflecting the reality, which captures the trade-offs that individuals make between income and asset. Unlike de Meza and Webb (2001), we consider the heterogeneity in preferences by utilizing a two-argument utility function that depends on income and asset. This realistic modification of the utility function is the foundation of a rationale for the existence of advantageous selection that the sensitive type may demand more insurance than the insensitive type while expending effort. In addition, we provide equilibrium results under full rationality, while Huang et al. (2010) and Spinnewijn (2013) introduce irrational (optimistic) individuals into their models. Furthermore, our model even shows that a complete risk pooling can occur in the market.

Moreover, this paper can provide a different theoretical foundation for future empirical studies investigating the relationship between risk and insurance demand. Heterogeneity in asset sensitivity can also explain the mixed results in empirical papers that find positive, negative, or even no correlation between risk and insurance coverage in some insurance markets. Furthermore, understanding the heterogeneity in preferences is important in that an insurer can design the insurance contract accordingly, and our analysis can therefore contribute to better underwriting performance.

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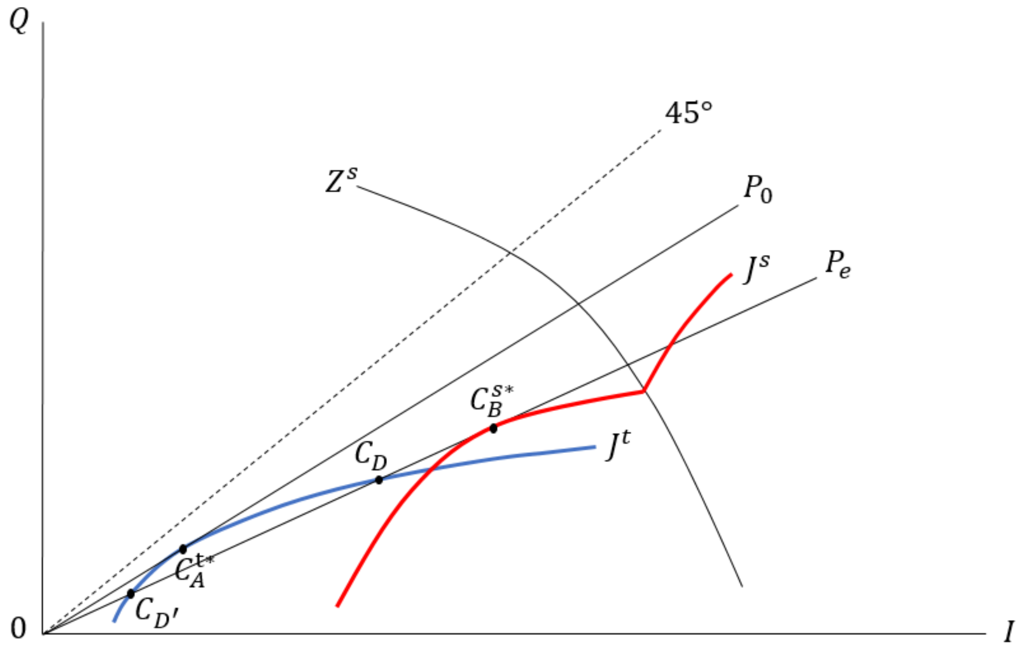

Figure 1. Separating equilibrium with advantageous selection (type t prefers contract $\mathrm{C_D}$ to $\mathrm{C_B^{s*}}$), when $\mathrm{U_{CA}} > 0$

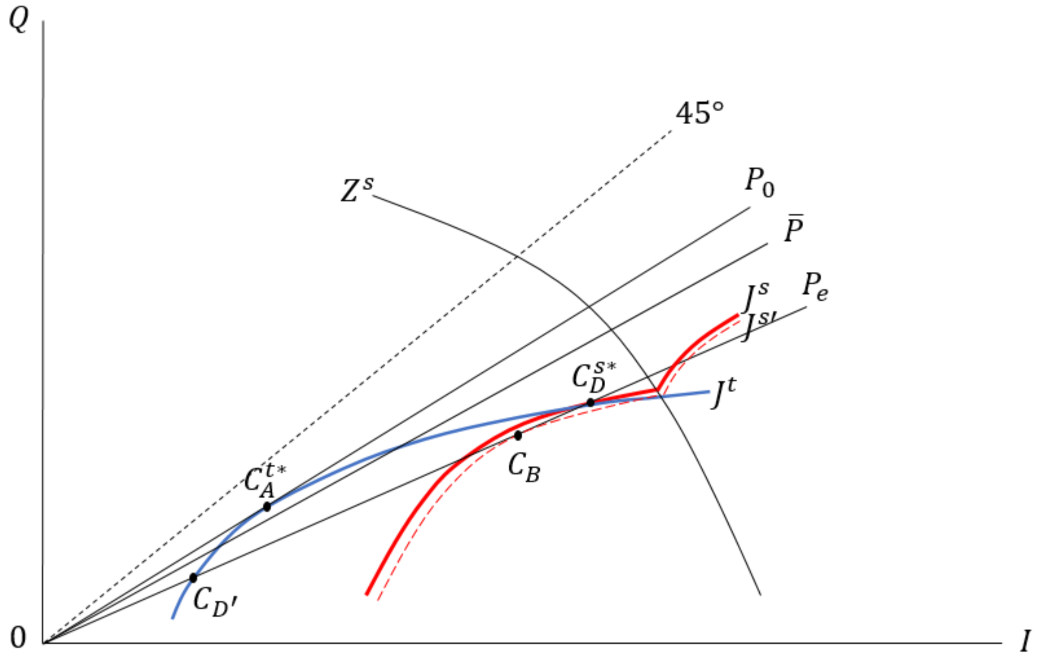

Figure 2. Separating equilibrium with advantageous selection (type t prefers contract $\mathrm{C_B}$ to $\mathrm{C_D^{s*}}$), when $\mathrm{U_{CA}} > 0$

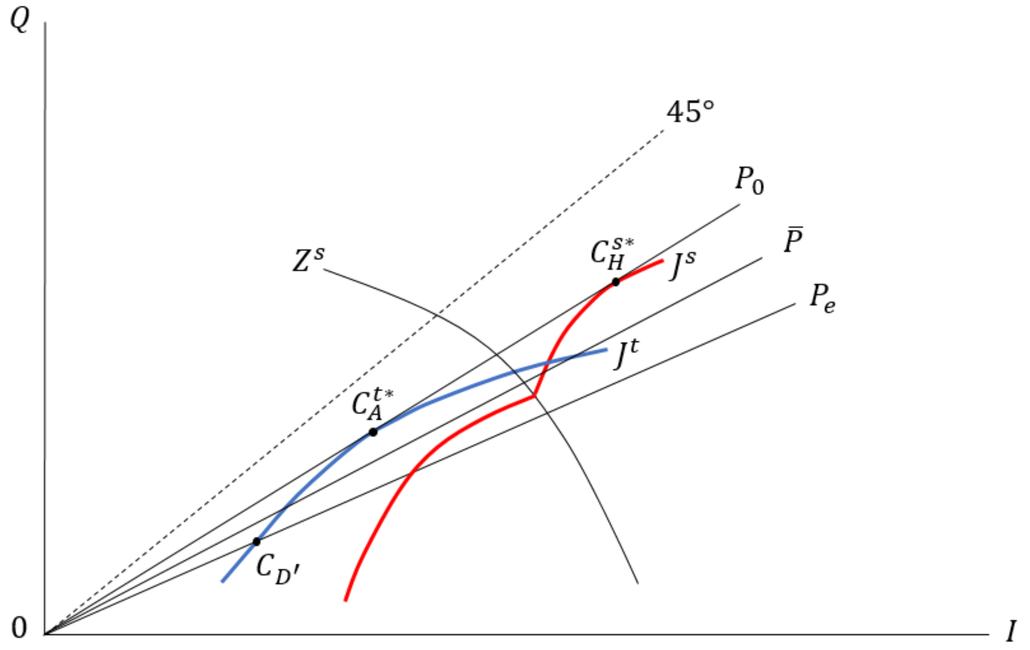

Figure 3. Separating equilibrium for a single premium rate under the single crossing property (SCP), when $U_{CA} > 0$

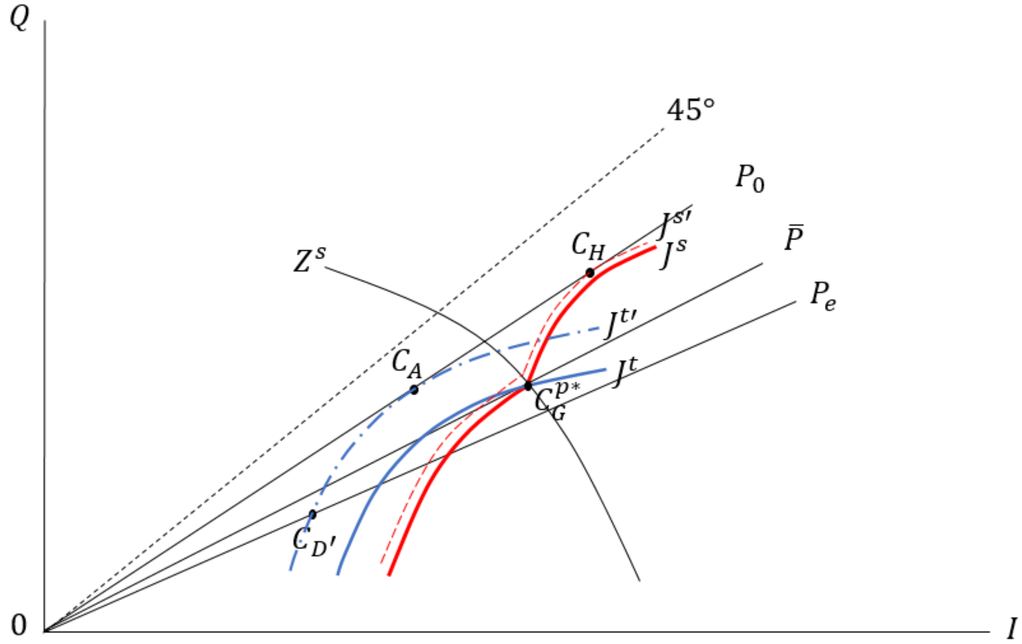

Figure 4. Pooling equilibrium under SCP, when $U_{CA} > 0$

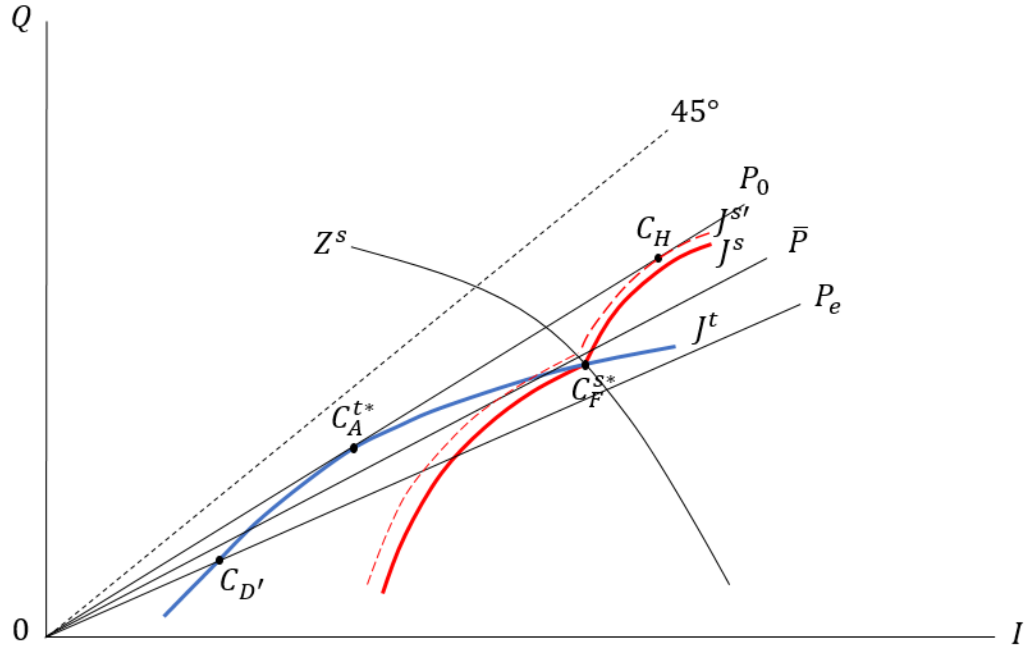

Figure 5. Partial pooling equilibrium under SCP, when $U_{CA} > 0$

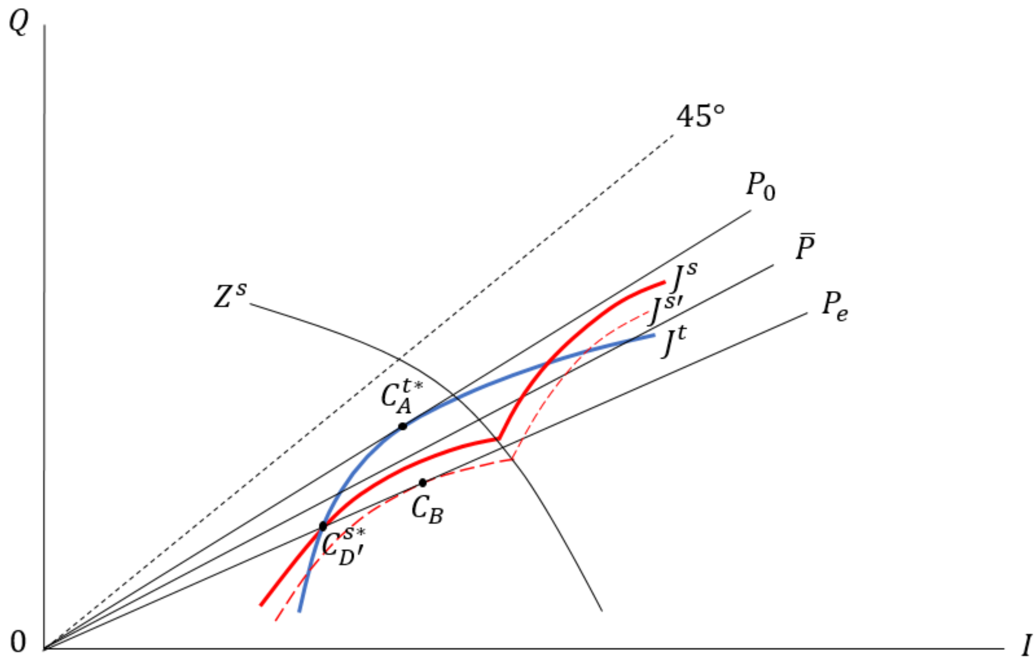

Figure 6. Adverse selection under the double crossing property (DCP), when $U_{CA} > 0$

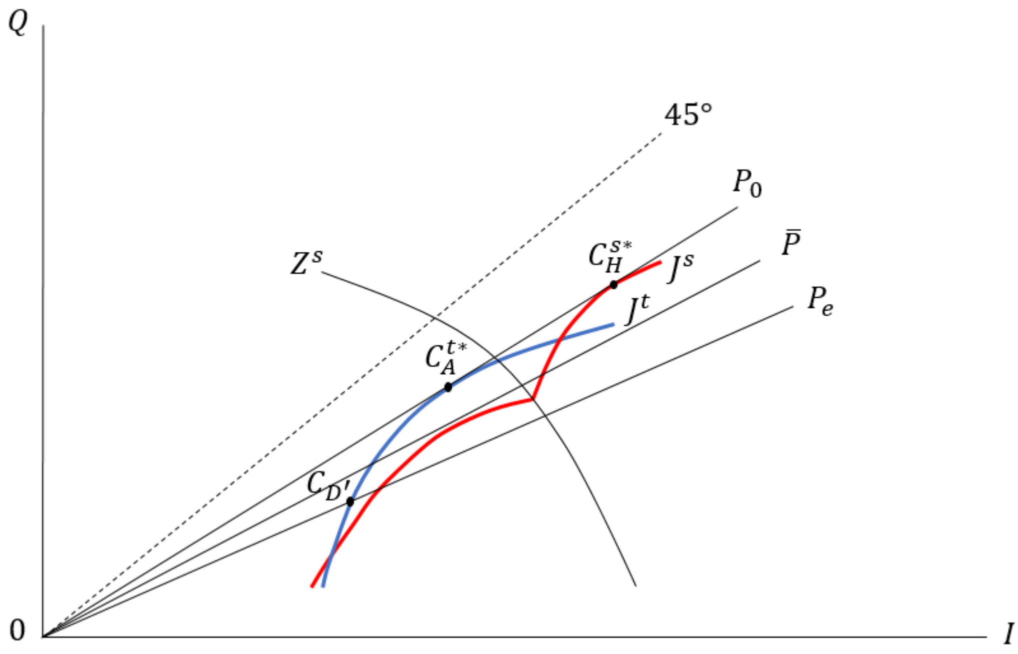

Figure 7. Separating equilibrium for a single premium rate under DCP, when $U_{CA} > 0$

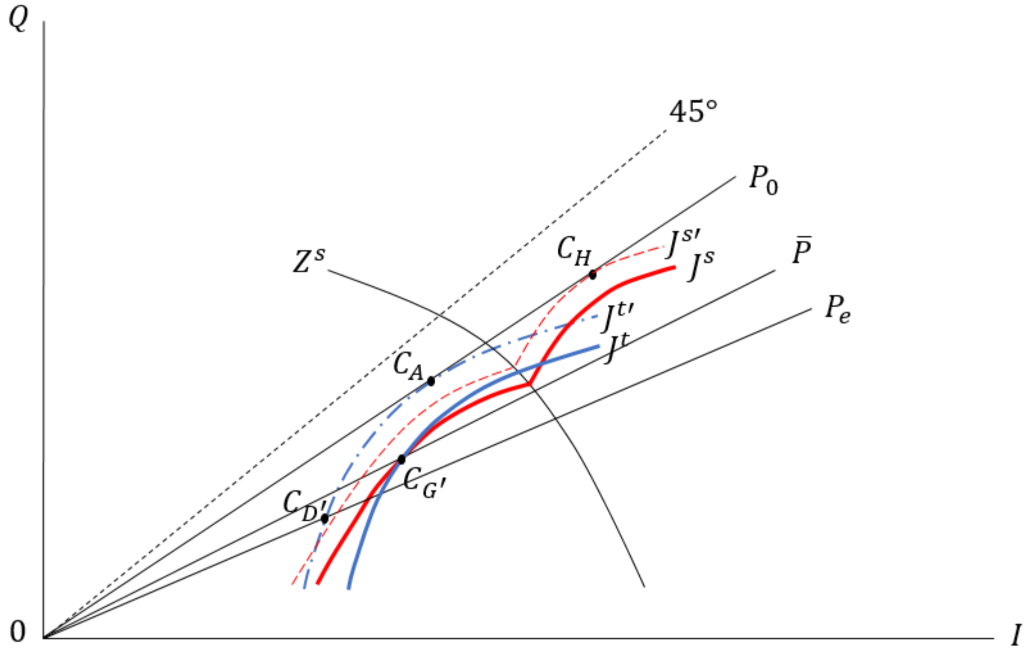

Figure 8. No pooling equilibrium under DCP, when $\mathrm{U_{CA}} > 0$

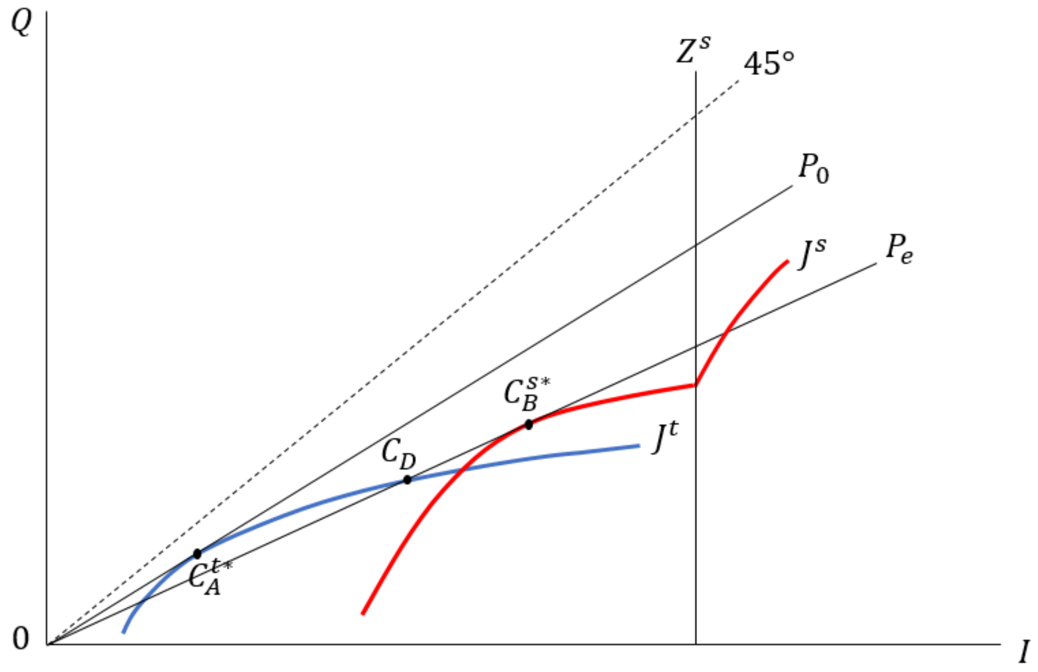

Figure A.1. Separating equilibrium with advantageous selection (type t prefers contract $\mathrm{C_D}$ to $\mathrm{C_B^{s*}}$), when $\mathrm{U_{CA}} = 0$

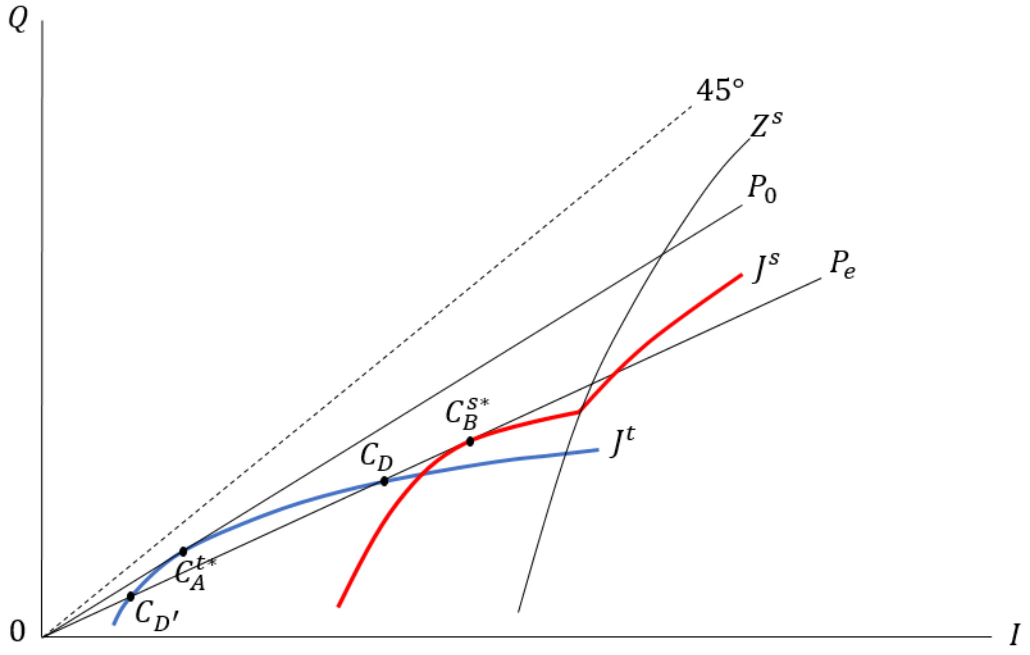

Figure A.2. Separating equilibrium with advantageous selection (type t prefers contract C_D to C_B^{s*}), when $U_{CA} < 0$

Appendix

1. Proof of Proposition 1

These propositions are depicted in Figures 1, 2, and 3, respectively. As we do not impose any limitations on the interaction between an income and an asset, we can consider all possible scenarios of equilibrium according to U_{CA}. One of the differences between the cases is the slope of Z^s, but the results of the equilibrium are similar. As we will see later in this section, the market equilibrium depends largely on asset sensitivity and $MRS^i(Q, I; p(\varepsilon))$, regardless of U_{CA}. Therefore, without loss of generality, we mainly discuss the cases in which $U_{CA} > 0$. Only for Proposition 1 (A), all the cases of $U_{CA} > 0$, $U_{CA} = 0$, and $U_{CA} < 0$ are illustrated by diagrams in Figures 1, A.1, and A.2, respectively.

(A) From Figure 1, which demonstrates Proposition 1 (A), it is trivial that a separating equilibrium exists (C_A^*, C_B^*). Suppose that an insurer offers a contract C_B' below P_e to attract type s only. Then C_B' makes a negative profit, and no insurers will provide such contract. Note that curve Z^s is downward sloping when $U_{CA} > 0$, as depicted in Figure 1. In Figure A.1, curve Z^s is vertical when $U_{CA} = 0$. Lastly, curve Z^s is upward sloping when $U_{CA} < 0$, as illustrated in Figure A.2. In all cases, however, the results are similar and the equilibrium occurs at (C_A^*, C_B^*). //

(B) In Figure 2, which describes Proposition 1 (B), the only way to attract type s who invests in self-protection is to provide a contract located below J^s where $\Delta^s(Q, I) \geq 0$ and above P_e. However, since this contract lies below $\overline{P}$, it cannot be an equilibrium contract that makes nonnegative profits for insurers. //

(C) Figure 3 illustrates Proposition 1 (C). To attract type s, an insurer should offer a contract that is located below J^s and above P_e. A contract located below J^s and above Z^s cannot induce type s to invest in self-protection. Therefore, the insurers will not deviate to offer a new contract, because P_0 lies above this area. By contrast, a contract located below J^s where $\Delta^s(Q, I) \geq 0$ and above P_e will attract both types of individuals and generate a negative profit. This is because the pooling price line $\overline{P}$ is located above this contract. //

2. Proof of Proposition 2

(A) In Figure 4, which demonstrates Proposition 2 (A), the pooling equilibrium lies at the intersection of the curve Z^s and the pooling price line $\overline{P}$. As we suppose that $\frac{p_e U_A^s(y-Q^G, w-D+I^G)}{p_e U_C^s(y-Q^G, w-D+I^G)+(1-p_e)U_C^s(y-Q^G, w)} > \bar{p}$, type s does not prefer any other pooling contract to C_G while expending effort. To attract type s, an insurer should offer a contract located below J^s and above P_e. However, any deviating contract located below J^s where $\Delta^s(Q, I) \geq 0$ and above P_e will also attract type t and make a negative profit. This is because $\overline{P}$ is located above this contract. //

(B) Figure 5 illustrates Proposition 2 (B). The only way to attract type s is to provide contracts located below J^s and above P_e. However, such a contract located in the area where $\Delta^s(Q, I) \geq 0$ attracts not only type s but also type t. Since this offer lies below the pooling price line $\overline{P}$, it is unprofitable, and thus cannot be an equilibrium contract. //

3. Proof of Proposition 3

(A) Figure 6 illustrates Proposition 3 (A). The only way to attract type s is to provide a contract located below J^s and above P_e. However, such a contract located in the area where $\Delta^s(Q, I) \geq 0$ is preferred by both type s and type t. Then, the insurer will earn a negative profit, because $\overline{P}$ is located above this contract. //

(B) Figure 7 illustrates Proposition 3 (B). The proof is similar to the proof of Proposition 1 (C). //

4. Proof of Proposition 4

Figure 8 illustrates Proposition 4. Suppose on the contrary that there exists a pooling contract $C_{G'}$ when DCP holds. To induce type s to expend effort, the pooling contract $C_{G'}$ should lie on the line $\overline{P}$ where $\Delta^s(Q, I) \geq 0$. That is, $EU^s(Q^{G'}, I^{G'}; e) \geq EU^s(Q^{G'}, I^{G'}; 0)$. In this region, the slope of the indifference curve of type t is steeper than that of type s by assumption. Then an insurer will deviate to offer a new contract located in the region below J^s where $\Delta^s(Q, I) \geq 0$, and surrounded by P_e and J^t. Since this contract lies above the line P_e and attracts type s only, it generates a positive profit. Therefore, a pooling equilibrium cannot occur when the DCP holds. //

5. Proof of Footnote 17

Insurers provide an insurance contract (Q, I), where $Q = p'\alpha D$, $I = \alpha D$, $0 \leq \alpha \leq 1$, and p' is the unit price of insurance. An individual will choose the efficient level of effort and insurance coverage to maximize his/her expected utility. Since the individual chooses whether to make an effort or not, she will compare the maximum expected utilities given $\varepsilon \in \{0, e\}$. Let us first consider the following maximization problem given an effort level of ε:

$$\mathrm{EU}\big(\alpha^*(\varepsilon)\big) \equiv \mathrm{Max}_{\alpha}\mathrm{EU}(\varepsilon)$$
$$= p(\varepsilon)U(y - Q, w - D + I) + \big(1 - p(\varepsilon)\big)U(y - Q, w) - c(\varepsilon), \quad \text{(A.1)}$$

subject to $Q = p'\alpha D$ and $I = \alpha D$.

Solving (A.1), the first-order condition is

$$p(\varepsilon)[U_C(y_1, w_1)(-p'D) + U_A(y_1, w_1)D] + \big(1 - p(\varepsilon)\big)U_C(y_1, w_0)(-p'D) = 0. \quad \text{(A.2)}$$

Then, the final solution would be $\{\alpha^*(\varepsilon), \varepsilon\}$ such that $\mathrm{EU}(\alpha^*(\varepsilon)) = \mathrm{Max}\{\mathrm{EU}(\alpha^*(0)), \mathrm{EU}(\alpha^*(e))\}$. Equation (A.2) implies that the optimal insurance coverage is determined such that $\mathrm{MRS}(Q, I; p(\varepsilon))$ equals the unit price of insurance, p'. Therefore, if $\mathrm{MRS}\big(0, 0; p(\varepsilon)\big) = \frac{p(\varepsilon)U_A(y,w-D)}{p(\varepsilon)U_C(y,w-D)+\big(1-p(\varepsilon)\big)U_C(y,w)} < p'$, an individual will exit the insurance market. As we are interested in the equilibrium contracts of each type, we want to exclude the obvious cases where one or all types of the insured decide to exit the market. Thus, we assume that the individuals have small enough $U_C(C, A)$ relative to $U_A(C, A)$ to guarantee the existence of sufficient contract space for the voluntary participation of the insured. //

6. Proof of Footnote 20

Consider the curvature of the indifference curve of type i:

$$\frac{d^2Q}{dI^2} = \frac{1}{\{E[U_C^i]\}^2}\left\{p(\varepsilon)\left[U_{CA}^i(y_1, w_1)\left(-\frac{dQ}{dI}\right) + U_{AA}^i(y_1, w_1)\right] \cdot E[U_C^i] - p(\varepsilon)U_A^i(y_1, w_1)\left[p(\varepsilon)\left[U_{CC}^i(y_1, w_1)\left(-\frac{dQ}{dI}\right) + U_{CA}^i(y_1, w_1)\right] + (1 - p(\varepsilon))U_{CC}^i(y_1, w_0)\left(-\frac{dQ}{dI}\right)\right]\right\}. \tag{A.3}$$

The sign of (A.3) coincides with that of the numerator. It is easy to prove that (A.3) is negative if $U_{CA} \geq 0$; thus J^i is increasing and concave. Now consider the case of $U_{CA} < 0$. The sign of (A.3) can be positive, negative or zero, and can be changed according to the given insurance contracts. We rewrite the numerator of $\frac{d^2Q}{dI^2}$ as

$$p(\varepsilon)\left[p(\varepsilon)U_A^i(y_1, w_1)\left\{-2U_{CA}^i + E[U_{CC}^i] \cdot \frac{U_A^i(y_1, w_1)}{E[U_c^i]}\right\} + U_{AA}^i(y_1, w_1)E[U_C^i]\right]. \tag{A.4}$$

The indifference curve in the (Q, I) plane is concave if (A.4) is negative, and its sign depends on the shape of the utility function. Note that the only positive term inside the bracket in (A.4) is $-2U_{CA}^i$. If the utility function is concave enough, i.e., $|U_{CC}^i|$ or $|U_{AA}^i|$ are large enough, (A.4) is negative. In addition, if income and insurable asset are almost independent of each other, that is, the absolute value of U_{CA} is small enough, indifference curves are concave. For simplicity, we assume that the indifference curves of both types are increasing and concave, regardless of the interaction between income and insurable asset. //

7. Existence of advantageous selection in Proposition 1 (A)

Consider the optimal expected utilities of type t and type s in Proposition 1 (A), as follows:

$$J^t(C_A) = p_0 U(y - p_0 I^A, w - D + I^A) + (1 - p_0) U(y - p_0 I^A, w),$$

$$J^s(C_B) = p_e U(y - p_e I^B, w - D + I^B) + (1 - p_e) U(y - p_e I^B, w) - c(e).$$

Note that type t is indifferent between choosing C_A and C_D, even though C_D is actuarially favorable for type t. In other words, type t cannot obtain higher expected utility than $EU^t(Q^A, I^A; 0)$ by choosing C_D, because he gives up too much of expected consumption of composite good in exchange for the higher expected loss recovery.

Now, we intend to show that $I^D < I^B$ such that $MRS^s(Q^B, I^B; p_e) = p_e$ and

$$\begin{aligned} EU^t(Q^D, I^D; 0) &= p_0 U(y - p_e I^D, w - D + I^D) + (1 - p_0) U(y - p_e I^D, w) \\ &> EU^t(Q^B, I^B; 0) \\ &= p_0 U(y - p_e I^B, w - D + I^B) + (1 - p_0) U(y - p_e I^B, w). \end{aligned}$$

If $I^D < I^B$, it is clear that $EU^t(Q^D, I^D; 0) > EU^t(Q^B, I^B; 0)$, because $MRS^t(Q^D, I^D; p_0) < p_e$. Thus, it suffices to show that type s demands higher insurance coverage (I^B) than I^D, while expending effort. If the degree of asset-sensitivity of type s is sufficiently large so that $\Delta^s(Q^B, I^B) \geq 0$, he/she will invest in self-protection, given insurance contract C_B. At the same time, if $MRS^t(Q^B, I^B; p_0)$ is much smaller than $MRS^s(Q^B, I^B; p_e) = p_e$, we can obtain $I^D < I^B$.

In summary, if not only U_A^s is large but also the difference between $MRS^t(Q, I; p_0)$ and $MRS^s(Q, I; p_e)$ is large enough, advantageous selection can occur in the insurance market in which both types obtain their first-best contracts. Notice that if we consider some appropriate utility functions of type i, all the assumptions and conditions in Proposition 1 (A) can be satisfied with a proper proportion of type s (θ). //

국문초록

피보험자산과 소득이 분리되는 경우의 도덕적 해이와 위험선택에 관한 연구

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경영학과 경영학 전공

본 연구는 소득과 피보험자산이 분리되는 경우 정보비대칭하에서의 최적보험계약을 분석하고 있다. 첫 번째 장에서는 소득과 피보험자산 간의 대체불가능성을 고려하여 2-요인 기대효용 모델의 필요성을 정당화였다.

두 번째 장에서는 보험자가 보험가입자의 예방적 노력 선택을 관찰하지 못하는 경우, 도덕적 해이 하에서의 최적보험계약을 분석한다. 도덕적 해이가 존재하지 않는 경우 최적 보험의 형태는 두 재화에 대한 개인의 선호에 의존하며, 무보험에서 초과보험까지 다양하게 나타난다. 반면, 도덕적 해이가 존재하는 경우 최적보험은 부분보험이고 보험담보의 크기는 도덕적 해이가 존재하지 않는 경우보다 작다. 최적 노력 수준도 도덕적 해이가 존재하는 경우에 더 낮게 나타난다. 한편, 소득에 대한 한계효용이 자산에 대한 한계효용보다 훨씬 큰 경우에는 보험료 지불에 따른 비효용(disutility)이 매우 크기 때문에 도덕적 해이의 존재 유무와 상관없이 무보험이 최적일 수 있다. 이와 같은 결과는 도덕적 해이 하에서 부분보험의 최적성을 주장하는 기존의 연구결과와 대비되는 것이다. 더불어 본 연구는 소득과 자산을 분리한

경우와 그렇지 않은 경우의 균형을 비교하였으며, 두 재화 간의 대체불가능성 정도와 두 재화 간의 상호관계에 따른 도덕적 해이의 상대적 중요도를 각각 살펴보았다.

세 번째 장에서는 정보 비대칭 하에서 개인의 위험 유형이 내생적으로 선택되는 위험 선택 모델을 분석한다. 본 연구는 2-요인 효용함수에 내재하는 자산민감도의 이질성을 가정함으로써, 민감한 개인은 균형에서 예방적 노력에 투자하는 반면 민감하지 않은 개인은 예방적 노력에 투자하지 않음을 보였다. Rothschild and Stiglitz (1976)의 표준 모형과 달리, 본 연구에서는 민감한 개인(저위험)이 민감하지 않은 개인(고위험)보다 더 많은 보험을 수요하는 순선택이 존재할 수 있다. 이 외에도 역선택, 단일 요율 균형, 부분풀링균형, 그리고 풀링균형이 존재할 수 있다. 본 연구는 기존 연구들과 대조적으로 소득과 자산 간의 트레이드오프(trade-off)를 반영한 균형결과를 제시하고 있다.

주요어 : 2-요인 효용함수, 소득, 피보험자산, 예방적 노력, 도덕적 해이, 순선택, 자산민감도

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