Defining Baseline Sales
In a Competitive Environment

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Abstract

The baseline ("normal" sales) has become the most commonly used procedure to measure the incremental effect of marketing variables, particularly promotions. Firms such as IRI and Nielsen provide their clients with baseline sales' estimates which they then use to determine the sales and profit effects of consumer and trade promotions and, in some cases, measure their brand equity. With the increasing importance of the baseline to the measurement of marketing effects, it is necessary to develop an accurate definition of the baseline and determine what it measures. This paper will define baseline sales and discuss its implications, show that the incorporation of competitive reaction is important to the definition of the baseline and discuss how to use this concept when a firm has a broad product line. In addition, we compare several baseline estimation methods using simulated data and show the importance of incorporating a competitive reaction component in baseline estimation. Finally, we apply the concept of competitive reaction to actual sales data and show that the baseline computed without a competitive reaction function is significantly different from the baseline with a competitive reaction function.

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I. INTRODUCTION

As firms are under greater pressure to demonstrate the economic return from marketing expenditures, the need for a new measurement standard has emerged. In the last five years, in part through Information Resources (IRI) and Nielsen, consumer product firms have begun using the baseline as a critical measurement tool. By studying whether marketing expenditures generate sales above the baseline, management evaluates whether its various marketing tactics are generating increases in sales and profitability. Part of the emerging prominence of the baseline is that market share has become a function of numerous marketing activities and is no longer a “clean” measure of a brand or product’s strength in the marketplace. Brand or product managers can increase market share through aggressive promotional programs which may not pay out economically. The rapid growth of promotional spending in the last decade has also focused firms on the “baseline” so that they can decide what “normal” sales are in the absence of highly aggressive short-term marketing spending.

The concept of a baseline is not new. In the late 50’s and early 60’s stochastic models (Herniter and Magee, 1961; Maffai, 1960) used Markov chains to predict normal sales in order to measure the impact of a change in the firm’s marketing tactics. Kuehn and Rohloff [1967] used the linear learning model to estimate normal (baseline) sales in the absence of promotions and then used them to analyze promotional effects. One of the reasons that these baseline did not gain prominence earlier was their inability to control for marketing factors during the base period. It was not until the advent of scanner data and related models which adjusted for promotional effects1) that the ability to measure the baseline, holding fixed other factors, became feasible.

Research that has focused on the baseline is not prevalent. Abraham and Lodish [1987] used a time-series model to estimate the baseline in the absence of promotional activities. Taking advantage of the idiosyncracies of

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1) For example, see Guadagni and Little (1983) who incorporated promotional factors into stochastic brand choice models and Blattberg and Wisniewski (1989) who estimated promotion effects for store level scanner data.
store-level data, Abraham and Lodish [1990, 1993] have improved their previous approach in various areas such as smoothing techniques, bias correction mechanism, etc. Numerous other models implicitly try to estimate incremental marketing effects above normal sales (i.e., Chintagunta, Jain, and Vilcassim, 1991) but the literature does not define "normal sales" and no attempt has been made to consider issues associated with defining and measuring the baseline. Given its current widespread use among package-goods manufacturers and its impact on marketing measurement, it is surprising more articles have not focused directly on this issue.

The purpose of our paper is to define baseline sales and discuss its implications, showing that the incorporation of competitive reaction is important in the estimation of baseline. The paper is organized as follows. Section 2 gives alternative definitions of the baseline: Section 3 shows, through a game-theoretic discussion, the importance of the competitive reaction (or its expectation) to the estimation of the baseline and discusses how a product manager having a broad product line can use our definition of the baseline: Section 4 uses a simulation to compare several baseline estimation methodologies, showing that the incorporation of competitive reaction is critical under certain market condition: Section 5 applies the concept of competitive reaction to store level data for three product categories, followed by the Section 6 which contains the future research direction, summary and the conclusions of the paper.

II. DEFINITION OF THE BASELINE

In defining the baseline we will draw parallels to the experimental design literature. The concept of a baseline is similar to a "control". A control is used in an experiment because it attempts to hold fixed all other factors. We will refer to the baseline as a "quasi-control" because the baseline uses statistical procedures to hold some factors fixed and to remove others, and hence, it is really a conditional control, with variables being set at fixed levels. This leads to the following definition.

The baseline is an estimate of sales after controlling for and/or removing the effects of specific marketing activities.
The baseline is designed to measure "base" sales in the absence of short-term marketing expenditures. It has been normal to remove promotional effects from sales but rarely to control for other marketing activities (such as regular price and media advertising) or competitive factors primarily because most baseline analyses are focused on measuring the impact of promotions. Our definition concentrates on both removing short-term marketing activities and on controlling other relevant marketing mix variables and competitive behavior. Generally, when a marketing activity is removed from the baseline, its level is set equal to zero. When a marketing activity or marketing mix variable is controlled for, its level is not set equal to zero but to the appropriate level given the time period being analyzed. For example, it does not make sense to set an item's regular price equal to zero (i.e., remove it) but rather to set it equal to the level at the time the baseline is being used to analyze the incremental effect of a given marketing activity.

It is always difficult to decide what to remove from the baseline. Generally, the rule regarding what to remove from the baseline appears to be marketing activities that its effect is only "short-term" and hence can be removed. Examples of these activities are:

- Retailer's temporary price reductions
- Coupons
- Special short-term purchase incentives
- Display and feature advertising
- Post or pre promotion troughs

Marketing activities which should be controlled for are those which are required to market the product and those which have a "long-term" effect. Examples of these activities are:

- Media advertising
- Regular shelf price
- New item introductions
- Product deletion decisions
- Changes in distribution
- Competitive behavior
No doubt there are numerous other marketing activities that could be included in this list, and, if important, should be controlled for or removed depending on whether their effect is short-term.

In deriving the baseline, key factors are held constant through statistical analysis. This is similar to the use of covariates or concomitant variables (Scheffe [1959]) in the analysis of variance. The reason covariates are included and an analysis of covariance is used is that the main effects in the model are affected by them. Scheffe ([1959], p. 195) states, “The analysis of covariance was introduced...as a device for simulating control factors not possible or feasible to control in an experiment.” Since it is difficult to use controlled experiments in the real world, baselines are usually estimated from sales data directly.

To better understand the concept of a baseline, it is useful to begin with a simple model:

\[ S_{jt} = \alpha_j + \beta_j \text{RP}_{jt} + \sum_k \gamma_k \beta_k \text{RP}_{kt} + \gamma_j \text{DDEPTH}_{jt} + \sum_k \gamma_j \gamma_k \text{DDEPTH}_{kt} + \epsilon_{jt} \]  

(2.1)

where \( S_{jt} \) is the unit sales for brand \( j \) at time \( t \), \( \text{RP}_{jt} \) is the regular price for brand \( j \) at time \( t \), and \( \text{DDEPTH}_{jt} \) is the depth of deal discount for brand \( j \) at time \( t \). Our goal is to analyze promotional effect on sales (e.g., incremental volume) so that the definition of the baseline is very important. What is the baseline for brand \( j \) from the demand function specified in equation (2.1)? Since the baseline is the sales without its own promotions (e.g., setting \( \text{DDEPTH}_{jt} \) to be zero), a possible candidate is:

\[ B_{jt} = \alpha_j + \beta_j \text{RP}_{jt} + \sum_k \gamma_k \beta_k \text{RP}_{kt} + \sum_k \gamma_j \gamma_k \text{DDEPTH}_{kt} \]  

(2.2)

where \( B_{jt} \) is the baseline of brand \( j \) at time \( t \). The controversial issue is whether to use the observed historical \( \text{DDEPTH}_{kt} \) in computing the baseline of brand \( j \). This depends upon what the decision maker believes is endogenous and exogenous. If one believes that competitor's promotions are exogenous, then it should be part of the controlled environment. If not, the historical \( \text{DDEPTH}_{kt} \) should not be used. Otherwise the baseline will be biased. For example, if competitors always match the promotion of the brand, \( \text{DDEPTH}_{kt} \) of zero (e.g., if the firm does not promote, competitors will not promote) should be used in computing the baseline of brand \( j \).
The above discussion illustrates that the definition of the baseline is complex. The definition of a control in an experiment is equally complex because one always assumes certain key factors are being held constant. We call the baseline a "quasi-control" because to hold fixed other factors, statistical models are used.

In determining which specific marketing activities to try to remove or control for, it is similar to the discussion that occurs when an econometric model is being built and the modeler must decide which terms belong in the model and which belong in the error term. By not including a variable in the model, one is assuming its effect is small and uncorrelated with the variables in the model. Hence, the criterion for not controlling for or removing a given marketing variable should be: "the marketing activity which has no (or very little) effect on the baseline."

II. INCORPORATING COMPETITIVE ACTIVITY INTO THE BASELINE

In this section, we will explain why the incorporation of competitive reaction is important in baseline estimation, mathematically define baseline sales in a competitive environment mathematically, suggest different baselines for different situations and discuss an important application of this concept to intra-brand competition.

3.1 Introduction

One of the most important issues that needs to be addressed in determining the baseline is the decision whether to incorporate competitive reaction into it. Many of the current procedures, such as time series analysis, do not directly include competitive reaction into the baseline because it is assumed "constant". For example, Abraham and Lodish [1993] define the baseline as:

"The baseline is an estimate for each store week of what the sales of the item would have been had only the item's promotion not been run. All other elements of the item's and the competitor's marketing mix are assumed ceteris paribus."
In other words, Abraham and Lodish are making an explicit assumption about competitive reaction—there is none. They treat the competitive promotions as exogenous so that the baseline of brand $j$ which has the demand function of equation (2.1) can be expressed by equation (2.2).

Abraham and Lodish argue that the usefulness of their baseline is to analyze specific promotional events for a subset of “stakeholders”, the sales force and the executional members of the marketing organization. When they analyze a promotional event, the effect being studied is the difference between sales and the baseline.

$$S_{jt} - B_{jt} = \gamma_j \text{DEPTH}_{jt} + \varepsilon_{jt}$$

where $S_{jt}$ is the sales of item $j$ at time $t$, $B_{jt}$ is the baseline for item $j$ at time $t$, and $\varepsilon_{jt}$ is a random error with mean zero. The promotional effect is $\gamma_j \text{DEPTH}_{jt}$. Implicitly they argue their stakeholders are more interested in understanding $\varepsilon_{jt}$ than $\gamma_j \text{DEPTH}_{jt}$ because they want to understand the success or failure of a given promotional execution. Their argument is that the team implementing promotions is not interested in the mean promotional response but in the increase over that mean because they design executional strategies for a general promotion. This translates into concentrating on using baseline to study $\varepsilon_{jt}$. This view of a baseline is clearly “short-run” and focuses on analyzing a specific promotional event.

The focus of our paper will be broader and will concentrate on analyzing the dynamic nature of the baseline. Specifically, we will analyze how actions taken by the firm in one week can affect the actions of their competitors and how this influences the accuracy of the baseline. For example, for the model given in (2.1) above, the issue we will address is the exogeneity of the promotional activity in the market. If it is not exogenous, then when one estimates the baseline, one must understand that the firm’s promotional action influences competitors’ actions.

Using Abraham and Lodish’s terminology, our approach to baselines will be useful for certain stakeholders such as the brand managers, planners in trade marketing, and strategists in the field organization. Abraham and Lodish’s focus is the front-line sales force and executional specialists within the marketing function. Our approach to estimating baselines incorporates
competitive response which is very important in planning annual or even quarterly promotional frequency and depth. By not incorporating competition into the estimation of the baseline, it would appear that promotions (and potentially other marketing actions) are more profitable than they actual are because most of the sales increases are caused by stealing customers from competition. If the firm and competition reduced promotional spending, profits would increase and sales would not decline significantly because customers would still buy in the category. Most current baseline procedures do not consider the endogeneity of competitive promotions, and hence, they overestimate the incremental impact of the firm's historical promotional events on its sales.

The remainder of this section will show how the inclusion or exclusion of a competitive promotion affects the height of the baseline and hence the incremental effect of promotional activity.

3.2. Game Theoretic View of the Baseline

For simplicity, it is assumed that there are two brands, brand 1 and 2, in the category and we are interested in estimating the baseline of brand 1. The demand (or sales) of brand 1, $S_1$, is expressed as

$$S_1 = f(D_1, D_2, z)$$

(3.1)

where $D_j$ ($j=1,2$) is the depth of deal discount for brand j and $z$ is a vector of variables (e.g., regular price of brand j) which affects the demand of brand 1 and should be included in the baseline itself. It is assumed that $\partial S_1 / \partial D_1 > 0$ and $\partial S_1 / \partial D_2 < 0$.

What should the baseline be under the above demand specification? We introduce two definitions of the baseline sales of brand 1, the "competitive reaction function" and "no competitive reaction function" baseline, in order to show the importance of incorporating competitive reactions.

$$S_{1\text{NCR}} = f(D_1 = 0, D_2, z)$$

(3.2a)

$$S_{1\text{CR}} = f(D_1 = 0, D_2 = 0, z)$$

(3.2b)
where $S_{1NCR}$ is the “no competitive reaction function” baseline sales of brand 1 and $S_{1CR}$ is the “competitive reaction function” baseline sales of brand 1. Notice that the “no competitive reaction function” baseline in equation (3.2a) implicitly assumes that the promotion of brand 1 does not influence the promotion of brand 2. It assumes a Nash behavior such that $\partial D_2 / \partial D_1 = 0$. In contrast, the “competitive reaction function” baseline in equation (3.2b) assumes a leader-follower relationship such that brand 2 will not promote its brand if brand 1 does not promote. We can generalize this notion of the “competitive reaction function” baseline by incorporating the reaction function of brand 2 to the depth of deal discount for brand 1. For example, $D_2 = g(D_2)$. Rewriting equation (3.1), the demand function of brand 1 can be written as

$$S_1 = f(D_1, g(D_2), z) \tag{3.3}$$

With this demand specification, the baseline sales of brand 1 can be defined as

$$S_1 = f(D_1 = 0, g(D_1 = 0), z) \tag{3.4}$$

Notice that the baseline sales computed from the equation (3.4) will be higher than that from the equation (3.2a) if $\partial g(D_1)/\partial D_1 > 0$ while it will be lower if $\partial g(D_1)/\partial D_1 < 0$. In other words, if brand 1 stops promoting its brand, brand 2 decreases its discount depth if $\partial g(D_1)/\partial D_1 > 0$ so that the baseline from the equation (3.4) becomes greater than $S_{1NCR}$ since $\partial S_1 / \partial D_2 < 0$ (e.g., the decrease of discount for brand 2 increases the demand of brand).  

Under these circumstances, therefore, if we include the reaction of the competitor (or competitors), the magnitude of the incremental volume induced by a promotion will be reduced because of the higher baseline sales. If firm

2) As a concrete example, suppose $S_1 = x + \beta D_1 + \gamma D_2$ where $\beta > 0$ and $\gamma < 0$. If the depth of discount for brand 2 is determined independently of the discount of brand 1, the baseline sales of brand 1 (e.g., no competitive reaction function baseline) is $x + \gamma D_2$. Now, assume the discount of brand 2 is determined by the reaction function, $D_2 = \delta_1 + \delta_2 D_1$, where $\delta_2 > 0$. Then, the baseline of brand 1 becomes larger since $D_2$ decreases when promotion for brand 1 is stopped and this decrease in the discount depth for brand 2 increases the baseline sales of brand 1.
1 reduces its promotional activity, brand 2 will reduce its promotional activity. This implies that the incentive to promote a brand is less than when we incorporate the effect of the competitor's reaction.

If our assumption about the competitive behavior is \( \frac{\partial g(D_1)}{\partial D_1} = 0 \), then the baseline from the equation (3.4) becomes equivalent to "no competitive reaction function" baseline from the equation (3.2a). If the competitive reaction function is likely to be \( \frac{\partial g(D_1)}{\partial D_1} < 0 \), which means that if the discount of brand 1 is increased, brand 2 will increase its discount even more, then the estimated baseline will be lower than the "no competitive reaction function" baseline from the equation (3.2a). Thus, if firm 1 reduces its promotional activity, firm 2 will increase its promotional activity.

By defining the baseline to be equation (3.4), we quickly see that the assumption we make about the reaction function is critical.

### 3.3 Intra-Brand Competition

The above baseline concept, which incorporates competitive reaction, is useful for a manufacturer which has a broad product line with multiple items and/or other brands. The "group product" manager is more concerned about the total profits of all brands rather than the profit for each item. Different from the inter-brand competition case, the group product manager knows or is able to control the reaction functions of the other items in the product line. Unless there is no cannibalization among the brands or items within a brand, the baseline sales of a given brand will change with the profitability of the promotions of each brands in the product line because of the argument given in the previous section. Therefore, in order to compute the profitability of a given promotion for a specific brand or item within the product line, it is important to compute baseline sales for each brand assuming the firm's other items do not promote.

As an example, assume a firm with two brands, brands A and B, wants to compute the incremental sales of brand A. Assume that the sales of brand A is determined by the following equation:

\[
S_{A_t} = 300 + 700 \text{DEPTH}_{A_t} - 100 \text{DEPTH}_{B_t}
\] (3.5)
where DDEPTH_A and DDEPTH_B are the depth of deal discounts of brand A and B during week t respectively. Further, assume that the firm alternates the promotions of its brands every week with brand A being promoted in weeks 1, 3, 5, ... and brand B being promoted in weeks 2, 4, 6, ... The depth of the discount is $1.00 (e.g., DDEPTH_A = $1.00 if the brand is promoted and DDEPTH_A = 0 if not). The sales of brand A is 200 units when it is not promoted because brand B is promoted during that week and it becomes 1,000 units when it is promoted. When we compute the profitability of the promotion, we should not use 800 units, 1,000 units minus 200 units, as the incremental sales of brand A from the promotion since the sale of 200 units of brand A includes the promotion of brand B. The incremental sales is upwardly biased since the baseline used is downwardly biased. Notice the "true" baseline is 300 units which is computed by assuming neither brand A and nor brand B promote. For the correct computation of the profitability of a given brand's promotion, we must use the baseline of the brand when the other brands (or items) in the product line do not promote.

IV. COMPARISON OF BASELINE ESTIMATION METHODS

In this section, we propose eight baseline estimation methodologies. Some methods are chosen because of their simplicity and support from previous literature while some are proposed on the basis of our intuition regarding potential candidates for baseline estimation. Because the true baseline is not known for "real data", we will use a simulation to evaluate the methods so that the true baseline is known. The performance of eight estimation methods are compared under 8 (e.g., 2×2×2 design factors) different market conditions which vary in terms of (1) whether the price promotion of a brand increases the sales of the category by a large amount or not, (2) whether the category is highly promoted or not, and (3) whether competitors make independent promotional decisions or not. These three aspects of market conditions will be explained in more detail later.

Once the data is generated through a simulation for a given market condition, each baseline estimation method will be compared on two performance criteria: mean absolute percentage error (MAPE) and bias. These are
computed by:

\[ \text{MAPE} = \frac{\sum |\text{True}_t - \text{Estim}_t| / \text{True}_t}{T} \]
\[ \text{Bias} = \frac{\sum (\text{True}_t - \text{Estim}_t) / \text{True}_t}{T} \]

where T is the total time period of the data, True\(_t\) is the true baseline for week \(t\) and Estim\(_t\) is the estimated baseline for week \(t\) from a given estimation model. Bias is considered since we want to study whether an estimation method overestimates or underestimates the true baseline, on average, over the entire simulation period.

4.1. Simulation

We assume a hypothetical market where there are three firms in the product category. Each firm produces one brand: firm \(i\) produces brand \(i\) \((i=1,2,3)\). We will focus on the baseline of brand 1 since the results for other brands are similar. The role of the retailer is ignored since our concern is the baseline of a brand. The incorporation of multiple retailers significantly increases the complexity of the simulation (e.g., we should consider consumer store choice decision as well as behavior of retailers) while it does not provide additional insight of our paper.\(^3\)

In the simulation 5,000 consumers were created who are heterogeneous in terms of their reservation prices, brand loyalties, price sensitivities, and mean purchase rates. For each week, consumers are assumed to make three purchase decisions, not necessarily sequential: (1) whether to buy the category, (2) which brand to buy, and (3) how much to buy. Chiang (1991) and Chintagunta (1993) have shown that the simultaneous treatment of the above three purchase decisions is superior to the sequential treatment. Our simulation will use their approach.

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3) If multiple retailers are incorporated into the simulation, there are two ways to compute the baseline for each brand. First, one can compute the baseline with aggregated data (e.g., sum of the sales and weighted average price across stores) at the market level or calculate the baseline of a given brand for each store and sum them across stores to compute the baseline of each brand. The baseline estimation method proposed in the paper can handle both cases without much difficulty.
4.1.1. Category Buying and Brand Choice Decision

For each week, the category buying decision of each consumer is assumed to be determined by the following equation. Consumer i's probability of buying the category at week t is

$$\text{Prob}(C_{it}=1)=1-\exp[-R_i\sum_j \exp(v_{ij})]$$

where $$v_{ij}=\alpha_{ij}-\beta_i\text{INV}_{it-1}-\ln(p_{ij})$$  \hspace{1cm} (4.1)

In the above equations, $$R_i$$ is the reservation price of consumer i, $$\text{INV}_{it-1}$$ is consumer i's inventory level for the category at week t-1, and $$p_{ij}$$ is the price of brand j at week k.

The brand choice decision for each consumer is similarly determined. The consumer i's probability of purchasing brand j at week t conditional on category purchase is

$$\text{Prob}(B_{it}=j|C_{it}=1)=\exp(v_{ij})/\sum_m \exp(v_{im})$$  \hspace{1cm} (4.2)

Notice that each consumer is heterogeneous in all parameters in equation (4.1) and (4.2) above. The reservation price, $$R_i$$, determines whether the category is expandable or not. For example, if most of consumers in a market have extremely low reservation prices (e.g., $$R_i<0$$ for all i) the price discount of a brand will not increase the category demand. For the simulated low category expansion market condition, $$R_i$$ is assumed to be distributed as log-normal with mean of 0.01 and variance of 0.012 across 5,000 households. For high category expansion, $$R_i$$ is distributed as log-normal with mean of 10 and variance of 0.042.

The inventory parameter, $$\beta_i$$, characterizes whether consumer has enough inventory space in her house to stockpile. For example, a consumer whose $$\beta_i$$ is close to zero may have a large house so that he or she will stockpile the promoted brand. In the simulation, $$\beta_i$$ is assumed to be distributed as log-normal with mean of 0.5 and variance of 0.22. Finally, consumers are also heterogeneous in terms of their brand preferences or loyalty. The distribution of $$\alpha_{ij}$$ will characterize this consumer specific loyalty pattern. We have assumed a market where there are three segments of consumers.

Alternatively, we can assume more complicated continuous distribution (e.g., three variate multivariate normal) for loyalty heterogeneity instead of the discrete distribution assumed here. We adopt the discrete distribution because of its simplicity and its popularity in marketing (Kamakura and Russell, 1989).
percent of consumers are assumed to be brand 1 loyal so that \([\alpha_{11}, \alpha_{12}, \alpha_{13}] = [1,0, -1]\). Another 30 percent of consumers are brand 2 loyal so that \([\alpha_{21}, \alpha_{22}, \alpha_{23}] = [0,1, -1]\). The remaining 40 percent are switchers so that \([\alpha_{31}, \alpha_{32}, \alpha_{33}] = [0,0,0]\).

4.1.2. Quantity Decision

Once the consumer chooses a brand, s/he decides on how many units to buy which is then determined using a truncated Poisson process. We use a "truncated" process because the quantity decision is made conditional on the brand choice decision. In our simulation, if a consumer makes a brand choice decision which is also conditional on the category buying decision, s/he buys at least one unit. Consumer i's probability of purchasing \(x(i,j,t)\) units of brand j at week t can be written as:

\[
\text{Prob}[X(i,j,t)=x(i,j,t)]=\lambda(i,j,t)\frac{e^{-\lambda(i,j,t)}}{x(i,j,t)!}
\]

where

\[
\lambda(i,j,t)=\lambda(i)\exp(\gamma\text{INV}_{t-1}+\delta p_{it})\text{ and }x(i,j,t)=1,2,...
\]

Note that \(\lambda(i)\) is consumer specific (time-invariant) mean purchase rate, \(\text{INV}_{t-1}\) is consumer i's inventory level of the category at week t-1, \(p_{it}\) is the price of brand j at week t.

In the simulation, we set the parameter value of \(\gamma = -1.0\) and \(d = -3.0\) which is constant across consumers and brands. However, consumers are heterogeneous in terms of their mean purchase rate, \(\lambda(i)\), which is assumed to be distributed as log-normal with mean of 10 and variance of 0.2. We have chosen the above parameter values based on whether the resulting mean purchase rate with and without promotions is reasonable. For example, the above parameter values implies that a household with mean \(\lambda(i)\) will buy 1.5 units of a product on average when there is no promotion which means a few households will buy two or three units at the regular price. However, the same household will buy 2.1 units of a product on average if there is a promotion.

4.1.3. Inventory Equation and Consumption

The inventory level at time t for consumer i is updated from the inventory level at time t-1 by adding the current purchase quantity and subtracting the current consumption quantity which implies \(\text{INV}_{it} = \text{INV}_{it-1} + \sum x(i,j,t) - c_{it}\) where \(c_{it}\) is consumer i's consumption rate at time t and \(x(i,j,t)\) is her
quantity purchased of brand j at time t. It is assumed that the consumption rate is $c_t = c$ when $\text{INV}_{t-1} + x(i,j,t) \geq c$ and $c_t = \text{INV}_{t-1} + x(i,j,t)$ when $\text{INV}_{t-1} + x(i,j,t) < c$ which means that the consumer will consume c when the inventory at time t is above c but will only consume the available inventory level if c is less than the on-hand inventory.

In the simulation, the initial inventory level is assumed to be zero for all consumers and $c=0.2$. However, because of the probabilistic nature of our simulation, the inventory level becomes heterogeneous across consumers. In the simulation, we generated 200 weekly sales observations (e.g., the time period t is week) for each consumer and dropped the first 100 weeks to stabilize the inventory level for each consumer and remove the effects of "initial" conditions. The brand's sales for each week is simply the summation of the individual brand purchases.

4.1.4. Promotional Decision of Each Firm

In addition to the high vs low category expansion conditions discussed above, we vary two other market conditions: (1) how frequently and how deep a temporary price reduction to offer, and (2) whether the promotional policy of competitors (brand 2 and 3) is independent of the promotional policy of firm 1.

We did not use the different response parameters for temporary price change and permanent (regular) price change since the regular price is fixed in our simulation. Therefore, the price parameter in our simulation model can be interpreted as the response to the temporary price change. The regular prices of all brands are fixed around $1.00.

Firms are assumed to make two promotional decisions for each quarter in our simulation model: promotional frequency decision (e.g., how many weeks per quarter will price discounts be given?) and promotional depth decision (e.g., if a price discount is given, how much will be offered?). For packaged grocery products, a promotional calendar which is prepared quarterly or annually is commonly used. At the beginning of each quarter, a firm makes promotional frequency and depth decisions which will not be changed in that quarter.

A. Promotional Frequency and Depth: It is generally believed that it is very difficult to estimate baselines if promotions are run frequently. With
frequent promotions, there will be few data points which are not contaminated with promotional effects. In order to see whether promotional frequency makes baseline estimation difficult, we used two promotion schedules, one high promotion frequency and the other low promotion frequency. For the high frequency condition, the (weekly) average promotional frequency of firm 1 is four a quarter (13 weeks) while for the low frequency condition, it is two a quarter. In the simulation, we determined the frequency of promotions for each quarter by generating the value from a Poisson distribution with a mean of 4 for the high frequency condition and with a mean of 2 for the low frequency conditions. Note that monthly promotions (about four promotions per quarter) are common for brands in heavily promoted categories. Similarly, the depth of promotional discount for each quarter is determined by generating the value from normal distribution with mean $=0.25$ and variance $=0.05^2$.

B. Competitive Reaction: We vary the promotional policy of firm 2 and 3 to see its effect on the baseline estimation. For the "independent" condition, firm 2 and 3 are assumed to make their quarterly promotional decisions independent of firm 1. For example, under the high frequency condition, the promotional frequency of firm 2 and 3 is four a quarter on average (also generated by Poisson with mean of 4) independent of the promotional frequency of firm 1. Their promotional depth of discount is also determined independent of the discount depth decision of firm 1.

For the "reaction" condition, however, it is assumed that firms 2 and 3 observe the promotional frequency and depth of firm 1 (both historical and current) and determine their promotional policy. We will use the following linear reaction function for promotional frequency and depth$^5$.

\[
\begin{align*}
\text{FREQ}_{ck} &= \alpha_c + \beta_c \text{FREQ}_{1k} + \epsilon_{ck} \quad c=2,3 \\
\text{ADEPTH}_{ck} &= \alpha_c + \beta_c \text{ADEPTH}_{1k} + \nu_{ck} \quad c=2,3
\end{align*}
\]

$^5$ Similar linear reaction functions have been widely used by political scientists (Axelrod, 1984) and economists (Kalai and Stanford, 1985). In addition, some non-linear reaction functions (e.g., semi-log) were used to simulate the data while linear models were applied to estimate the reaction function. The main results do not change. In order to estimate the baseline accurately, the incorporation of competitive reaction is critical while the choice of functional form for the reaction function is less critical.
where \( \text{FREQ}_{ck} \) is the promotional frequency of firm \( c \) (\( c=2,3 \)) in quarter \( k \) and \( \text{ADEPT}_{ck} \) is the firm \( c \)'s average depth of discount in quarter \( k \). \( \varepsilon_{ck} \) is assumed to be distributed as normal with mean \( =0 \) and variance \( =1 \) and \( \nu_{ck} \) is normal with mean \( =0 \) and variance \( =0.1^2 \). In the simulation reported, \( \alpha_1 \) and \( \alpha_2 \) are assumed to be \( 0 \) and \( \beta_{c1} \) and \( \beta_{c2} \) are \( 1 \). With this specification of parameter values, we implicitly are assuming that competitors tend to match the promotional frequency and depth of firm 1 each quarter.\(^6\)

4. 2. Baseline Estimation Methods

4.2.1. Time Series Based Methods

A. Simple Mean (MEAN): This method estimates the baseline by simply averaging the actual sales across time. This method will work well if a firm is a monopolistic firm and there is no category expansion caused by its promotions. Under this market condition, the additional sales made during promotional weeks comes entirely from future sales or stockpiling since there is neither category expansion (e.g., either consumption increase or additional consumers) nor brand substitution from a lowered price: that is, the additional sales (or stockpiling amount) made during the promotional period are exactly the same as the loss of sales after a promotion.

B. Exponential Smoothing with All Data (EXPOA): The baseline is computed using a first-order exponential smoothing method applied to all data points. The baseline at time \( t \) is computed from \( \text{EB}_t = \alpha S_{t-1} + (1-\alpha) \text{EB}_{t-1} \) where \( S_{t-1} \) is the sales of time \( t-1 \), \( \text{EB}_t \) is the estimated baseline at time \( t \) and \( \alpha \) is a smoothing constant. The market condition under which this method performs well is similar to that of MEAN. For each application, the smoothing constant \( \alpha \) is chosen to minimize \( \sum (\text{EB}_t - S_t)^2 \) to estimate the baseline.

C. Exponential Smoothing without Promotional Weeks (EXPON): The baseline is computed by exponentially smoothing the data after removing any weeks contaminated with the brand's (or item's) own promotions. The elimination of promotional weeks produces many missing data points which make the direct application of exponential smoothing impossible. Therefore,\(^6\)

\(^6\) The main results did not change with different values of \( \alpha_{c1}, \alpha_{c2}, \beta_{c1}, \) and \( \beta_{c2} \).
if the given week is the promoted week, its sales will be replaced by the mean sales of the previous non-promoted and the next non-promoted week. The first-order exponential smoothing method is applied to this modified sales data.

D. Variable Window Weighted Moving Average (VWWMA): This method is a simplified version of the model (PROMOTER) suggested by Abraham and Lodish (1987). This method computes the baseline by first eliminating the points contaminated with promotions and forming a centered window of width p (e.g., p=6 weeks) for the remaining data. The baseline sales for the given week is the mean for this window. Since the promoted weeks are eliminated from the data, the time period used to compute the mean for a given point will be longer than the width p.

Conceptually, VWWMA and EXPON are similar in that both methods eliminate the data points contaminated with the item's own promotion. These methods will work well if competitors make independent pricing decisions or the additional sales from promotions come completely from category expansion.

4.2.2. Regression Based Methods
A. Ordinary Least Square (OLS): The baseline of brand j at time t is computed by first fitting the following regression model to the data.

\[ S_{jt} = \alpha_j + \beta_j R_{jt} + \gamma_j D\text{DEPTH}_{jt} \]  

where \( S_{jt} \) is the unit sales of brand j at time t, \( R_{jt} \) the regular shelf price of brand j at time t, and \( D\text{DEPTH}_{jt} \) is brand j's depth of discount at time t. The baseline, by definition, is equal to \( \alpha_j + \beta_j R_{jt} \) by setting \( D\text{DEPTH}_{jt} \) to be zero. Conceptually, OLS is similar to EXPON and VWWMA because it eliminates the promotional weeks by setting \( D\text{DEPTH}_{jt} \) to be zero.

B. OLS with Lags of Deal Discounts (OLSL): The baseline is computed by

---

7) The complete version of PROMOTER is an iterative procedure which can handle various aspect of sales data such as trend and seasonality. However, VWWMA without iteration may approximate their procedure since our simulation data do not have trend and seasonality.
first fitting the regression model (4.6), and setting DDEPTH\(_{jt-r}\) (r=0,1,2,...) to be zero.

\[
S_{jt} = \alpha_j + \beta_jRP_{jt} + \sum_{r} \gamma_{jt}DDEPTH_{jt-r} 
\]  

(4.6)

The lagged deal discounts are added in this model since sales for the immediate periods after the promotional period can be contaminated when consumers stockpile the brand during the promotional period. One or two lags are applied since more than two lags are not significant for our simulated data. However, the number of lags would be determined empirically.

C. OLSL with Competitive Price Effects (OLSC): The baseline is computed by fitting the regression model (4.7), with setting DDEPTH\(_{jt-r}\) (r=0,1,2,...) to be zero for the own deal discounts and plugging the observed DDEPTH\(_{kt}\) for competitive deal discounts.

\[
S_{jt} = \alpha_j + \beta_jRP_{jt} + \sum_{r} \gamma_{jt}DDEPTH_{jt-r} + \sum_{k} \delta_{k} DDEPTH_{kt} 
\]  

(4.7)

This method appears more sensible than the previous methods because it incorporates competitive promotions in the model. Sales of the brand are influenced by competitive promotional activities as well as own promotions.

D. OLSC with Competitive Reaction Function (OLSR): This model is the same as OLSC except that the competitive depth of discounts, DDEPTH\(_{kt}\) (k=2,3), are replaced by the values calibrated from the reaction function described below. In order to estimate the reaction equation for each brand, we first count the number of price promotion for each quarter and calculate the average depth of discount (when there is a price reduction) in a given quarter.

With the frequency of promotion and the average depth for each brand, the following regressions are applied for brand 2 and 3 (note that we are

8) There are several studies available in estimating reaction function (Lambin, Naert and Bultez, 1975; Hanssens, 1980; Leeflang and Wittink, 1991). However, (weekly) cross correlation analysis proposed in these studies may not be appropriate to identify the pattern of competitive reaction among brands in frequently purchased grocery products. Our experience in this area indicates that we can observe quarterly frequency and depth reaction even though it is very difficult to see the weekly promotional reactions among brands.
interested in the baseline of brand 1) to identify the pattern of competitive reactions.

\[
\text{FREQ}_{ck} = \alpha_{c1} + \beta_{c1} \text{FREQ}_{1k} \quad (4.8a)
\]
\[
\text{ADEPTH}_{ck} = \alpha_{c2} + \beta_{c2} \text{ADEPTH}_{1k} \quad \text{where } c = 2 \text{ and } 3 \quad (4.8b)
\]

From the fitted reaction regression above, we can calculate the quarterly promotional frequency \((\alpha_{c1})\) and average depth of discount \((\alpha_{c2})\) for firm \(c\) when firm 1 does not promote (i.e., \(\text{FREQ}_{1k} = 0\) and \(\text{ADEPTH}_{1k} = 0\)). However, we do not know the exact promotion schedule without the promotion of brand 1. We cannot determine which specific weeks firm \(c\) will offer the deal from the quarterly promotional frequency \((\alpha_{c1})\). Therefore, we introduce a concept of expected \(\text{DDEPTH}_{ck}\) which will be computed by multiplying the weekly probability of deal \((\alpha_{c1}/13)\) and the average depth of discount if dealt \((\alpha_{c2})\).9

OLSR is different from OLSC in that it utilizes the pattern of competitive reaction. It calibrates the expected depth of discounts for competitive brands when the own brand, brand 1, does not promote. Therefore, if competitors make independent promotional decisions, OLSR becomes the same as OLSC. However, if competitors react to the promotional decision of our brand, the expected depth of discounts for competitive brands determined in the above equation will be different from \(\text{DDEPTH}_{kt}\) so that these expected depth of discounts should be used in computing baseline sales of brand 1. Note that the baseline of a brand is the sales when the brand is not promoted. When the brand does not promote, competitors will react and hence the expected depth of discounts for competitive brands should be used to compute the baseline of brand 1.

9) Alternatively, the promotion schedule can be simulated with the weekly probability of deal occurred of \(\alpha_{c1}/13\) and the depth of discount of \(\alpha_{c2}\). For each simulation, we generate one possible promotion schedule of firm 2 and firm 3, and estimate the baseline of firm 1. The final baseline can be calculated by averaging over the baselines computed from a number of simulated promoted schedule. This simulated procedure (100 simulations used) has produced very similar baseline to the baseline computed by the expected \(\text{DDEPTH}_{kt}\) above.
4.3. Simulation Results

As mentioned, eight baseline estimation methods are compared for 8 different market conditions. Tables 1 and 2 show the performance of each method for each market assumption. Moreover, Figure 1 to 4 show the actual sales of firm 1, corresponding true and estimated baselines for a subsample of various market conditions. Given the space limitation, we will report the estimation results for firm 1. The computation and result of the other firms are similar to those of firm 1.

Comparing the high and low frequency promotion conditions, Tables 1 and 2 show that all methods produce more accurate baselines when there are few promotions. Both the bias and MAPE are smaller. This result intuitively makes sense since more data are contaminated with promotions under the high frequency promotional condition resulting in less "normal periods" to estimate the baseline. At the extreme, when there are no promotions during the entire data period, the actual sales become the baseline.

Second, all methods tend to work better when there is low category expansion. In our simulation framework, promotions increase a brand's sales

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<th>Low Category Expansion</th>
<th>High Category Expansion</th>
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<tr>
<td></td>
<td>High Freq</td>
<td>Low Freq</td>
</tr>
<tr>
<td>MEAN</td>
<td>0.37^1</td>
<td>0.22</td>
</tr>
<tr>
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^1 The top value (0.37) represents MAPE and the value on the bottom (0.35) represents Bias.
Table 2
Performance of Each Estimation Method When Competitors React

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<th>High Category Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Freq</td>
<td>Low Freq</td>
</tr>
<tr>
<td>MEAN</td>
<td>0.10(^1)</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.04</td>
</tr>
<tr>
<td>EXPONA</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.03</td>
</tr>
<tr>
<td>EXPON</td>
<td>0.21</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>-0.21</td>
<td>-0.09</td>
</tr>
<tr>
<td>VWWMA</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>-0.21</td>
<td>-0.09</td>
</tr>
<tr>
<td>OLS</td>
<td>0.22</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>-0.22</td>
<td>-0.09</td>
</tr>
<tr>
<td>OLSL</td>
<td>0.18</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>-0.18</td>
<td>-0.07</td>
</tr>
<tr>
<td>OLSR</td>
<td>0.23</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>-0.21</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

\(^1\) The top value (0.10) represents MAPE and the value on the bottom (0.10) represents Bias.

Through: (1) brand switching, (2) stockpiling, and (3) category expansion or additional new buyers. The ideal baseline estimation procedure would estimate these three effects separately so that it can compute the baselines correctly. However, it is very difficult to separate these components with market-level aggregate data. Under the low category expansion condition, the baseline is easier to estimate since promotional sales increases come mainly from two sources (e.g., brand switching and stockpiling) instead of three.

Third, **OLSR tends to be the best estimation method under all 8 market conditions.** The unique component of OLSR is its consideration of the competitor's reaction function. It uses the expected competitors' promotions given the promotional schedule of firm 1. Note also that when competitors make their promotional decisions independently, the performance of OLSC is almost identical to that of OLSR. When firm 2 and 3 make their promotional decision independently, the expected competitive deal depth given no price discount of firm 1 is almost the same as the currently observed deal depth.

Fourth, MEAN and EXPOA work well when there is **low category expansion.**
sion and competitors react. We have assumed a competitor's reaction function such that competitors tend to match the promotional frequency and depth of firm 1. Therefore, when firm 1 does not promote (baseline of firm 1), the other firms tend to charge the regular price because of the above reaction function. This reaction function with no category expansion explains why MEAN and EXPOA performs well. The (cooperative) optimal strategy for all firms when there is no market expansion is to avoid promoting. When competitors react, the average sales becomes constant because in the "no category expansion" case, the average (across weeks) sales of firm 1 without any promotions is the same as its sales with promotion (also, see Figure 1). MEAN and EXPOA estimate the baseline with reasonable accuracy. In contrast, when there is large category expansion with promotion and competitors do not react, these two methods become the worst. They do, however, help improve our intuition about how baselines change as market characteristics change.

Fifth, EXPON, VWWMA, OLS, and OLSL are conceptually similar to each other because they eliminate the data points contaminated with promotions. These methods do not work well for various market conditions. When competitors do not react. Notice that when competitors make their promotional decisions independent of the promotions of firm 1, they will continue to promote even if firm 1 stops promoting. Therefore, when firm 1 promotes, its volume is incremental and traditional methods such as OLS capture the baseline. This intuitive explanation is confirmed by the result that these methods do not perform well when competitors react but do well when competitors do not react.

Sixth, OLSC turns out to be the one of the worst methods when competitors react because it always underestimates the baseline. However, it works well when competitors do not react. OLSC assumes that competitors will not stop promoting even when brand 1 stops promoting. This suggests that when competitors react, it is not recommended to include competitor's prices in the baseline estimation model unless the reaction function is also used. This is an interesting and surprising result.

The overwhelming performance of OLSR over other methods described above can be seen more clearly on Figures 1 to 4. The figures show the cases in which promotional frequency is high while they differ in terms of two
Figure 1. Low Category Expansion and Reaction

Actual sales vs. Baseline

True vs Estimated Baseline
Figure 2. High Category Expansion and Reaction

Actual sales vs. Baseline

True vs Estimated Baseline
Figure 3. Low Category Expansion and No Reaction

Actual sales vs. Baseline

True vs Estimated Baseline
Figure 4. High Category Expansion and No Reaction

Actual sales vs. Baseline

True vs Estimated Baseline
market conditions, category expansion and the existence of competitive reaction. Figure 1 has two plots: the top represents simulated actual sales and the corresponding true baseline and the bottom shows the true baseline with estimated baselines from OLS, OLSR and VWWMA. Notice that OLSR estimates the true baseline very accurately while other methods significantly underestimate it\textsuperscript{10}. Figure 2 shows the case in which the market is expanded and competitors react. The results are similar to Figure 1. Notice that the sales during promotion is much higher than the previous figure. The additional sales during promotional period comes from category expansion as well as brand switching. Notice also OLSR approximates the true baseline reasonable well.

Figure 3 and 4 show the results when competitors do not react. Note that the true baseline itself is very volatile when competitors do not react. The ups and downs of the true baseline come from competitor's promotional activities. OLS misses these ups and downs causing high MAPE but low bias. However, both OLSC and OLSR recovers these ups and downs by including competitive variables in the estimation model. The performance of OLSC and OLSR is almost identical because there are no competitive reactions. Figure 4 shows similar results under high expansion condition.

V. APPLICATION: COMPUTING STORE-LEVEL BASELINE LINES

As shown in the simulation, one of the key issues in estimating baselines is the reaction function of competitors to a firm's promotional decision. In this section, we will show there exist positive promotional reactions among competitors in a real market so that the baseline using methods without considering competitive reactions (OLS or OLSL) is significantly underestimated. The data set to be analyzed has been supplied by Dominick's Finer Food Co., a major grocery chain in Chicago which owns more than 80 stores in metropolitan Chicago area. To simplify the reporting of the results, a sub-

\textsuperscript{10} It is interesting to observe that VWWMA underestimates the true baseline for the entire period and the magnitude of bias is positively correlated with the frequency of promotions (e.g., VWWMA's very low baseline during the end of the data period with very frequent promotions).
urban store is chosen for the analysis\textsuperscript{11}. Three product categories which cover 156 weeks will be studied: bathroom tissue, frozen orange juice, and refrigerated orange juice.

5.1 Baseline for Bathroom Tissue

There are four major brands for bathroom tissue category: Northern, Cottonelle, Charmin, and White Cloud. We focus our attention on Northern, the market leader. A simple method of estimating the baseline for Northern is OLS. The result of the estimation with standard errors in parenthesis is

$$S_{\text{Nor},t}=212.6+1925.1 \text{ DDEPTH}_{\text{Nor},t}$$

$$\text{Adj-}R^2=0.51$$

where $S_{\text{Nor},t}$ is the unit sales of Northern in week $t$, DDEPTH$_{\text{Nor},t}$ is the depth of deal discount for Northern in week $t$. Note that we did not include the regular price of Northern and the lagged values of discount depth since they are not significant. The OLS baseline of Northern is 212.6 for the entire period which is computed by setting DDEPTH$_{\text{Nor},t}$ to be zero.

To determine the baseline of Northern using OLSR, the first step is to estimate a model in which sales of Northern are related to the discount depths of other brands as well as Northern. The estimation result is written as

$$S_{\text{Nor},t}=246.4+1905.7 \text{ DDEPTH}_{\text{Nor},t}-408.8 \text{ DDEPTH}_{\text{Cott},t}$$

$$\text{Adj-}R^2=0.59$$

where DDEPTH$_{\text{Cott},t}$ is the depth of deal discount for Cottonelle in week $t$. Note that we did not include the depth of deal discount for White Cloud and Charmin since they were not significant. As explained in simulation section, OLSR baseline for Northern can be computed by setting DDEPTH$_{\text{Nor},t}$ to be

\textsuperscript{11} The same approach can be applied to the chain-wide data by using dummy variables for the intercept of each store.
zero for the entire period and replacing $DDEPTH_{COTT,t}$ by the expected depth of Cottonelle discount at week $t$. If Cottonelle makes an independent promotional decision relative to Northern, we may have to use the historically observed depth of discount for $DDEPTH_{COTT,t}$. However, we should use the adjusted value of $DDEPTH_{COTT,t}$ if Cottonelle changes its promotional decision based on the promotional decision of Northern.

To determine the interrelationship between Cottonelle's promotional activity and Northern's, simply correlating when the promotions occur does not prove informative because retailer's do not run promotions for two brands in the same week. To overcome this problem we counted the number of price deals for Cottonelle and Northern for each quarter as well as computed their average deal depth in order to see whether Cottonelle reacts to Northern in its promotional decisions. The following simple regressions show the positive reaction of Cottonelle to Northern in terms of both the frequency of the price deals and the average depth of these deals.

\[
\begin{align*}
FREQ_{COTT,k} &= 0.81 + 0.62 \, FREQ_{NOR,t} \\
\text{Adj-}R^2 &= 0.43 \quad (5.3a) \\
FREQ_{COTT,k} &= 0.01 + 1.33 \, ADEPTH_{NOR,t} \\
\text{Adj-}R^2 &= 0.75 \quad (5.3b)
\end{align*}
\]

where $FREQ_{i,k}$ is the frequency of temporary price reduction of brand $i$ ($i=$ Cottonelle, Northern) in quarter $k$ and $ADEPPTH_{i,k}$ is the average depth of discount of brand $i$ in quarter $k$ when the brand $i$ is offered a price discount. Clearly the promotion for Cottonelle and Northern are correlated.

What causes each manufacturer to react to the other's promotional decision (e.g., temporary price reduction)? The answer is the lead time offered by the sales force to the buyers so that the buyers can "forward buy" when a promotion occurs. The sales force provides this information to avoid adverse reaction from the retailer's buyers. Because the buyer obtains this information in advance, they also communicate it to competitors in order to learn if they are planning similar promotions. Because of this communication, which occurs before the promotions are offered to the consumer, the manufacturer learns of competitor's promotion and matches them because it is optimal to avoid being in the off-diagonal cell in the "prisoners' dilemma". In
Figure 5. OLS and OLSR Baseline for Northern Bathroom Tissue
addition, a promotional schedule is usually set quarterly basis so that the weekly (buying) promotions are for the given quarter once the promotional schedule is made.

The above reaction equations suggest that if Northern does not promote the quarterly frequency of Cottonelle's promotion is 0.81 and the average depth is only 0.01 when a promotion is offered. In order to compute the OLSR baseline, which assumes that Northern does not promote, we use the weekly expected deal depth for Cottonelle (DDEPTHTCOTT) which is 0.0007 (0.81x0.01813). OLSR baseline of Northern is 246.1 which is 15 percent higher than the OLS baseline given in equation (5.1) with DDEPTHNOR set equal to zero. This underestimation of OLS can be seen more clearly on Figure 5 where OLS and OLSR baselines are plotted with the actual sales of Northern. OLS baseline is underestimated so that the incremental volume computed is overestimated.

5.2 Baseline for Refrigerated Orange Juice

There are three major national brands for refrigerated orange juice category: Tropicana, Minute Maid and Citrus Hill. We focus our attention on Tropicana, the market leader. The result of OLSL baseline estimation equation is

\[ S_{TROP,t} = 156.6 - 41.6R_{TROP,t} + 249.4DDEPTHTROP_t - 89.1DDEPTHTROP_{t-1} \]

(5.3) (20.7) (21.5) (21.6)

\[ Adj-R^2 = 0.47 \]

(5.4)

where \( R_{TROP,t} \) is the regular price of Tropicana in week \( t \). We use OLSL in which the first lag of deal depth is included. Differing from the case of bathroom tissue, the equation has the significant regular price coefficient. The OLSL baseline of Tropicana for week \( k \) can be determined by setting the value of deal depth at week \( t \) and \( t-1 \) to be zero and using the observed regular price at week \( t \), which is 156.6 - 41.6 \( R_{TROP,t} \). Notice the baseline changes weekly depending upon the regular price level for this product category.

The OLSR baseline can similarly be determined except that sales of
Tropicana are related to the deal depth of other competitive brands as well as Tropicana. The estimation result is written as

\[
S_{TROP,t} = 138.6 - 25.1 R_{TROP,t} + 246.4 D_{DEPTHTROP,t} - 70.4 D_{DEPTHTROP,t-1} - 70.4 D_{DEPTHTROP,t-2}
\]

(52.3) (22.5) (20.9) (21.6) (21.5)

\[\text{Adj}-R^2 = 0.51 \quad (5.5)\]

where \(D_{DEPTHTROP,t}\) is the depth of deal discount for Minute Maid in week \(t\). Note that we did not include the depth of deal discount for Citrus Hill since it was not significant. The next step is to calibrate the adjusted value of \(D_{DEPTHTROP,t}\), which depends on whether Minute Maid reacts to the promotional decisions of Tropicana. The reaction equations for the quarterly frequency and the average deal depth are estimated as

\[
\begin{align*}
\text{FREQ}_{MM,k} &= 3.01 + 0.52 \text{FREQ}_{TROP,k} \quad \text{Adj}-R^2 = 0.21 \quad (5.6a) \\
\text{ADEPTH}_{MM,k} &= 0.18 + 0.60 \text{ADEPTH}_{TROP,k} \quad \text{Adj}-R^2 = 0.51 \quad (5.6b)
\end{align*}
\]

(1.44) (0.27) (0.10) (0.20)

The above reaction equations suggest that the quarterly frequency of Minute Maid's promotion is 3.01 if Tropicana does not promote and the average depth when promoted is 0.18. Therefore, the weekly expected deal depth or the adjusted \(D_{DEPTHTROP,t}\), is 0.04 \((3.01 \times 0.18 ÷ 13)\). OLSR baseline of Tropicana is 135.8 - 25.1 \(R_{TROP,t}\). As shown in figure 6, the OLSR baseline is always higher than the OLSL baseline. On average regular price of $2.39, for example, the OLSL baseline is 56.7 which is 35 percent lower than the OLSR baseline of 75.8. Again, the OLSL baseline, which does not incorporate competitive reaction, is underestimated.

5.3 Baseline for Frozen Orange Juice

The third product category investigated is frozen orange juice which has Tropicana, Minute Maid and Citrus Hill as major national brands. We focus our attention on Tropicana, the market leader. The result of OLSL baseline estimation equation is
Similar to the refrigerated orange juice, we use OLSL in which the first lag of deal depth is significant. However, the regular price is not significant so that it is not included in the regression. The OLSL baseline of Tropicana for week k is 57.9 when the value of deal depth at week t and t-1 is set to be zero in computing the baseline.

The OLSR baseline was similarly be determined except that sales of Tropicana are related to the deal depth of other competitive brands as well as Tropicana. However, none of the deal depth of other competitor (Minute Maid and Citrus Hill) were significant, implying that the promotions of competitors do not influence the unit sales of Tropicana. This implies that the baseline of Tropicana stays the same whether other competitors react to the promotional decisions of Tropicana or not. In other words, OLSR baseline is the same as the OLSL baseline for this product category.

VI. SUMMARY

In this paper we have defined baseline sales and discussed its implications, showing that the incorporation of competitive reaction function is crucial to the measurement of the baseline. In addition, we discussed how a brand manager who manages a broad product line can use our estimation procedure of baseline sales to properly measure the incremental profitability of a brand’s promotion.

In a real market, it is not unusual to observe that each brand monitors and reacts to the promotions of other brands. As shown in the previous section, competitors react to the promotional decisions of the firm for all three product categories. When a firm increases the quarterly frequency of promotions, competitors will also increase their frequency. Moreover, this

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12) In fact, both Minute Maid and Citrus Hill positively react to Tropicana in terms of the frequency of deal discount. However, they do not react in terms of the average of deal depth.
positive reaction among competitors is observed in terms of the depth of the deal discount.

The importance of incorporating a competitor’s reaction function is shown by a simulation in which several different baseline estimation methods are compared under various market conditions. In the simulation, we have shown that the competitive reaction function is critical to accurately measuring the baseline. We have applied our concept of competitive reaction to real data and showed that the baseline results of incorporating competitive reaction into baseline estimation results in significantly different estimates than methods which do not incorporate the competitive reaction function.

When setting promotions for a brand with key items which cannibalize other items within the brand or from other brands controlled by the firm, the baseline should be computed assuming that the promotions of cannibalizing items or brands in the product line are set equal to zero. Otherwise, the incremental profits from promotions will be too high and again the firm will misallocate resources.

In summary, baseline estimation poses many interesting modeling and game theoretic issues. The art and science of baseline estimation is in its infancy and we hope this paper will help other researchers working on this problem. For real-world practitioners, this paper has identified issues associated with the case of baselines and has emphasized the importance of understanding the assumptions underlying baseline estimation and the potential pitfalls of not recognizing potential biases in the baselines currently being estimated.

REFERENCES


