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# A Theory of the Term Structure of Interest Rates under Non-expected Intertemporal Preferences*

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## Abstract

This paper presents general equilibrium term structure models under a non-expected intertemporal utility function, in which two disparate preference elements — intertemporal substitution and risk aversion — are disentangled. One major finding is that in a risk averse production economy, bond prices are independent of intertemporal substitution and thus separating the two preference components becomes totally irrelevant. The models produce several other results that are contrasted with those found in the existing literature.

## 1. Introduction

Using a non-expected recursive utility function, this paper studies the term structure of real interest rates in three-date general equilibria. In particular, the paper revisits the issues addressed in Cox, Ingersoll, and Ross [CIR, henceforth] (1981) who have contributed significantly to the term structure literature by re-examining several traditional hypotheses in the modern perspective. The main portion of their paper explores the conditions under which each hypothesis holds in continuous-

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time general equilibria One feature of CIR's (1981) model is the use of time-additive power expected utility for the specification of agents' preferences. As noted by others (e.g., Lucas (1978)), this utility function cannot distinguish between two disparate attitudes of economic agents towards riskiness and temporal unevenness of their consumption profiles For example, when the power function is chosen for one-period utility, the elasticity of intertemporal substitution is necessarily constrained to be the inverse of the risk aversion coefficient. This implies that some results in CIR's study may well be affected by this constraint inherent in agents' preferences.

The main purpose of this paper is to explore implications of separating intertemporal substitution and risk aversion for studying the term structure of interest rates. To do so, I present three-date term structure models using the non-expected utility function developed by Epstein and Zin (1989) [EZ utility, henceforth], in which the two preference components are specified independently. Duffie and Epstein (1992) have used a continuous-time version of EZ utility to study the term structure. By deriving closed-form solutions for interest rates in the case of unit elasticity of substitution, they generalize the formula presented in CIR's 1985 paper. By comparison, this article, without constraining the elasticity of substitution to unity, is concerned with the substance contained in CIR's 1981 paper. Related work also includes LeRoy (1982, 1983), Woodward (1983), Benninga and Protopapadakis (1986), Campbell (1986), Gilles and LeRoy (1986), and Sun (1992).

The setup of the model presented below follows Cho (1998) who also uses EZ utility to study the connection between the expectations hypothesis of the term structure and risk neutrality. The difference is that models in this paper make distributional assumptions on the process output produced in the economy. This allows us to produce several additional results by parameterizing all the state variables.

Major findings of this paper are as follows: (i) In a risk averse pure exchange economy, increasing aversion to intertemporal substitution magnifies (reduces) the size of term premia if the mean (variance) effect is dominant. (ii) In a risk averse production economy where linear productivity shocks are assumed to be identically lognormally distributed, bond prices do not depend on intertemporal substitution, and thus disentangling the two preference components becomes totally irrelevant. Hence, there is no advantage of using EZ utility over time-additive expected utility in this case. (iii) What CIR (1981) call the return-to-maturity expectations hypothesis can be sustained in a rational expectations equilibrium, if consumption is perfectly negatively autocorrelated. While this result appears contrary to CIR's (1981) conclusion that only the local expectations hypothesis is sustainable in equilibrium, the two seemingly contradicting results stem from the difference in assumptions on the stochastic process of state variables. Meanwhile, unlike CIR's (1981) continuous-time model, the model (in the risk averse production economy) does not require logarithmic utility for the local expectations hypothesis to hold under local certainty. (iv) Contrary to Campbell's (1986) prediction, the size of term premia does not necessarily increase with risk aversion or uncertainty in both exchange and production economies.

The remainder of this article proceeds as follows: Section 2 presents notation and definitions to be used later. Section 3 presents a term structure model in a pure exchange economy where output growth rates over time are identically lognormally distributed. In section 4, the model is considered in an economy where linear technology shocks over time are identically lognormally distributed. Section 5 summarizes and concludes the article

## 2. Definitions of Term Premia[1)]

Let ${}_tr_{t+n}$ denote the $n$-period annual real (gross) interest rate prevailing at time $t$. Suppose that at time 0, an investor plans to hold bonds for one period. If the investor invests in one-period bonds, the (gross) return on these bonds will be ${}_0r_1$ for sure. If he buys two-period pure discount bonds to be sold at time 1, the return on this strategy will be ${}_0r_2^2/{}_1\tilde{r}_2$, where ${}_1\tilde{r}_2$ denotes the future one-period spot rate and is uncertain at time 0. The

1) Definitions below are given in three-date context While the results in this paper can be extended beyond three dates with little difficulty, all the basic insights are captured in this simple model

expected excess return on this strategy is $E[{}_0r_2^2/{}_1\tilde{r}_2] - {}_0r_1$.

Suppose that the investment horizon is two periods. If the investor buys one-period bonds successively, the holding period return will be ${}_0r_1 \quad E[{}_1\tilde{r}_2]$. Despite his liquidity preference, the investor may hold two-period bonds rather than rolling over one-period bonds. This will yield a sure return of ${}_0r_2^2$, and thus the expected excess return on this strategy is ${}_0r_2^2 - {}_0r_1 \quad E[{}_1\tilde{r}_2]$.

These expected excess returns are referred to as the holding period premium [the *HP*, henceforth] and the rolling premium [the *RP*, henceforth], respectively (see Campbell (1986)):

$$HP \equiv E[{}_0r_2^2/{}_1\tilde{r}_2] - {}_0r_1 \tag{1}$$
$$\mathrm{RP} \equiv {}_0r_2^2 - {}_0r_1 \quad E[{}_1\tilde{r}_2]. \tag{2}$$

The sign of these term premia will form a basis for the theory of the term structure of interest rates. In particular, if the *HP* is zero, what CIR (1981) call the local expectations hypothesis holds. If the *RP* is zero, what they call the return-to-maturity expectations hypothesis is valid.[2,3] If the *RP* is positive (negative), the liquidity preference (aversion) hypothesis will hold. I examine below the sign and magnitude of these term premia in general equilibria where the interest rates are endogenously determined as functions of various state variables Two types of economy are considered: a pure exchange economy where consumption goods are perishable, and a production economy where capital is accumulated to produce future output.

## 3. A Pure Exchange Economy

Consider a three-date ($t$ = 0,1,2) pure exchange economy in which non-storable consumption goods produced by capital are exogenous and stochastic (see Lucas (1978)). Suppose that the

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2) In the three-date case, what CIR call the unbiased expectation hypothesis will also hold if the *RP* is zero

3) By Jensen's inequality, if the *RP* is positive, the *HP* must be positive If the *HP* is negative, the *RP* must be negative (see Woodward (1983) for further details) Moreover, a positive (negative) *RP* implies that one-period forward rate is greater (less) than the expected future spot rate, and thus the liquidity preference (aversion) hypothesis will hold

amount of output over time follows a geometric random walk of the form:

$$\tilde{y}_{t+1} = \tilde{g}_{t+1} \ y_t \quad \text{for} \quad t = 0,1, \tag{3}$$

where $y_t$ is the output at time $t$, and $\tilde{g}_{t+1}$ is the (gross) output growth rate from time $t$ to $t+1$. $\tilde{g}_{t+1}$ is uncertain at time $t$ but will be known at $t+1$

A representative consumer in this economy maximizes the following quantity:

$$U_0 = [c_0^{1-\rho} + \delta \ \hat{U}_1^{1-\rho}]^{1/(1-\rho)} \tag{4}$$

where

$$\hat{U}_1 \equiv \{E_0 \tilde{U}_1^{1-\gamma}\}, \ \ \tilde{U}_1 \equiv [\tilde{c}_1^{1-\rho} + \delta \ \hat{\tilde{c}}_2^{1-\rho}]^{1/(1-\rho)}, \tag{5}$$

$c_t$ and $U_t$ are, respectively, consumption and utility at time $t$ ($t = 0,1$), $\delta$ is the discount factor, $E_t$ is the expectation operator conditional on all information up to time $t$. $\gamma$ ($0 \leq \gamma \neq 1$) and $\rho$ ($0 < \rho \neq 1$) are risk aversion coefficient and intertemporal substitution parameter, respectively (the elasticity of substitution is $1/\rho$) This objective function is the three-date version of EZ utility, which is a parametric representation of Kreps and Porteus (1978) non-expected recursive preferences. It is a CES (Constant Elasticity of Substitution) aggregator of current consumption and certainty equivalent of future utility, denoted by $\hat{U}_1$. Henceforth, the hat ( ˆ ) notation is used to indicate the certainty equivalent of any random variable, and it is computed in the same manner as in equation (5), where the larger the risk aversion coefficient is, the smaller the certainty equivalent is. Note that $\hat{c}_2$ is a random variable at time 0 because it is a function of a conditional expectation formed at time 1. EZ utility is characterized by a multi-period generalization of the time-additive power expected utility in the sense that it identifies risk aversion and intertemporal substitution independently (see Epstein and Zin (1989) for specific details).

Under the utility function in (4), the value function at time 0, $V_0$, may be written as:

$$V_0 \equiv \max_{\{c_t\}_{t=0}^{2}} U_0 = [c_0^{*1-\rho} + \delta \quad \hat{V}_1^{1-\rho}]^{1/(1-\rho)} \tag{6}$$

where

$$\hat{V}_1 \equiv \{E_0 \tilde{V}_1^{1-\gamma}\}^{1/(1-\gamma)} \quad \text{and} \quad \tilde{V}_1 = \max_{c_1, c_2} \tilde{U}_1 = [\tilde{c}_1^{*1-\rho} + \delta \quad \tilde{c}_2^{*1-\rho}]^{1/(1-\rho)}, \tag{7}$$

and the asterisk (*) notation is used to indicate optimal solutions. Since no savings are allowed in this economy, the following must hold in equilibrium

$$c_t^* = y_t \quad \text{for} \quad t = 0, 1, 2 \tag{8}$$

In this economy, the price of an $n$-period zero-coupon bond in period $t$, ${}_t d_{t+n}$, which will pay one unit of consumption good in $n$ periods, is determined as follows:

$${}_t d_{t+n} = ({}_t r_{t+n})^{-n} = E_t[MRS_t^{t+n}]^* \tag{9}$$

where ${}_t r_{t+n}$ is as defined earlier, and $MRS_t^{t+n}$ is the marginal rate of substitution of consumption between $t + n$ and $t$ The asterisk (*) notation now indicates that all the values are evaluated in equilibrium. After computing the value function at each time, Cho (1998) derives the following bond price formulas based on equation (9):

$${}_0 d_1 = \delta \times \hat{\varphi}_1^{\gamma-\rho} \times E_0[\Theta_1^{\rho-\gamma} \quad \tilde{g}_1^{-\gamma}], \tag{10}$$

$${}_0 d_2 = \delta^2 \times \hat{\varphi}_1^{\gamma-\rho} \times E_0[\Theta_1^{\rho-\gamma} \quad \hat{g}_2^{\gamma-\rho}] \times E_0[\tilde{g}_1^{-\gamma} \quad \tilde{g}_2^{-\gamma}], \tag{11}$$

$${}_1 \tilde{d}_2 = M\tilde{R}S_1^{2*} = \delta \times \hat{g}_2^{\gamma-\rho} \times \tilde{g}_2^{-\gamma}, \tag{12}$$

where $\varphi_1 \equiv \Theta_1 \quad g_1$ and $\Theta_1 \equiv [1 + \delta \quad \hat{g}_2^{1-\rho}]^{1/(1-\rho)}$. Note that $\Theta_1$ is a random variable at time 0 since it is a function of a conditional expectation $\hat{g}_2$. By the inverse relationship between bond prices and interest rates, ${}_0 r_1 = ({}_0 d_1)^{-1}$, ${}_0 r_2 = ({}_0 d_2)^{-1/2}$, and ${}_1 \tilde{r}_2 = ({}_1 \tilde{d}_2)^{-1}$ With $\rho$ equal to $\gamma$, equations in (10)-(12), will be reduced to the familiar bond price formulas under time-additive power expected utility (see equation (4) of Sun (1992)).

### 3.1. Local Certainty and the Local Expectations Hypothesis

In continuous time, CIR (1981) shows that in a risk averse exchange economy, the local expectations hypothesis holds (the *HP* as defined in (1) is zero) if consumption is locally certain. This result can be verified in the discrete-time model here (also see Gilles and LeRoy (1986) for discrete-time expositions). In equations (10)-(12), with uncertainty in the immediate future resolved, $\tilde{g}_1$, $\hat{g}_2$, and $\Theta_1$ become non-random variables. A computation taking this into account will show that the *HP* is zero. Note that although the next period's consumption is certain, the future bond price is still stochastic (in equation (12), $\tilde{g}_2$ is random).

### 3.2. Identical Distributions and Term Premia

In order to gain insights into the sign and magnitude of the premia under risk aversion, let us assume that $\tilde{g}_1$ and $\tilde{g}_2$ are identically distributed but not necessarily independent. By this assumption, neither $\hat{g}_2$ nor $\Theta_1$ in equations (10)-(12) is a random variable,[4] so that each term can be taken out of the expectation operator, $E_0$. Note that $\hat{\varphi}_1$ is now written as $\Theta_1 \; \hat{g}_1$. As a result, the pricing formulas in equations (10) and (11) are reduced as follows:

$$ {}_0d_1 = \delta \times \hat{g}_1^{\gamma-\rho} \times E_0[\tilde{g}_1^{-\gamma}] \tag{13} $$

$$ {}_0d_2 = \delta^2 \times \hat{g}_1^{\gamma-\rho} \times \hat{g}_2^{\gamma-\rho} \times E_0[\tilde{g}_1^{-\gamma} \;\; \tilde{g}_2^{-\gamma}] \tag{14} $$

Note that as time 2 is the terminal date in this model, equation (12) is not affected by the identical distribution assumption. Using (13), (14), and (12) will show that under risk neutrality ($\gamma = 0$), both the *HP* and the *RP* are uniformly zero and thus the expectations hypothesis holds. However, since $\hat{g}_2$ in (12) is no longer a random variable at time 0 by assumption, these results are trivial in the sense that the future interest rate is

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4) While $\hat{g}_2$ and $\Theta_1$ are respectively an expectation and its function conditional on the information at time 1, they are known at time 0 due to the identical distribution assumption

degenerate. Note that this is true regardless of the degree of $\rho$.

Now let us assume that at time $t$, the (gross) growth rate $\tilde{g}_{t+1}$ is lognormally distributed:

$$\ln \tilde{g}_{t+1} \sim N(m, \sigma^2) \quad \text{for} \quad t = 0, 1. \tag{15}$$

Assume further that $\tilde{g}_1$ and $\tilde{g}_2$ are jointly lognormally distributed. Then, by the joint lognormality assumption, the last component of ${}_0d_2$ in (14) may be written as:[5]

$$\begin{aligned} E_0[\tilde{g}_1^{-\gamma} \ \tilde{g}_2^{-\gamma}] &= E_0[\tilde{g}_1^{-\gamma}] \times E_0[\tilde{g}_2^{-\gamma}] \times \exp[\mathrm{Cov}(-\gamma \ \ln \tilde{g}_1, \ -\gamma \ \ln \tilde{g}_2)] \\ &= E_0[\tilde{g}_1^{-\gamma}] \times E_0[E_1 \tilde{g}_2^{-\gamma}] \times \exp[\phi \gamma^2 \sigma^2], \end{aligned} \tag{16}$$

where $\phi$ denotes the autocorrelation coefficient between the two successive growth rates. To obtain the second equality, the law of iterated expectations is used. Applying the well-known lognormality property[6] to equations (12)-(14), and (16) yields the following closed-form formulas for the interest rates:

$${}_0r_1 = \delta^{-1} \ \bar{g}_1^{\rho} \ \exp[-1/2 \ (1+\rho)\gamma\sigma^2] \tag{17}$$

$${}_0r_2^2 = \delta^{-2} \ \bar{g}_1^{\rho} \ \bar{g}_2^{\rho} \ \exp[-(1+\rho)\gamma\sigma^2] \ \exp[-\phi\gamma^2\sigma^2] \tag{18}$$

$$E_0[1/{}_1\tilde{r}_2] = \delta \ \bar{g}_2^{-\rho} \ \exp[1/2 \ (1+\rho)\gamma\sigma^2] \tag{19}$$

$$E_0[{}_1\tilde{r}_2] = \delta^{-1} \ \bar{g}_2^{\rho} \ \exp[\{-1/2 \ (1+\rho)\gamma + \gamma^2\} \ \sigma^2], \tag{20}$$

where $\bar{g}_t \equiv \exp[m + 1/2\sigma^2]$, for $t = 1,2$. Equations (17)[7] and (18) show that increasing risk aversion (higher $\gamma$) will lower the one-period interest rate, but not necessarily the two-period rate. The two-period rate will decrease with risk aversion unless the autocorrelation of consumption is sufficiently negative. This follows from the fact that the effect of risk aversion on the price of the two-period bond is partly determined by its value as a hedging instrument. The hedging argument will be introduced

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5) If random variables $X$ and $Y$ are jointly lognormally distributed, then $E[X \ Y] = E[X] \ E[Y] \ \exp[\mathrm{Cov}(\ln X, \ln Y)]$

6) If $X$ is a lognormal random variable, $E[X^t] = \exp[tE[\ln X] + 1/2t^2\mathrm{Var}[\ln X]]$

7) Kandel and Stambaugh (1991) also obtains this one-period interest rate formula

later in this section. It is noted that this result is consistent with that predicted by the continuous-time model of Duffie and Epstein (1992).

Bond prices are positively related to the discount factor, $\delta$ and negatively related to the mean growth rate, $m$ since increasing $\delta(m)$ will raise (lower) the expected marginal rate of substitution. Hence interest rates are negatively related to $\delta$ and positively related to $m$. Reflecting the demand for precautionary savings, increased variance (a mean-preserving spread) will push up the bond price and thus lowers interest rates (see Sun (1992) also).

Equations (17) and (19) indicate that one-period bond prices in this economy follow a martingale since $E_0[{}_1\tilde{d}_2] = {}_0d_1$. As a result, the yield curve is upward-sloping (flat, downward-sloping) if and only if the sign of the $HP$ is positive (zero, negative).[8] Note that in this case, one-period interest rates will follow a submartingale since $E_0[{}_1\tilde{r}_2] > {}_0r_1$.

A computation based on (17)-(20) will produce the following expressions for the $HP$ and the $RP$, respectively:

$$HP = \delta^{-1} \; \bar{g}_1^{\rho} \; \exp[-1/2 \; (1+\rho)\gamma\sigma^2] \; \{\exp[-\phi\gamma^2\sigma^2] - 1\} \tag{21}$$

$$RP = \delta^{-2} \; \bar{g}_1^{\rho} \; \bar{g}_2^{\rho} \; \exp[-2(1+\rho)\gamma\sigma^2)] \; \{\exp[-\phi\gamma^2\sigma^2] - \exp[\gamma^2\sigma^2]\}. \tag{22}$$

The last component of each formula indicates that under risk aversion ($\gamma > 0$) and uncertainty ($\sigma > 0$), the sign of each premium depends critically on how the consumption growth rates are serially correlated. As for the sign of the $HP$, explanations are as follows: If the growth rates are positively (negatively) correlated, the two-period bond is a good (bad) hedge against time 1 consumption uncertainty. That is, at time 1, the two-period bond price will be low when the consumption level is high and will be high when the consumption level is low. Therefore the two-period bond must be sold at a premium (discount) and thus its return is low (high) (see Woodward (1983), Benninga and Protopapadakis (1986), and Sun (1992)).

The $RP$ is negative unless future growth rates are perfectly negatively autocorrelated ($-1 < \phi \leq 1$). In this case, the future

8) Due to the martingale property, the $HP$ can be written as follows
$HP \equiv E[{}_0r_2^2/{}_1\tilde{r}_2] - {}_0r_1 = {}_0r_2^2 \;\; E[{}_1\tilde{d}_2] - {}_0r_1 = {}_0r_2^2 \;\; {}_0d_1 - {}_0r_1 = {}_0d_1 \;\; ({}_0r_2^2 - {}_0r_1^2)$

short-term bond is an imperfect hedge against future consumption risk and thus the investor will be reluctant to take the risky roll-over strategy without being properly compensated. As a result, the liquidity aversion hypothesis (see Stiglitz (1970)) holds although the model does not incorporate the liquidity risk formally. When $\phi = -1$, the *RP* is zero and thus the return-to-maturity expectations hypothesis is valid, indicating that the short-term bond is a perfect hedge against consumption risk and so no premium is required on the risky roll-over strategy Note that the traditional liquidity preference hypothesis is never upheld in the present model unless the liquidity risk is formulated. This result makes sense in view of the fact that under risk aversion, the expected return on a risky investment strategy should be greater than that on a safe strategy.

The case where the *RP* is zero may serve as a counter example to CIR's (1981) conclusion that among various versions of the expectations hypothesis, only the local expectations hypothesis are sustainable in equilibrium. These seemingly contradicting results stem from different assumptions on the stochastic process of state variable(s). While the present model allows two successive growth rates to be autocorrelated with each other, CIR's model does not. Their model assumes a Wiener process for state variables, the key property of which is serial independence. Hence, CIR's model may be considered as a special case of the model here and their result is not inconsistent with the above result. Indeed, when the growth rates are serially uncorrelated ($\phi = 0$), the return-to-maturity expectations hypothesis is not compatible with equilibrium here, either

As for non-zero premia, their magnitudes (the absolute values) are affected by various economic parameters. Taking derivatives of expressions in (21) and (22) with respect to $\rho$ will show that increasing aversion to intertemporal substitution magnifies (reduces) each premium if $m$ is greater (less) than $1/2\ (\gamma - 1)\sigma^2$ (the mean (variance) effect is dominant, in Sun's (1992) terminology). Thus, given the means and variances of the log growth rates, unless the investor is sufficiently risk averse, each premium will increase with the degree of $\rho$.[9] Meanwhile,

9) During the period 1889-1978, the mean and the standard deviation of the U S annual consumption growth rates are 1 8% and 3 6%, respectively (see

increasing either risk aversion or uncertainty (a mean-preserving spread) magnifies the last component of each premium and reduces the rest. The overall effect on the magnitude is thus ambiguous. As in Campbell (1986), if the term premium is defined by the log of the ratio of the expected returns, all the components other than the last one are disguised, and as a result, the effect would appear to be positive for both premia.[10]

## 4. A Production Economy

Let us get into a three-date one-good production economy in which the production technology is represented by a stochastic constant-returns-to-scale so that capital (or wealth) is accumulated by the following process.

$$\tilde{k}_{t+1} = (k_t - c_t)\ \tilde{s}_{t+1}, \ \text{for} \ \ t = 0,1, \tag{23}$$

where $k_t$ denotes capital at time $t$ ($k_0$ is given) and $\tilde{s}_{t+1}$ represents the random productivity of capital

Suppose that the representative consumer maximizes the objective function given in (4), subject to the constraint in (23). Working backwards will give the optimal consumption rule and the value function at each time. Using these solutions and the bond pricing equation in (9), Cho (1998) obtains the following bond price formulas:

$$_0d_1 = \hat{\Omega}_1^{\gamma-1} \times E_0[\theta_1^{1-\gamma}\ \ \tilde{s}_1^{-\gamma}], \tag{24}$$

$$_0d_2 = \hat{\Omega}_1^{\gamma-1} \times E_0[\theta_1^{1-\gamma}\ \ (1-\lambda_1)^{\gamma}\ \ \tilde{s}_2^{\gamma-1}] \times E_0[(1-\lambda_1)^{-\gamma}\ \ \tilde{s}_1^{-\gamma}\ \ \tilde{s}_2^{-\gamma}], \tag{25}$$

$$_1\tilde{d}_2 = \tilde{s}_2^{\gamma-1} \times \tilde{s}_2^{-\gamma} \tag{26}$$

where $\Omega_1 \equiv \theta_1\ \ s_1$, $\theta_1 \equiv \lambda_1^{\rho/(\rho-1)}$, $\lambda_1 \equiv [1 + \delta^{1/\rho}\ \ \hat{s}_2^{(1-\rho)/\rho}]^{-1}$. Again, by setting $\rho$ equal to $\gamma$ in these equations, one would obtain the formulas under time-additive power expected utility.

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Mehra and Prescott (1985)) These numbers translate into $m = 0\ 017215$ and $\sigma^2 = 0\ 001250$ In this case, the critical value of $\gamma$ is about 29

10) Each premium takes the form of $A\ \ (X - Y) = A\ \ X - A\ \ Y$ Following the log definition, the premium becomes $\ln X - \ln Y$

### 4.1. Local Certainty and the Local Expectations Hypothesis

CIR (1981) suggest that in a risk averse production economy, the local expectations hypothesis holds only if consumption is locally certain and the utility function is logarithmic (also see Gilles and LeRoy (1986)). While using equations (24)-(26) (with $\tilde{s}_1$, $\hat{s}_2$, $\lambda_1$, and $\theta_1$ non-random) will verify this, it turns out that the local certainty assumption alone is sufficient for the zero *HP*. Hence, unlike CIR's continuous-time model, logarithmic utility is not a necessary condition for the local expectations hypothesis to hold under local certainty.

### 4.2. Identical Distributions and Term Premia

As before, let us assume that the $\tilde{s}_1$ and $\tilde{s}_2$ are identically distributed. By this assumption, neither $\hat{s}_2$ nor $\theta_1$ (a function of $\hat{s}_2$) is a random variable and so each term can be taken out of the expectation operator, $E_0$, with $\tilde{\Omega}_1$ written as $\theta_1$ $\hat{s}_1$. As a result, the pricing formulas in (24)-(26) can be reduced as follows:

$$ {}_0d_1 = E_0[MRS_0^1]^* = \{E_0\tilde{s}_1^{1-\gamma}\}^{-1} \times E_0[\tilde{s}_1^{-\gamma}] \tag{30}$$

$$ {}_0d_2 = E_0[MRS_0^2]^* = \{E_0\tilde{s}_1^{1-\gamma}\}^{-1} \times \{E_1\tilde{s}_2^{1-\gamma}\}^{-1} \times E_0[\tilde{s}_1^{-\gamma} \quad \tilde{s}_2^{-\gamma}] \tag{31}$$

$$ {}_1\tilde{d}_2 = MRS_1^{2*} = \{E_1\tilde{s}_2^{1-\gamma}\}^{-1} \times \tilde{s}_2^{-\gamma}. \tag{32}$$

It is remarkable that contrasted with the result in the pure exchange economy, the bond prices are now independent of the intertemporal substitution parameter ($\rho$), and the discount factor ($\delta$) as well. The discrepancy between the results in the two economies is explained as follows: In the exchange economy where the mean (variance) effect is dominant [$m >$ ($<$) $1/2\ (\gamma - 1)\sigma^2$], higher $\rho$ will lower (raise) the marginal utility of future consumption, and thus the incentive to increase (decrease) current consumption must decrease (increase) bond prices and increase (decrease) interest rates in order for the agent not to consume more than his endowment. The higher $\delta$ will also increase bond prices and decrease interest rates (see Gilles and LeRoy (1986), and Sun (1992)). Hence, $\rho$ and $\delta$ appear in the pricing formulas. In the production economy, higher $\rho$ not only increases current consumption but also lowers future consumption by reducing current savings and thus future wealth. As a result, the lower marginal utility of current consumption, along with the higher marginal utility of future consumption, will have a positive effect on bond prices. On the other hand, higher $\rho$ also increases the future consumption rate ($\lambda_1$ increases with $\rho$), and thus will have a negative effect on bond prices. It turns out that the two effects exactly cancel each other out so that bond prices are independent of $\rho$.[11)] The impact of changing $\delta$ can be explained similarly. Hence, the disappearance of $\rho$ in the production economy illustrates another case where disentangling intertemporal substitution and risk aversion becomes totally irrelevant.[12)] Had the problem been analyzed in the time-additive power expected utility, the same bond price formulas would have been obtained. Put differently, for the purpose of the present analysis, time-additive expected power utility bypasses its constraint of itself.

Using the lognormality assumptions on $\tilde{s}_1$ and $\tilde{s}_2$ will yield closed-form formulas for interest rates and term premia as well. Comparative static analyses based on these formulas will produce results consistent with those obtained in the exchange economy except for those involving $\rho$ and $\delta$.[13)]

## 5. Summary and Conclusion

Using the non-expected recursive utility function of Epstein and Zin (1989), this article has presented three-date general

11) This result may hold due to the constant returns to scale production technology The two effects may not be offset exactly when other assumptions (e g , decreasing returns to scale) are made

12) Cho (1992) shows that the equity premium is independent of intertemporal substitution in a production economy In the observational irrelevancy case of Kocherlakota (1990), asset prices still depend on intertemporal substitution in the second-order sense This can be seen in the bond price formulas in section II of this paper Thus, the irrelevancy case here is tronger than that of Kocherlakota (1990)

13) When production technology is concave and i i d over time, Benninga and Protopapadakis (1986) shows that the *HP* is positive in a complete market, but not necessarily in an incomplete market if the utility function exhibits decreasing risk aversion

equilibrium term structure models in both exchange and production economies. Each model explores implications of separating intertemporal substitution and risk aversion for the sign and magnitude of the holding period and rolling premia

Disentangling the two preference components proves to be relevant in the following sense: In a pure exchange economy, increasing aversion to intertemporal substitution magnifies (reduces) each premium if the mean (variance) effect is dominant. One would not obtain this result under the time-additive power expected utility function.

Whereas, the distinction between the two preference components is irrelevant in a risk averse production economy. Assuming that the production technology exhibits constant returns to scale and that distributions of future productivity are identical, it is shown that bond prices are independent of the degree of intertemporal substitution (i.e., depend solely on risk aversion) and thus time-additive expected utility circumvents its constraint of itself.

The models in this paper lead to some additional results which differ from those predicted elsewhere: Under risk aversion, the return-to-maturity expectations hypothesis can be sustained in equilibrium. Moreover, in the risk averse production economy, logarithmic utility is not required to obtain the local expectations hypothesis under local certainty (compare to CIR (1981)). Contrary to quick intuition, the effect of increasing risk aversion or uncertainty on the magnitude of each premium is not definitive in both exchange and production economies (compare to Campbell (1986)).

## References

Backus, David K., Allan W. Gregory, and Stanley E. Zin, 1989, Risk Premiums in the Term Structure Evidence from Artificial Economies, *Journal of Monetary Economics* 24, 371-399

Benninga, Simon, and Aris Protopapadakis, 1986, General equilibrium properties of the term structure of interest rates, *Journal of Financial Economics* 16, 389-410

Campbell, John Y , 1986, Bond and stock returns in a simple exchange model, *Quarterly Journal of Economics* 101, 785-803

Campbell, John Y , and Robert J Shiller, 1984, A simple account of the behavior of long-term interest rates, *American Economic Review* 74, 44-48

Cho, Jaeho, 1992, The stock market premium, production, and relative risk aversion, *Economics Letters* 40, 193-196

Cho, Jaeho, 1998, On the Connection between the Expectations Hypothesis of the Term Structure and Risk Neutrality, Working Paper, College of Business Administration, Seoul National University

Cox, John C , Jonathan E Ingersoll, Jr , and Stephen A Ross, 1981, A re-examination of the traditional hypotheses of the term Structure of interest rates, *Journal of Finance* 36, 769-799

Cox, John C , Jonathan E Ingersoll, Jr , and Stephen A Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 385-407

Duffie, Darrell, and Larry G Epstein, 1992, Asset pricing with stochastic differential utility, *Review of Financial Studies* 5, 411-436

Epstein, Larry G., and Stanley E Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns A theoretical framework, *Econometrica* 57, 937-969

Gilles, C , and Stephen F LeRoy, 1986, A note on the local expectations hypothesis. A discrete-time exposition, *Journal of Finance* 41, 975-979

Gregory, Allan W , and Graham M Voss, 1991, The term structure of interest rates· Departures from time-separable expected utility, *Canadian Journal of Economics* 24, 923-939

Kandel, Shmuel, and Robert F Stambaugh, 1991, Asset returns and intertemporal preferences, *Journal of Monetary Economics* 27, 39-71.

Kocherlakota, Narayana R , 1990, Disentangling the coefficient of relative risk aversion from the elasticity of intertemporal substitution. An irrelevance result, *Journal of Finance* 45, 175-190.

Kreps, David M , and E L Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185-200

LeRoy, Stephen F , 1982, Risk aversion and the term structure of real interest rates, *Economics Letters* 10, 355-361

LeRoy, Stephen F , 1983, Risk aversion and the term structure of real interest rates Correction, *Economics Letters* 12, 339-340.

Lucas, Robert E , Jr., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1426-1446

Sun, Tong-sheng, 1992, Real and nominal interest rates A discrete-time model and its continuous-time limit, *Review of Financial Studies* 5, 581-611

Woodward, Susan, 1983, The liquidity premium and the solidity premium, *American Economic Review* 73, 348-361