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# A Valuation of Bond Guarantees by the Risky Guarantor Using Contingent Claims Analysis

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## Abstract

This paper examines a model to estimate the value of bond guarantees by employing the risk neutral valuation approach. We examine the valuation of bond guarantees both by a risky guarantor and by a riskless guarantor. We discuss the valuation of bond guarantees by a bank as an example of guarantees by a risky guarantor. As an example of bond guarantees by a riskless guarantor, we present the valuation of bond guarantees by government. Numerical examples are shown to gain insight into the relative effects of changes in the various parameters on the values of bond guarantees.

## 1. Introduction

The valuation of guarantees has received considerable attention from financial economists. By issuing a standby letters of credit, the bank agrees to repay the beneficiary upon the notice of default or nonperformance. However, the generally accepted accounting principles do not require banks to recognize them on the balance sheet. These contingent liabilities do not represent actual liabilities of the banks, but they do represent potential risks. Loan guarantees by the government are not included in the government budget, although they represent a contingent liability of the government. Few federal loan

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guarantee programs require any evaluation of the nature of this contingent liability.

Considerable debate centers on the determination of the magnitudes of these contingent liabilities. Merton(1977), Jones and Mason(1980), and Sosin(1980) have evaluated certain loan guarantees by using option pricing techniques. They have shown that option pricing theory can be applied to determine the value of loan guarantees. They explained that the payoff structure of the loan guarantee is analogous to that of a put option. Recent researches on valuing some kinds of guarantees include Pennacchi and Lewis(1994), Hsieh, Chen and Ferris(1994).

All these earlier works, however, have dealt with the valuation of guarantees in the continuous time approach of Black-Scholes(1973) and Merton(1973) analysis. In addition, they have analyzed only the case of guarantees by the government, which is a riskless guarantor. No other previous studies have analyzed the valuation of guarantees by a risky guarantor. The value of guarantees by a risky guarantor is less than that of guanantees by a riskless guarantor. For example, the Seoul Guarantee Insurance Co.(SGIC), which is a major guarantor in Korea, has been in financial distress, and the bondholders with the bond guaranteed by the SGIC could not be paid in full when the borrowing firm could not pay the promised payment.

Risk neutral valuation relationships for contingent claims have been derived from two general classes of model. The first is in the spirit of the Black-Scholes(1973) and Merton(1973) model, which places no restrictions on investors' preferences beyond the assumption of nonsatiety, but involves the formation by investors of a riskless hedge portfolio in continuous time. The second approach, developed by Rubinstein(1976) and Brennan (1979), restricts investors' preferences to eliminate the need for construction and maintenance of a riskless hedge.

Stapleton and Subrahmanyam(1984) have generalized the analysis of Brennan to the case of complex contingent claims, where the payoffs are dependent upon two or more stochastic variables. Turnbull and Milne(1991), Amin and Ng(1993) also use an extention of the equilibrium framework of Rubinstein (1976) and Brennan(1979) to derive an option valuation formula.

The purpose of this study is to estimate the value of bond guarantees by employing the risk neutral valuation framework of

Rubinstein and Brennan. We will analyze both cases of bond guarantees: bond guarantees by a risky guarantor and bond guarantees by a riskless guarantor.

## 2. The Valuation of Bond Guarantee by a Risky Guarantor: the Case of Guarantee by a Bank

With appropriate restrictions upon investor's preferences, the contingent claim can be valued by discounting an appropriately adjusted expected value of its payoffs at the riskless rate. The adjustment is explained below. Following Rubinstein and Brennan, we have developed a model under the following assumptions:

(A1) Single-price law of markets: All securities or portfolios of securities with identical payoffs sell at the same price.
(A2) Capital market is perfectly competitive with no transaction costs, no taxes, and equal access to information by all investors.
(A3) The conditions for aggregation are met so that securities are priced as though all investors had the same characteristics as a representative investor.
(A4) The borrowing firm's asset value, the bank's asset, and aggregated wealth are multivariate normally distributed.[1]
(A5) The representative investor exhibits constant absolute risk aversion.
(A6) For simplicity, we assume a single period model.

(Model Notation)

$A_0$ = the beginning-of-period asset value of the firm which issues corporate bond
$A_1$ = the end-of-period asset value of the firm which issues corporate bond
$R_0$ = the asset value of the bank, a risky guarantor, at the beginning of the period
$R_1$ = the asset value of the bank at the end of the period
$B$ = the promised payments of corporate bond when it matures

1) A model to value guarantee may be derived under the assumption of multivariate lognormality and constant proportional risk aversion.

$r_f$ = the riskless interest rate

We note that the value of corporate bond guaranteed by a bank, $V_{RG}$, can be decomposed into the value of corporate bond without guarantee, $V_{NG}$, and the value of the bank guarantee, $G_{RG}$. In other words, $G_{RG} = V_{RG} - V_{NG}$. Thus, to estimate the value of the bank guarantee, the following steps are taken: (1) First, we estimate the value of corporate bond without guarantee and (2) then, we estimate the value of corporate bond with bank guarantee, and finally (3) we take the difference between the two values to get the value of the bank guarantee.

### 2.1 The Valuation of Corporate Bond without Guarantee

Suppose there are no other existing debts except the debt of corporate bond in the borrowing firm. At the maturity of the corporate bond at the end of the period, if the asset value of the borrowing firm is equal to or greater than the promised payment, then the investor will receive full payment, $B$. If the asset value of the borrowing firm is less than the promised payment, then the investor will receive only as much as the asset value. We assume the limited liability of the shareholders in the borrowing firm. Then, the end of period payoff to the bondholder, $g(A_1)$, is represented as

$$g(A_1) = \text{Min}\,[A_1, B] \qquad (1)$$

Let $f(A_1)$ denote the density function of the end of the period asset value of the borrowing firm, $A_1$. Also define $\hat{f}(A_1)$ as a density function whose location parameter is chosen so that the mean of $A_1$ is equal to $A_0(1+r_f)$, while the other parameters are identical to those of $f(A_1)$. If a risk neutral valuation relationship exists, the value of the corporate bond may be valued as though investor preferences were risk neutral.

Under risk neutrality, the appropriate density function of $A_1$ will be $\hat{f}(A_1)$, and the value of the corporate bond may be represented by discounting its expected terminal value at the riskless rate. Rubinstein and Brennan prove assumptions (A1) through (A6) are sufficient for risk-neutral valuation. Then, $V_{NG}$, the value of the corporate bond without guarantee, may be written as follows.

$$V_{NG} = \frac{1}{1+r_f} \int_{-\infty}^{\infty} g(A_1)\hat{f}(A_1)dA_1 \tag{2}$$

Under the assumption of normality, $\hat{f}(A_1)$ is given by

$$\hat{f}(A_1) = (\sigma\sqrt{2\pi})^{-1} \quad \exp\{-(A_1 - \hat{\mu})^2 / 2\sigma^2\} \tag{3}$$

where, $\sigma$ = standard deviation of $A_1$

$\hat{\mu} = A_0(1 + r_f)$

Note that $\hat{\mu}$ is the appropriately-adjusted mean discussed earlier.

Here, we assume that the borrowing firm's asset value, $A_1$, and the promised payment, $B$, have positive values. We define $\hat{f}$ $(A_1)$ so that

$\hat{f}'(A_1) = 0$ if $A_1 < 0$

$\hat{f}'(A_1) = \hat{f}(A_1)/[\{1\text{-}F(0)]$ where $F(\cdot)$ is the cumulative normal density function.

Then, Equation (2) may be rewritten as

$$V_{NG} = \frac{1}{1+r_f}[\int_0^B A_1\hat{f}'(A_1)dA_1 + \int_B^{\infty} B\hat{f}'(A_1)dA_1] \tag{4}$$

Using Equation (3), Equation (4) may be rewritten as

$$V_{NG} = \frac{k}{1+r_f}\left[\int_0^B A_1 \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-(A_1 - \hat{\mu})^2}{2\sigma^2}]dA_1 + \right.$$

$$\left. \int_B^{\infty} B \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-(A_1 - \hat{\mu})^2}{2\sigma^2}]dA_1 \right] \tag{5}$$

where, $\kappa = \frac{1}{1-N(-\hat{\mu}/\sigma)}$, and $N(\cdot)$ is the cumulative standard normal density function.

Here, Equation (5) may be rewritten using standardized normal variable, $Z=(A_1 - \hat{\mu})/\sigma$

$$V_{NG} = \frac{\kappa}{1+r_f}\left[\int_{-\hat{\mu}/\sigma}^{(B-\hat{\mu})/\sigma} (\hat{\mu}+\sigma Z)\frac{1}{\sqrt{2\pi}}\exp[-\frac{Z^2}{2}]dZ + \right.$$

$$\left. \int_{(B-\hat{\mu})/\sigma}^{\infty} B\frac{1}{\sqrt{2\pi}}\exp[-\frac{Z^2}{2}]dZ\right] \tag{6}$$

In Equation (6), let

$$\int_{-\hat{\mu}/\sigma}^{(B-\hat{\mu})/\sigma} (\hat{\mu}+\sigma Z)\frac{1}{\sqrt{2\pi}}\exp[-\frac{Z^2}{2}]dZ = M$$

$$M = \int_{-\hat{\mu}/\sigma}^{(B-\hat{\mu})/\sigma} \hat{\mu}\frac{1}{\sqrt{2\pi}}\exp[-\frac{Z^2}{2}]dZ + \int_{-\hat{\mu}/\sigma}^{(B-\mu)/\sigma} \sigma Z\frac{1}{\sqrt{2\pi}}\exp[-\frac{Z^2}{2}]dZ$$

$$= \hat{\mu}\int_{-\hat{\mu}/\sigma}^{(B-\hat{\mu})/\sigma} \frac{1}{\sqrt{2\pi}}\exp[-\frac{Z^2}{2}]dZ +$$

$$\sigma\left[\frac{1}{\sqrt{2\pi}}\exp[\frac{-\hat{\mu}^2}{2\sigma^2}] - \frac{1}{\sqrt{2\pi}}\exp[\frac{-(B-\hat{\mu})^2}{2\sigma^2}]\right] \tag{7}$$

When simplified using Equation (7), Equation (6) on the value of corporate bond may be rewritten in the following form.

$$V_{NG} = \frac{\kappa}{1+r_f}[\hat{\mu}[N(\frac{B-\hat{\mu}}{\sigma}) - N(\frac{-\hat{\mu}}{\sigma})] + \sigma[n(\frac{\hat{\mu}}{\sigma})$$

$$- n(\frac{B-\hat{\mu}}{\sigma})] + B\cdot N(\frac{\hat{\mu}-B}{\sigma})] \tag{8}$$

where, $N(\cdot)$ is the cumulative standard normal density function and $n(\cdot)$ is the standard normal density function.

## 2.2 The Valuation of Corporate Bond with Bank Guarantee

Suppose a bank guarantees the payment of corporate bond for a borrowing firm. We assume that there are no other debts except corporate bond in the borrowing firm, and in the bank that is a guarantor. We also assume that the borrowing firm value, $A_1$, and the bank value, $R_1$, and the promised payment, $B$, have positive values. At the maturity of the corporate bond, if the sum of the firm value and the bank value, $A_1+R_1$, is equal to or greater than the promised payment, $B$, then the investor will receive full payment. However, if the amount, $A_1+R_1$, is less than the promised payment, the investor will receive only as much as $A_1+R_1$. We assume the limited liability of the shareholders of the borrowing firm and the bank.

Then, the end of period payoff to the investor, $k(A_1, R_1)$, is represented as

$$k(A_1, R_1) = \mathrm{Min}[B, A_1 + R_1] \tag{9}$$

Let $f(A_1, R_1)$ denote the density function of the end of period asset value of the borrowing firm($A_1$) and the bank($R_1$). Also we define $\hat{f}(A_1, R_1)$ as the density function whose adjusted location parameters are chosen so that the mean of $A_1$ is equal to $A_0(1+r_f)$ and the mean of $R_1$ is equal to $R_0(1+r_f)$, while the other parameters are identical to those of $f(A_1, R_1)$. The value of the corporate bond with bank guarantee can now be written as

$$V_{RG} = \frac{1}{1+r_f}\left[\int_0^{\infty}\int_0^{\infty} k(A_1, R_1)\ \hat{f}'(A_1, R_1)\,dR_1\,dA_1\right] \tag{10}$$

where, we define $\hat{f}'(A_1, R_1)$ so that
$\hat{f}'(A_1, R_1) = 0$ if $A_1 < 0,\ R_1 < 0$
$\hat{f}'(A_1, R_1) = \hat{f}'(A_1, R_1) / [1 - F(0,0)]$
where $F(\cdot)$ is the cumulative bivariate normal density function.

Here we note a property of the multivariate normal distribution: a linear combination of any of the variables in a multivariate normal distribution has a normal distribution. If variables $A_1$ and $R_1$ are bivariate normally distributed, then $V_1$, which is the sum of $A_1$ and $R_1$ is univariate normally distributed with the mean and variance as

$$\mu_V = \mu_A + \mu_R$$
$$\sigma_V^2 = \sigma_A^2 + \sigma_R^2 + 2\rho\sigma_A\sigma_R \tag{11}$$

where $\mu_V, \mu_A, \mu_R$ = the means of $V_1, A_1, R_1$, respectively
$\sigma_V, \sigma_A, \sigma_R$ = the variances of $V_1, A_1, R_1$, respectively
$\rho$ = the correlation coefficient between $A_1$ and $R_1$

Then Equation (10) can be represented as follows in a simple form.

$$V_{RG} = \frac{w}{1+r_f}[\int_0^B V_1 \hat{f}'(V_1)dV_1 + \int_B^\infty B\hat{f}'(V_1)dV_1] \tag{12}$$

where,

$$\hat{f}'(V_1) = w(\sigma_V\sqrt{2\pi})^{-1} \cdot \exp\{-(V_1 - \hat{\mu}_V)^2 / 2\sigma_V{}^2\}$$

$W = \frac{1}{1-F(0)}$ where $F(\cdot)$ is the cumulative normal density function. When simplified in the same way in Equation (8), Equation (12) may be rewritten in the following form.

$$V_{RG} = \frac{w}{1+r_f}\left[\hat{\mu}_V[N(\frac{B-\hat{\mu}_V}{\sigma_V}) - N(\frac{-\hat{\mu}_V}{\sigma_V})] + \sigma_V[n(\frac{\hat{\mu}_V}{\sigma_V}) \right.$$
$$\left. -n(\frac{B-\hat{\mu}_V}{\sigma_V})] + B \cdot N(\frac{\mu_V - B}{\sigma_V})\right] \tag{13}$$

## 2.3 The Valuation of a Bank Guarantee

As shown earlier, the value of a bank guarantee, $G_{RG}$, is the difference between the value of the corporate bond with a bank guarantee, $V_{RG}$, and the value of the corporate bond without the guarantee, $V_{NG}$.

$$\begin{aligned} G_{RG} &= V_{RG} - V_{NG} \\ &= \frac{1}{1+r_f}[\int_0^B V_1 \hat{f}'(V_1)dV_1 + \int_B^\infty B\hat{f}'(V_1)dV_1] \\ &- \frac{1}{1+r_f}[\int_0^B A_1 \hat{f}'(V_1)dV_1 + \int_B^\infty B\hat{f}'(A_1)dA_1] \end{aligned} \tag{14}$$

Using Equation (13) and Equation (8), Equation (14) may be simplified.

## 3. The Valuation of Bond Guarantee by a Riskless Guarantor: the Case of Guarantee by Government

In this section we will estimate the value of the bond guarantee by government, which is a riskless guarantor. We assume the government guarantees the payment on corporate bond. The value of the government guarantee is the difference between the value of corporate bond with the government guarantee and the value of corporate bond without the government guarantee. With the government guarantee, the corporate bond is riskless, and the value of the bond at the beginning of the period, VGG, may be represented as

$$V_{GG} = \frac{B}{1+r_f} \tag{15}$$

The value of the corporate bond without the guarantee is derived in Equation (4). Then the value of the government guarantee, $G_{GG}$, is represented as

$$\begin{aligned} G_{GG} &= V_{GG} - V_{NG} \\ &= \frac{1}{1+r_f}[B - (\int_0^B A_1 \hat{f}'(A_1)dA_1 + \int_B^\infty B\hat{f}'(A_1)dA_1)] \end{aligned} \tag{16}$$

Using Equation (8), Equation (16) may be rewritten as follows.

$$\begin{aligned} G_{GG} = \frac{1}{1+r_f}\Bigg[ & B - \kappa[\hat{\mu}\left(N(\frac{B-\hat{\mu}}{\sigma}) - N(\frac{-\hat{\mu}}{\sigma})\right) \\ & + \sigma\left(n(\frac{\hat{\mu}}{\sigma}) - n(\frac{B-\hat{\mu}}{\sigma})\right) + B\cdot N(\frac{\hat{\mu}-B}{\sigma})]\Bigg] \end{aligned} \tag{17}$$

## 4. Numerical Examples

We now present some numerical examples to gain insight into the relative effects of changes in the various parameters on the

**Table 1. Sensitivity of the Value of Guarantees to the Standard Deviation of the Borrowing Firm's Asset Value**

$r_f$ = 10%, $B$ = \$1,000, $R_0$ = \$10,000, $\sigma_R$ = \$3,000, $\rho$= 0.9

| $A_0$ | $\sigma_A$ | $\sigma_A/A_0$ | $G_{RG}$ | $G_{GG}$ |
|---|---|---|---|---|
| 5000 | 1500 | 0.30 | 0.3200 | 0.3688 |
| 5000 | 1750 | 0.35 | 1.3360 | 1.4271 |
| 5000 | 2000 | 0.40 | 3.2112 | 3.3663 |
| 5000 | 2250 | 0.45 | 5.7097 | 5.9543 |
| 5000 | 2500 | 0.50 | 8.4596 | 8.8220 |
| 5000 | 2750 | 0.55 | 11.1514 | 11.6608 |
| 5000 | 3000 | 0.60 | 13.5939 | 14.2791 |
| 5000 | 3250 | 0.65 | 15.6970 | 16.5847 |
| 5000 | 3500 | 0.70 | 17.4370 | 18.5516 |
| 5000 | 3750 | 0.75 | 18.8284 | 20.1905 |

Notes:
$A_0$ = the beginning-of-period asset value of the firm which issues corporate bond
$R_0$ = the asset value of the bank, a risky guarantor, at the beginning of the period
$B$ = the promised payments of corporate bond when it matures
$r_f$ = the riskless interest rate
$\mu_A$, $\mu_R$ = the means of $A_1$, $R_1$ respectively.
$\sigma_A$, $\sigma_R$ = the variances of $A_1$, $R_1$ respectively.
$\rho$= the correlation coefficient between $A_1$ and $R_1$.
$G_{RG}$ = the value of the bank guarantee
$G_{GG}$ = the value of the government guarantee

value of bond guarantees. Table 1 shows the effect of changes in the standard deviation of the borrowing firm's asset value ($\sigma_A$) on the value of bond guarantees. Fixing other parameters except $\sigma_A$, we examine the behavior of the value of bond guarantees as $\sigma_A$ takes on some range of values.

In Table 1 the amount of the promised payment of corporate bond is assumed to be \$1,000. The risk-free rate is assumed to be 10%; the asset value of the bank at the beginning of the period is assumed to be \$10,000; the standard deviation of bank asset value is assumed to be \$3,000. In addition the correlation coefficient between the borrowing firm's asset value and the bank's asset value is assumed to be 0.9.

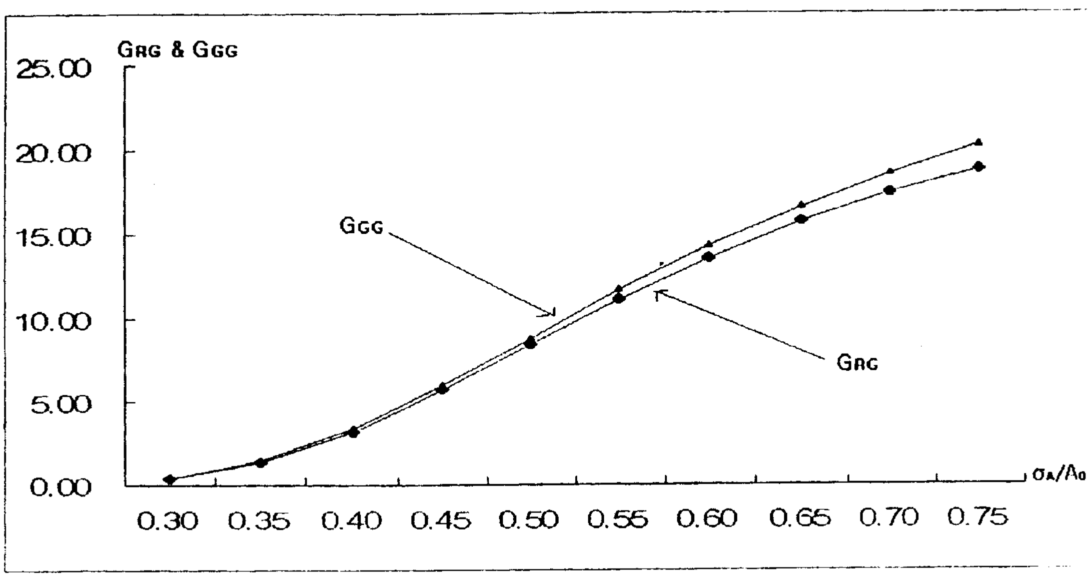


**Figure 1. Sensitivity of the Value of Guarantees to the Standard Deviation of the Borrowing Firm's Asset Value**

The results shown in Table 1 indicate that the value of the bank guarantee and the value of the government guarantee are very sensitive to, and increasing functions of, the standard deviation of the borrowing firm's asset value. These patterns are revealed in Figure 1 which depicts the value of guarantees as a function of $\sigma_A/A_0$, the ratio of the standard deviation of the borrowing firm's asset value to the initial asset value of the borrowing firm. Clearly, the value of guarantees increases as the ratio $\sigma_A/A_0$ increases.

From the given numerical examples in Table 1, we can see that in the case of a firm with the initial asset value of $5,000, the values of bank guarantees range from as low as 0.032% of the promised payments to as high as the 1.883% of the promised payments, as the standard deviation of the firm's asset value varies from 30% to 75% of the firm's asset value. In addition we may compare the value of guarantees by a bank, which is a risky guarantor, with the value of guarantees by the federal government, which is an example of a riskless guarantor. As expected, the government's guarantees show higher values than the values of bank guarantees in each level of the standard deviation of the borrowing firm's asset value.

The value of the bank guarantee is also an increasing function of the ratio of the promised payment to the borrowing firm's

**Table 2. Sensitivity of the Value of Guarantees with respect to the Ratio of the Promised Payment to the Value of the Borrowing Firm's Asset**

$r_f$ = 10%, $R_0$ = \$10,000, $\sigma_R$ = \$3,000, $\rho$= 0.9

| $B$ | $A_0$ | $\sigma_A$ | $B/A_0$ | $G_{RG}$ | $G_{GG}$ |
|---|---|---|---|---|---|
| 50 | 5000 | 2000 | 0.01 | 0.0050 | 0.0053 |
| 100 | 5000 | 2000 | 0.02 | 0.0205 | 0.0218 |
| 200 | 5000 | 2000 | 0.04 | 0.0860 | 0.0911 |
| 300 | 5000 | 2000 | 0.06 | 0.2031 | 0.2148 |
| 400 | 5000 | 2000 | 0.08 | 0.3792 | 0.4006 |
| 500 | 5000 | 2000 | 0.10 | 0.6226 | 0.6568 |
| 600 | 5000 | 2000 | 0.12 | 0.9426 | 0.9931 |
| 700 | 5000 | 2000 | 0.14 | 1.3494 | 1.4199 |
| 800 | 5000 | 2000 | 0.16 | 1.8545 | 1.9489 |
| 900 | 5000 | 2000 | 0.18 | 2.4705 | 2.5930 |
| 1000 | 5000 | 2000 | 0.20 | 3.2112 | 3.3663 |
| 1100 | 5000 | 2000 | 0.22 | 4.0921 | 4.2845 |
| 1200 | 5000 | 2000 | 0.24 | 5.1296 | 5.3646 |
| 1300 | 5000 | 2000 | 0.26 | 6.3422 | 6.6250 |

Notes: See the notes for Table 1.

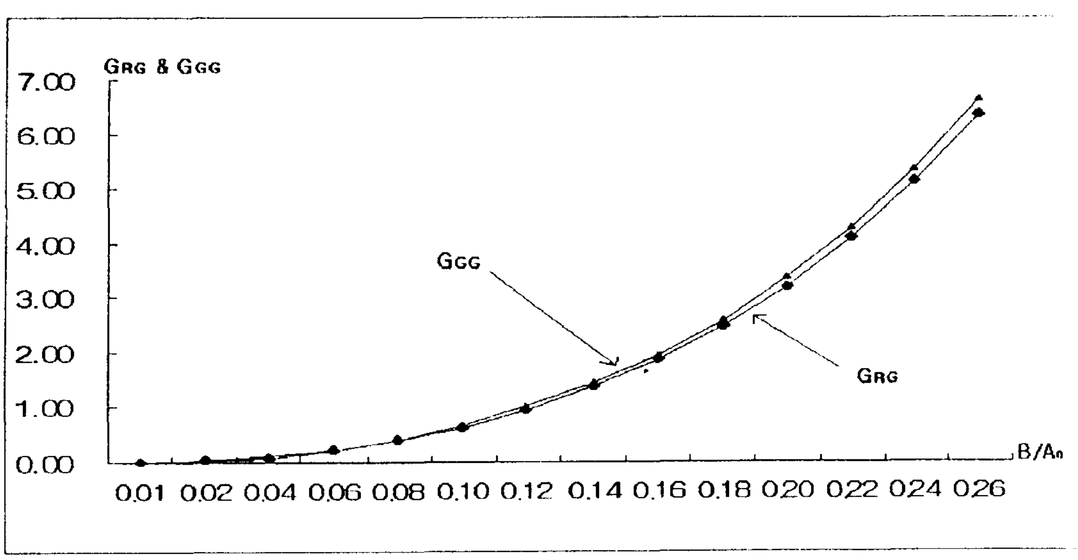


**Figure 2. Sensitivity of the Value of Guarantees with respect to the Ratio of the Promised Payment to the Value of the Borrowing Firm's Asset**

**Table 3. Sensitivity of the Value of Bank Guarantees to the Standard Deviation of the Bank's Asset Value**

$B$ = \$1,000, $r_f$ = 10%, $A_0$ = \$5,000, $\sigma_A$ = \$2,000, $\rho$= 0.9

| $R_0$ | $\sigma_R$ | $\sigma_R/R_0$ | $G_{RG}$ |
|---|---|---|---|
| 10000 | 500 | 0.05 | 3.3663 |
| 10000 | 800 | 0.08 | 3.3663 |
| 10000 | 1000 | 0.10 | 3.3663 |
| 10000 | 1500 | 0.15 | 3.3656 |
| 10000 | 2000 | 0.20 | 3.3576 |
| 10000 | 2500 | 0.25 | 3.3191 |
| 10000 | 3000 | 0.30 | 3.2112 |
| 10000 | 3500 | 0.35 | 2.9988 |
| 10000 | 4000 | 0.40 | 2.6679 |

Notes: See the notes for Table 1.

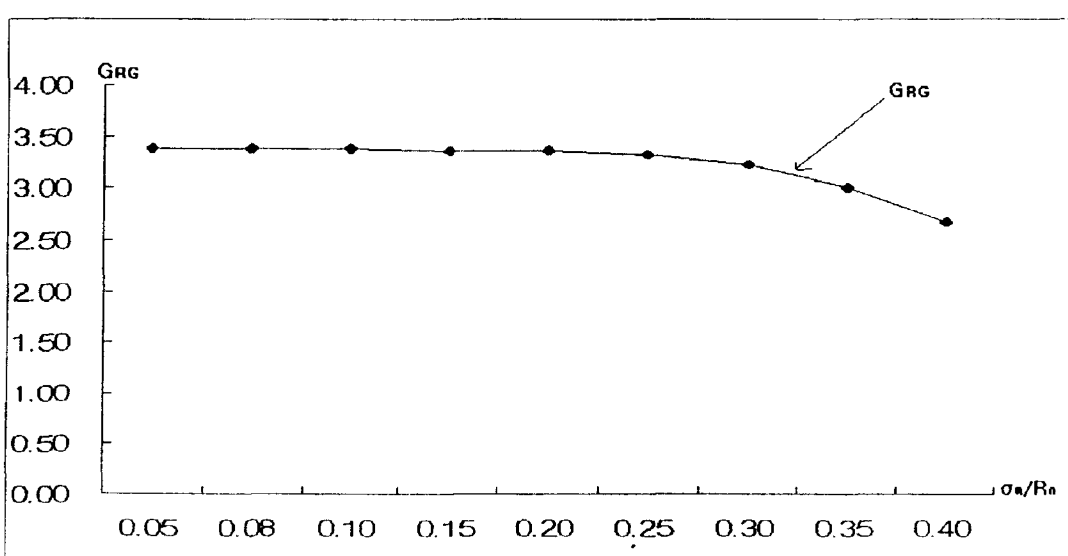


**Figure 3. Sensitivity of the Value of Bank Guarantees to the Standard Deviation of the Bank's Asset Value**

asset value. Table 2 shows that the value of the bank guarantee is quite sensitive to the ratio of the promised payment to the borrowing firm's asset value. Clearly, in Figure 2, an increase in the ratio of the promised payment to the borrowing firm's asset value ($B/A_0$) leads to an increase in the value of guarantees. Also, Table 1 and Table 2 show that the value of government guarantees are quite sensitive to the changes in the standard

**Table 4. Sensitivity of the Value of Bank Guarantees to the Correlation Coefficient between the Borrowing Firm's Asset Value and the Bank's Asset Value**

$B$ = \$1,000, $r_f$ = 10%, $A_0$ = \$5,000, $\sigma_A$ = \$2,000, $R_0$ = \$10,000, $\sigma_R$ = \$3,000

| $\rho$ | $G_{RG}$ |
|---|---|
| 0.05 | 3.3630 |
| 0.10 | 3.3614 |
| 0.20 | 3.3566 |
| 0.30 | 3.3491 |
| 0.40 | 3.3380 |
| 0.50 | 3.3228 |
| 0.60 | 3.3029 |
| 0.70 | 3.2779 |
| 0.80 | 3.2473 |
| 0.90 | 3.2112 |

Notes: See the notes for Table 1.

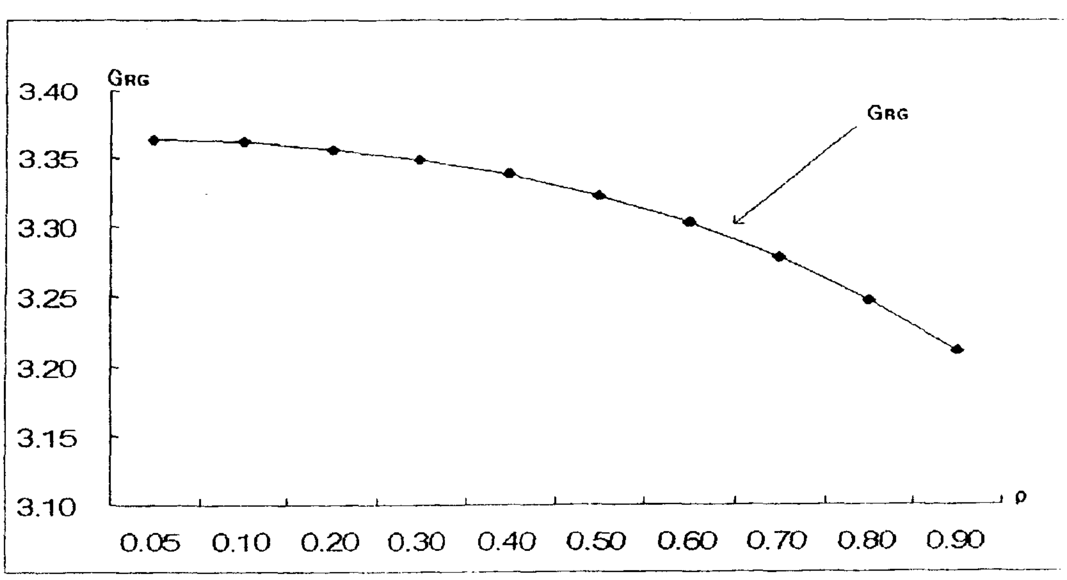


**Figure 4. Sensitivity of the Value of Bank Guarantees to the Correlation Coefficient between the Borrowing Firm's Asset Value and the Bank's Asset Value**

deviation of the borrowing firm's asset value and to the ratio of the promised payment to the value of the borrowing firm's asset.

In Table 3 we present the effect of changes in the standard

**Table 5. Sensitivity of the Value of Guarantees with respect to the Ratio of the Promised Payment to the Value of the Bank's Asset**

$B$ =\$1,000, $r_f$= 10%, $A_0$ = \$5,000, $\sigma A$ = \$3,000, $\sigma_R$ =30%, $\rho$= 0.9

| $R_0$ | $\sigma_R$ | $B/R_0$ | $G_{RG}$ |
|---|---|---|---|
| 5000 | 1500 | 0.200 | 2.9550 |
| 10000 | 3000 | 0.100 | 3.2112 |
| 15000 | 4500 | 0.067 | 3.2798 |
| 20000 | 6000 | 0.050 | 3.3084 |
| 25000 | 7500 | 0.040 | 3.3235 |
| 30000 | 9000 | 0.033 | 3.3326 |
| 35000 | 10500 | 0.029 | 3.3387 |
| 40000 | 12000 | 0.025 | 3.3429 |
| 45000 | 13500 | 0.022 | 3.3461 |
| 50000 | 15000 | 0.020 | 3.3485 |

Notes: See the notes for Table 1.

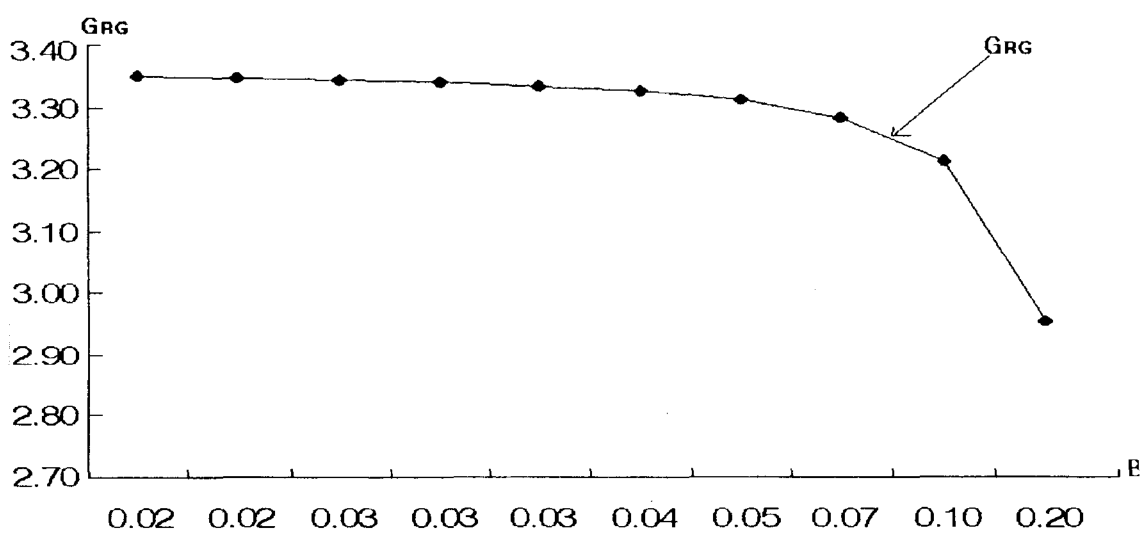


**Figure 5. Sensitivity of the Value of Guarantees with respect to the Ratio of the Promised Payment to the Value of the Bank's Asset**

deviation of the bank's asset value on the bank guarantees. The results in Table 3 show that the value of a bank guarantee is very sensitive to, and a decreasing function of, the standard deviation of the bank's asset value. As the standard deviation of the bank's asset value varies from 5% to 40% of the bank's asset value, the value of a bank guarantee on the corporate bond ranges from as high as 0.337% of the promised payments, to as low as 0.267% of the promised payments. The results show that as the riskiness of a bank increases, the value of the bank guarantee to the beneficiary decreases. These patterns are clearly portrayed in Figure 3. Table 4 examines the effects of changes in the correlation coefficient between the borrowing firm's asset value and the bank's asset value on the value of guarantees. As Table 4 and Figure 4 indicate, the effect of an increase in the correlation coefficient decreases the value of bank guarantees.

Finally, in Table 5 we examine the effect of changes in the ratios of the promised payment to the value of the Bank's asset. The effect of an increase in the ratio of the promised payment to the value of the Bank's asset decreases the value of bank guarantees.

## 5. Summary and Conclusion

This paper presents a model to estimate the value of bond guarantees by employing contingent claims analysis in discrete time. We examine the valuation of bond guarantees both by a risky guarantor and by a riskless guarantor. We discuss the valuation of bond guarantees by a bank as an example of guarantees by a risky guarantor. As an example of bond guarantees by a riskless guarantor, we present the valuation of bond guarantees by government.

As the given numerical examples demonstrate, the value of bank guarantee is highly sensitive to the characteristics of such parameters as the standard deviation of the borrowing firm's asset value, the ratio of the promised payments to the mean value of the borrowing firm's assets, and the standard deviation of the assets value of the bank that is a risky guarantor. Also, numerical examples show that the value of the bank guarantees is also sensitive to the changes in the other parameters: the correlation coefficient between the borrowing firm's asset value, and the bank's asset value, and the ratio of the promised payments to the value of the bank's asset.

Further, this study has indicated that the value of government guarantees is quite sensitive to the value of the standard deviation of the borrowing firm's asset value and the ratio of the promised payments to the mean value of the borrowing firm's assets. In comparison of the value of bank guarantees, the value of government guarantee is higher than the value of the bank guarantees. The models developed in this study might be applied in the valuation of other guarantees: the valuation of loan guarantees for subsidiary companies by their parent companies, the valuation of guarantees for a firm by an insurance company.

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