Fixed vs. Usage-Based Pricing: Choice of Pricing Schemes and Optimal Profit Allocation in the Online Content Industry

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Abstract

One of the core debates of the online economy has been whether a fixed, one-time access fee or a usage-dependent fee would yield greater return for each player of the online economy. This paper develops a simple model of the online economy consisting of a single Content Provider (CP) and a single Network Provider (NP) in order to provide an insight to this issue.

Three characteristics related to the type of content are identified as major determinants of level of demand; marginal utility of a unit content consumption for the marginal consumer, concavity of utility functions, and denseness of type distribution. Higher utility of consumption, coupled with less dense type distribution, leads to price insensitivity of the demand function in terms of number of subscribers. However, a highly concave utility function is required for price insensitivity of consumer consumption amount. These content characteristics need to be considered in designing an optimal pricing strategy for an online player.

A numerical example illustrates that all fixed pricing might not be optimal, not only for the purpose of social welfare maximization, but also for the purpose of consumers’ surplus maximization. It also highlights the importance of the profit sharing rule in order for a price alliance to be reached.

Keywords: fixed pricing, usage-based pricing, profit sharing rule

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INTRODUCTION

Since the rise of the online economy, pricing of various online services such as database access, webzines, online gaming, and network access has been the focal issue for commercial entities, scholars, and ultimately, consumers. One of the core debates has been whether a fixed, one-time access fee or some form of usage-dependent fee such as a connection-time based fee or packet-consumption based fee would yield greater return for each player of the online economy (Varian 1993; Fishburn, Odlyzko and Siders 1997). In the case of network infrastructure pricing, scholars have long taken the position that some form of usage-based pricing is socially more efficient than a fixed pricing practice, since it results in more efficient usage of limited network resources (Varian 1993). Nevertheless, market forces have led to a prevalence of fixed-pricing practice. The dominance of the fixed-pricing format observed in the online industry is attributed to two forces: almost unanimous preference by the consumer population (Odlyzko 1996) and fierce competition in both the network access market and digital contents and services markets.

Proponents of a fixed pricing format over a usage-based format have argued that usage-based pricing schemes will discourage service usage of online consumers for a number of reasons. First, online consumers are accustomed to fixed and free pricing schemes for historical reasons and high market competition, which make them highly price sensitive. Second, in an open network such as the Internet, consumers cannot foresee the amount of expected usage accurately, which can be perceived as a risk factor. Such discouragement of potential online consumers will lead to a slowdown in the growth of the online economy. As a result, imposition of a usage-based pricing scheme will reduce consumers’ surplus due to discouraged usage and higher perceived risk.

In this paper, a comparison between fixed and usage-based pricing in the online industry context is presented. The online industry is assumed to consist of content and network layers. The network layer represents the connection service for users whose service is provided by the network provider (NP) whereas
the content layer consists of players that provide various value-added services on top of the network layer, whose service is to be provided by the content provider (CP). In this setting, a vertical price alliance between a CP and an NP can cause a difference in efficiency for each pricing format. The pricing schedules of the CP and the NP determine the total cost consumers have to incur in content consumption activity. With a price alliance, two players can coordinate the level of respective prices in a way that can maximize the joint profit. Without such coordination, each player will set the respective price so as to maximize the individual profit. A comparison between alliance and non-alliance cases will also be explored.

In order for two players to agree and to reach price alliance agreement, a proper rule for profit allocation needs to be developed to avoid at least the following two risks. First, a risk of ambiguity as to profit sharing is always a source of conflict. Second, one of the participants of the alliance might receive a lower profit level than the pre-alliance level. A numerical example illustrates that utilizing concepts of core and Shapley value can prevent such risks.

The Model section presents the consumer utility model of this paper, which closely follows literature on telecommunications demand. The Analysis section explores qualitative differences among market mechanisms in four market environments depending on whether the content and network service are priced based on fixed or usage-dependent pricing. Then, Numerical Analysis section presents a numerical example providing additional results that complement those of Analysis section. It also addresses differences in consumer and total surplus according to the price scheme of each player. One possibility suggested here is that an all-fixed pricing environment (in which both content and network service are charged by fixed prices) does not yield the maximum level of consumer surplus compared to that from other pricing cases involving usage-based price component. This result directly conflicts with the claim by many online users that usage-based pricing undermines consumer benefits. This section also derives profit allocation corresponding to core and Shapley value rules, which are needed for an alliance to be sustainable. Discussion section examines a few representative online market and discusses pricing practices
in each market in association with this paper’s analytical results. Finally, Conclusion and Future Research section provides a summary and discusses limitations and possible extensions of the paper.

THE MODEL

This paper assumes one of the simplest possible forms of an online industry structure consisting of only a single content provider (CP) and a single network provider (NP). Each player is assumed to have a choice of either fixed or usage-based charge, leading to a total of four combinations of pricing schemes. Analysis of such a simplified market structure can be useful in the following context. First, a specialized content network may be dedicated to servicing a particular CP. Examples of such specialized content delivery networks include the Video-on-Demand (VoD) service network tested by major telcos in the early to mid 1990s (Johnson 1993; Han 1996) and specialized database networks such as LEXIS-NEXIS. Second, there may be a single provider of collection of contents and the CP imposes a single pricing scheme that depends only on the amount of content consumption by users. Then, the basket of contents can be treated as a single product provided by a single CP.

The unit of charge in the usage-based pricing can differ according to the market context and the feasibility of the billing system. The most common practice of usage-based pricing is to charge a fee proportional to the amount of usage time. In other cases, the content usage may be measured according to different criteria such as the number of articles users access. The difference among these units for charge under the usage-based pricing scheme can usually be accommodated trivially by converting the units for charging under different schemes. In this paper, the usage-based pricing case is examined with time-based usage charges for both network service and content consumption.

Following a widely employed assumption in literature on two-part tariffs (Coase 1946; Oi 1971; Littlechild 1975; Schmalensee 1981), a general model of consumer decisions on content consumption activity is presented in this section. The
heterogeneity of consumers is represented by a one-dimensional continuum of *type parameter* $v$. Consumers of the same type group are assumed to exhibit the same content consumption pattern.

Each consumer is further assumed to determine whether to subscribe to the network service and to purchase the content, as well as the optimal amount of usage according to the following two-stage model. In the first stage, each consumer observes whether the expected net benefit at the optimal content consumption amount from the second stage is positive. If this net benefit is positive, the consumer subscribes to the network service and purchases content, and vice versa. In the second stage, with the assumption that the consumer has already subscribed to the network service, each consumer decides on the optimal amount of content consumption so as to maximize the net utility.

Let $x$ be the amount of time consumers spend in the content consumption activity (and therefore, the amount of time spent online). A consumer belonging to type $v$ has a utility function, $U(x, v)$. Each consumer type $v$ maximizes the net benefit $U(x, v) - p(x)$, where $p(x)$ is the total consumer’s cost associated with the content usage level of $x$. We denote the optimal level of content usage from the net benefit maximization problem by $x^*$. Then, $x^*$ will be dependent on the usage-based portion of the consumers’ cost function, $p(x)$. Let $p_f^c$ and $p_u^c$ represent fixed and usage-based price, respectively, charged by the CP and $p_f^N$ and be those charged by the NP. For example, if the content is charged by the amount of usage whereas the network service price is fixed so that $p(x) = p_f^c + p_f^N$, the type $v$ consumer’s usage demand function will be $x^*(p_c^v, v)$ As long as there is any usage-dependent component in the total consumer’s price, each consumer type’s optimal usage level will be dependent on the usage-based component of the total price.

The following assumptions on the properties of the utility function have been frequently made in the telecommunications pricing literature (Wilson 1993).

**Assumption 1.** The type parameter $v$ is distributed according to the distribution function $F(v)$, which is continuous and differentiable in $[0, \bar{v}]$. 

Assumption 2. Utility increases as more content is consumed and diminishing marginal return of content usage applies, i.e., \( \frac{\partial U(x,v)}{\partial x} > 0 \) and \( \frac{\partial^2 U(x,v)}{\partial x^2} < 0 \).

Assumption 3. Higher type consumer derives higher utility and higher marginal utility from content consumption at all levels of consumption, i.e., \( \frac{\partial U(x,v)}{\partial v} > 0 \) and \( \frac{\partial^2 U(x,v)}{\partial x \partial v} > 0 \).

Assumption 1 and assumption 2 establish that each consumer’s utility function is continuous, differentiable, and concave. Assumption 3 says that consumers with a higher type parameter receive higher utility from content consumption activity at any level of usage so that the utility functions of different types of consumers do not cross at any ranges of usage level. This no-crossing property of the utility function guarantees that the consumer with the higher type parameter will demand more than the lower type consumer at all price schedules, as figure 1 illustrates (Wilson 1993). For simplicity, the responsiveness of the utility function to the change in the type parameter, \( \frac{\partial U(x,v)}{\partial v} \), is referred to as the “denseness of utility functions” in the sense that if utility functions across types of consumers are more densely distributed (i.e., if \( \partial U(x,v)/\partial v \) is small), an increase in the

Figure 1. Determination of Optimal Usage Level.
type parameter will cause a small increase in the utility level at the same usage level. In other words, if the incremental utility at a given consumption level between successive users along the type parameter is small, the market is characterized by a “dense” utility function. In fig. 1, such incremental utility can be represented by a vertical distance between $U(x, v)$ and $U(x, v')$.

In figure 1, each customer determines the usage level so as to maximize the net benefit, $U(x, v) - p(x)$. At this optimal usage point, the slope of $U$ and $p(x)$ are equal, following the rule that marginal utility equals marginal cost.

**ANALYSIS**

**Fixed Content and Fixed Network Pricing**

When both prices are fixed, a consumer’s net benefit maximization problem will not be bounded by the price constraint as long as the marginal utility from content consumption is nonnegative. It will, however, be bounded by other resource constraints faced by consumers, such as a time constraint. As long as there exists any binding resource constraint (which will always be the case in reality), the net benefit maximization problem will yield a finite usage level, $x^*$, for each consumer. Such a “loosely bounded” benefit maximization problem is likely to yield a much larger optimal usage amount than implied by other pricing environments that include any usage-based components. Nevertheless, demand will still be dependent on the type parameter due to assumption 3. On the other hand, the fixed price component will not affect the customer’s usage optimization problems. Thus, the customer type $v$’s demand function will only be dependent on the type parameter, and thus can be expressed as $x^*(v)$.

A marginal consumer $\hat{v}$ who is indifferent between subscribing and not subscribing and faces a fixed content and network price is determined by the relationship

$$U(x^*(\hat{v}), \hat{v}) = p_{C} + p_{N}$$

Consumers with positive net benefit at the offered price levels
will subscribe to the network service and purchase the content. The fraction of the total population that subscribes to both the content and the network service is therefore obtained by aggregating consumers with positive net benefit. Let $D_C$ and $D_N$ be the fraction of the total population that subscribes to the content and the network service, respectively. Then,

$$D_C = D_N = \int_{\hat{v}}^{\infty} dF(v) = G(\hat{v})$$  \hspace{1cm} (2)$$

where $G(v) = 1 - F(v)$.

Then, the actual amount of network usage in the market is derived by aggregating usage levels of individuals.

$$X = \int_{\hat{v}}^{\hat{v}} x^*(v) \ dF(v).$$  \hspace{1cm} (3)$$

The determination of the rank order of the marginal consumer as per (1) affects the demand size for the CP and the NP.

**Proposition 1.** Under all fixed pricing environment, a unit increase in price will cause large reduction in number of subscribers when consumer utility function is densely distributed.

**Proof.** All proofs are in Appendix A.

Proposition 1 says that if utility functions are “dense” in a sense that consumer distribution density is high around the given level of type parameter, an increase in the price level will cause a rather large change in the marginal consumer’s rank, thereby leading to large decrease in number of subscribers. Therefore, when utility functions are densely distributed along the type parameter so that an incremental utility between higher type consumer and a lower type one is small, players should be careful in increasing the price level since it can trigger a large increase in the rank of the marginal consumer.

With fixed content and network pricing, the respective profit maximization problems for the CP and the NP are

CP’s problem: $\max_{\pi_c} \pi_c = p_C^f G(\theta(p_C^f, p_N^f))$  \hspace{1cm} (4)
NP’s problem: \[ \max_{\hat{\pi}_N} \pi_N = \hat{p}_N G(\hat{v}(p_C, p_N)) \] (5)

where \( \hat{v}(p_C, p_N) \) is implicitly determined by (1).

Taking the first-order conditions of (4) and (5) and substituting (A1) yields

\[ \frac{\partial \pi_C}{\partial p_C} = G(\hat{v}) + p_C^f G' \frac{1}{\partial U(x^*(\hat{v}), \hat{v}) / \partial \hat{v}} \] (6)

and

\[ \frac{\partial \pi_N}{\partial p_N} = G(\hat{v}) + p_N^f G' \frac{1}{\partial U(x^*(\hat{v}), \hat{v}) / \partial \hat{v}} . \] (7)

From (6) and (7), the CP and the NP set respective prices such that the rate of increase in the profit due to the higher price is offset by the decrease in the profit due to the forfeiture of those consumers dropping their subscriptions. The rate at which consumers withdraw subscription is inversely proportional to the change in the utility attained at the optimal usage level of the marginal consumer. Again, less dense (or “sparse”) utility functions will make the magnitude of the second terms in (6) and (7) small, resulting in higher optimal price for both players.

The CP and the NP involved in a price alliance will solve the following joint profit maximization problem.

\[ \max_{p_C, p_N} \pi_J = (p_C^f + p_N^f) G(\hat{v}(p_C^f, p_N^f)) \] (8)

Optimality conditions with respect to \( p_C^f \) and \( p_N^f \) yield the following redundant equations.

\[ \frac{\partial \pi_J}{\partial p_C^f} = \frac{\partial \pi_J}{\partial p_N^f} = G(\hat{v}) + (p_C^f + p_N^f) G' \frac{1}{\partial U(x^*(p_C^f + p_N^f), \hat{v}) / \partial \hat{v}} \equiv 0 \] (9)

**Proposition 2.** Consumers face lower total cost of content consumption under price alliance arrangement when both the CP and the NP employ fixed pricing schemes. As a result, total consumers’ and producers’ surplus will both increase under
price alliance in all fixed pricing environment.

Notice, however, that which of the two players will generate more profit from the alliance case is uncertain without a predetermined profit sharing rule.

Usage-Based Content Pricing and Usage-Based Network Pricing

Under usage-based pricing for both content and network service, the marginal consumer satisfies

\[
U(x_*(p^u_C, p^u_N, \bar{v}), \bar{v}) = (p^u_C + p^u_N)x^*(p^u_C, p^u_N, \bar{v}) \tag{10}
\]

and the total content consumption by all subscribers, which coincides with the total network service usage, is

\[
X = \int_{0}^{\bar{v}} x^*(p^u_C, p^u_N, v)dF(v). \tag{11}
\]

**Proposition 3.** Under all usage-based pricing environment, a unit increase in price will cause large reduction in number of subscribers when consumer utility function is densely distributed and when subscribers consume large amount of content.

The numerator of (A2) can be seen as a calibrating factor that explains how the marginal consumer determination under usage-based pricing is different from that of fixed pricing. The numerator represents the change in the customer’s cost due to the change in the optimal usage level triggered by the increase in price. With a unit price increase, customers face higher cost by the amount of the optimal usage level. It represents the change in the marginal consumer’s net benefit in the event of a price increase. If the extent of fall in the marginal consumer’s net benefit in response to the price increase is large, the rank order of the marginal consumer will respond sensitively to the change in price. From this, the impact of a change in usage-based price on number of subscribers is large if consumers’ optimal usage amount is large. On the other hand, the effect of the denseness of utility function to usage-based price component can be interpreted similarly as in the all-fixed pricing environment.
Proposition 4. Under all usage-based pricing environment, a unit increase in price will cause large reduction in number of subscribers when the extent of decrease in marginal utility of an additional unit of content consumption is small.

The change in the optimal usage level of each consumer in response to a change in the usage-based price is dependent on the concavity of the utility function. If the extent of decrease in marginal utility of an additional unit of content consumption is large (i.e., if it is highly concave), the sensitivity of consumers' usage levels to a price change will be small.

Proposition 5. Under all usage-based pricing environment, a unit increase in price will cause large reduction in total amount of content consumption when consumer utility function is densely distributed, when subscribers consume large amount of content and when the extent of decrease in marginal utility of an additional unit of content consumption is small.

With usage-based content and network pricing, the respective profit maximization problems are

CP's Problem: \[ \max_{p_C^u} \pi_C = p_C^u X(p_C^u, p_N^u, \hat{v}) \] (12)

NP's Problem: \[ \max_{p_N^u} \pi_C = p_N^u X(p_C^u, p_N^u, \hat{v}) \] (13)

where \( \hat{v}(p_C^u, p_N^u) \) is implicitly determined by (11).

Substituting (A6) and (A7) into the first-order conditions for (12) and (13) results in the following.

\[
\frac{\partial \pi_C}{\partial p_C^u} = X(p_C^u, p_N^u, \hat{v}) + p_C^u \left[ \int_{\bar{v}} \frac{\partial x^*(p_C^u, p_N^u, v)}{\partial p_C^u} dF(v) \right] \tag{14}
\]

\[
\left| - x^*(p_C^u, p_N^u, \hat{v}) f(\hat{v}) \frac{\partial \hat{v}}{\partial p_C^u} \right| = 0
\]

\[
\frac{\partial \pi_C}{\partial p_N^u} = X(p_C^u, p_N^u, \hat{v}) + p_C^u \left[ \int_{\bar{v}} \frac{\partial x^*(p_C^u, p_N^u, v)}{\partial p_N^u} dF(v) \right] \tag{15}
\]
Again, less concave and denser utility functions will lead to a larger reduction in the amount of usage in response to a price increase, leading to a lower optimal price level.

In the alliance case, the joint profit maximization problem is

\[
\text{Allying firm's problem: } \max_{p_C, p_N} \pi_J = (p_C^u + p_N^u)X(p_C^u, p_N^u, \hat{v})
\]

(16)

The first-order conditions are as follows.

\[
\frac{\partial \pi_J}{\partial p_C^u} = X(p_C^u, p_N^u, \hat{v}) + (p_C^u + p_N^u) \frac{dX}{dp_C^u} \equiv 0
\]

(17)

\[
\frac{\partial \pi_J}{\partial p_N^u} = X(p_C^u, p_N^u, \hat{v}) + (p_C^u + p_N^u) \frac{dX}{dp_N^u} \equiv 0
\]

(18)

**Proposition 6.** As in the all fixed pricing environment, consumers face lower total cost of content consumption under price alliance arrangement when both the CP and the NP employ usage-based pricing schemes. As a result, total consumers’ and producers’ surplus will both increase under price alliance in all usage-based pricing environment.

As noted in all fixed pricing case, when two players use the same pricing scheme, the optimality conditions in the alliance case yields a redundant equation. Therefore, the optimality condition will be satisfied by a collection of price pairs, \((p_C^u, p_N^u)\). Again, this results in an ambiguity as to which of these price pairs the two players should agree to charge.

**Fixed Content Pricing and Usage-Based Network Pricing**

In a single-content environment, the usage-based network and fixed content pricing case yield qualitatively the same result as the fixed network and usage-based pricing case. Therefore, this section concentrates on one of these two possible asymmetric pricing cases. Under the usage-based network and fixed content
pricing schedule, the total cost for the consumer is \( p(x) = p_C^f + p_N^f x \). The marginal consumer then satisfies

\[
U(x^*(p_N^u, \  \tilde{v}), \ \tilde{v}) = p_C^f + p_N^f x^*(p_N^u, \  \tilde{v}). \tag{19}
\]

The fraction of the total population subscribing to the content and the network service (which is the demand function for the CP) is

\[
D_C = \int_0^{\tilde{v}} dF(v) = G(\tilde{v}) \tag{20}
\]

and the total amount of usage (which is the demand function for the NP) is

\[
X = \int_0^{\tilde{v}} x^*(p_N^u, \  v) dF(v). \tag{21}
\]

Differentiating (19) with respect to \( p_C^f \) and \( p_N^f \), applying Envelope Theorem and rearranging yields

\[
\frac{\partial \tilde{v}}{\partial p_C^f} = \frac{1}{\frac{\partial U(x^*(\tilde{v}), \ \tilde{v})}{\partial \tilde{v}}} \quad \text{and} \quad \frac{\partial \tilde{v}}{\partial p_N^f} = \frac{x^*(p_N^u, \  \tilde{v})}{\frac{\partial U(x^*(\tilde{v}), \ \tilde{v})}{\partial \tilde{v}}} \tag{22}
\]

Implications of (22) are readily deducible from proposition 1 and proposition 3.

Non-allying CP and NP solve the following profit maximization problems:

CP’s problem: \[
\max_{p_C^f} \pi_C = p_C^f G(\tilde{v}) \tag{23}
\]

NP’s problem: \[
\max_{p_N^u} \pi_N = p_N^u X(p_N^u, \ \tilde{v}) \tag{24}
\]

First-order conditions of (23) and (24) are

\[
\frac{\partial \pi_C}{\partial p_C^f} = G(\tilde{v}) + p_C^f G' \frac{d\tilde{v}}{dp_C^f} \equiv 0 \tag{25}
\]

and

\[
\frac{\partial \pi_N}{\partial p_N^u} = X(p_N^u, \ \tilde{v}) + p_N^u \frac{dX}{dp_N^u} \equiv 0. \tag{26}
\]
The allying firm’s profit maximization problem is

Allying firm’s problem: \( \max_{p_C^f, p_N^u} \pi_J = p_C^f G(\bar{v}(p_C^f, p_N^u)) + p_N^u D_N(p_N^u, \bar{v}). \) \( (27) \)

The optimality conditions with respect to \( p_C^f \) and \( p_N^u \) are

\[
\frac{\partial \pi_J}{\partial p_C^f} = G(\bar{v}) + p_C^f \frac{\partial G}{\partial \bar{v}} \frac{\partial \bar{v}}{\partial p_C^f} + p_N^u \frac{\partial D_N}{\partial \bar{v}} \frac{\partial \bar{v}}{\partial p_C^f} \equiv 0 
\]

and

\[
\frac{\partial \pi_J}{\partial p_N^u} = p_C^f \frac{\partial G}{\partial \bar{v}} \frac{\partial \bar{v}}{\partial p_N^u} + D_N + p_N^u \left[ \frac{\partial D_N}{\partial p_N^u} + \frac{\partial D_N}{\partial \bar{v}} \frac{\partial \bar{v}}{\partial p_N^u} \right] \equiv 0. \] \( (29) \)

**Proposition 7.** As in the all fixed and all usage-based pricing environments, consumers face lower total cost of content consumption under price alliance arrangement when the CP employs fixed pricing and the NP employs usage-based pricing scheme. As a result, total consumers’ and producers’ surplus will both increase under price alliance in this mixed pricing environment.

From (28) and (29), players under the alliance take into account the effect of own price on the change in the partner’s profit. With this consideration, both players will set lower prices than those implied by (25) and (26). Unlike the symmetric pricing cases, an alliance under asymmetric pricing will generally yield a unique optimal price pair that satisfies (28) and (29). However, as will be shown in Numerical Analysis section, this optimal cooperative price pair might lead to a decrease in one player’s profit. The reason for this is that if one player’s demand function is too sensitive to a change in the other player’s price change, the optimal price under the alliance attempts to compensate for this effect by keeping the other player’s price low. This effect might be significant enough to make the profitability of that player lower than his non-alliance level. Under certain situations, the gaining player must compensate the losing player for its loss in order for the alliance to be successfully reached and sustained.
A NUMERICAL EXAMPLE

For a more concrete illustration of the insights from Analysis section and a demonstration of the profit sharing issue, the following numerical example is utilized. Let \( U(x, v) = vx - \frac{1}{2}x^2 \) and type parameter \( v \) be uniformly distributed in \([0, 1]\). Furthermore, assume that each consumer has a resource constraint that renders the maximum amount of content consumption for the consumer type \( v \) as \( v \). The calculation processes of relevant numerical results are included in Appendix B.

Table 1 summarizes outcomes of interest from this numerical example. An examination of table 1 yields several observations and insights. First, the joint profits of the CP and the NP are maximized under the asymmetric pricing environment for both non-alliance and alliance cases. With the price alliance agreement in the asymmetric pricing environment, consumers essentially face a two-part tariff. A two-part tariff is more effective in extracting consumer surplus than a tariff consisting of only a fixed or a usage-based component. Via observation of the same result in the absence of a price alliance, it is conjectured that when fixed and usage-based components are both present, players are more efficient in extracting consumer surplus even without the price alliance arrangement.

As expected, the market penetration, represented by \( \hat{v} \), and the total amount of content consumption, are the highest under all fixed pricing environments. Therefore, it is confirmed that the fixed-pricing only environment encourages subscription and content consumption. However, consumer surplus is not the highest in all fixed pricing environments with or without a price alliance. In the long debate between proponents of the adoption of fixed pricing and those favoring usage-based pricing, the major argument of the former has been that usage-based pricing will greatly undermine consumer benefits. Here, this claim is shown not to hold. Fixed pricing environments are not only the least efficient in terms of total surplus in both alliance and non-alliance cases, they are also not the most efficient one in terms of consumers’ surplus alone. This result highlights the importance of providing incentives to the market in terms of growing the total market size. Compared to the all fixed pricing
environments, other pricing environments are more efficient in achieving the higher market efficiency due to more optimized consumer consumption behavior induced by usage-based portion of the tariff they face. As a result, contrary to conventional conjecture, consumers are better off under pricing environment with some usage-based component in comparison to the situation with all fixed pricing scheme.

The total surplus, defined as the sum of consumer and producer surplus, is maximized under asymmetric pricing when there is no price alliance. When players form a price alliance, on the other hand, the total surplus is maximized under all usage-based pricing cases.

The join profit maximization effort results in a higher aggregate profit level compared to the non-alliance level in all four cases. Therefore, a coalitional form representation of these scenarios is superadditive.\(^1\) From an individual player’s perspective, however, cooperative pricing yields either an obscure pricing decision or an incentive incompatibility. From the symmetric pricing environment in which both the CP and the NP employ either fixed or usage-based pricing policies, the optimal pricing decision under the alliance yields only a linear relationship between price levels of the CP and the NP. Some of these feasible optimal price levels might leave one of the players worse off compared to the non-alliance pricing case. Restricting the price levels to be inside the core of the game will resolve such issues. Under the asymmetric pricing environment in which one player employs fixed pricing and the other player exercises usage-based pricing, joint profit maximization yields the result that the player with the fixed-pricing receives a lower profit level than the one from a non-alliance. In this case, a side payment from the player with the usage-based fee to the one with the fixed fee is necessary in order to avoid breakdown of the alliance. The amount of the side payment, of course, needs to be set such that both players are inside the core. The specific ranges of payoff division and side payment amount corresponding to the core, as well as particular amount of payoff division and side payment corresponding to Shapley allocation corresponding to the number example of this section are calculated and illustrated in

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\(^1\) For an overview of cooperative game theory, refer to Eichberger (1993) and Weber (1994).
## Table 1. Summary of Outcomes in Various Pricing Environments

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<th>Symmetric Pricing</th>
<th>Asymmetric Pricing</th>
<th>Fixed</th>
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<td>Fixed</td>
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<td><strong>Content Pricing</strong></td>
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<td><strong>Network Pricing</strong></td>
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<td>Content Price</td>
<td>0.16(fixed)</td>
<td>0.25(per unit)</td>
<td>0.1361(fixed)</td>
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<tr>
<td>Network Service Price</td>
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<td>0.25(per unit)</td>
<td>0.2174(per unit)</td>
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<td>Marginal Consumer Rank</td>
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<td>0.5</td>
<td>0.7391</td>
</tr>
<tr>
<td>Total Content Usage</td>
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<td>0.125</td>
<td>0.1701</td>
</tr>
<tr>
<td>Content Demand³</td>
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<td>0.125</td>
<td>0.2609</td>
</tr>
<tr>
<td>Network Demand³</td>
<td>0.2</td>
<td>0.125</td>
<td>0.1701</td>
</tr>
<tr>
<td>CP’s Profit</td>
<td>0.032</td>
<td>0.03125</td>
<td>0.0355</td>
</tr>
<tr>
<td>NP’s Profit</td>
<td>0.032</td>
<td>0.03125</td>
<td>0.0369</td>
</tr>
<tr>
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<td>0.0625</td>
<td>0.0725</td>
</tr>
<tr>
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<td>0.0208</td>
<td>0.0207</td>
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<tr>
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<td>0.0833</td>
<td>0.0932</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Price-Alliance</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Content Price</td>
<td>0.222⁴</td>
<td>0.333(per unit)⁴</td>
<td>0.08(fixed)</td>
</tr>
<tr>
<td>Network Service Price</td>
<td>0.6667</td>
<td>0.333</td>
<td>0.6</td>
</tr>
<tr>
<td>Marginal Consumer Rank</td>
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<td>0.24</td>
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<tr>
<td>Total Surplus</td>
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<td></td>
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</tr>
</tbody>
</table>

² The usage-based content, fixed network pricing scenario yields symmetrically equivalent result and therefore, is eliminated from Table 1.

³ Content and network demand are expressed as the fraction of the total population when the price is fixed and as the total amount of usage by all subscribers when the price is usage-based.

⁴ In symmetric pricing cases, the optimal price under a price alliance is expressed in terms of the sum. Thus, the respective optimal prices for the CP and the NP only need to summate to the designated level to maximize the joint profit.

table 2.
DISCUSSION

In order to facilitate the discussion of various pricing schemes of real-world online content services, Table 3 has been constructed. In the first four services, services are provided on top of “open” Internet infrastructure so that there is no price alliance between content and network service providers. On the other hand, mobile Internet service is regarded as a “closed” content market (Oh 2007), in which only subscribers of the distribution NP have access to the content provided by a CP. Consequently, due to exclusive nature of business relationship between these players, it is likely that pricing of content and network services are coordinated at least in an implicit manner.

Online movie or music service sites such as Blockbuster Online and Apple’s iTunes Music store employ fixed charge schemes as in table 3. Online news service such as Wall Street Journal Online also employs fixed charge scheme although a fixed price is applied to a bundle of both online and offline subscription services. Network infrastructure for these three content services is Internet which is normally provided by broadband Internet service providers these days. Thus, we can regard these three cases to correspond to the fixed content, fixed network pricing situation. In this situation, as proposition 1 indicated, denseness of distribution of utility functions of
consumers determines price sensitivity of demands for these content services. For general content services such as music and movie services, it is likely that utility functions are highly dense since these contents probably attract large consumer pool so that incremental utility between successive consumers is likely to be small. Thus, demand functions for these services are likely to be rather price sensitive, requiring careful examination of demand characteristics in determining optimal price level. On the other hand, specialized content such as Wall Street Journal articles probably have smaller consumer base. In this case, the demand is likely to be price sensitive and the optimal price can be set rather high. This line of reasoning conforms to our intuition that associates a lower price with general contents and a premium price with specialized and focused contents (Chae and Flores 1998.)

The Lexis-Nexis service database service is an example of usage-based content pricing and fixed network pricing context. In this situation, proposition 3, proposition 4 and proposition 5 indicates that not only the denseness of utility functions of

consumers, but also the optimal usage level of each consumer as well as concavity of utility functions affects the price sensitivity of demand in terms of both number of subscribers and total amount of content usage. In line with the argument in the preceding paragraph, the specialized and focused nature of Lexis-Nexis articles probably renders its demand in terms of both number of subscribers and amount of usage to be price insensitive. Also, the optimal usage level of typical Lexis-Nexis users is conjectured to be rather low compared to the usage level of general content services which contributes to price insensitivity of its demand. On the other hand, the extent of decrease in marginal utility of additional content is conjectured to be large due to specific purpose and focus of these articles for their consumers, which is suggestive of price insensitivity. From these, demand for article service of Lexis-Nexis in terms of number of users and amount of usage is likely to be highly price insensitive.

Mobile Internet service is an example of all usage-based pricing with price alliance case. In this situation, as illustrated by proposition 6, consumers are likely to face lower cost of content consumption compared to the case in which content and network service providers are not involved in exclusive business relationship.

**CONCLUSIONS AND FUTURE RESEARCH**

In the setup of the two-stage consumers’ behavior model of this chapter, three characteristics of utility functions of consumers — the marginal utility of a unit content consumption for the marginal consumer, concavity of utility functions, and denseness of type distribution — are identified as primary factors determining the price sensitivity of number of subscribers and the total amount of usage of consumers. Higher utility of consumption, coupled with sparse type distribution, leads to price insensitivity of the demand function in terms of number of subscribers. However, a highly concave utility function is required for price insensitivity of the consumers’ consumption amount. Thus, a balance among these characteristics during the content product design phase as well as a suitable pricing
strategy in accordance with the product design is needed.

These content product characteristics can be linked to the type of contents under consideration. A requirement for a large marginal benefit implies that a CP needs to make efforts to provide updated value to consumers who have already experienced the content offering in order to command a high price premium. On the other hand, a content offering highly focused on a specific consumer segment is likely to exhibit a large “gap” between the low willingness-to-pay group and the high willingness-to-pay group. If the current price is set around the region between the low valuation group and the high valuation group, the price can be increased without much reduction in the number of subscribers.

Depending on the combination of usage-based and fixed pricing schemes between the CP and the NP, the actual determination of the subscriber set and the usage level is a complicated and is an inter-linked process. When both the CP and the NP employ fixed pricing, the pricing schedule is not a binding constraint for the consumer’s second-stage optimal usage determination. Therefore, the usage level determination is subject to other resource limitations, such as time constraints. The first-stage subscription decision is determined by the sensitivity of the rank of the marginal consumer to the price changes. This sensitivity was shown to be inversely proportional to the sensitivity of the utility function to the type parameter. In turn, this sensitivity depends on both the marginal utility of the marginal consumer and the denseness of the type parameter distribution at the marginal consumer’s rank.

With the existence of a usage-based component in the total price schedule for consumers, the description of the subscriber set and usage level determination is even more complicated. The complication arises from the fact that, in determining the marginal consumer, a change in the usage price affects the optimal usage level of the marginal consumer. Extra attention to the benefit and the cost due to this changed optimal usage level needs to be paid in describing the subscriber set determination process. Nevertheless, the influence of the aforementioned three characteristics on the demand sensitivity in terms of number of subscribers and consumption level is still seen to be the same when the usage-based pricing component is present.
In a symmetric pricing scenario of all fixed or all usage-based pricing environments, a price alliance between the CP and the NP yields only a linear relationship between the two elements of the optimal price pair. This indeterminate description of the optimal price pair is a source for dispute between potential alliance partners in that profit levels of participants of the alliance depend on which specific price pair the allying parties agree to exercise. On the other hand, in an asymmetric pricing scenario, a unique optimal price pair exists for alliance partners as long as the joint profit function is a well-defined concave function. However, as shown in Numerical Analysis section, a price alliance can lead to a decrease in the profit of one of the alliance partners. The aforementioned two potential obstacles for an alliance formation can be resolved by an application of core and Shapley allocation rules, which are demonstrated in Numerical Analysis section.

This paper models a pricing behavior and its consequences in a hypothetical online content market in which there is one CP and one NP. One of the significant modeling limitations of this paper is that only a single CP is included in the market. Incorporation of multiple CP case will undoubtedly facilitate a much richer discussion of a more realistic picture of online content markets. For example, nature of relationship among multiple contents in terms of substitutability can possibly either strengthen or weaken the effectiveness of prices alliance between NP and CP’s. Furthermore, under such environment, price competition and alliance can occur both horizontally and vertically and such various market configurations will influence the effectiveness of pricing schemes and its determinants. Although this paper intends to focus on vertical relationship between CP and NP in assessing implications of various pricing scheme arrangements, inclusion of multiple CP’s is deemed to be a necessary direction for extension of this paper.

Another model limitation worth mentioning stems from the assumption that \( p_C \) and \( p_N \) are simultaneously determined. In reality, depending on specific market situation, either one of the price levels might have been determined by the time of determination of the other. Such price leadership normally entails significant strategic implications. It is conjectured that in many cases, NP is likely to possess price leadership due to its
physical precedence over content services as well as its scale and scope economics. Incorporation of price leadership is expected to affect results of this paper which is another possible topic of further research.

APPENDIX A. Proofs of Propositions

Proof of Proposition 1.
Differentiating (1) by \( p_C^l \) and \( p_N^l \), applying Envelope Theorem and rearranging yields

\[
\frac{\partial \hat{v}}{\partial p_C^l} = \frac{\partial \hat{v}}{\partial p_N^l} = \frac{1}{\partial \hat{v}} \frac{\partial U(x^*(\hat{v}), \hat{v})}{\partial \hat{v}} \tag{A1}
\]

From (A1), an increase in either the CP or the NP’s fixed price causes large increase in the rank of the marginal consumer, \( \hat{v} \), thereby leading to a large decrease in number of subscribers when the utility function is densely distributed.

Proof of Proposition 2.
From (9), the CP and the NP can choose any combination of price pairs that satisfy (9). Comparing (9) with (6) and (7), the first-order conditions with respect to either \( p_C^l \) or \( p_N^l \) are imposed on the sum \( p_C^l + p_N^l \) under the joint profit maximization. Consequently, the optimal price levels of \( p_C^l \) and \( p_N^l \) under the price alliance will be lower, resulting in higher consumer surplus than that in the non-allying case. Since joint profit maximization also yields higher producer surplus compared to non-allying case, the total welfare will be enhanced in the joint profit maximization case.

Proof of Proposition 3.
Differentiating (10) with respect to \( p_C^u \) and \( p_N^u \), applying Envelope Theorem and rearranging yields

\[
\frac{\partial \hat{v}}{\partial p_C^u} = \frac{\partial \hat{v}}{\partial p_N^u} = \frac{x^*(p_C^u, p_N^u, \hat{v})}{\partial \hat{v}} \frac{\partial U(x^*(p_C^u, p_N^u, \hat{v}), \hat{v})}{\partial \hat{v}} \tag{A2}
\]
From (A2), an increase in either the CP or the NP’s usage-based price causes large increase in the rank of the marginal consumer, $\hat{v}$, thereby leading to a large decrease in number of subscribers when the utility function is densely distributed and when the optimal content consumption level of the marginal consumer is large.

**Proof of Proposition 4.**
Each consumer’s utility function maximization yields

$$\frac{\partial U(x^*, v)}{\partial x^*} = p^u_C + p^u_N. \quad (A3)$$

Further differentiating with respect to $p^u_C$,

$$\frac{\partial^2 U(x^*, v)}{\partial x^* \partial p^u_C} \frac{\partial x^*}{\partial p^u_C} = 1. \quad (A4)$$

Rearranging, we get

$$\frac{\partial x^*}{\partial p^u_C} = \frac{1}{\frac{\partial^2 U}{\partial x^* \partial ^2}}. \quad (A5)$$

Hence, the responsiveness of each consumer’s optimal usage level to a change in usage-based price is inversely proportional to the concavity of the utility function.

**Proof of Proposition 5.**
The effect of changes in the usage-based price on the total amount of content consumption can be seen by differentiating (11) with respect to $p^u_C$ and $p^u_N$.

$$\frac{dX}{dp^u_C} = \int_0^\infty \frac{\partial x^* (p^u_C, p^u_N, \hat{v})}{\partial p^u_C} dF(v) - x^* (p^u_C, p^u_N, \hat{v}) f(\hat{v}) \frac{d\hat{v}}{dp^u_C} \quad (A6)$$

$$\frac{dX}{dp^u_N} = \int_0^\infty \frac{\partial x^* (p^u_C, p^u_N, \hat{v})}{\partial p^u_C} dF(v) - x^* (p^u_C, p^u_N, \hat{v}) f(\hat{v}) \frac{d\hat{v}}{dp^u_N} \quad (A7)$$

The first terms of R.H.S. of (A6) and (A7) are large when
concavity of utility function is small by Proposition 4. Given that $\frac{d\phi}{dp_C}$ and $\frac{d\phi}{dp_N}$ are negative, the second terms are positive and large when $\frac{d\phi}{dp_C}$ and $\frac{d\phi}{dp_N}$ are large and $x^*(p^*_C, p^*_N, \hat{v})$ is large. By Proposition 3, such condition is satisfied when $x^*(p^*_C, p^*_N, \hat{v})$ is large and when $\frac{\partial U(x^*(p^*_C, p^*_N, \hat{v}), \hat{v})}{\partial \hat{v}}$ is small.

**Proof of Proposition 6.**

From (17) and (18), which are identical since $\frac{dX}{dp_C} = \frac{dX}{dp_N}$, the CP and the NP can choose any combination of price pairs that satisfy (17) and (18). Comparing (17) and (18) with (14) and (15), the first-order conditions with respect to either $p^*_C$ or $p^*_N$ are imposed on the sum $p^*_C + p^*_N$ under the joint profit maximization. Consequently, the optimal price levels of $p^*_C$ and $p^*_N$ under the price alliance will be lower, resulting in higher consumer surplus than that in the non-allying case. Since joint profit maximization also yields higher producer surplus compared to non-allying case, the total welfare will be enhanced in the joint profit maximization case.

**Proof of Proposition 7.**

Comparing (25) and (26) with (28) and (29), the first-order conditions with respect to either $p^*_C$ or $p^*_N$ are imposed on the sum $p^*_C + p^*_N$ under the joint profit maximization. Consequently, the optimal price levels of $p^*_C$ and $p^*_N$ under the price alliance will be lower, resulting in higher consumer surplus than that in the non-allying case. Since joint profit maximization also yields higher producer surplus compared to non-allying case, the total welfare will be enhanced in the joint profit maximization case.

**APPENDIX B. Derivation of Numerical Analysis Results**

**Fixed Content and Network Pricing**

Since there is no usage-based cost associated with the content consumption activity, each user will consume the maximum amount of content that yields the individual usage demand function of $x^*(v) = v$. The marginal consumer satisfies the condition, $v^2 - \frac{1}{2}v^2 = p^*_C + p^*_N$. From this,
\[ \hat{v} = \sqrt{2(p_C^J + p_N^J)} \]

\[ D_C = D_N = \int_1^{p_C^J} \frac{1}{2(p_C^J + p_N^J)} dF(v) = 1 - \sqrt{2(p_C^J + p_N^J)} \]

\[ CS = \int_1^{p_C^J + p_N^J} (U(x^*, v) - p_C^J - p_N^J) dv = \int_1^{p_C^J + p_N^J} \left( \frac{v^2}{2} - p_C^J - p_N^J \right) dv \]

Total content usage by subscribers is

\[ \int_1^{p_C^J + p_N^J} v \cdot 1 dv = \frac{1}{2} - p_C^J - p_N^J. \]

The first-order conditions yield the following price reaction functions:

\[ \tilde{p}_C^J(p_N^J) = \frac{1}{9} \left( \frac{2}{3} p_N^J + \frac{\sqrt{1 + 6p_N^J}}{9} \right). \]

and

\[ \tilde{p}_N^J(p_C^J) = \frac{1}{9} \left( \frac{2}{3} p_C^J + \frac{\sqrt{1 + 6p_C^J}}{9} \right). \]

In order to distinguish between optimal price and profit levels from the price alliance and those from a non-alliance and to discern optimal variable levels from general variable designation, the following changes in the variable naming convention are adopted. The superscript \( J \) designates variables from price alliance whereas no superscript denotes variables from non-alliance. An asterisk next to a variable denotes the optimal level derived from the example of Numerical Analysis section.

The demand under the joint profit maximization is

\[ D(p_C^J, p_N^J) = 1 - \sqrt{2(p_C^J + p_N^J)} = 1 - \sqrt{2(p_C^J + \left( \frac{2}{9} - p_N^J \right) \cdot \frac{2}{9}} = \frac{1}{3}. \]

The Profit Possibility Frontier (PPF)\(^6\) is defined by the relationship between the optimal price pair, \( p_C^J + p_N^J = \frac{2}{9} \). Since

\(^6\) PPF is a set of profit pairs that results from a successful collusion (Slade and Jacquemin, 1992).
Thus, the PPF is defined by the set of profit pairs

$$\pi_C^J = \frac{1}{3}, \quad \pi_N^J = p_N^J \cdot \frac{1}{3} = \left(\frac{2}{9} - p_C^J\right) \cdot \frac{1}{3} = \frac{2}{27} - \pi_C^J.$$  Thus, the PPF is defined by the set of profit pairs

$$0 \leq \pi_C^J \leq \frac{2}{27} \quad \text{and} \quad \pi_N^J = \frac{2}{27} - \pi_C^J.$$

The core is defined by the following two conditions:

$$\pi_C^J(p_C^J^*, p_N^J^*) = p_C^J \cdot \frac{1}{3} \geq 0.032$$

$$\pi_N^J(p_C^J^*, p_N^J^*) = p_N^J \cdot \frac{1}{3} \geq 0.032$$

which yield the set of price pairs

$$0.096 \leq p_C^J^*, p_N^J^* \quad \text{and} \quad p_C^J^*, p_N^J^* = \frac{2}{9}$$

The resulting profit range corresponding to the core on the PPF is

$$0.032 \leq \pi_C^J \leq 0.0420741 \quad \text{and} \quad \pi_N^J = \frac{2}{27} - \pi_C^J.$$

On the other hand, the Shapley value rule yields the relationship

$$p_C^J^* \cdot \frac{1}{3} - 0.032 = p_N^J^* \cdot \frac{1}{3} - 0.032.$$

Thus, \( (p_C^J^*, p_N^J^*) = \left(\frac{1}{9}, \frac{1}{9}\right) \) implies a price pair congruent to the Shapley value rule. The profit pair corresponding to the Shapley value rule is \( (\pi_C^J^*, \pi_N^J^*) = \left(\frac{1}{27}, \frac{1}{27}\right) \).

**Usage-Based Content and Network Pricing**

Consumer’s benefit maximization yields the following optimal usage demand function:

$$x^*(p_C^u, p_N^u, v) = v - p_C^u - p_N^u.$$  The marginal consumer \( \hat{v} \) satisfies \( U(\hat{v} - p_C^\hat{v} - p_N^\hat{v}) = (p_C^\hat{v} + p_N^\hat{v}) (\hat{v} - p_C^\hat{\nu} - p_N^\hat{\nu}), \) which, in turn, yields the following results:
\[\hat{v} = p_C^u + p_N^u\]
\[D_C(p_C^u, p_N^u) = D_N(p_C^u, p_N^u)\]
\[= \int_{p_C^u + p_N^u}^{1} (v - p_C^u - p_N^u) dv\]
\[= \frac{(1 - p_C^u - p_N^u)^2}{2}\]
\[CS = \int_{p_C^u + p_N^u}^{1} (U(v - p_C^u - p_N^u) - (p_C^u + p_N^u)(v - p_C^u - p_N^u)) dv\]
\[= \frac{1}{6} (1 - p_C^u - p_N^u)^3\]

The reaction functions from first-order conditions on profit functions are

\[\tilde{p}_C(p_N^u) = \frac{1 - p_N^u}{3}\]
and

\[\tilde{p}_N(p_C^u) = \frac{1 - p_C^u}{3}\]

From the optimal price relationship, \( p_C^j + p_N^j = \frac{1}{3} \), PPF is

\[0 \leq \pi_C^j \leq \frac{2}{27} \text{ and } \pi_N^j \leq \frac{2}{27} - \pi_C^j.\]

The core is derived from conditions \( p_C^j \geq 0.140625 \) and \( p_N^j = 0.03125 \), which yields

\[p_C^j \geq 0.140625 \text{ and } p_N^j \geq 0.03125 \text{ and } p_C^j + p_N^j = \frac{1}{3}.\]

From this, the core is

\[0.0312 \leq \pi_C^j \leq 0.074 \text{ and } \pi_N^j = \frac{2}{27} - \pi_C^j.\]

The shaply value is derived from \( p_C^j \geq 0.03125 = p_N^j \geq 0.03125 \text{ and } p_C^j + p_N^j = \frac{1}{3} \), which yields
(p_C^f, p_N^f) = \left( \frac{1}{6}, \frac{1}{6} \right) \text{ and } (\pi_C^f, \pi_N^f) = \left( \frac{1}{27}, \frac{1}{27} \right).

Fixed Content and Usage-Based Network Pricing

The consumer maximizes the benefit function, \( u x - \frac{1}{2} x^2 - p_C^f - p_N^f x. \) From this,

\[ x^*(p_N^u, v) = v - p_N^u. \]

The marginal consumer satisfies \( \dot{v}(v - p_N^u) = \frac{1}{2}(v - p_N^u)^2 = p_C^f + p_N^u \) \( (v - p_N^u) \) From this,

\[ \dot{v} = p_N^u \pm \sqrt{2p_C^f}. \]

\[ D_C(p_C^f, p_N^u) = \int_{p_N^u + \sqrt{2p_C^f}}^{1} 1d\dot{v} = 1 - (p_N^u + \sqrt{2p_C^f}) \]

\[ D_N(p_C^f, p_N^u) = \int_{p_N^u + \sqrt{2p_C^f}}^{1} (v - p_N^u) \cdot 1d\dot{v} = (v - p_N^u)(1 - (p_N^u + \sqrt{2p_C^f})) \]

\[ CS = \int_{p_N^u + \sqrt{2p_C^f}}^{1} \left( \int_{p_N^u}^{v} (v - x)dx - p_C^f \right) d\dot{v} \]

\[ = \frac{2\sqrt{2}}{3} p_C^f \sqrt{p_C^f} - p_C^f(1 - p_N^u) + \frac{1}{6}(1 - p_N^u)^3 \]

The reaction functions are

\[ \tilde{p}_C^f(p_N^u) = \frac{2}{9}(1 - 2p_N^u + p_N^{u^2}) \]

and

\[ \tilde{p}_N^u(p_C^f) = \frac{2 - \sqrt{1 + 6p_C^f}}{3}. \]

From table 1, the profit for the CP decreases after the alliance and, therefore, the NP needs to make up for this loss by making a side payment to the CP. Let \( T \) be the amount of this side payment. The PPF is defined by the amount of the side payment from the NP to the CP. Since \( \pi_N = 0.048, 0 \leq T \leq 0.048 \). Thus, the PPF is the set of profit pairs satisfying
\[ \pi_C = 0.032 + T, \pi_N = 0.048 - T, \ 0 \leq T \leq 0.048 \]

The core is defined by \( \pi_C^* + T \geq \pi_C^* \) and \( \pi_N^* - T \geq \pi_N^* \) from which the relationship \( \pi_C^* - \pi_C^* \leq T \leq \pi_N^* - \pi_N^* \) results. From this,

\[ 0.0035 \leq T \leq 0.0111 \]

and the resulting profit ranges are

\[ 0.0355 \leq \pi_C^I \leq 0.0431, \ 0.0369 \leq \pi_N^I \leq 0.0445 \]

\[ \pi_C^I = \pi_C^I + \pi_N^I = 0.08. \]

The Shapley value rule tells us that \( (\pi_N^I - T) - \pi_N^I = (\pi_C^I + T) - \pi_C^I \) from which,

\[ T = \frac{(\pi_N^I - \pi_N^I) - (\pi_C^I - \pi_C^I)}{2} \]

results. Thus,

\[ T = 0.0073, (\pi_C^I, \pi_N^I) = (0.0393, 0.0407). \]

**REFERENCES**


389.