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Analysis of Sensitivity Patterns for MT Response Functions and its Application to Inversion

MT 반응함수들의 민감도 패턴 분석 및 역산에의 적용

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Abstract

Analysis of Sensitivity Patterns for MT Response Functions and its Application to Inversion

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Magnetotelluric (MT) exploration measures the electric and magnetic fields generated by natural sources over time at the surface. The measured electromagnetic fields are converted into various MT response functions in the frequency domain, which are used in inversion to image subsurface structures. Inversion results are dependent on types of model parameters and input data. There have been studies on characteristics of multi-parameters using the sensitivity matrix in geophysical inversion. Also, there have been studies showing that different inversion results are obtained according to different input MT response functions. However, those studies did not examine which factors of the MT response functions cause different inversion results.

In this thesis, sensitivity patterns are analyzed and applied to characterize major MT response functions (impedance tensor, apparent resistivity, phase, tipper, effective impedance, and phase tensor) in inversion. Because the sensitivity pattern represents changes of data in space due to a small change in a model parameter, it represents the features of MT response functions in the inverse procedure that matches the modeling MT responses for the model parameter vector with the observed data. This thesis describes both 3D sensitivity patterns and 2D sensitivity patterns on the surface where MT data are acquired. Then, a total of 22 MT response functions are classified into six groups. MT response functions with similar surface-sensitivity patterns (i.e., playing a similar role in inversion) are classified into the same group. MT response functions in the different groups can have complementary roles in inversion.

In synthetic situations for 1D, 2D, and 3D interpretation of MT data, it is investigated how observed data and inversion results for the MT response functions differ according to their sensitivity patterns. Through these synthetic examples, it is demonstrated that 1) the effective impedance is optimal when considering the dimensionality error in 1D interpretation; 2) Transverse magnetic (TM) mode MT response functions are recommended in general 2D interpretation, but the tipper is superior for a receiver array that cannot use data recorded right above a target 2D structure; and 3) 3D inversion results can be improved when the impedance tensor is selectively used in a specific case where two anomalies exist vertically. The examples are summarized in one table that recommends the input MT response functions for inversion in the given exploration situations.

To investigate the feasibility of applying sensitivity patterns to inversion of MT field data, four strategies of selecting MT response functions are considered for the field data of the Utah Frontier Observatory for Research in Geothermal Energy (FORGE). The case study represents that the MT response functions selected from

the sensitivity patterns can enhance inversion results. Considering the MT field situation where receivers are widely distributed along the y-axis of the target structure, it can be inferred that the responses of the target structure may be mainly contained in the y-component of the tipper whose sensitivity patterns are in the shape of two petals in the y-axis direction. The model inverted by the data selection strategy including the y-component of the tipper clearly represents the target body. Several error analyses indicate that the y-component of the tipper makes a significant contribution for imaging of the target structure.

This thesis provides a guideline for selecting the optimal MT response functions in various MT inversions using the sensitivity patterns.

Keyword: Magnetotelluric, MT response function, Sensitivity pattern, Inversion, Utah FORGE
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Chapter 1. Introduction

1.1. Background of the study

Magnetotellurics (MT), which is one of the passive geophysical electromagnetic methods, was independently introduced by Japanese (Rikitake 1948), Russian (Tikhonov 1950), and French (Cagniard 1953) geophysicists. The terminology 'magnetotellurics' consists of two parts, 'magneto' and 'telluric', which imply the magnetic fields and electric currents in the earth, respectively (Cagniard 1953). As can be inferred from the two words, the MT exploration measures the electromagnetic fields at the surface generated by natural sources to estimate the earth's electrical properties, such as electrical resistivity or its reciprocal, electric conductivity.

The dependence of the MT method on the natural sources brings both the pros and cons. Because the natural sources contain low-frequency components that are difficult to artificially generate, the MT exploration has a great depth of penetration. On the other hand, the weak natural electromagnetic fields make the MT survey more susceptible to noise. The interaction of the solar wind (a plasma stream ejected from the upper atmosphere of the sun) with the ionosphere and magnetosphere around the earth generates low-frequency components of the measured electromagnetic fields (< 10 Hz). High-frequency components of the measured data come from some electromagnetic energy that is generated by the worldwide thunderstorm activities and travels bounded between the surface and the ionosphere of the earth (Chave & Jones 2012). The natural electromagnetic fields can be assumed as plane waves because they travel far distances before reaching the surface of the earth. Due to the large resistivity differences between the air and the earth, most of the natural electromagnetic sources are reflected at the surface, while a small amount of the energy propagates into the earth. Since the velocity of electromagnetic waves in the conductive earth is much smaller than in the almost dielectric air, the transmitted electromagnetic waves propagate vertically into the earth satisfying Snell's law (Vozoff 1972; Vozoff 1991). The MT method is based on the two important assumptions that the natural sources are plane-polarized electromagnetic waves, and they impinge on the earth as near-vertical incidence.

The MT method goes through four procedures: data acquisition, data processing, imaging, and interpretation. In the data acquisition stage, five components of the electromagnetic fields are measured over time. Two horizontal components of the electric fields (E_x and E_y) and magnetic fields in all directions (H_x , H_y , and H_z) are acquired at the surface using electrodes connected with cables and magnetometers (e.g., induction coils and/or fluxgate magnetometers), respectively. Especially, the remote reference method, which simultaneously acquires additional MT data at a station remote from the main measurement site, is used to remove local electromagnetic noises (Gamble et al. 1979; Simpson & Bahr 2005). The data obtained by the remote reference method are utilized in the next data processing procedure. The measured electromagnetic fields can be processed in the time domain for better quality.

In the data processing procedure, the measured and processed electromagnetic fields in the time domain are converted to other MT response functions in the frequency domain. The first step of the procedure is a Fourier transform of the electromagnetic fields from the time to the frequency domains. Next, the data in the format of the electromagnetic fields are transformed into other MT response functions in the frequency domain (in this step remote reference data, especially the magnetic fields, can be used). Various MT response functions such as impedance tensor, apparent resistivity, phase, and vertical magnetic transfer function (i.e., tipper) were introduced (Tikhonov 1950; Cagniard 1953; Cantwell 1960; Vozoff 1991; Chave & Jones 2012). Because it is difficult to know the exact information about the natural sources for the MT exploration, the MT response functions are defined as the ratio of the measured electromagnetic fields, so that they are not affected by the amplitude of the natural sources. Furthermore, to analyze subsurface features such as dimensionality, directionality, and galvanic distortion, derived MT response functions (e.g., inhomogeneity parameter, skew, and phase tensor) and techniques dealing with those response functions (e.g., polar diagram, induction arrow, and groom-bailey distortion decomposition) have also been studied (Simpson & Bahr 2005; Berdichevsky & Dmitriev 2008; Berdichevsky & Dmitriev 2010; Chave & Jones 2012). After the data processing, the noise-removing process in the frequency domain may be applied using the apparent resistivity curves or Nyquist diagrams (Egbert 1997; Yang et al. 2019; Uhm et al. 2021).

For imaging the subsurface electrical properties from the processed MT data, a number of numerical forward modeling and inversion schemes have been developed. The representative modeling schemes are the integral equation (IE) method (Wannamaker et al. 1984; Newman & Hohmann 1988; Wannamaker 1991), the finite difference method (FDM) (Pek & Verner 1997; Siripunvaraporn et al. 2005b), and the finite element method (FEM) (Nam et al. 2007; Liu et al. 2008; Ren et al. 2013). For the inversion scheme, Constable et al. (1987) introduced Occam's inversion that yields simple and smooth models by considering the model roughness in an objective function, and then many studies adopted their idea to image subsurface structures from MT data (deGroot-Hedlin & Constable 1990; Ogawa & Uchida 1996; Key 2016). A number of inversion schemes related to model constraint, data weighting, roughness matrix, etc., (Avdeeva 2008; Abubakar et al. 2009; Usui 2015; Key 2016) were also proposed to increase the convergence and stability of the inversion process. In addition, after some free MT inversion software packages, such as MARE2DEM (Key 2016), ModEM (Kelbert et al. 2014), and WSINV3DMT (Siripunvaraporn et al. 2005b), were released, imaging technology using inversion has been conveniently and commonly applied.

Finally, in the interpretation procedure, subsurface structures are interpreted from an inverted model according to the purposes of MT exploration. One of the main purposes of MT exploration is to reveal geological structures; Wei et al. (2001) represented the crust structures of the Tibetan plateau, Becken & Ritter (2012) studied the San Andreas Fault zone, and Naif et al. (2013) imaged the lithosphereasthenosphere boundary (LAB) beneath the edge of the Cocos plate at the Middle America trench offshore of Nicaragua. Also, MT method is applied for engineering purposes, e.g., geothermal systems (Newman et al. 2008), mineral deposits (Farquharson & Craven 2009), and hydrocarbons (Patro 2017). Moreover, some projects for mapping the country's geological structures using MT data have been conducted such as 'SinoProbe' that is for deep surveys in China including MT exploration (Dong et al. 2013), 'US Array MT data' acquired across the continental USA (Meqbel et al. 2014), and 'Australian Lithospheric Architecture Magnetotelluric Project (AusLAMP)' (Kirkby et al. 2020).

1.2. Research objective

As mentioned in Chapter 1.1, there are many MT response functions, and inversion is the main technique for imaging subsurface structures from MT data. The important point is that the inversion result depends on both model parameters and input data. In geophysical methods, for multi-parameter inversion retrieving anisotropic properties, the sensitivity matrix (i.e., Jacobian matrix) has been investigated to analyze the characteristics of each model parameter in the inverse problem (Ramananjaona et al. 2011; Operto et al. 2013; Oh & Alkhalifah 2016). Also, some studies showed that inversion results rely on which MT response function is used for the inverse process: Siripunvaraporn et al. (2005a) represented that each component of the impedance tensor yields different inversion results; Wang et al. (2019) and Luo et al. (2020) showed that meaningful changes occur in inverted models when the tipper data are included; Pedersen & Engels (2005) compared inversion results using the effective impedance with those obtained by transverse electric (TE) or transverse magnetic (TM) mode impedance; and Patro et al. (2013) described 3D models imaged by the impedance and phase tensors. However, unlike the studies on different model parameters, the studies on different types of MT data do not examine how various MT response functions affect inversion results, but simply compare the models inverted from the MT response functions with each other.

In this study, to investigate the main characteristics of various MT response functions for inversion, sensitivity patterns based on the Jacobian matrix are analyzed. Then, the MT response functions are classified into several groups. The MT response functions in the same group possess similar sensitivity patterns, and therefore produce similar inversion results. On the other hand, the MT response functions in other groups whose sensitivity patterns are rarely overlapped spatially with each other can play a complementary role in the inverse procedure. Furthermore, synthetic examples and a case study show that the MT response functions selected from the sensitivity patterns can improve inversion results. Consequentially, this study provides a guideline on which MT response function is better to use for inversion according to the situation of MT exploration using their sensitivity patterns.

1.3. Outline

In Chapter 2, the theories of 3D MT modeling, 3D MT inversion, and major MT response functions (impedance tensor, apparent resistivity, phase, tipper, effective impedance, and phase tensor) are described. In Chapter 3, the sensitivity patterns for the major MT response functions are presented, and their features are analyzed. The various MT response functions are divided into six groups according to the features of the sensitivity patterns. In Chapter 4, synthetic examples for 1D, 2D, and 3D interpretation of MT data are provided to explain how observed data and inversion results obtained in some specific structures differ according to the characteristics of the sensitivity patterns. Chapter 5 shows a case study establishing a strategy to select MT response functions in consideration of the sensitivity patterns and field environment, and examining the difference in the inverted models.

Chapter 2. Theory

2.1. 3D MT modeling

For 3D MT modeling, Maxwell's equations are solved by numerical simulation for a given electrical conductivity (or electrical resistivity) model with boundary conditions. Maxwell's equations, the numerical simulation using the edge-based finite element method, and the boundary conditions for MT method are described in Chapters 2.1.1, 2.1.2, and 2.1.3, respectively.

2.1.1. Maxwell's equations

Maxwell's equations are a set of four fundamental equations that describe behaviors of the electromagnetic fields. Maxwell's equations are expressed in general differential forms as (Ward & Hohmann 1987):

$$\nabla \cdot \mathbf{d} = \rho_{v} \quad , \tag{2-1}$$

$$\nabla \cdot \mathbf{b} = 0 \quad , \tag{2-2}$$

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t} \quad , \tag{2-3}$$

$$\nabla \times \mathbf{h} = \frac{\partial \mathbf{d}}{\partial t} + \mathbf{j} \quad , \tag{2-4}$$

where **d** and **b** are the electric (C/m²) and magnetic (Wb/m²) flux density,

respectively; **e** and **h** are the electric (V/m) and magnetic (A/m) fields, respectively; ρ_{ν} is the electric volume charge density (C/m³); and **j** is the electric current density (A/m²). $\nabla \cdot$ and $\nabla \times$ are the divergence and curl operators, respectively. Equations (2-1) and (2-2) are Gauss's law for electricity and magnetism, respectively; equation (2-3) is Faraday's law; and equation (2-4) is Ampère-Maxwell's law.

Maxwell's equations in the frequency domain with the time convention $e^{-i\omega t}$ (where ω is the angular frequency) can be written as:

$$\nabla \cdot \mathbf{D} = \rho_{v} \quad , \tag{2-5}$$

$$\nabla \cdot \mathbf{B} = 0 \quad , \tag{2-6}$$

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B} \quad , \tag{2-7}$$

$$\nabla \times \mathbf{H} = i\omega \mathbf{D} + \mathbf{J} \quad , \tag{2-8}$$

where **D** and **B** are the electric and magnetic flux density in the frequency domain, respectively; **E** and **H** are the electric and magnetic fields in the frequency domain, respectively; and **J** is the electric current density in the frequency domain. In order to express Maxwell's equations only with **E** and **H**, the constitutive relations describing the macroscopic properties of the medium are considered (Ward & Hohmann 1987), which are written below:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad , \tag{2-9}$$

$$\mathbf{B} = \boldsymbol{\mu} \mathbf{H} \quad , \tag{2-10}$$

$$\mathbf{J} = \boldsymbol{\sigma} \mathbf{E} \quad , \tag{2-11}$$

where ε , μ , and σ are the dielectric permittivity (F/m), the magnetic permeability (H/m), and the electrical conductivity (S/m), respectively.

The third and fourth formulae (equations 2-7 and 2-8) depict the propagation of the electric and magnetic fields. Introducing the constitutive relations (equations 2-9 to 2-11), equations (2-7) and (2-8) can be rearranged as follows:

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H} \quad , \tag{2-12}$$

$$\nabla \times \mathbf{H} = (i\omega\varepsilon + \sigma)\mathbf{E} \quad . \tag{2-13}$$

Among the material properties of ε , μ , and σ in equations (2-12) and (2-13), the effect of dielectric permittivity (ε) is negligible because $\omega\varepsilon \ll \sigma$ in the frequency range of MT exploration (about $10^{-4} \sim 10^5$ Hz). This means that in the periods of MT survey (about $10^{-5} \sim 10^4$ s), the electric displacement current density ($\partial \mathbf{d}/\partial t$ in equation 2-4) can be ignored, i.e., the quasi-static approximation can be adopted in the MT method. Variations in the magnetic permeability (μ) of rocks can be also neglected compared with variations in the conductivity (σ) of bulk rocks. Therefore, μ is assumed as a constant of the magnetic permeability at the free space ($\mu_0 = 4\pi \times 10^{-7}$ H/m) (Chave & Jones 2012). Substituting equation (2-13) into the curl of equation (2-12) leads to the governing equation of 3D MT modeling consisting of only the electric fields (i.e., the vector Helmholtz equation of **E**) with the single variable of the electrical conductivity (σ), which can be expressed as follows:

$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu_0 \sigma \mathbf{E} = 0 \quad . \tag{2-14}$$

By using vector identity and assuming $\nabla \cdot \mathbf{e} = 0$ inside the earth, equation (2-14) can be rewritten in the time domain as follows:

$$\nabla \times \nabla \times \mathbf{e} = \nabla (\nabla \cdot \mathbf{e}) - \nabla^2 \mathbf{e} = -\nabla^2 \mathbf{e} = -\mu_0 \sigma \frac{\partial \mathbf{e}}{\partial t} \quad , \tag{2-15}$$

$$\therefore \nabla^2 \mathbf{e} = \mu_0 \sigma \frac{\partial \mathbf{e}}{\partial t} \quad , \tag{2-16}$$

where ∇^2 is the vector Laplacian operator. Because equation (2-16) is in the form of a diffusion equation, the electromagnetic waves generated from the MT sources diffuse in the earth and dissipate exponentially.

A skin depth is defined as the depth at which the amplitude of the electromagnetic fields decays to $1/e \approx 37\%$ of the amplitude at the surface in a homogeneous half-space medium (Chave & Jones 2012). In such a medium, there are no vertical components of the electromagnetic fields (i.e., $E_z = 0$ and $H_z = 0$), and only E_x and H_y (or E_y and H_x) are relevant that only vary with depth (i.e., $\partial \mathbf{E}/\partial x = 0$, $\partial \mathbf{H}/\partial x = 0$, $\partial \mathbf{E}/\partial y = 0$, and $\partial \mathbf{H}/\partial y = 0$). Considering these conditions, equation (2-14) can be written as follows:

$$\frac{\partial^2 E_x}{\partial z^2} = i\omega\mu_0 \sigma E_x \quad . \tag{2-17}$$

Because there is no resistivity interface in the homogeneous half-space medium (i.e., there is no reflection), the elementary solution of equation (2-17) can be written as

follows:

$$E_{x} = A \exp(-kz); k = \sqrt{i\omega\mu_{0}\sigma} \quad , \qquad (2-18)$$

$$k = \sqrt{i\omega\mu_0\sigma} = \alpha + i\alpha; \ \alpha = \sqrt{\frac{\omega\mu_0\sigma}{2}}$$
, (2-19)

$$\therefore E_x = A \exp(-\alpha z) \exp(-i\alpha z) \quad . \tag{2-20}$$

In equation (2-20), the term of $\exp(-\alpha z)$ represents the amplitude attenuation, while the term of $\exp(-i\alpha z)$ describes the harmonic motion. According to the definition of the skin depth δ (m), the following formula must be satisfied:

$$\frac{\exp(-\alpha\delta)}{\exp(0)} = \exp(-1) \quad . \tag{2-21}$$

If equation (2-21) is rearranged, the skin depth for MT survey is calculated as follows:

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2\rho}{\left(2\pi f\right)\left(4\pi \times 10^{-7}\right)}} \approx 503\sqrt{\frac{\rho}{f}} \quad , \tag{2-22}$$

where f is the frequency (Hz); and ρ is the electrical resistivity (Ω m). The skin depth is an important factor in MT method because it implies the penetration depth of the electromagnetic waves.

2.1.2. Edge-based finite element method

There are various ways to find numerical solutions of equation (2-14) derived from Maxwell's equations for a given 3D electrical conductivity model. Initially the integral equation (IE) approach (Wannamaker et al. 1984; Newman & Hohmann 1988; Wannamaker 1991) was applied. The IE approach requires a small number of unknowns, but is restricted to simple background models (Farquharson & Miensopust 2011). As computational technology has advanced, the finite difference method (FDM) or the finite element method (FEM) has been widely used for complex 3D models. However, numerical solutions obtained by the FDM that discretizes models with strong topography in stair-stepped grids may be incorrect in narrow period bands (Müller & Haak 2004). The FEM can be an appropriate method to incorporate topography (Nam et al. 2007; Ren et al. 2013; Usui 2015) and is used in this study.

The FEM is a practical technique for obtaining approximate solutions by numerically solving the partial differential equations by subdividing an entire domain into small and simple local elements. The solution of the governing equation is approximated by a linear combination of shape functions defined within each element. By assembling the governing equation for each local element into the entire computational domain, a global matrix equation is constructed. By solving this matrix equation, the approximate solution for the entire domain can be obtained. The characteristics of the FEM vary depending on which shape function is used. Shape functions of the edge-based FEM are vector basis that assigns degrees of freedom to the edges of each element (Whitney 1957; Nédélec 1980). Therefore, the edge-based FEM approximates the electric fields at an arbitrary position within the local element as follows:

$$\mathbf{E}^{e} = \sum_{j=1}^{n_{edge}} E_{j}^{e} \mathbf{\Phi}_{j}^{e} \quad , \qquad (2-23)$$

$$E_j^e = \int_{I_j} \mathbf{E} \cdot \mathbf{d} \mathbf{l} \quad , \tag{2-24}$$

where \mathbf{E}^{e} is the electric fields in the element; E_{j}^{e} is a scalar coefficient defined at the *j*-th edge; Φ_{j}^{e} is the vector shape function associated with the *j*-th edge; n_{edge} is the number of edges of the element; and l_{j} represents the length of the *j*th edge.

The vector shape function of the edge-based FEM has two important features. The first feature is that $\nabla \cdot \mathbf{\Phi}_{j}^{e} = 0$ in the local element (Jin 2002). Because this feature leads to the formula as below:

$$\nabla \cdot \mathbf{E}^{e} = \sum_{j=1}^{n_{edge}} E_{j}^{e} \ \nabla \cdot \mathbf{\Phi}_{j}^{e} = 0 \quad , \qquad (2-25)$$

the edge-based FEM makes the divergence-free condition of the electric field (i.e., $\nabla \cdot \mathbf{E} = 0$) satisfied in the earth. Another feature is that at any position on a surface of the local element, the tangential components exist only in the shape functions associated with the edges constituting the surface. In other words, even if the tangential electric fields on the surface are calculated at two adjacent elements, they are identically expressed as the linear combination of the shape functions and the

scalar coefficients related to the edges constituting the surface (Jin 2002). Therefore, the edge-based FEM naturally satisfies one of the boundary conditions that the tangential component of the electric fields must be continuous at the interface. Because of these two features of the vector shape function, the edge-based FEM is advantageous for avoiding the spurious solutions (Webb 1993; Jin 2002). In this thesis, the edge-based FEM with the direct solver (Chung et al. 2014; Usui 2015) is adopted to simulate the electric fields in the MT method.

To derive the variational formula of the FEM for the governing equation (2-14), the Galerkin's method (one of the weighted residual methods) is used, and its residual (\mathbf{r}) is defined as follows:

$$\mathbf{r} = \nabla \times \nabla \times \mathbf{E} + i\omega\mu_0 \sigma \mathbf{E} \quad . \tag{2-26}$$

In this method, the shape functions are applied as a weighting, and the coefficients for the shape functions are obtained to minimize the variational formula (the dot product of the residual and the weighting) for the entire domain. Thus, the variational formula for the Galerkin's method is defined as follows:

$$\sum_{e=1}^{N_e} \sum_{i=1}^{n_{edge}} \int_{\Omega_e} \mathbf{r} \cdot \mathbf{\Phi}_i^e dV = 0 \quad , \tag{2-27}$$

where N_e is the total number of elements; and Ω_e is the volume of the each local element. Substituting the residual (equation 2-26) into the integral term of the variational formula (equation 2-27) yields

$$\int_{\Omega_e} \mathbf{r} \cdot \mathbf{\Phi}_i^e dV = \int_{\Omega_e} \left(\nabla \times \nabla \times \mathbf{E} \right) \cdot \mathbf{\Phi}_i^e dV + \int_{\Omega_e} i\omega\mu_0 \sigma \mathbf{E} \cdot \mathbf{\Phi}_i^e dV \quad .$$
(2-28)

From the vector calculus identity, $(\nabla \times \mathbf{a}) \cdot \mathbf{b} = \nabla \cdot (\mathbf{a} \times \mathbf{b}) + \mathbf{a} \cdot (\nabla \times \mathbf{b})$ where $\mathbf{a} = \nabla \times \mathbf{E}$ and $\mathbf{b} = \mathbf{\Phi}_i^e$, the first term on the right-hand side of equation (2-28) is rearranged as follows:

$$\int_{\Omega_e} \left(\nabla \times \nabla \times \mathbf{E} \right) \cdot \mathbf{\Phi}_i^e dV = \int_{\Omega_e} \nabla \cdot \left\{ \left(\nabla \times \mathbf{E} \right) \times \mathbf{\Phi}_i^e \right\} dV + \int_{\Omega_e} \left(\nabla \times \mathbf{E} \right) \cdot \left(\nabla \times \mathbf{\Phi}_i^e \right) dV \quad .$$
(2-29)

If the divergence theorem is applied to the first term on the right-hand side of equation (2-29), it can be written as follows:

$$\int_{\Omega_e} \nabla \cdot \left\{ \left(\nabla \times \mathbf{E} \right) \times \mathbf{\Phi}_i^e \right\} dV = \oint_{\partial \Omega_e} \mathbf{n} \cdot \left\{ \left(\nabla \times \mathbf{E} \right) \times \mathbf{\Phi}_i^e \right\} dS \quad , \tag{2-30}$$

where $\partial \Omega_e$ is the surface surrounding the local element Ω_e ; and **n** is a unit normal vector pointing outward of Ω_e on the surface. When the local variational formula of each element is assembled into the entire domain following equation (2-27), values for the right-hand side of equation (2-30) are canceled out between the elements sharing the same surface except for the values at the boundaries of the computational domain. However, the remaining values do not need to be considered when the Dirichlet boundary condition is applied to MT modeling. Considering equations (2-29) and (2-30), equation (2-28) is rearranged as follows:

$$\int_{\Omega_e} \mathbf{r} \cdot \mathbf{\Phi}_i^e dV = \int_{\Omega_e} \left(\nabla \times \mathbf{E} \right) \cdot \left(\nabla \times \mathbf{\Phi}_i^e \right) dV + \int_{\Omega_e} i \omega \mu_0 \sigma \mathbf{E} \cdot \mathbf{\Phi}_i^e dV \quad . \quad (2-31)$$

By substituting equation (2-23) into equation (2-31) and rearranging equation (2-27), the equation containing the scalar coefficient (E_j^e) can be written as follows:

$$\sum_{e=1}^{N_e} \left[\sum_{i=1}^{n_{edge}} \sum_{j=1}^{n_{edge}} \left\{ \left(K_{ij}^e + i\omega\mu_0 \sigma M_{ij}^e \right) E_j^e \right\} \right] = 0 \quad , \tag{2-32}$$

where

$$K_{ij}^{e} = \int_{\Omega_{e}} \left(\nabla \times \mathbf{\Phi}_{i}^{e} \right) \cdot \left(\nabla \times \mathbf{\Phi}_{j}^{e} \right) dV \quad , \qquad (2-33)$$

$$M_{ij}^{e} = \int_{\Omega_{e}} \mathbf{\Phi}_{i}^{e} \cdot \mathbf{\Phi}_{j}^{e} dV \quad .$$
(2-34)

 K_{ij}^{e} and M_{ij}^{e} are the components of the elementary stiffness and mass matrices, respectively. By assembling the elementary stiffness and mass matrices over the entire domain using the global edge number, equation (2-32) can be expressed as the following global matrix equation:

$$\mathbf{A}\mathbf{x} = \mathbf{0} \quad , \tag{2-35}$$

where $\mathbf{A} \in \mathbb{R}^{N_{edge} \times N_{edge}}$ (N_{edge} is the total number of edges for the entire elements) is the coefficient matrix; and $\mathbf{x} \in \mathbb{R}^{N_{edge}}$ is the unknown vector composed of the scalar coefficients (E_e) following the order of edges for the whole elements. Because $K_{ij}^e = K_{ji}^e$ and $M_{ij}^e = M_{ji}^e$ in equations (2-33) and (2-34), the coefficient matrix (\mathbf{A}) consisting of the elementary stiffness and mass matrices is symmetric. An appropriate source vector for the right-hand side of equation (2-35) is derived when the Dirichlet boundary condition is considered (will be described in the next section).

In this study, tetrahedral or structured hexahedral elements are used to discretize the entire 3D model, and their shape functions, curl of shape functions, elementary stiffness matrix, and elementary mass matrix required for 3D MT modeling are described in Appendix A.

2.1.3. Boundary conditions

The source term in the matrix equation (2-35) is generated by applying the Dirichlet boundary condition to the boundaries of the 3D model. Generally, the Cartesian coordinate system is applied and the entire 3D model is assumed as a cuboid for 3D MT modeling. It is also assumed that the boundaries of the computational domain are sufficiently far away from the domain of interest so that the structures within the target area do not affect the electric fields at the boundaries. At the boundaries of the 3D model, the tangential components of the electric field at the interface should be continuous:

$$\mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_{\partial \Omega} \quad , \tag{2-36}$$

where **n** is a normal unit vector pointing outward the boundary; $\partial \Omega$ is the outermost boundary of the 3D model; and $\mathbf{E}_{\partial\Omega}$ is the electric fields at the boundary. By dividing the boundaries of the coboid 3D model into top, bottom, and side boundaries, equation (2-36) can be considered as follows:

$$\mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_{Top}$$
 at the top boundary , (2-37)

$$\mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_{Bot}$$
 at the bottom boundary , (2-38)

$$\mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_{2D}$$
 at the side boundaries . (2-39)

For 3D MT modeling, the electromagnetic fields generated from perpendicularly incident plane waves polarized in x- and y-directions are required. The boundary conditions of equations (2-37) to (2-39) depend on the direction in which the source is polarized.

For the E_x source, also called xy-polarization (Nam et al. 2007), $\mathbf{E}_{Top} = (1,0,0)$ in equation (2-37) and $\mathbf{E}_{Bot} = \mathbf{0}$ in equation (2-38) considering the sufficiently deep bottom boundary (at least 3 times of the maximum skin depth). In the two yz-planes whose strike direction is parallel to the x-axis among the side boundaries for \mathbf{E}_{2D} in equation (2-39), the E_x source is equivalent to TE mode, and $\mathbf{n} \times \mathbf{E}_{2D} = \mathbf{0}$ because only E_x , H_y , and H_z components exist in the TE mode (McNeill and Labson, 1991) and the direction of the normal vector (\mathbf{n}) is the x-axis. On the other hand, the two xz-planes have the strike along the y-axis, which is perpendicular to the E_x source (i.e., TM mode). In this TM mode, E_x , E_z , and H_y components only exist (McNeill and Labson, 1991) and the direction of the normal vector (\mathbf{n}) is the y-axis, therefore $\mathbf{n} \times \mathbf{E}_{2D} \neq \mathbf{0}$ and \mathbf{E}_{2D} should be calculated through 2D MT modeling with 2D elements extracted from the 3D elements at the boundaries of the two xz-planes. Similar to 3D MT modeling, the governing equation and the boundary conditions for 2D MT modeling are as follows:
$$\nabla_t \times \nabla_t \times \mathbf{E}_{2D} + i\omega\mu_0 \sigma \mathbf{E}_{2D} = 0 \quad , \tag{2-40}$$

$$\mathbf{n} \times \mathbf{E}_{2D} = \mathbf{n} \times \mathbf{E}_{2DTop}$$
 at the 2D top boundary , (2-41)

$$\mathbf{n} \times \mathbf{E}_{2D} = \mathbf{n} \times \mathbf{E}_{2DBot}$$
 at the 2D bottom boundary , (2-42)

$$\mathbf{n} \times \mathbf{E}_{2D} = \mathbf{n} \times \mathbf{E}_{1D}$$
 at the side boundaries , (2-43)

where ∇_t is the transverse del operator. The transverse del operator at the xz-plane is $(\partial/\partial x, \partial/\partial y = 0, \partial/\partial z)$. $\mathbf{E}_{2DTop} = \mathbf{E}_{Top} = (1,0,0)$ and $\mathbf{E}_{2DBot} = \mathbf{E}_{Bot} = \mathbf{0}$ in equations (2-41) and (2-42), respectively. The direction of the vector (**n**) at the side of the xz-plane is the x-axis, and \mathbf{E}_{1D} includes only E_x component for the E_x source, thus $\mathbf{n} \times \mathbf{E}_{1D} = \mathbf{0}$ in equation (2-43). At the xz-plane side boundaries for the E_x source, \mathbf{E}_{2D} is obtained by solving a matrix equation based on equation (2-40) with the Dirichlet boundary conditions (equations from 2-41 to 2-43) and it is applied in equation (2-39).

Similarly, for the E_y source also called yx-polarization (Nam et al. 2007), $\mathbf{E}_{Top} = (0,1,0)$ in equation (2-37); $\mathbf{E}_{Bot} = \mathbf{0}$ in equation (2-38); $\mathbf{n} \times \mathbf{E}_{2D} = \mathbf{0}$ at the xz-plane side boundaries in equation (2-39); and \mathbf{E}_{2D} at the yz-plane side boundaries in equation (2-39) is calculated through the 2D MT modeling.

The electric fields in equations (2-37) to (2-39) are projected along each edge of the elements at the boundaries. Then, the Dirichlet boundary conditions are applied to make the projected value be a solution of the unknown scalar coefficient (E_e in equation 2-35). When the Dirichlet boundary conditions are applied to equation (2-35), both the coefficient matrix (**A**) and the vector on the right-hand side are modified. As an example of equation (2-35), the 3 by 3 matrix equation before applying the boundary condition can be expressed as follows:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} .$$
(2-44)

When the Dirichlet boundary condition ($E_1 = E_s$) is applied, the matrix equation (2-44) are transformed as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} E_s \\ -A_{21}E_s \\ -A_{31}E_s \end{bmatrix} .$$
(2-45)

As in equations (2-44) and (2-45), the modified matrix equation can be obtained by incorporating the Dirichlet boundary conditions (equations 2-37 to 2-39) into equation (2-35):

$$\mathbf{\hat{A}x} = \mathbf{b} \quad , \tag{2-46}$$

where $\tilde{\mathbf{A}} \in \mathbb{R}^{N_{edge} \times N_{edge}}$ is the modified coefficient matrix; and $\mathbf{b} \in \mathbb{R}^{N_{edge}}$ is the modified vector serving as a source term. In this study, because the modified coefficient matrix ($\tilde{\mathbf{A}}$) is a symmetric sparse matrix, the matrix equation (2-46) is solved by PARDISO contained in Intel MKL, which is one of the sparse direct solvers.

Finally, the electric and magnetic fields at the receivers are obtained through

the scalar coefficients (E_e) solved by equation (2-46) with equations (2-23) and (2-12), which are expressed as follows:

$$\mathbf{E}\left(\mathbf{r}_{g}\right) = \sum_{i=1}^{n_{edge}} E_{i}^{e_{r}} \mathbf{\Phi}_{i}^{e_{r}}\left(\mathbf{r}_{l}\right) , \qquad (2-47)$$

$$\mathbf{H}(\mathbf{r}_{g}) = -\frac{1}{i\omega\mu_{0}} \sum_{i=1}^{n_{edge}} E_{i}^{e_{r}} \left(\nabla \times \mathbf{\Phi}_{i}^{e_{r}} \left(\mathbf{r}_{l} \right) \right) , \qquad (2-48)$$

where \mathbf{r}_{g} and \mathbf{r}_{l} are the location of the receiver in the global coordinates and corresponding local coordinates of the element, respectively; and e_{r} is the element that contains the location of the receiver. If the electromagnetic fields in the receivers are defined as a vector $\mathbf{u} \in \mathbb{R}^{n_{d}}$ where n_{d} is the number of data, it can be represented by the following relation using the solution vector (\mathbf{x}) in equation (2-46) (Heo 2022):

$$\mathbf{u} = \mathbf{P}\mathbf{x} \quad , \tag{2-49}$$

where $\mathbf{P} \in \mathbb{R}^{n_d \times N_{edge}}$ is the projection matrix based on equations (2-47) and (2-48). Then, the electromagnetic fields at the receivers can be converted into various MT response functions (discussed in detail in Chapter 2.3).

2.2. 3D MT inversion

MT modeling is a technique for calculating MT responses from a given model parameter vector, while MT inversion is a technique for creating a subsurface model from given MT data. In other words, MT inversion is a process of finding such a model parameter vector that its modeling results fit well with the observed data. Therefore, the relationship between the model parameter vector, $\mathbf{m} \in \mathbb{R}^{n_m}$ where n_m is the number of model parameters, and the modelled data $\mathbf{d}(\mathbf{m}) \in \mathbb{R}^{n_d}$ is important. It is expressed by the following Jacobian matrix ($\mathbf{J} \in \mathbb{R}^{n_d \times n_m}$) (Nocedal & Wright 2006):

$$\mathbf{J}(\mathbf{m}) = \frac{\partial \mathbf{d}(\mathbf{m})}{\partial \mathbf{m}} \quad . \tag{2-50}$$

The model parameter vector and the modelled data are not in a linear relationship. To solve the nonlinear inverse problem in an iterative manner, the Gauss-Newton method, which is one of the Newton-type optimization methods assuming the linearity based on the Taylor series, has been widely used (Constable et al. 1987; Zhdanov 2002; Sasaki 2004). Especially, Occam's inversion proposed by Constable et al. (1987) yields smoothly inverted models by introducing the roughness of a given model in an objective function. Many studies adopted Occam's inversion scheme to stabilize and improve the inverse process. In this study, the inversion algorithm based on Occam's inversion is also adopted.

2.2.1. Occam's inversion with the Gauss-Newton method

Inversion is an optimization problem finding a model that minimizes the objective function (or the cost function) based on the difference between observed data and modeling results. The objective function $O(\mathbf{m}) \in \mathbb{R}$ of Occam's inversion consists of the data misfit $O_d(\mathbf{m}) \in \mathbb{R}$ and the model roughness $O_m(\mathbf{m}) \in \mathbb{R}$ as follows:

$$O(\mathbf{m}) = O_d(\mathbf{m}) + \lambda O_m(\mathbf{m}) \quad , \tag{2-51}$$

where $\lambda \in \mathbb{R}$ is the Lagrange multiplier. If the data misfit and the model roughness are defined using the l_2 -norms, equation (2-51) can be rewritten as follows:

$$O(\mathbf{m}) = \left\{ \sum_{nf} \left\| \mathbf{W}_{f} \left(\mathbf{d}_{f, obs} - \mathbf{d}_{f} \left(\mathbf{m} \right) \right) \right\|^{2} \right\} + \lambda \left\| \mathbf{Rm} \right\|^{2} , \qquad (2-52)$$

where nf is the number of frequencies; $\mathbf{W}_f \in \mathbb{R}^{n_d \times n_d}$ is a diagonal weighting matrix for each frequency; $\mathbf{d}_{f, obs} \in \mathbb{R}^{n_d}$ and $\mathbf{d}_f(\mathbf{m}) \in \mathbb{R}^{n_d}$ are the observed and modelled data for each frequency, respectively; and $\mathbf{R} \in \mathbb{R}^{n_m \times n_m}$ is the roughness matrix. For the Newton-type optimization method, the second-order Taylor expansion of the objective function around $\mathbf{m}_k \in \mathbb{R}^{n_m}$, which is a model parameter vector at the *k*-th iteration (set an initial model parameter vector to \mathbf{m}_1), is developed as follows:

$$O(\mathbf{m}_{k} + \Delta \mathbf{m}_{k}) \approx O(\mathbf{m}_{k}) + \mathbf{g}^{T} \Delta \mathbf{m}_{k} + \frac{1}{2} \Delta \mathbf{m}_{k}^{T} \mathbf{H} \Delta \mathbf{m}_{k} , \qquad (2-53)$$

where $\Delta \mathbf{m}_k \in \mathbb{R}^{n_m}$ is the increments vector at the *k*-th iteration. $\mathbf{g} \in \mathbb{R}^{n_m}$ and $\mathbf{H} \in \mathbb{R}^{n_m \times n_m}$ are the gradient vector and Hessian matrix of $O(\mathbf{m})$, respectively. Because the right-hand side of equation (2-53) is in the form of a convex function for $\Delta \mathbf{m}_k$, its minimum can be found with the condition that the partial derivative is zero as follows:

$$\frac{\partial O(\mathbf{m}_k + \Delta \mathbf{m}_k)}{\partial \Delta \mathbf{m}_k} = 0 \quad . \tag{2-54}$$

Substituting $\mathbf{m}_k + \Delta \mathbf{m}_k$ into equation (2-52) yields:

$$O\left(\mathbf{m}_{k} + \Delta \mathbf{m}_{k}\right)$$

$$= \left\{ \sum_{nf} \left\| \mathbf{W}_{f} \left(\mathbf{d}_{f, obs} - \mathbf{d}_{f} \left(\mathbf{m}_{k} + \Delta \mathbf{m}_{k} \right) \right) \right\|^{2} \right\} + \lambda \left\| \mathbf{R} \left(\mathbf{m}_{k} + \Delta \mathbf{m}_{k} \right) \right\|^{2} \quad (2-55)$$

For the Gauss-Newton method, it is assumed that $\mathbf{d}_f(\mathbf{m}_k)$ has a linear relationship with \mathbf{m}_k for small $\Delta \mathbf{m}_k$, which is expressed by

$$\mathbf{d}_{f}\left(\mathbf{m}_{k}+\Delta\mathbf{m}_{k}\right)\approx\mathbf{d}_{f}\left(\mathbf{m}_{k}\right)+\mathbf{J}_{f}\Delta\mathbf{m}_{k} \quad , \qquad (2-56)$$

and the difference between the observed and modelled data ($\mathbf{e}_f \in \mathbb{R}^{n_d}$) is defined as follows:

$$\mathbf{e}_{f} = \mathbf{d}_{f, obs} - \mathbf{d}_{f} \left(\mathbf{m}_{k} \right) \quad . \tag{2-57}$$

Using equations (2-56) and (2-57), equation (2-55) is rearranged as follows:

$$O\left(\mathbf{m}_{k} + \Delta \mathbf{m}_{k}\right) = \left\{\sum_{nf} \left\|\mathbf{W}_{f}\left(\mathbf{e}_{f} - \mathbf{J}_{f}\Delta \mathbf{m}_{k}\right)\right\|^{2}\right\} + \lambda \left\|\mathbf{R}\left(\mathbf{m}_{k} + \Delta \mathbf{m}_{k}\right)\right\|^{2} .$$
(2-58)

Substituting equation (2-58) into the condition (2-54), the formula is written as follows:

$$-2\sum_{nf} \left[\left(\mathbf{W}_{f} \mathbf{J}_{f} \right)^{T} \left\{ \mathbf{W}_{f} \left(\mathbf{e}_{f} - \mathbf{J}_{f} \Delta \mathbf{m}_{k} \right) \right\} \right] + 2\lambda \mathbf{R}^{T} \mathbf{R} \left(\mathbf{m}_{k} + \Delta \mathbf{m}_{k} \right) = 0 \quad .$$

$$(2-59)$$

If equation (2-59) is rearranged with $\Delta \mathbf{m}_k$, the normal equation is derived as follows:

$$\left[\left\{\sum_{nf} \left(\mathbf{W}_{f}\mathbf{J}_{f}\right)^{T} \mathbf{W}_{f}\mathbf{J}_{f}\right\} + \lambda \mathbf{R}^{T}\mathbf{R}\right] \Delta \mathbf{m}_{k} = \sum_{nf} \left(\mathbf{W}_{f}\mathbf{J}_{f}\right)^{T} \mathbf{W}_{f}\mathbf{e}_{f} - \lambda \mathbf{R}^{T}\mathbf{R}\mathbf{m}_{k}.$$
(2-60)

In equation (2-60), the Hessian matrix and negative gradient vector are expressed by

$$\mathbf{H} = \left[\left\{ \sum_{nf} \left(\mathbf{W}_{f} \mathbf{J}_{f} \right)^{T} \mathbf{W}_{f} \mathbf{J}_{f} \right\} + \lambda \mathbf{R}^{T} \mathbf{R} \right] , \qquad (2-61)$$

$$-\mathbf{g} = \sum_{nf} \left(\mathbf{W}_{f} \mathbf{J}_{f} \right)^{T} \mathbf{W}_{f} \mathbf{e}_{f} - \lambda \mathbf{R}^{T} \mathbf{R} \mathbf{m}_{k} \quad , \qquad (2-62)$$

respectively. To be precise, double of the right-hand side of equations (2-61) and (2-62) are the Hessian and negative gradient of $O(\mathbf{m})$, respectively, and the right-

hand side of equation (2-61) is an approximated Hessian matrix of $O(\mathbf{m})$. The approximated Hessian for the Gauss-Newton method is caused by the assumption of equation (2-56). The model parameter vector can be iteratively updated during the inversion process through the normal equation.

Equations (2-52) to (2-60) describe the 'jumping' method that uses $\|\mathbf{Rm}\|^2$, whereas the 'creeping' method uses $\|\mathbf{R}\Delta\mathbf{m}\|^2$ as the model roughness term. $O_c(\mathbf{m}_k + \Delta\mathbf{m}_k)$, which is the objective function for $\mathbf{m}_k + \Delta\mathbf{m}_k$, and the normal equation of the creeping method are defined as follows:

$$O_{c}\left(\mathbf{m}_{k}+\Delta\mathbf{m}_{k}\right) = \left\{\sum_{nf} \left\|\mathbf{W}_{f}\left(\mathbf{d}_{f, obs}-\mathbf{d}_{f}\left(\mathbf{m}_{k}+\Delta\mathbf{m}_{k}\right)\right)\right\|^{2}\right\} + \lambda \left\|\mathbf{R}\Delta\mathbf{m}_{k}\right\|^{2},$$
(2-63)

$$\left[\left\{\sum_{nf} \left(\mathbf{W}_{f} \mathbf{J}_{f}\right)^{T} \mathbf{W}_{f} \mathbf{J}_{f}\right\} + \lambda \mathbf{R}^{T} \mathbf{R}\right] \Delta \mathbf{m}_{k} = \sum_{nf} \left(\mathbf{W}_{f} \mathbf{J}_{f}\right)^{T} \mathbf{W}_{f} \mathbf{e}_{f} \quad . \quad (2-64)$$

2.2.2. Jacobian matrix

As can be seen from the normal equations (2-60) and (2-64), the Jacobian matrix plays a key role in the inversion procedure. Furthermore, because constructing the Jacobian matrix involves the time-consuming process of solving the modified coefficient matrix equation (2-46) several times, it is important to calculate the Jacobian matrix efficiently in inversion algorithms. In this section, the Jacobian matrix ($\mathbf{J}_{EH,f} \in \mathbb{R}^{n_d \times n_m}$) for the model parameter vector consisting of the electrical

conductivity ($\mathbf{\sigma} \in \mathbb{R}^{n_m}$) and the electromagnetic fields at the receivers (\mathbf{u}_f in equation 2-49) for each frequency is defined as:

$$\mathbf{J}_{\mathbf{EH},f} = \frac{\partial \mathbf{u}_f}{\partial \boldsymbol{\sigma}} \quad . \tag{2-65}$$

There are three main methods of calculating the Jacobian matrix: the perturbation, sensitivity-equation, and adjoint-equation approaches (McGillivray & Oldenburg 1990; McGillivray et al. 1994). In this study, the perturbation approach is used when calculating sensitivity patterns, and the sensitivity-equation approach is used in the inversion algorithm.

In the perturbation approach, the *i*-th row and the *j*-th column of the Jacobian matrix in equation (2-65) is obtained through the following formula:

$$J_{\mathbf{EH},f}(i,j) = \frac{\partial u_{f,i}(\boldsymbol{\sigma})}{\partial \sigma_{j}} \approx \frac{\Delta u_{f,i}}{\Delta \sigma_{j}} = \frac{u_{f,i}(\boldsymbol{\sigma} + \Delta \boldsymbol{\sigma}_{j}) - u_{f,i}(\boldsymbol{\sigma})}{\Delta \sigma_{j}} \quad , \quad (2-66)$$

where $u_{f,i}$ is the *i*-th component of \mathbf{u}_f ; σ_j is the *j*-th component of $\mathbf{\sigma}$; and $\Delta \mathbf{\sigma}_j$ is the model parameter vector whose *j*-th component only has a non-zero value $\Delta \sigma_j$. To obtain all the components of the Jacobian matrix through the perturbation approach, $u_f(\mathbf{\sigma} + \Delta \mathbf{\sigma}_j)$ from j = 1 to $j = n_m$ (n_m : the number of model parameters) and $u_f(\mathbf{\sigma})$ are required, therefore a total of $n_m + 1$ times of modeling for each frequency should be performed.

In the sensitivity-equation approach, the Jacobian matrix is constructed using the matrices of equations (2-46) and (2-49), which are calculated during the modeling process. Substituting the relation of equation (2-49) into equation (2-65), the Jacobian matrix is arranged as follows:

$$\mathbf{J}_{\mathbf{EH},f} = \mathbf{P}_f \, \frac{\partial \mathbf{x}_f}{\partial \boldsymbol{\sigma}} \quad . \tag{2-67}$$

Taking the partial derivative with respect to σ in equation (2-46) yields

$$\frac{\partial \tilde{\mathbf{A}}_{f}}{\partial \boldsymbol{\sigma}} \mathbf{x}_{f} + \tilde{\mathbf{A}}_{f} \frac{\partial \mathbf{x}_{f}}{\partial \boldsymbol{\sigma}} = \frac{\partial \mathbf{b}_{f}}{\partial \boldsymbol{\sigma}} \quad .$$
(2-68)

From equation (2-32), $\partial \tilde{\mathbf{A}}_f / \partial \boldsymbol{\sigma}$ in equation (2-68) for the *i*-th element is defined as follows:

$$\frac{\partial \tilde{\mathbf{A}}_{f}}{\partial \sigma_{i}} = i\omega\mu_{0}\mathbf{M}^{i\text{-th element}} \quad .$$
(2-69)

 $\mathbf{M}^{i\text{-th element}} \in \mathbb{R}^{N_{edge} \times N_{edge}}$ has non-zero values only in rows and columns of the global edge numbers constituting the *i*-th element, and the values are the elementary mass matrix (equation 2-34) of the corresponding local edge numbers. Because of the Dirichlet boundary conditions, the components of $\partial \mathbf{\tilde{A}}_f / \partial \mathbf{\sigma}$ corresponding to the edges at the boundary of the model are zero, and the right-hand side of equation (2-68) is **0**. Therefore, the partial derivative of \mathbf{x}_f with respect to $\mathbf{\sigma}$ is written as follows:

$$\frac{\partial \mathbf{x}_f}{\partial \boldsymbol{\sigma}} = \left(\tilde{\mathbf{A}}_f\right)^{-1} \left(-\frac{\partial \tilde{\mathbf{A}}_f}{\partial \boldsymbol{\sigma}} \mathbf{x}_f\right) .$$
(2-70)

The Jacobian matrix is represented by substituting equation (2-70) into equation (2-67) as follows:

$$\mathbf{J}_{\mathbf{EH},f} = \mathbf{P}_f \left(\tilde{\mathbf{A}}_f \right)^{-1} \left(-\frac{\partial \tilde{\mathbf{A}}_f}{\partial \boldsymbol{\sigma}} \mathbf{x}_f \right) .$$
(2-71)

From equation (2-71), the transpose of the Jacobian matrix is defined as follows:

$$\left(\mathbf{J}_{\mathbf{EH},f}\right)^{T} = \left(-\frac{\partial \tilde{\mathbf{A}}_{f}}{\partial \boldsymbol{\sigma}} \mathbf{x}_{f}\right)^{T} \left\{\left(\tilde{\mathbf{A}}_{f}\right)^{-1}\right\}^{T} \left(\mathbf{P}_{f}\right)^{T} .$$
(2-72)

Because $\tilde{\mathbf{A}}_{f}$ is symmetric, equation (2-72) is rearranged as follows:

$$\left(\mathbf{J}_{\mathbf{EH},f}\right)^{T} = \left(-\frac{\partial \tilde{\mathbf{A}}_{f}}{\partial \boldsymbol{\sigma}} \mathbf{x}_{f}\right)^{T} \left(\tilde{\mathbf{A}}_{f}\right)^{-1} \left(\mathbf{P}_{f}\right)^{T} .$$
(2-73)

In equation (2-73), since the number of columns of $(\mathbf{P}_f)^T$ is n_d (i.e., the number of data), a total of n_d times of modeling for each frequency is required to construct the Jacobian matrix by the sensitivity-equation approach. In inversion algorithms, the sensitivity-equation approach is more efficient than the perturbation approach, because n_d is generally much smaller than $n_m + 1$.

2.2.3. Techniques for inversion

In this section, some techniques to improve the efficiency, stability, and quality of inversion are described. Blocky parameterization, model parameterization, data weighting method, roughness matrix, Lagrange multiplier, and line search technique are explained in Chapters 2.2.3.1 to 2.2.3.6, respectively. Considering the normal equation (equation 2-60 or 2-64), the blocky parameterization and model parameterization are related to \mathbf{J}_f , $\Delta \mathbf{m}_k$, and \mathbf{m}_k ; the data weighting method is about \mathbf{W}_f ; the roughness matrix is related to constructing \mathbf{R} ; the Lagrange multiplier is related to setting λ ; and the line search technique is for determining a step length of $\Delta \mathbf{m}_k$.

2.2.3.1. Blocky parameterization

Because MT exploration applies a diffusion equation with a relatively low frequency, the spatial resolution of the inverted model is not high. Therefore, to improve the computational efficiency and obtain stable inversion results, a blocky parameterization technique (Shin et al. 1999) of merging several modeling elements into an inversion block is used in MT inversion. The relationship between the model parameter vectors composed of the electrical conductivity for the modeling elements $(\boldsymbol{\sigma}_{mod} \in \mathbb{R}^{N_e})$ and the inversion blocks $(\boldsymbol{\sigma}_{block} \in \mathbb{R}^{n_{block}})$ can be defined as follows:

$$\boldsymbol{\sigma}_{mod} = \bar{\mathbf{M}}_1 \boldsymbol{\sigma}_{block} \quad , \tag{2-74}$$

$$\boldsymbol{\sigma}_{block} = \bar{\mathbf{M}}_2 \boldsymbol{\sigma}_{mod} \quad , \tag{2-75}$$

where $\bar{\mathbf{M}}_1 \in \mathbb{R}^{N_e \times n_{block}}$ and $\bar{\mathbf{M}}_2 \in \mathbb{R}^{n_{block} \times N_e}$ are the mapping matrices (N_e and n_{block} are the numbers of modeling elements and inversion blocks, respectively). In this study, the modeling and inversion meshes are constructed with generally small

and large elements, respectively, and the inversion blocks to be practically used for inversion are constructed by merging modeling elements whose centers belong to identical inversion elements. Therefore, if the *i*-th modeling element $(1 \le i \le N_e)$ is included in the *j*-th inversion block ($1 \le j \le n_{block}$), $\overline{\mathbf{M}}_1(i, j) = 1$ and $\overline{\mathbf{M}}_2(j, i) = V_{i\text{-th modeling element}}/V_{j\text{-th inversion block}}$, otherwise $\overline{\mathbf{M}}_1(i, j) = 0$ and $\overline{\mathbf{M}}_2(j, i) = 0$. Fig. 2.1 shows a modeling mesh composed of triangular elements, an inversion mesh composed of rectangular elements, and an example of making inversion blocks with the modeling and inversion meshes.



Fig. 2.1. Example for blocky parameterization: (a) modeling mesh, (b) inversion mesh, (c) inversion mesh superimposed on the modeling elements, and (d) inversion blocks.

Constructing inversion blocks based on this standard has the advantages that the resolution of the surface topography in the modeling mesh can be preserved in the inversion blocks and there is a high degree of freedom in setting the inversion mesh. Furthermore, the Jacobian matrix for the inversion blocks ($\mathbf{J}_{block} \in \mathbb{R}^{n_d \times n_{block}}$) can be easily calculated from the Jacobian matrix for the modeling elements ($\mathbf{J}_{mod} \in \mathbb{R}^{n_d \times N_e}$) with the mapping matrix in equation (2-74) as follows:

$$\mathbf{J}_{block} = \mathbf{J}_{mod} \mathbf{M}_1 \quad . \tag{2-76}$$

As a result of equation (2-76), the number of columns of the Jacobian matrix is greatly reduced from the number of columns of \mathbf{J}_{mod} (N_e) to the number of columns of \mathbf{J}_{block} (n_{block}). In other words, the size of the matrix to be solved in the normal equation greatly decreases.

2.2.3.2. Model parameterization

In this thesis, a model parameterization refers to a method of using a parameterized variable instead of the electrical conductivity as a component of the model parameter vector in the inverse process. The model parameterization allows the model parameter vector to have an appropriate updating scale, and makes the electrical conductivity to be bounded to the geophysical range. Therefore, the inversion procedure can be stabilized and the prior information can be considered through the model parameterization. The electrical conductivity of subsurface media has a large range about $10^{-5} \sim 10^{1}$ S/m, and the subsurface model is usually represented by the electrical resistivity $(10^{-1} \sim 10^{5} \ \Omega \text{ m})$ on the logarithmic scale. Therefore, the electrical conductivity is first parameterized as the electrical resistivity on the logarithmic scale as follows:

$$r = \log_{10}(\rho) = \log_{10}\left(\frac{1}{\sigma}\right) = -\log_{10}(\sigma)$$
 (2-77)

Through the conversion in equation (2-77), the electrical conductivity is updated on the appropriate logarithmic scale.

An additional parameter is required to constrain the variable r within the lower bound l and the upper bound u (i.e., $10^{-u} < \sigma < 10^{-l}$) in the process of updating the model parameter vector. The variable r can be parameterized to m_{para} through the formula defined as follows (Kim & Kim 2008):

$$m_{para} = \frac{1}{n} \ln \left(\frac{r-l}{u-r} \right); \quad \begin{array}{c} l < r < u \\ -\infty < m_{para} < \infty \end{array}, \quad (2-78)$$

where *n* is a positive constant. The following formulae are used to convert m_{para} to *r* and *r* to σ :

$$r = \frac{l + u \exp\left(n \cdot m_{para}\right)}{1 + \exp\left(n \cdot m_{para}\right)} \quad , \tag{2-79}$$

$$\sigma = 10^{-r}$$
 . (2-80)

The Jacobian matrix for the model parameter vector $\mathbf{m}_{para} \in \mathbb{R}^{n_{block}}$, which is composed of m_{para} in the inversion blocks, is computed as follows:

$$\mathbf{J}_{f} = \frac{\partial \mathbf{u}_{f}}{\partial \mathbf{m}_{para}} = \frac{\partial \mathbf{u}_{f}}{\partial \boldsymbol{\sigma}} \frac{\partial \boldsymbol{\sigma}}{\partial \mathbf{r}} \frac{\partial \mathbf{r}}{\partial \mathbf{m}_{para}} , \qquad (2-81)$$

where $\mathbf{\sigma} \in \mathbb{R}^{n_{block}}$ and $\mathbf{r} \in \mathbb{R}^{n_{block}}$ are the model parameter vectors composed of rand σ in the inversion blocks, respectively. $\partial \mathbf{u}_f / \partial \mathbf{\sigma}$ in the right-hand side of equation (2-81) can be computed by equations (2-73) and (2-76). From equations (2-77) to (2-80), the partial derivative of σ with respect to r and the partial derivative of r with respect to m_{para} can be calculated as follows:

$$\frac{\partial \sigma}{\partial r} = -\ln(10) \cdot \sigma \quad , \tag{2-82}$$

$$\frac{\partial r}{\partial m_{para}} = \frac{n(u-l)\exp(n \cdot m_{para})}{\left\{1 + \exp(n \cdot m_{para})\right\}^2} \quad . \tag{2-83}$$

Key (2016) proposed the band pass filter transfer function to constrain r between the two bounds (i.e., l and u), and the model parameterization from r to m_{para} is defined as follows:

$$m_{para} = \frac{1}{c} \ln \left(\frac{\exp\{c(r-l)\} - 1}{1 - \exp\{c(r-u)\}} \right) + l; \quad l < r < u \\ -\infty < m_{para} < \infty \quad , \tag{2-84}$$

$$r = \frac{1}{c} \ln \left(\frac{1 + \exp\{c(l - m_{para})\}}{1 + \exp\{c(u - m_{para})\}} \right) + u \quad , \tag{2-85}$$

$$\frac{\partial r}{\partial m_{para}} = \frac{1 - \exp\{c(l-u)\}}{\left[1 + \exp\{-c(m_{para} - l)\}\right] \left[1 + \exp\{c(m_{para} - u)\}\right]} \quad .$$
(2-86)

The positive constant c is used as 15/(u-l) in his paper. This model parameterization makes r and parameterized m_{para} have similar values between l and u so that $\partial r/\partial m_{para}$ scaling the Jacobian matrix (equation 2-81) is close to 1, and the roughness values of \mathbf{r} and \mathbf{m}_{para} are considered similarly. Examples for the two schemes of the model parameterization (equations 2-78 and 2-84) in Fig. 2.2 show the features for the model parameterization using the band pass filter transfer function.



Fig. 2.2. Examples for (a) model parameterization from r to m_{para} and (b) $\partial r / \partial m_{para}$ when the lower bound (l) is -1 and the upper bound (u) is 5. The red lines are for equations (2-78) and (2-83) with n=1 (Kim & Kim 2008) and the blue lines are for equations (2-84) and (2-86) with c=15/(u-l) (Key 2016).

2.2.3.3. Data weighting matrix

It is possible to adjust the influence of data according to the receivers and frequencies on the inversion procedure by making the n_d by n_d diagonal data weighting matrix for each frequency. Therefore, the components of the data weighting matrix should be basically set according to the reliability of the data. The data weighting matrix can also play a role in balancing the magnitude of the data for stations and frequencies.

The standard deviation of the observed data, which can be obtained in the process of converting data in the time to the frequency domains, is most commonly used in the data weighting techniques (Constable et al. 1987; Sasaki 2004; Key 2016). The *i*-th component of the data weighting matrix for each frequency, $W_f(i,i)$, is defined as follows using the inverse of the standard deviation of the data:

$$W_f(i,i) = \frac{1}{s_{f,i}}; \ i = 1 \sim n_d$$
, (2-87)

where $s_{f,i}$ is the standard deviation of the *i*-th data at the frequency. When the data weighting matrix of equation (2-87) is applied, the weighted data misfit is calculated as follows:

$$\left\|\mathbf{W}_{f}\mathbf{e}_{f}\right\|^{2} = \sum_{i=1}^{n_{d}} \left(\frac{e_{f,i}}{s_{f,i}}\right)^{2} , \qquad (2-88)$$

where $e_{f,i}$ is the *i*-th data of \mathbf{e}_f . The data weighting matrix of equation (2-87) has the effect of increasing the influence of data with a small standard deviation (i.e.,

high reliability) and decreasing the influence of data with a large standard deviation (i.e., low reliability). Moreover, assuming that the noise included in the data follows the Gaussian distribution, this method has an advantage that the normalized data misfit should converge to 1 in the inverse problem. However, the weighting technique using the standard deviation does not reflect the results of post-processing for data in the frequency domain (Uhm et al. 2021). Also, this technique does not fully consider the magnitude of data for each frequency.

The Jacobian matrix can be used to construct the data weighting matrix. 'Jacobian weighting' proposed by Abubakar et al. (2009) simply corrects the magnitude of data for frequencies and types of MT response functions. Through the Jacobian matrix of the initial model $\mathbf{J}_{f_{-init}}$, the data weighting matrix is defined as follows:

$$\mathbf{W}_{f} = \left\{ diag \left(\mathbf{J}_{f_init} \mathbf{J}_{f_init}^{T} \right) \right\}^{-\frac{1}{2}} .$$
 (2-89)

A weighting technique proposed by Avdeeva (2008) is a method for applying an impedance tensor as an input MT response function to inversion. The weighting matrix is defined as follows:

$$W_f(i,i) = \sqrt{\frac{2}{tr\left(\mathbf{Z}_{obs,i}^T \mathbf{Z}_{obs,i}\right)}}; \ i = 1 \sim n_d \quad , \tag{2-90}$$

where $\mathbf{Z}_{obs, i}$ is the observed impedance tensor at the receiver of the *i*-th data; and $tr(\cdot)$ is the trace of the matrix. This weighting technique of equation (2-90) balances the contributions to the magnitude of the data over receivers.

Through a weighting technique, it is possible to exclude some data of specific receivers and frequencies from the inversion process. For example, if do not want to use the data of the *a*-th station and the *b*-th frequency for inversion, the weighting matrix can be set as follows:

$$W_{f=j}(i,i) = 1; \ i = 1 \sim n_d, \ j = 1 \sim nf$$

$$W_{f=j}(i,i) = 0; \ \text{if} \ i = a \ \text{and} \ j = b$$
 (2-91)

Another weighting technique proposed by Abubakar et al. (2009) is applied after making \mathbf{W}_{f} by the methods mentioned above. For each frequency, the constant weighting factor (W_{f}^{norm}) is defined as follows:

$$W_f^{norm} = \frac{1}{\left\| \mathbf{W}_f \mathbf{d} \right\|^2} \quad . \tag{2-92}$$

Then, $W_f^{norm} \cdot \mathbf{W}_f$ is used as the data weighting matrix. This method not only balances the contribution of data over frequencies, but also corrects the differences in the number of observed data used for inversion over frequencies.

Basically, it is good to use the weighting technique using the standard deviation of equation (2-87). However, if bad data are excluded and the remaining data are reliable after the frequency-domain post-processing for data, it is also a good strategy to construct the data weighting matrix using equations (2-90) to (2-92) together.

2.2.3.4. Roughness matrix

The roughness of the model is the contrary concept of the smoothness of the model, and literally quantifies how roughly the model changes. In the inversion process, the roughness of model parameter vector $\mathbf{m} \in \mathbb{R}^{n_{block}}$ composed of m_{para} in the inversion blocks is expressed as follows:

$$O_m(\mathbf{m}) = \|\mathbf{Rm}\|^2 \quad , \tag{2-93}$$

where $\mathbf{R} \in \mathbb{R}^{n_{block} \times n_{block}}$ is the roughness matrix. The roughness varies depending on the standard for making the roughness matrix and the norm to measure the value of **Rm**. In this study, the roughness matrix **R** is set based on the second derivatives and **Rm** is measured using the l_2 -norm.

In MT inversion, adding the roughness of the model to the objective function (equation 2-51) prevents the model roughness value from being too large, which means applying a smoothness constraint that makes the inverted model parameter vector change smoothly in the model domain. Moreover, the roughness term plays a role in alleviating the non-uniqueness problem of inversion. The number of model parameters is generally greater than the number of data, and some components of the model parameter vector have little effect on the data (e.g., the components around the boundary of the deep subsurface). Therefore, $\sum_{nf} (\mathbf{W}_f \mathbf{J}_f)^T \mathbf{W}_f \mathbf{J}_f$ in the Hessian matrix (equation 2-61) is a singular matrix, which does not have the inverse matrix, and it leads to the non-uniqueness problem of inversion. The roughness matrix contributes to mitigating the non-uniqueness problem and stabilizes the

inverse process, because the Hessian matrix becomes non-singular as $\lambda \mathbf{R}^T \mathbf{R}$ is added to $\sum_{nf} (\mathbf{W}_f \mathbf{J}_f)^T \mathbf{W}_f \mathbf{J}_f$.

For the inversion blocks of the structured hexahedral elements, the spatial change of the *i*-th inversion block (m_i) is defined as follows by the finite difference equation of the second derivative:

$$\partial^2 m_i \approx \frac{m_i^B + m_i^F + m_i^R + m_i^L + m_i^D + m_i^U - 6m_i}{6} \quad , \tag{2-94}$$

where m_i^B , m_i^F , m_i^R , m_i^L , m_i^D , and m_i^U are the inversion blocks located at back, front, right, left, down, and up side of the central *i*-th inversion block m_i , respectively. Therefore, the roughness of equation (2-93) is obtained by considering equation (2-94) to all inversion blocks as follows:

$$O_m(\mathbf{m}) = \|\mathbf{Rm}\|^2 = \sum_{i=1}^{n_{block}} \left(\frac{m_i^B + m_i^F + m_i^R + m_i^L + m_i^D + m_i^U - 6m_i}{6}\right)^2 \quad .(2-95)$$

The components of the roughness matrix in equation (2-95) are $R_{ii} = -1$ where $i = 1 \sim n_{block}$; $R_{ij} = 1/6$; $j \in \{j^B, j^F, j^R, j^L, j^D, j^U\}$ where m_i^B , m_i^F , m_i^R , m_i^L , m_i^D , and m_i^U are the j^B , j^F , j^R , j^L , j^D , and j^U -th inversion blocks; and the rest of the components are 0.

Similar to equation (2-94), for the inversion blocks of the unstructured tetrahedral elements, $\partial^2 m_i$ and $O_m(\mathbf{m})$ are defined as follows (Usui 2015):

$$\partial^2 m_i \approx \frac{1}{N_{Face}^i} \sum_{j=1}^{N_{Face}^i} \left(m_j - m_i \right) \quad , \tag{2-96}$$

$$O_m(\mathbf{m}) = \|\mathbf{Rm}\|^2 = \sum_{i=1}^{n_{block}} \left\{ \frac{1}{N_{Face}^i} \sum_{j=1}^{N_{Face}^i} (m_j - m_i) \right\}^2 , \qquad (2-97)$$

where N_{Face}^{i} is the total number of faces of the *i*-th inversion block. The roughness matrix can also be constructed in the same way.

2.2.3.5. Lagrange multiplier

As can be seen in equation (2-51), the Lagrange multiplier λ determines a trade-off between the terms of the data misfit $O_d(\mathbf{m})$ and roughness $O_m(\mathbf{m})$. If λ is too large, the contribution of $O_m(\mathbf{m})$ increases, and the observed data are not properly considered in the inversion result. If λ is too small, the contribution of $O_m(\mathbf{m})$ decreases, and the inverse problem becomes unstable.

The L-curve is conventionally used to determine the Lagrange multiplier λ (Hansen 1992; Farquharson & Oldenburg 2004). To plot the L-curve, several $O_d(\mathbf{m})$ and $O_m(\mathbf{m})$ according to the Lagrange multiplier should be calculated within the range $\gamma_{\min} \leq \lambda \leq \gamma_{\max}$ where γ_{\min} and γ_{\max} are the smallest and biggest generalized singular values of the Hessian matrix, respectively. The graph for the pairs $(O_m(\mathbf{m}), O_d(\mathbf{m}))$ on the logarithmic-logarithmic scales has an L-shape, which is called the L-curve, and λ corresponding to the corner of the L-curve is used. However, drawing the L-curve requires too much additional computational cost. Fig. 2.3 shows an example of the L-curve and its corner.



Fig. 2.3. Example of the L-curve and its corner.

The ratio of the data misfit $O_d(\mathbf{m})$ and model roughness $O_m(\mathbf{m})$ can be another criterion for determining the Lagrange multiplier (Van den Berg & Abubakar 2001; Kim et al. 2013), and λ can be defined as follows:

$$\lambda = \gamma \frac{O_d(\mathbf{m})}{O_m(\mathbf{m})} \quad , \tag{2-98}$$

where γ is a user-defined positive constant. Because in the objective function of equation (2-51), $O_d(\mathbf{m})$ and $O_m(\mathbf{m})$ mean the amount of the contribution of data misfit and model roughness term, respectively, equation (2-98) is an intuitive and simple criterion. Moreover, as inversion proceeds, the Lagrange multiplier naturally decreases from the criterion, because $O_d(\mathbf{m})$ decreases and $O_m(\mathbf{m})$ increases compared to the initial values. It leads to an increase in the influence of $O_d(\mathbf{m})$ and a decrease in the influence of $O_m(\mathbf{m})$ in the later iteration of inversion.

Grayver et al. (2013) tried to set the Lagrange multiplier λ considering the relationship between $\sum_{nf} (\mathbf{W}_f \mathbf{J}_f)^T \mathbf{W}_f \mathbf{J}_f$ and $\mathbf{R}^T \mathbf{R}$ in the Hessian matrix (equation 2-61) (mentioned in Chapter 2.2.3.4), which is determined as follows:

$$\lambda = \gamma \frac{\left\| \sum_{nf} \left(\mathbf{W}_{f} \mathbf{J}_{f} \right)^{T} \mathbf{W}_{f} \mathbf{J}_{f} \right\|_{p}}{\left\| \mathbf{R}^{T} \mathbf{R} \right\|_{p}} \quad , \tag{2-99}$$

where γ is a user-defined positive constant; and $\|\cdot\|_p$ is the p-norm of the matrix. In particular, $\|\cdot\|_2$ means the largest singular value of the matrix. The largest singular values are similar to the maximum values of the diagonal components for the two matrices, $\sum_{nf} (\mathbf{W}_f \mathbf{J}_f)^T \mathbf{W}_f \mathbf{J}_f$ and $\mathbf{R}^T \mathbf{R}$, because they are diagonally dominant matrices. Therefore, using the two maximum diagonal values the Lagrange multiplier can be set as follows:

$$\lambda = \gamma \; \frac{\max\left\{ diag\left(\sum_{nf} \left(\mathbf{W}_{f} \mathbf{J}_{f}\right)^{T} \mathbf{W}_{f} \mathbf{J}_{f}\right)\right\}}{\max\left\{ diag\left(\mathbf{R}^{T} \mathbf{R}\right)\right\}} \; , \qquad (2-100)$$

where γ is a user-defined positive constant. Equation (2-100) requires much smaller computational cost than equation (2-99), and is similar to the regularization method used in full-waveform inversion (FWI) of seismic data (Shin et al. 2001; Shin & Min 2006).

In the inversion algorithm for field data of this study, λ is determined as a smaller value of Lagrange multiplier values in equations (2-98) and (2-100) when γ is 0.005. At this time, if the new λ is less than 0.4 times of previous λ , the Lagrange multiplier is replaced with the value of 0.4 times of previous λ .

The above techniques focus on setting the Lagrange multiplier to an optimal scalar value. Yi et al. (2003) proposed the active constraint balancing (ACB) method to define the Lagrange multiplier as a spatial variable vector at the location of inversion blocks. To apply the ACB method, the model resolution matrix is calculated, and the spread function (Menke 1984) values are obtained for the rows of the matrix. Because the spread function implies the resolving power of the inversion block, the Lagrange multiplier can be set by determining λ to have a linear relationship with the spread function on the logarithmic-logarithmic scales. Through the ACB method, a small Lagrange multiplier is determined for an

inversion block with strong resolving power near the receivers to increase the contribution of the data misfit term, while a large Lagrange multiplier is set for an inversion block with weak resolving power around the boundary of the deep subsurface to increase the contribution of the model roughness term. Especially, for 3D MT inversion with a large model the ACB method helps to improve the inversion results, but it incurs additional cost in calculating the model resolution matrix.

Uhm et al. (2018) presented the sensitivity-based constraint balancing (SCB) method that requires little additional computational cost while maintaining the advantage of the ACB method of defining the Lagrange multiplier as a spatial variable vector. The SCB method uses the diagonal integrated sensitivity matrix (\mathbf{S}) (Kaputerko et al. 2007) defined as follows:

$$\mathbf{S} = diag \left(\sum_{nf} \left(\mathbf{W}_f \mathbf{J}_f \right)^T \mathbf{W}_f \mathbf{J}_f \right)^{\frac{1}{2}} .$$
 (2-101)

By replacing the spread function of the ACB method with the inverse of S, the Lagrange multiplier is determined as follows in the SCB method:

$$\log_{10}(\lambda_{i}) = \log_{10}(\lambda_{\min}) + \frac{\log_{10}(\lambda_{\max}) - \log_{10}(\lambda_{\min})}{\log_{10}(S_{\min}^{-1}) - \log_{10}(S_{\max}^{-1})} \{ \log_{10}(S_{i}^{-1}) - \log_{10}(S_{\max}^{-1}) \},$$
(2-102)

where λ_i is the Lagrange multiplier of the *i*-th inversion block; λ_{\min} and λ_{\max} are the minimum and maximum of the Lagrange multipliers defined by user,

respectively; S_{\min} and S_{\max} are the minimum and maximum components of **S**; and S_i is the *i*-th component of **S**. Because **S** is obtained in the process of calculating the Hessian matrix in equation (2-61), the SCB method does not need additional computational cost. The SCB method can set a small Lagrange multiplier for an inversion block with a large integrated sensitivity value near the receivers to enhance the contribution of the data misfit term, and a large Lagrange multiplier for an inversion block with a small integrated sensitivity value around the boundary of the deep subsurface to strengthen the contribution of the model roughness term similar to the ACB method. The SCB method is almost identical to the method of applying weighting to the roughness matrix presented by Kordy et al. (2016).

2.2.3.6. Line search

The line search methods (Nocedal & Wright 2006) determine the optimal positive scalar step length for a search direction. In the inverse problem, the model parameter vector is updated as follows:

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \Delta \mathbf{m}_k \quad , \tag{2-103}$$

where α_k is the step length for $\Delta \mathbf{m}_k$ in equation (2-60) or (2-64).

The objective function for \mathbf{m}_{k+1} can be defined as a function (ϕ) for the step length α as follows:

$$\phi(\alpha) = O(\mathbf{m}_{k+1}) = O(\mathbf{m}_k + \alpha \Delta \mathbf{m}_k) \quad . \tag{2-104}$$

An exact line search finds an ideal step length α_{ideal} , which allows $\phi(\alpha_{ideal})$ to have a global minimum value (Fig. 2.4). However, the exact line search method is too expensive to identify the global minimum value in inversion. As a practical technique, an inexact line search determines a step length that provides sufficient decrease of the objective function with moderate cost. In this study, the inexact line search technique goes through two stages. The first stage is to set an appropriate stopping condition for the objective function, and the second stage is a backtracking approach in which the step length is reduced from an initial value until the condition of the first stage is satisfied.

For the first stage, the Armijo condition is used in this thesis, which is expressed as follows:

$$\phi(\alpha) = O(\mathbf{m}_{k} + \alpha \Delta \mathbf{m}_{k}) \le O(\mathbf{m}_{k}) + c\alpha \nabla O(\mathbf{m}_{k})^{T} \Delta \mathbf{m}_{k} \quad , \qquad (2-105)$$

where c is a small positive constant (generally $c = 10^{-4}$). The right-hand side of equation (2-105) is a linear function with respect to α , which can be written as follows:

$$l(\alpha) = c\nabla O(\mathbf{m}_k)^T \Delta \mathbf{m}_k \alpha + O(\mathbf{m}_k) \quad .$$
 (2-106)

The slope of the linear function is negative as shown in the following equation:

$$c\nabla O(\mathbf{m}_k)^T \Delta \mathbf{m}_k = -c\nabla O(\mathbf{m}_k)^T \mathbf{H}^{-1} \nabla O(\mathbf{m}_k) < 0 \quad . \tag{2-107}$$

The ranges of the step length that satisfy the Armijo condition of equation (2-105) are shown in Fig. 2.4. The line search technique with only the Armijo condition cannot ensure that the step length is always properly defined because a very small step length that does not update the model parameter vector also satisfies this stopping condition.



Fig. 2.4. Example of the ideal step length and the acceptable ranges of the step length for the Armijo condition.

To prevent too small step lengths from being selected, the backtracking approach is applied for the second stage. The initial step length (α_0) for the backtracking approach is required and set to 1 for the Gauss-Newton method (Nocedal & Wright 2006). If the initial step length ($\alpha_0 = 1$) satisfies the Armijo condition of equation (2-105), α_k in equation (2-103) will be determined as $\alpha_0 = 1$. Otherwise, the next step length α_1 should be defined between 0 and $\alpha_0 = 1$. To set α_1 , $\phi(\alpha)$ is assumed as a quadratic function $\phi_q(\alpha)$ satisfying the three conditions: $\phi_q(0) = \phi(0) = O(\mathbf{m}_k)$, $\phi'_q(0) = \phi'(0) = \nabla O(\mathbf{m}_k)^T \Delta \mathbf{m}_k$, and $\phi_q(\alpha_0) = \phi(\alpha_0) = O(\mathbf{m}_k + \Delta \mathbf{m}_k)$. Then, $\phi_q(\alpha)$ is expressed as follows:

$$\phi_{q}(\alpha) = \left(\frac{\phi(\alpha_{0}) - \phi'(0)\alpha_{0} - \phi(0)}{\alpha_{0}^{2}}\right)\alpha^{2} + \phi'(0)\alpha + \phi(0) \quad . \quad (2-108)$$

 α_1 is defined by the minimizer of $\phi_q(\alpha)$ in equation (2-108), and is expressed as follows:

$$\alpha_{1} = -\frac{\phi'(0)\alpha_{0}^{2}}{2\{\phi(\alpha_{0}) - \phi'(0)\alpha_{0} - \phi(0)\}}; \ 0 \le \alpha_{1} \le \alpha_{0} = 1 \quad .$$
 (2-109)

If the Armijo condition of equation (2-105) is satisfied at α_1 , the line search will be terminated. Otherwise, the next step length α_2 should be set between 0 and α_1 . To determine α_2 , $\phi(\alpha)$ is assumed as a cubic function $\phi_c(\alpha)$ satisfying the four conditions: $\phi_c(0) = \phi(0) = O(\mathbf{m}_k)$, $\phi'_c(0) = \phi'(0) = \nabla O(\mathbf{m}_k)^T \Delta \mathbf{m}_k$, $\phi_c(\alpha_0) = \phi(\alpha_0) = O(\mathbf{m}_k + \Delta \mathbf{m}_k)$, and $\phi_c(\alpha_1) = \phi(\alpha_1) = O(\mathbf{m}_k + \alpha_1 \Delta \mathbf{m}_k)$. Then, $\phi_c(\alpha)$ is expressed as follows:

$$\phi_c(\alpha) = a\alpha^3 + b\alpha^2 + \phi'(0)\alpha + \phi(0) \quad , \tag{2-110}$$

where

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\alpha_0^2 \alpha_1^2 (\alpha_1 - \alpha_0)} \begin{bmatrix} \alpha_0^2 & -\alpha_1^2 \\ -\alpha_0^3 & \alpha_1^3 \end{bmatrix} \begin{bmatrix} \phi(\alpha_1) - \phi(0) - \phi'(0) \alpha_1 \\ \phi(\alpha_0) - \phi(0) - \phi'(0) \alpha_0 \end{bmatrix}$$

Under the two conditions: $\phi_c(\alpha_2) = 0$ and $0 \le \alpha_2 \le \alpha_1$, the minimizer α_2 of $\phi_c(\alpha)$ is expressed by

$$\alpha_2 = \frac{-b + \sqrt{b^2 - 3a\phi'(0)}}{3a}; \ 0 \le \alpha_2 \le \alpha_1 \quad . \tag{2-111}$$

Until α_n satisfies the Armijo stopping condition of equation (2-105), repeat the process to determine α_{i+1} $(1 \le i \le n-1)$ using $\phi_c(\alpha)$ with the four conditions: $\phi_c(0) = \phi(0)$, $\phi'_c(0) = \phi'(0)$, $\phi_c(\alpha_{i-1}) = \phi(\alpha_{i-1}) = O(\mathbf{m}_k + \alpha_{i-1}\Delta\mathbf{m}_k)$, and $\phi_c(\alpha_i) = \phi(\alpha_i) = O(\mathbf{m}_k + \alpha_i\Delta\mathbf{m}_k)$. To prevent α_{i+1} from being too small or too close to α_i , this backtracking approach needs an additional rule defined as follows:

$$\alpha_{i+1} = 0.5\alpha_i$$
; if $v_1\alpha_i \le \alpha_{i+1} \le v_2\alpha_i$ or $\alpha_{i+1} < v_3\alpha_i$, (2-112)

where v_1 , v_2 , and v_3 are the positive constants determined by user. In this study, v_1 , v_2 , and v_3 are fixed as 0.9, 1.1, and 0.1, respectively (Abubakar et al. 2009). In the inversion algorithm, the number of repetitions of the explained line search technique is limited to a maximum of 5 times.

2.3. MT response functions

The electric and magnetic fields at the receivers calculated through 3D MT modeling in Chapter 2.1 should be converted into other MT response functions that are not affected by the amplitude of the source. Among the MT response functions, impedance tensor, apparent resistivity, phase, tipper, effective impedance, and phase tensor that are commonly used as input data for MT inversion are explained.

2.3.1. Impedance tensor

Tikhonov (1950) introduced a scalar impedance that represents the ratio of the horizontal electric field to the orthogonal horizontal magnetic field in the frequency domain (i.e., E_x/H_y or E_y/H_x). Later, Cantwell (1960) extended the concept to an impedance tensor. The impedance tensor is the most used type among the MT response functions, and many different types of MT response functions are derived from the impedance tensor. The frequency-domain impedance tensor is defined as follows:

$$\begin{pmatrix} E_x^{xy} & E_x^{yx} \\ E_y^{xy} & E_y^{yx} \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x^{xy} & H_x^{yx} \\ H_y^{xy} & H_y^{yx} \end{pmatrix}, \qquad (2-113)$$

where E_x^{xy} , E_y^{xy} , H_x^{xy} , and H_y^{xy} are the horizontal electric and magnetic fields for the xy-polarization source, respectively; E_x^{yx} , E_y^{yx} , H_x^{yx} , and H_y^{yx} are the horizontal electric and magnetic fields for the yx-polarization source, respectively; and Z_{xx} , Z_{xy} , Z_{yx} , and Z_{yy} are the components of the impedance tensor. From equation (2-113), the components of the impedance tensor can be written as follows:

$$Z_{xx} = \frac{E_x^{xy} H_y^{yx} - E_x^{yx} H_y^{xy}}{H_x^{xy} H_y^{yx} - H_x^{yx} H_y^{xy}}$$

$$Z_{xy} = -\frac{E_x^{xy} H_x^{yx} - E_x^{yx} H_x^{xy}}{H_x^{xy} H_y^{yx} - H_x^{yx} H_y^{xy}}$$

$$Z_{yx} = \frac{E_y^{xy} H_y^{yx} - E_y^{yx} H_y^{xy}}{H_x^{xy} H_y^{yx} - H_x^{yx} H_y^{xy}}$$

$$Z_{yy} = -\frac{E_y^{xy} H_y^{yx} - E_y^{yx} H_x^{xy}}{H_x^{xy} H_y^{yx} - H_x^{yx} H_y^{yy}}$$
(2-114)

Equation (2-113) can be expressed in matrix form as follows:

$$\mathbf{E}_{xy} = \mathbf{Z}\mathbf{H}_{xy} \quad , \tag{2-115}$$

where $\mathbf{Z} \in \mathbb{C}^{2 \times 2}$ is the impedance tensor.

The impedance tensor is sometimes rotated according to the direction of the axes. Fig. 2.5 shows the new x' - and y' -axes rotated by θ in the clockwise direction from the original x- and y-axes. The rotation matrix for the coordinate system $\mathbf{R}(\theta)$ is written as follows:

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} .$$
 (2-116)

Using the rotation matrix $\mathbf{R}(\theta)$, equation (2-115) can be developed as follows:

$$\mathbf{E}_{xy}(\theta) = \mathbf{R}(\theta)\mathbf{E}_{xy} = \mathbf{R}(\theta)\mathbf{Z}\mathbf{H}_{xy}$$

= $\mathbf{R}(\theta)\mathbf{Z}\mathbf{R}(-\theta)\mathbf{R}(\theta)\mathbf{H}_{xy} = \mathbf{R}(\theta)\mathbf{Z}\mathbf{R}(-\theta)\mathbf{H}_{xy}(\theta)$, (2-117)

where \mathbf{E}_{xy} , \mathbf{H}_{xy} , and \mathbf{Z} are for the original x- and y-axes; and $\mathbf{E}_{xy}(\theta)$ and $\mathbf{H}_{xy}(\theta)$ are for the rotated x' - and y' -axes in Fig. 2.5. From equation (2-117), the rotated impedance tensor $\mathbf{Z}(\theta)$ for the new x' - and y' -axes is expressed as follows:

$$\mathbf{Z}(\theta) = \mathbf{R}(\theta)\mathbf{Z}\mathbf{R}(-\theta) \quad . \tag{2-118}$$



Fig. 2.5. Rotation of axes from original x- and y-axes to new x' - and y'-axes.
The partial derivative of equation (2-115) with respect to σ is expressed as follows:

$$\frac{\partial \mathbf{E}_{xy}}{\partial \sigma} = \frac{\partial \mathbf{Z}}{\partial \sigma} \mathbf{H}_{xy} + \mathbf{Z} \frac{\partial \mathbf{H}_{xy}}{\partial \sigma} \quad . \tag{2-119}$$

From equation (2-119), the partial derivative of \mathbf{Z} with respect to σ is calculated as follows:

$$\frac{\partial \mathbf{Z}}{\partial \sigma} = \left(\frac{\partial \mathbf{E}_{xy}}{\partial \sigma} - \mathbf{Z} \frac{\partial \mathbf{H}_{xy}}{\partial \sigma}\right) \mathbf{H}_{xy}^{-1} \quad . \tag{2-120}$$

Equation (2-120) is used to calculate the Jacobian matrix for the impedance tensor.

2.3.2. Apparent resistivity and phase

The apparent resistivity ρ_a and phase φ introduced by Cagniard (1953) are also representative MT response functions. In particular, MT data for each receiver are usually plotted by the apparent resistivity and phase. They are calculated from the impedance components as follows:

$$\rho_{a, ij} = \frac{1}{\omega \mu_0} \left| Z_{ij} \right|^2 , \qquad (2-121)$$

$$\varphi_{ij} = \tan^{-1} \left\{ \frac{Z_{ijI}}{Z_{ijR}} \right\} \quad , \tag{2-122}$$

where the subscript 'ij' represents an arbitrary combination of x and y; and the subscripts 'R' and 'I' mean the real and imaginary parts of the complex number. The apparent resistivity implies volumetrically averaged resistivity over the penetration distance. For instance, in a uniform half space, the apparent resistivity represents the electrical resistivity of the medium, and the phase has 45 degrees.

The partial derivatives of equations (2-121) and (2-122) with respect to σ are calculated as follows:

$$\frac{\partial \rho_{a, ij}}{\partial \sigma} = \frac{2}{\omega \mu_0} \left(Z_{ijR} \frac{\partial Z_{ijR}}{\partial \sigma} + Z_{ijI} \frac{\partial Z_{ijI}}{\partial \sigma} \right) , \qquad (2-123)$$

$$\frac{\partial \varphi_{ij}}{\partial \sigma} = \frac{1}{Z_{ijR}^{2} + Z_{ijI}^{2}} \left(Z_{ijR} \frac{\partial Z_{ijI}}{\partial \sigma} - Z_{ijI} \frac{\partial Z_{ijR}}{\partial \sigma} \right) .$$
(2-124)

Equations (2-123) and (2-124) are used to calculate the Jacobian matrices for the apparent resistivity and phase.

2.3.3. Tipper

One of the MT response functions that correlates the vertical and horizontal magnetic fields is called the tipper or the magnetic transfer function (Vozoff 1991). The tipper for each frequency is defined as follows:

$$\begin{pmatrix} H_z^{xy} \\ H_z^{yx} \\ H_z^{yx} \end{pmatrix}^T = \begin{pmatrix} T_x \\ T_y \end{pmatrix}^T \begin{pmatrix} H_x^{xy} & H_x^{yx} \\ H_y^{xy} & H_y^{yx} \\ H_y^{xy} & H_y^{yx} \end{pmatrix} , \qquad (2-125)$$

where H_z^{xy} and H_z^{yx} are the vertical magnetic fields for the xy-polarization and yx-polarization sources, respectively. T_x and T_y are the x- and y-components of the tipper vector, respectively. From equation (2-125), the components of the tipper vector can be written as follows:

$$T_{x} = \frac{H_{z}^{xy}H_{y}^{yx} - H_{z}^{yx}H_{y}^{xy}}{H_{x}^{xy}H_{y}^{yx} - H_{x}^{yx}H_{y}^{xy}}$$

$$T_{y} = -\frac{H_{z}^{xy}H_{x}^{yx} - H_{z}^{yx}H_{x}^{xy}}{H_{x}^{xy}H_{y}^{yx} - H_{x}^{yx}H_{y}^{xy}}$$
(2-126)

Equation (2-125) is represented in matrix form as follows:

$$\mathbf{H}_{z}^{T} = \mathbf{T}^{T} \mathbf{H}_{xy} \quad , \tag{2-127}$$

where $\mathbf{T} \in \mathbb{C}^2$ is the tipper vector. In the 1D structure, both T_x and T_y are **0** because $\mathbf{H}_z = \mathbf{0}$. The size of the tipper, $|\mathbf{T}| = (|T_x|^2 + |T_y|^2)^{1/2}$, is always less than 1 (Zonge & Hughes, 1991).

For the rotation of the tipper, equation (2-127) can be developed as follows:

$$\mathbf{H}_{z}^{T}(\theta) = \mathbf{H}_{z}^{T} = \mathbf{T}^{T}\mathbf{H}_{xy} = \mathbf{T}^{T}\mathbf{R}(-\theta)\mathbf{R}(\theta)\mathbf{H}_{xy} = \mathbf{T}^{T}\mathbf{R}(-\theta)\mathbf{H}_{xy}(\theta) , \qquad (2-128)$$

where \mathbf{H}_{z} , \mathbf{H}_{xy} , and \mathbf{T} are for the original x- and y-axes in Fig. 2.5. $\mathbf{H}_{z}(\theta)$ and $\mathbf{H}_{xy}(\theta)$ are for the rotated x' - and y'-axes in Fig. 2.5. As can be seen in equation (2-128), the transpose of the rotated tipper $\mathbf{T}^{T}(\theta)$ for the new x' - and y'-axes is written as follows:

$$\mathbf{T}^{T}(\boldsymbol{\theta}) = \mathbf{T}^{T} \mathbf{R}(-\boldsymbol{\theta}) \quad . \tag{2-129}$$

The partial derivative of equation (2-127) with respect to σ can be arranged as follows:

$$\frac{\partial \mathbf{T}^{T}}{\partial \sigma} = \left(\frac{\partial \mathbf{H}_{z}^{T}}{\partial \sigma} - \mathbf{T}^{T} \frac{\partial \mathbf{H}_{xy}}{\partial \sigma}\right) \mathbf{H}_{xy}^{-1} \quad .$$
(2-130)

Equation (2-130) is used to calculate the Jacobian matrix for the tipper.

2.3.4. Effective impedance

The original and rotated values for the rotational invariants of the impedance tenser are the same (Berdichevsky & Dmitriev 2008). In other words, the rotational invariants have the same value regardless of the orientation of the measured electromagnetic fields. The complex 2 by 2 impedance tensor has eight independent real values, and the maximum number of the real independent rotational invariants for the impedance tensor is seven (Szarka & Menvielle 1997). The effective impedance Z_{eff} is a complex rotational invariant, and has been used for 2D MT inversion (Pedersen & Engels 2005; Wang et al. 2020), which is defined as follows:

$$Z_{eff} = \sqrt{Z_{xx} Z_{yy} - Z_{xy} Z_{yx}} \quad . \tag{2-131}$$

The partial derivative of Z_{eff} with respect to σ can be written as follows:

$$\frac{\partial Z_{eff}}{\partial \sigma} = \frac{1}{2Z_{eff}} \left(\frac{\partial Z_{xx}}{\partial \sigma} Z_{yy} + Z_{xx} \frac{\partial Z_{yy}}{\partial \sigma} - \frac{\partial Z_{xy}}{\partial \sigma} Z_{yx} - Z_{xy} \frac{\partial Z_{yx}}{\partial \sigma} \right) . \quad (2-132)$$

Equation (2-132) is used to calculate the Jacobian matrix for the effective impedance.

2.3.5. Phase tensor

The measured electric fields can be distorted by local near-surface inhomogeneities, and this phenomenon is called the galvanic distortion. Caldwell et al. (2004) proposed the phase tensor that is not distorted by the galvanic distortion. The phase tensor $\mathbf{\Phi} \in \mathbb{R}^{2\times 2}$ is defined as follows:

$$\boldsymbol{\Phi} = \mathbf{Z}_R^{-1} \mathbf{Z}_I \quad , \tag{2-133}$$

where \mathbf{Z}_{R} and \mathbf{Z}_{I} are matrices composed of the real and imaginary impedance components (i.e., $\mathbf{Z} = \mathbf{Z}_{R} + i\mathbf{Z}_{I}$). Equation (2-133) can be written as follows:

$$\begin{pmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{yx} & \Phi_{yy} \end{pmatrix} = \frac{1}{\det(\mathbf{Z}_{R})} \begin{pmatrix} Z_{yyR} Z_{xxI} - Z_{xyR} Z_{yxI} & Z_{yyR} Z_{xyI} - Z_{xyR} Z_{yyI} \\ Z_{xxR} Z_{yxI} - Z_{yxR} Z_{xxI} & Z_{xxR} Z_{yyI} - Z_{yxR} Z_{xyI} \end{pmatrix},$$
(2-134)

where det (\mathbf{Z}_R) is the determinant of \mathbf{Z}_R (i.e., $Z_{xxR}Z_{yyR} - Z_{xyR}Z_{yxR}$).

The partial derivative of $\, \Phi \,$ with respect to $\, \sigma \,$ can be developed as follows:

$$\frac{\partial \mathbf{\Phi}}{\partial \sigma} = \mathbf{Z}_{R}^{-1} \left(\frac{\partial \mathbf{Z}_{I}}{\partial \sigma} - \frac{\partial \mathbf{Z}_{R}}{\partial \sigma} \mathbf{\Phi} \right) .$$
(2-135)

Equation (2-135) is used to compute the Jacobian matrix for the phase tensor.

The symbols of components of MT response functions used in this study are described in Table 2.1. Components of a complex number are denoted by the symbols separated by real and imaginary parts.

Symbol	Description				
Z_{xxR}	Z _{xxR} Real part of xx-component of impedance tensor				
Z_{xxI}	Imaginary part of xx-component of impedance tensor				
Z_{xyR}	Z_{xyR} Real part of xy-component of impedance tensor				
Z_{xyI}	Imaginary part of xy-component of impedance tensor				
Z_{yxR}	Real part of yx-component of impedance tensor				
Z_{yxI}	Z _{yx1} Imaginary part of yx-component of impedance tensor				
Z_{yyR}	Real part of yy-component of impedance tensor				
Z_{yyI}	Imaginary part of yy-component of impedance tensor				
$ ho_{a, xy}$	xy-component of apparent resistivity				
$ ho_{a, yx}$	yx-component of apparent resistivity				
$arphi_{xy}$	xy-component of phase				
φ_{yx}	yx-component of phase				
T_{xR}	Real part of x-component of tipper				
T_{xI}	Imaginary part of x-component of tipper				
T_{yR}	Real part of y-component of tipper				
T_{yI}	Imaginary part of y-component of tipper				
$Z_{e\!f\!f\!R}$	Real part of effective impedance				
$Z_{e\!f\!f\!I}$	Imaginary part of effective impedance				
Φ_{xx}	xx-component of phase tensor				
Φ_{xy}	xy-component of phase tensor				
Φ_{yx}	yx-component of phase tensor				
Φ_{yy}	yy-component of phase tensor				

Table. 2.1. List of symbols of MT response functions used in this thesis.

Chapter 3. Sensitivity patterns for MT response functions

Sensitivity patterns are defined by the variations of the MT responses due to the perturbation of one element of the model parameter vector (i.e., one column of the Jacobian matrix). In other words, the sensitivity patterns imply some locations where the MT responses by an anomaly are strong. Thus, in this section, sensitivity patterns are analyzed to investigate the features of MT response functions in inversion, and they are divided into six groups according to their sensitivity patterns, so that MT response functions with similar roles in inversion belong to the same group.

The sensitivity patterns for major MT response functions in Table 2.1 (impedance, apparent resistivity, phase, tipper, effective impedance, and phase tensor) with the model parameter of the electrical conductivity are considered. Subsurface MT response functions are calculated using the same formulae defined at the surface. The sensitivity patterns are calculated in the homogeneous model with a resistivity of 100 Ω m (i.e., 10^{-2} S/m) at a frequency of 1 Hz through the perturbation approach in equation (2-66) with two times of 3D MT modeling using structured hexahedral elements. 3D sensitivity patterns are described in the 8 $(-4 \sim 4) \times 8 (-4 \sim 4)$ x 4 (0~4) km (i.e., 1.6δ x 1.6δ x 0.8δ where δ is the skin depth in equation 2-22), and their values are computed at intervals of 100 m (i.e., $81 \times 81 \times 41 =$ 269,001 points). The perturbed element whose size is 100 x 100 x 100 m (i.e., $\delta/50$ x $\delta/50$ x $\delta/50$) is located at the center of the 3D space, i.e., (0, 0, 2 km) = (0, 0, 0, 0, 0) 0.4δ). The change of the model parameter of electrical conductivity is 10^{-4} S/m (i.e., 1/100 of the background value). 2D sensitivity patterns at z=0 km are also plotted because the MT survey is conducted on the surface. They are briefly called

'surface-sensitivity patterns' in this thesis.

When designing an MT survey, it is important to consider the ratio of the horizontal distance between target and receiver to the target depth, which is called the 'survey ratio' in this thesis. The positions where the survey ratio is 1 or 2 ($\pm 0.4\delta$ or $\pm 0.8\delta$) with the reference to the perturbed depth (i.e., 2 km = 0.4δ) are marked by the black open squares in the both 3D sensitivity patterns and surface-sensitivity patterns. The 3D sensitivity patterns are displayed by two isosurfaces corresponding to 1-st (negative, red color) and 99-th (positive, blue color) percentiles of the total distribution of the sensitivity values. Only the 3D sensitivity patterns of the phase and the diagonal components of the phase tensor are depicted by 3-rd and 97-th percentiles to represent slightly weaker patterns. For the significant surface-sensitivity patterns, only values larger than 1/5 of the maximum absolute value are shown. The sensitivity patterns are plotted using the Voxler of Golden Software Inc.

3.1. Sensitivity patterns for impedance tensor

Fig. 3.1 shows the sensitivity patterns of the real components of the impedance tensor (i.e., Z_{xxR} , Z_{xyR} , Z_{yxR} , and Z_{yyR}). In Figs. 3.1(a) and 3.1(b), the 3D sensitivity pattern and surface-sensitivity pattern of Z_{xxR} are in the shape of four diagonal petals and significant values of the surface-sensitivity pattern spread beyond the boundary with the survey ratio of 1. Because the strong sensitivity regions widely appear in all four quadrants, Z_{xxR} can play an important role in 3D MT inversion. The 3D sensitivity pattern of Z_{xyR} shown in Fig. 3.1(c) is composed of two parts: The first part in red has a doughnut shape in the yz-plane, and the other

in blue has a shape of two petals penetrating the first part along the x-axis. In Fig. 3.1(d), the surface-sensitivity pattern of Z_{xyR} has an elliptical shape with the major axis along the y-axis. The elliptical shape at the surface comes from the doughnutshaped 3D sensitivity pattern that contains intensive sensitivities in the upward direction from the perturbation point. The significant surface-sensitivity pattern is distributed within the boundary with the survey ratio of 1. Because Z_{xyR} has the meaningful sensitivity pattern along the y-axis at the surface, it is well suited for imaging 2D structures whose strike is along the y-axis. The 3D sensitivity pattern and surface-sensitivity pattern of Z_{vxR} in Figs. 3.1(e) and 3.1(f) have such a form that can be acquired by rotating the sensitivity patterns for Z_{xyR} by 90 degrees around the z-axis, while their signs are reversed. Thus, Z_{xxR} is suitable for inverting 2D structures with their strike along the x-axis. Compared with Figs. 3.1(a) and 3.1(b), both the 3D sensitivity pattern and surface-sensitivity pattern of Z_{yyR} in Figs. 3.1(g) and 3.1(h) have similar forms to those of Z_{xxR} with their signs reversed, which means that they possess similar features in the inverse problem.

Fig. 3.2 represents the sensitivity patterns of the imaginary components of the impedance tensor (i.e., Z_{xxl} , Z_{xyl} , Z_{yxl} , and Z_{yyl}). Compared to the 3D sensitivity patterns for the real components of the impedance tensor (Figs. 3.1a, 3.1c, 3.1e, and 3.1g), the 3D sensitivity patterns for the imaginary components (Figs. 3.2a, 3.2c, 3.2e, and 3.2g) have similar aspects, but they are slightly drooping down. The surface-sensitivity patterns of the imaginary components of the impedance tensor (Figs. 3.2b, 3.2d, 3.2f, and 3.2h) are distributed over narrower ranges than those of the real components. However, the overall sensitivity patterns of the real and imaginary parts of each impedance component are similar.



Fig. 3.1. Sensitivity patterns of the real components of the impedance tensor: (a, b) Z_{xxR} , (c, d) Z_{yyR} , (e, f) Z_{yxR} , and (g, h) Z_{yyR} . (a,

c, e, g) and (b, d, f, h) are the 3D sensitivity patterns and surface-sensitivity patterns, respectively.



Fig. 3.2. Sensitivity patterns of the imaginary components of the impedance tensor: (a, b) Z_{xxl} , (c, d) Z_{yyl} , (e, f) Z_{yxl} , and (g, h) Z_{yyl} . (a, c, e, g) and (b, d, f, h) are the 3D sensitivity patterns and surface-sensitivity patterns, respectively.

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3.2. Sensitivity patterns for apparent resistivity and phase

In Fig. 3.3, the sensitivity patterns for the xy- and yx-components of the apparent resistivity (i.e., $\rho_{a, xy}$ and $\rho_{a, yx}$) and the xy- and yx-components of the phase (i.e., φ_{xy} and φ_{yx}) are displayed. Because the 3D sensitivity pattern of $\rho_{a, xy}$ (Fig. 3.3a) resembles that of Z_{xy} , the surface-sensitivity pattern of $\rho_{a, xy}$ (Fig. 3.3b) also has similar features to that of Z_{xy} . Both the 3D sensitivity pattern and surface-sensitivity pattern of $\rho_{a, yx}$ (Figs. 3.3c and 3.3d) are analogous with those of Z_{yx} except for the signs.

The 3D sensitivity patterns of φ_{xy} (Fig. 3.3e) and φ_{yx} (Fig. 3.3g) have strong energy in five directions, which are like a butterfly shape. They are rotated by 90 degrees around the z-axis to each other. In the surface-sensitivity patterns of φ_{xy} (Fig. 3.3f) and φ_{yx} (Fig. 3.3h), both the positive (blue) and negative (red) parts of the 3D sensitivity patterns appear, but this thesis focuses on the dominant positive parts. Although the 3D sensitivity pattern of φ_{xy} is different from those of Z_{xy} and $\rho_{a, xy}$, the xy-components for the MT response functions (i.e., Z_{xy} , $\rho_{a, xy}$, and φ_{xy}) commonly have the strong surface-sensitivity patterns along the y-axis. The yx-components of the MT response functions (i.e., Z_{yx} , and φ_{yx}) also have in common that they show the intensive sensitivity patterns along the x-axis at the surface.



Fig. 3.3. Sensitivity patterns of the xy- and yx-components of the apparent resistivity and phase: (a, b) $\rho_{a, xy}$, (c, d) $\rho_{a, yx}$, (e, f) ϕ_{xy} , and (g, h) ϕ_{yx} . (a, c, e, g) and (b, d, f, h) are the 3D sensitivity patterns and surface-sensitivity patterns, respectively.

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3.3. Sensitivity patterns for tipper

Fig. 3.4 shows the sensitivity patterns of the real and imaginary components of the tipper (i.e., T_{xR} , T_{xI} , T_{yR} , and T_{yI}). The 3D sensitivity pattern of T_{xR} (Fig. 3.4a) is like two petals along the x-axis. In the surface-sensitivity pattern of T_{xR} (Fig. 3.4b), the significant sensitivities are not only confined around the x-axis but spread widely as the shape of the two petals is preserved. Similar to the relationship between the sensitivity patterns for the real and imaginary components of the impedance tensor, T_{xI} has the slightly drooping 3D sensitivity pattern (Fig. 3.4c) and the narrower surface-sensitivity pattern (Fig. 3.4d) than those of T_{xR} . Because the real and imaginary x-components of the tipper (T_{xR} and T_{xI}) possess the sensitivity patterns with similar characteristics, they will have similar functions in inversion. The 3D sensitivity patterns and surface-sensitivity patterns of T_{yR} and T_{yI} (Figs. 3.4 e, 3.4f, 3.4g, and 3.4h) are obtained by rotating those of T_{xR} and T_{xI} 90 degrees clockwise around the z-axis. Because the tipper contains the vertical component of the magnetic fields (H_z) that is not related to the impedance-based MT response functions (i.e., impedance, apparent resistivity, and phase), the new sensitivity patterns are observed in the tipper, and therefore the tipper can complement the other MT response functions during the inverse process. The tipper is suitable for both 3D and 2D MT inversion, because the surface-sensitivity patterns have wide distribution along a specific direction. Furthermore, the tipper can play an important role in a specific acquisition case, which will be described in more detail in Chapter 4.2.



Fig. 3.4. Sensitivity patterns of the real and imaginary components of the tipper: (a, b) T_{xR} , (c, d) T_{xI} , (e, f) T_{yR} , and (g, h) T_{yI} . (a, c, e, g) and (b, d, f, h) are the 3D sensitivity patterns and surface-sensitivity patterns, respectively.

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3.4. Sensitivity patterns for effective impedance

The sensitivity patterns of the real and imaginary components for the effective impedance (i.e., Z_{effR} and Z_{effT}) are described in Fig. 3.5. Their 3D sensitivity patterns (Figs. 3.5a and 3.5c) include energy concentrated along the z-axis. Therefore, the corresponding surface-sensitivity patterns (Figs. 3.5b and 3.5d) appear as a circle around the center. The effective impedance has the smallest survey ratio among all the MT response functions considered in this thesis. These features make the effective impedance suitable for 1D interpretation of MT data.



Fig. 3.5. Sensitivity patterns of the real and imaginary components of the effective impedance: (a, b) Z_{effR} and (c, d) Z_{effI} . (a, c) and (b, d) are the 3D sensitivity patterns and surface-sensitivity patterns, respectively.

3.5. Sensitivity patterns for phase tensor

Fig. 3.6 shows the sensitivity patterns of the components for the phase tensor (i.e., Φ_{xx} , Φ_{yy} , Φ_{yx} , and Φ_{yy}). The sensitivity patterns for Φ_{xx} and Φ_{yy} (Figs. 3.6a, 3.6b, 3.6g, and 3.6h) are similar to those of the yx- and xy-components of the phase (i.e., φ_{yx} and φ_{xy}), respectively. The 3D sensitivity patterns of Φ_{xy} and Φ_{yx} (Figs. 3.6c and 3.6e) are in the shape of eight petals. Compared with the xx- and yy-components of the impedance tensor (i.e., Z_{xx} and Z_{yy}), Φ_{xy} and Φ_{yx} have different 3D sensitivity patterns, but the surface-sensitivity patterns (Figs. 3.6d and 3.6f) are similar. Therefore, Φ_{xx} and Φ_{yy} can replace the yxcomponents of the MT response functions (i.e., Z_{yx} , $\rho_{a, yx}$, and φ_{yx}) and the xycomponents of the MT response functions (i.e., Z_{xy} , $\rho_{a, xy}$, and φ_{xy}), respectively, while Φ_{xy} and Φ_{yx} can play a similar role to Z_{xx} and Z_{yy} in MT inversion. If the phase tensor, which is not affected by the galvanic distortion, is used as the input data for inversion instead of the impedance tensor, then the available real numbers are halved from eight (i.e., Z_{xxR} , Z_{xxI} , Z_{xyR} , Z_{xyI} , Z_{yxR} , Z_{yxI} , Z_{yyR} , and Z_{yyI}) to four (i.e., Φ_{xx} , Φ_{xy} , Φ_{yx} , and Φ_{yy}) for each receiver. However, subsurface structures may be similarly interpreted, because the sensitivity patterns of the four components of the phase tensor contain all information given by those of the eight real numbers for the impedance tensor.



Fig. 3.6. Sensitivity patterns of the components of the phase tensor: (a, b) Φ_{xx} , (c, d) Φ_{yy} , (e, f) Φ_{yx} , and (g, h) Φ_{yy} . (a, c, e, g) and

(b, d, f, h) are the 3D sensitivity patterns and surface-sensitivity patterns, respectively.

3.6. Classification of MT response functions

In Table 3.1, the MT response functions covered in this study are classified into six groups according to the characteristics of their surface-sensitivity patterns. In MT inversion, MT response functions in the same group may provide similar results, and MT response functions in different groups can play complementary roles with each other because they may contain different information on the same subsurface structures. Group 1 contains Z_{xxR} , Z_{xxI} , Z_{yyR} , Z_{yyI} , Φ_{xy} , and Φ_{yx} whose surface-sensitivity patterns are in the shape of four petals dominantly in diagonal directions. Z_{xyR} , Z_{xyI} , $\rho_{a, xy}$, φ_{xy} , and Φ_{yy} , which have strong surfacesensitivity patterns along the x-axis, belong to Group 2. Z_{yxR} , Z_{yxI} , $\rho_{a, yx}$, φ_{yx} , and Φ_{xx} whose surface-sensitivity patterns are concentrated along the x-axis, are included in Group 3. The x-components of the tipper (i.e., T_{xR} and T_{xI}) and ycomponents of the tipper (i.e., T_{yR} and T_{yI}) possess two petals-shaped surfacesensitivity patterns dominant along the x- and y-axes, and they belong to Groups 4 and 5, respectively. Finally, Group 6 has the real and imaginary components of the effective impedance (i.e., Z_{effR} and Z_{effI}) showing small circle-shaped surfacesensitivity patterns.

Table.	3.1.	Classification	of MT	response	functions	according	to	the	surface-
sensitiv	vity p	atterns.							

Classification	MT response functions		
Group 1	$Z_{xxR}, Z_{xxI}, Z_{yyR},$		
Diagonal 4 petals-shaped pattern	$Z_{yyI}, \Phi_{xy}, \Phi_{yx}$		
Group 2	$Z_{xyR}, \ Z_{xyI}, \ ho_{a,\ xy},$		
Linear pattern along the y-axis	$arphi_{_{xy}}, \ \Phi_{_{yy}}$		
Group 3	$Z_{yxR}, Z_{yxI}, \rho_{a, yx},$		
Linear pattern along the x-axis	$\varphi_{_{yx}}, \ \Phi_{_{xx}}$		
Group 4	T_{xR} , T_{xI}		
Linear 2 petals-shaped pattern along the x-axis			
Group 5	ТТ		
Linear 2 petals-shaped pattern along the y-axis	$\mathbf{I}_{yR}, \mathbf{I}_{yI}$		
Group 6	7 7		
Small circle-shaped pattern	\mathcal{L}_{effR} , \mathcal{L}_{effI}		

3.7. Surface-sensitivity patterns for other frequencies

In Chapters 3.1 to 3.5, the sensitivity patterns of the different MT response functions (i.e., impedance, apparent resistivity, phase, tipper, effective impedance, and phase tensor) were examined. The sensitivity patterns were shown in the 100 Ω m homogeneous cuboid space of (-4 or 4, -4 or 4, 0 or 4 km) nodes, i.e., (-0.8 δ or 0.8δ , -0.8δ or 0.8δ , 0 or 0.8δ) nodes where δ is the skin depth in equation (2-22), to represent the survey ratio up to 2 when the perturbed depth was 2 km (i.e., 0.4δ) for data at 1 Hz. In this section, surface-sensitivity patterns for higher and lower frequencies on different spatial scales considering the same range normalized by the skin depth (δ) in the 100 Ω m homogeneous model are shown. Fig. 3.7 shows the surface-sensitivity patterns of all the MT response functions for the higher frequency at 100 Hz on the 1/10 scale than those for 1 Hz, which means the surface consists of (-0.4 or 0.4, -0.4 or 0.4 km) nodes, i.e., the same (-0.8δ or 0.8δ , -0.8δ or 0.8δ) nodes, and the position of (0, 0, 0.2 km), i.e., the same (0, 0, 0.2 km)0, 0.4δ), is perturbed. In Fig. 3.8, the surface-sensitivity patterns of the same MT response functions for the lower frequency at 0.01 Hz are illustrated on the dimension of 80 x 80 km (i.e., the same $1.6\delta \times 1.6\delta$) with the perturbation point of $(0, 0, 20 \text{ km}) = (0, 0, 0.4\delta)$, which is 10 times the previous dimension for the same range expressed in δ . The surface-sensitivity patterns for each MT response function in Figs. 3.7 and 3.8 have the same shape as those in Figs. 1 to 6 regardless of frequency, but their spread-distance changes in proportion to the skin depth at the frequency.



Fig. 3.7. Surface-sensitivity patterns of all the MT response functions in Table 2.1 for the perturbation point of $(0, 0, 0.2 \text{ km}) = (0, 0, 0.4\delta)$ up to the survey ratio of 2 (i.e., ± 0.4 km and $\pm 0.8\delta$) at a frequency of 100 Hz in the 100 Ω m homogeneous model.



Fig. 3.8. Surface-sensitivity patterns of all the MT response functions in Table 2.1 for the perturbation point of $(0, 0, 20 \text{ km}) = (0, 0, 0.4\delta)$ up to the survey ratio of 2 (i.e., $\pm 40 \text{ km}$ and $\pm 0.8\delta$) at a frequency of 0.01 Hz in the 100 Ω m homogeneous model.

Chapter 4. Synthetic Examples

The sensitivity patterns for MT response functions analyzed and classified in Chapter 3 can be used to make a strategy for inversion of real field data because different inversion results are produced according to the characteristics of the sensitivity patterns. Before applying to the inverse process of field data, in this section, it is investigated how the sensitivity patterns are reflected in observed data and inversion results for synthetic cases of 1D, 2D, and 3D interpretation of MT data, from which the optimal MT response functions for each case are recommended. For the synthetic examples, the modeling and inversion algorithms in Table 4.1 are applied. Because the synthetic examples contain many inversion results of various MT response functions, the inverted models are simply presented with normalized root-mean-square (NRMS) error, which is normalized by the difference between maximum and minimum values. The inverted models are displayed by the Voxler of Golden Software Inc.

3D MT modeling with edge-based FEM					
Modeling mesh (Chapter A.2)	Structured hexahedral elements				
3D MT inversion					
Objective function (Chapter 2.2.1)	Jumping method in equation (2-52)				
Jacobian calculation (Chapter 2.2.2)	Sensitivity equation approach in equation (2-73)				
Blocky parameterization (Chapter 2.2.3.1)	Structured hexahedral inversion mesh				
Model parameterization (Chapter 2.2.3.2)	Equations (2-77) and (2-78)				
Data weighting matrix (Chapter 2.2.3.3)	N/A or equation (2-89)				
Roughness matrix (Chapter 2.2.3.4)	Equation (2-95)				
Lagrange multiplier (Chapter 2.2.3.5)	Equation (2-100)				
Line search (Chapter 2.2.3.6)	N/A				

Table. 4.1. Settings of 3D MT modeling and inversion for the synthetic examples.

4.1. Synthetic examples for 1D interpretation of MT data

When the sensitivity patterns of MT response functions are analyzed for 1D interpretation of MT data, two points should be considered: The first is whether the MT response functions respond to the 1D model, and the second is how strongly the MT response functions are affected by the dimensionality error. The dimensionality error occurs when the dimension considered by the inversion algorithm is lower than the dimension of the true subsurface structures, e.g., 1D inversion of data acquired over 2D or 3D structures and 2D inversion of data acquired over 3D structures (Chave & Jones 2012).

Because 1D interpretation of MT data aims to describe vertical variations of electrical properties assuming no lateral variations, appropriate MT response functions for 1D inversion should have strong 3D sensitivity patterns along the z-axis from the perturbation location (i.e., the surface-sensitivity patterns should be concentrated around the center right above the perturbation position). According to this criterion, Groups 1, 4, and 5 in Table 3.1 are not suitable for 1D inversion, while Groups 2, 3, and 6 can be used for the interpretation of 1D structures.

Next, the dimensionality error for the MT response functions available for 1D inversion (i.e., Groups 2, 3, and 6) is considered. Groups 2 and 3 have linear surface-sensitivity patterns along the y- and x-axes, respectively, whereas Group 6 shows a point-shaped surface-sensitivity pattern. These patterns imply that Groups 2 and 3 can be more affected by the dimensionality error than Group 6 is, when the subsurface structures are not perfectly 1D.

Figure 4.1 shows a synthetic model for 1D interpretation of MT data composed

of a background medium (100 Ω m) and a conductive anomalous body (1 Ω m) along the positive y-axis near the surface, which can cause the dimensionality error. One receiver indicated by a red circle is located at the origin. In this case of the 1D interpretation of MT data, it can be expected that the MT response functions with the linear surface-sensitivity pattern along the y-axis (Group 2) are more distorted by the dimensionality error than the MT response functions possessing the surfacesensitivity pattern of the small circle (Group 6). To confirm this, the modeling results of the real part of the effective impedance (Z_{effR} of Group 6) and the xy-component of the apparent resistivity ($\rho_{a,xy}$ of Group 2), which are representative MT response functions used for 1D inversion, are compared.



Fig. 4.1. Synthetic model for 1D interpretation of MT data: (a) plan view and (b) yz cross-section. Resistivity of the background medium and the hexahedral anomalous body is 100 and 1 Ω m, respectively. Receiver position is represented by the red circles.

Fig. 4.2 shows observed modeling data of Z_{effR} (blue circles) and $\rho_{a,xy}$ (red triangles) for the anomalous-body-included model shown in Fig. 4.1 at frequencies ranging from 0.01 to 100 Hz. In order to compare the dimensionality error, modeling results in the homogeneous model without the anomaly (black squares) are also presented. To clearly describe the influence of the dimensionality error, percent error between the modeling results for the two models are computed. Fig. 4.2(a) represents that there is little difference in two modeling results of Z_{effR} , while in Fig. 4.2(b), for the anomalous-body-included model is lower than that for the $\rho_{a,xy}$ homogeneous model due to the low resistivity anomalous body. Therefore, the percent error of $\rho_{a,xy}$ is higher than that of Z_{effR} particularly over frequencies lower than 10 Hz in Fig. 4.2(c). For the two MT response functions, the observed modeling data and the characteristics of the surface-sensitivity patterns related to the dimensionality error mentioned above are consistent. These results also indicate that even under the same condition (model, frequency, receiver position, etc.) interpretation may be different depending on the data type of MT response functions. From both the sensitivity pattern analysis and the numerical example for 1D interpretation of MT data, it can be concluded that the data type of effective impedance (Z_{eff}) can be a method of choice for 1D inversion of MT data.



Fig. 4.2. Modeling results of (a) real effective impedance (Z_{effR}) and (b) xy-apparent resistivity ($\rho_{a,xy}$) for both the synthetic model shown in Fig. 4.1 and its background homogeneous model. (c) shows percent error between the two modeling results for Z_{effR} and $\rho_{a,xy}$.

4.2. Synthetic examples for 2D interpretation of MT data

To examine features of MT response functions for 2D interpretation of MT data, the 3D modeling and inversion with the 2D survey geometry are performed as Siripunvarapron et al. (2005a) did. The characteristics of the MT response functions are investigated for an ideal case of interpreting a complete 2D structure with a general receiver array (one line across a 2D structure). Furthermore, two specific cases are considered: The first case is that receivers right above a target structure cannot be used, and the second case is for an off-plane error. To compare modeling and inversion results for the second specific case with those for the ideal case, two models are assumed in Fig. 4.3: One (Fig. 4.3a) has only a square-shaped 2D structure (1 Ω m) whose strike is the y-axis, and the other (Fig. 4.3b) has not only the 2D structure but an off-plane anomalous body (1 Ω m). Their homogeneous background medium is 100 Ω m. For the first specific case, two receiver arrays in the general 2D model (Fig. 4.3a) are considered: One is the general receiver array marked by the red lines in Figs. 4.3(a) and 4.3(b) that a total of 21 receivers are uniformly located in the perpendicular direction to the strike at intervals of 0.5 km covering from -5 to 5 km, and the other receiver array indicated by the blue lines in Fig. 4.3(a) is that five receivers right above the target 2D structure covering from -3 to -1 km are excluded due to poor data-quality, inaccessible field conditions, etc.



Fig. 4.3. Two synthetic models for 2D interpretation of MT data composed of the background medium of 100 Ω m and 2D structure of 1 Ω m (a) without and (b) with an off-plane structure of 1 Ω m. The strike direction of the 2D structure is the y-axis: (a) is a xz cross-section and (b) is a plan view. The red and blue lines indicate the fully-covered and partially-missing cases of receiver arrays, respectively.

For the ideal case of interpreting the complete 2D structure whose strike is along the y-axis with the general receiver array in the direction perpendicular to the strike, the MT response functions belonging to Groups 1 and 5 in Table 3.1 are zero in all the receivers. Because their sensitivity patterns are symmetric about the x-axis and the signs of both the symmetrical parts are reverse to each other. It means that responses of the MT response functions due to the structure extended along the positive y-axis are cancelled out with those due to the structure extended along the negative y-axis. The remaining Groups 2, 3, 4, and 6 differently respond to the 2D structure. To compare characteristics of the four groups, the modeling and inversion results of $\rho_{a,xy}$ (Group 2), $\rho_{a,yx}$ (Group 3), T_{xR} (Group 4), and Z_{effR} (Group 6), which are mainly used for 2D MT inversion, are analyzed based on their sensitivity patterns.

In Fig. 4.4, the responses of $\rho_{a, xy}$, $\rho_{a, yx}$, T_{xR} , and Z_{effR} at a frequency of 1 Hz for the general 2D model (Fig. 4.3a) (blue circles), the model with off-plane structure (Fig. 4.3b) (red triangles), and their homogeneous background model (black squares) are represented. For reference, the boundaries of the 2D structure and off-plane anomalous body are indicated by the black and red dotted lines, respectively.

First, the modeling results in the general 2D model (Fig. 4.3a) are compared with those in the homogeneous background medium. In Fig. 4.4(a), the responses of $\rho_{a,xy}$ have low values only above the 2D structure (i.e., between the black dotted lines) because the surface-sensitivity pattern is strong in the same direction as the strike. In contrast, the modeling results of $\rho_{a,yx}$ in Fig. 4.4(b) show low values over a relatively wider range along the direction of receiver array because $\rho_{a,yx}$ has the strong surface-sensitivity pattern in the direction perpendicular to the strike (i.e., parallel to the receiver array). In the modeling results of Z_{effR} represented in Fig. 4.4(d), the responses to the 2D structure are slightly wider than those of $\rho_{a,xy}$, but more consistent with the boundaries of the 2D structure than those of $\rho_{a,yx}$. These three MT response functions show noticeable differences between the modeled data for the homogeneous and general 2D models in the receivers above the 2D structure. On the other hand, the model responses of T_{xR} in Fig. 4.4(c) appear large in the receivers located apart from the 2D structure, because the surface-sensitivity pattern has large values not near but at some distance apart from the right above the perturbed point. Because of this feature of the surface-sensitivity pattern, T_{xR} can be effectively used in the first specific case that receivers right above the 2D structure cannot be used.

Next, in Fig. 4.4, the off-plane error for each MT response function is examined by comparing the modeling results in the model including the off-plane anomaly (Fig. 4.3b) and in the general 2D model (Fig. 4.3a). In Fig. 4.4(a), $\rho_{a,xy}$ is severely affected by the off-plane structure within the boundaries of the off-plane structure (i.e., between the red dotted lines), whereas $\rho_{a,yx}$, T_{xR} , and Z_{effR} are relatively less affected in Figs. 4.4(b), 4.4(c), and 4.4(d), respectively. Because the surfacesensitivity pattern of $\rho_{a,xy}$ is strong along the y-axis, the responses of $\rho_{a,xy}$ are most distorted by the off-plane structure existing away from the 2D plane along the y-axis.



Fig. 4.4. Modeling results of (a) xy-apparent resistivity ($\rho_{a, xy}$), (b) yx-apparent resistivity ($\rho_{a, yx}$), (c) real part of x-component of tipper (T_{xR}), and (d) real effective impedance (Z_{effR}) at 1 Hz for the general 2D model (Fig. 4.3a) with the blue circles, the model with off-plane structure (Fig. 4.3b) with the red triangles, and their homogeneous background model with the black squares. The black and red dotted lines represent boundaries of the 2D and off-plane structures in Fig. 4.3, respectively.

To investigate whether the sensitivity patterns of the four MT response functions ($\rho_{a, xy}$, $\rho_{a, yx}$, T_{xR} , and Z_{effR}) are properly reflected in inverted models, inversion is performed for the three cases mentioned above (Fig. 4.3), which are the ideal case, the first specific case related to the receiver array, and the second specific case for the off-plane error. The homogeneous model of 100 Ω m is assumed as an initial model, and a total of 9 frequencies sampled at a uniform interval on the logarithmic scale ranging from 1 to 100 Hz are considered. Fig. 4.5 shows inversion results of $\rho_{a, xy}$, $\rho_{a, yx}$, T_{xR} , and Z_{effR} with the boundaries of the 2D and offplane structures (Fig. 4.3) indicated by the white and red dotted lines, respectively.

In the inversion results obtained for the ideal case (i.e., for the general 2D model with the full receiver array) represented in the second column of Fig.4.5, the inverted model of $\rho_{a,xy}$ (TM mode in this case) matches well with the boundaries of the 2D structure, whereas low resistivity in the inversion result of $\rho_{a,yx}$ (TE mode in this case) extends beyond the 2D structure. Inversion result of T_{xR} retrieves the 2D structure well, but the resistivity of the background medium is slightly higher than the true value in the positive x-axis. The inverted model of Z_{effR} is similar to that of $\rho_{a,xy}$, but the low resistivity zone due to the 2D structure extends to the bottom of the true model.

The inverted models of $\rho_{a,xy}$, $\rho_{a,yx}$, and Z_{effR} for the specific case of receiver array where some receivers are missing right above the 2D structure in the first column of Fig. 4.5 are worse around the anomalous body than those for the full receiver array in the second column of Fig. 4.5. However, the inversion result of T_{xR} obtained with the partially-missing receiver array describes the 2-D structure well, which is comparable to that obtained with the fully-covered receiver array.

Finally, the inversion results of $\rho_{a,xy}$, $\rho_{a,yx}$, T_{xR} , and Z_{effR} obtained for the model with an off-plane anomalous body (the third column of Fig. 4.5) are compared with those obtained for the general 2D model (the second column of Fig. 4.5). Footprints of the off-plane structure are severe in the inversion results of $\rho_{a,xy}$ in the zone between the red dotted lines in Fig. 4.5. However, the inversion results of the other three MT response functions have smaller footprints. In particular, remarkable differences in the inversion results of T_{xR} are hardly observed.

In conclusion, the inversion results of the different MT response functions $(\rho_{a,xy}, \rho_{a,yx}, T_{xR}, \text{ and } Z_{effR})$ in Fig. 4.5 agree well with their sensitivity patterns and modeling results described above in some situations for 2D interpretation of MT data. From these results, it can be concluded that MT response functions with a strong surface-sensitivity pattern in a strike (i.e., TM mode MT response functions) are optimal for the ideal 2D MT inversion imaging complete 2D structures with sufficient data acquired in the direction transverse to the strike; it is helpful to use the tipper when the data cannot be used around the target structure; and it is a good strategy to use the tipper or effective impedance when uninterested anomalous bodies such as off-plane structures exist.


Fig. 4.5. Inversion results of the xy-apparent resistivity ($\rho_{a,xy}$), yx-apparent resistivity ($\rho_{a,yx}$), real part of x-component of tipper (T_{xR}), and real effective impedance (Z_{effR}) for the ideal case with the general 2D structure and receiver array (second column), the specific case with missing receivers right above the 2D structure (first column), and the specific case with the off-plane structure (third column) shown in Fig. 4.3. The white and red dotted lines indicate the boundaries of the 2D and off-plane structures in Fig. 4.3, respectively.

4.3. Synthetic examples for 3D interpretation of MT data

Some MT response functions do not show meaningful responses to 1D or 2D structures, while all the MT response functions dealt with in this thesis respond to 3D structures. Therefore, all the types of MT response functions can be considered as input data for 3D MT inversion. In this section, all the components of the impedance tensor (i.e., Z_{xx} , Z_{xy} , Z_{yx} , and Z_{yy}) belonging to Groups 1, 2, and 3 in Table 3.1 are mainly covered because they are usually used for 3D MT inversion; inversion results of the components of the phase tensor (i.e., Φ_{xx} , Φ_{yy} , Φ_{yx} , and Φ_{yy}) are discussed briefly; and the MT response functions belonging to Groups 4, 5, and 6 (i.e., tipper and effective impedance) are not considered because they were already handled in Chapters 4.1 and 4.2 and their features are similar in 3D cases.

Before describing examples, features of the components of the impedance tensor for 3D interpretation of MT data are demonstrated based on their sensitivity patterns. In Figs. 2.1 and 2.2, the surface-sensitivity patterns of diagonal (Z_{xx} and Z_{yy}) and off-diagonal (Z_{xy} and Z_{yx}) components do not overlap each other, which means that they will play different and complementary roles in 3D MT inversion. Thus, it would be commonly good to use all the components of the impedance tensor for 3D MT inversion. However, the components may have an adverse effect on the imaging of specific structures depending on their sensitivity patterns, as the diagonal impedance components cannot image 2D structures mentioned in Chapter 4.2. There is also a specific case that off-diagonal components may deteriorate inversion results. Fig. 4.6 represents surface-sensitivity patterns only for the larger values than 4/5 of the maximum absolute value of Z_{xxR} (Figs. 4.6a and 4.6c) and Z_{xyR} (Figs. 4.6b and 4.6d) at a frequency of 1 Hz when the perturbed position is $(0, 0, 1 \text{ km}) = (0, 0, 0.2\delta)$ (Figs. 4.6a and 4.6b) and $(0, 0, 3 \text{ km}) = (0, 0, 0.6\delta)$ (Figs. 4.6c and 4.6d). In Figs. 4.6(a) and 4.6(c), the sensitivities of Z_{xxR} due to the two different perturbation depths do not overlap each other, whereas the strongest responses of Z_{xyR} appear at the same central position in Figs. 4.6(b) and 4.6(d). Therefore, it is difficult to distinguish the responses due to structures at different depths using the off-diagonal components in a limited frequency range.



Fig. 4.6. Surface-sensitivity patterns of the values above 4/5 of the maximum absolute value for (a, c) Z_{xxR} and (b, d) Z_{xyR} on the same scale in Figs. 3.1 to 3.6 when the perturbed position is at (a, b) (0, 0, 1 km) = (0, 0, 0.2\delta) and (c, d) (0, 0, 3 km) = (0, 0, 0.6\delta).

To further examine the aforementioned characteristics of the impedance components, 3D MT inversion is performed for the synthetic model shown in Fig. 4.7. In the model, a cross-shaped anomalous body (1000 Ω m) composed of two long structures along the x- and y-axes exists in the shallow depth, and a cube-shaped anomalous body (1 Ω m) is located below the cross-shaped anomaly. A total of 81 receivers are located at a uniform interval of 1 km covering from -4 to 4 km in the x- and y-axes. A total of 13 frequencies sampled at a uniform interval on the logarithmic scale ranging from 0.1 to 100 Hz are considered. Inversion is conducted for three frequency ranges: high frequencies (10~100 Hz), low frequencies (0.1~1.778 Hz), and all frequencies (0.1~100 Hz). Inverted models are represented with the boundaries of the true structures.



Fig. 4.7. Synthetic model for 3D interpretation of MT data: (a) plan view and (b) xz cross-section. Resistivity values of the background medium (sky-blue), the two long anomalous bodies (blue), and the cube anomalous body (red) are 100, 1000, and 1 Ω m, respectively.

Fig. 4.8 shows the inversion results of the components of the impedance tensor for the high-frequency range (10~100 Hz). Because the high-frequency data include information of only the shallow structures in Fig. 4.7, the inversion models are constructed up to 1.5 km along the z-axis, and are displayed on the xy-plane crossing at z=0.75 km, which corresponds to the central depth of the cross-shaped structure. In Fig. 4.8(a), the inversion result of Z_{xx} is a cross shape, but it extends beyond the true boundaries. Fig. 4.8(b) shows that Z_{xy} recovers the long structure well along the y-axis, whereas do not properly reconstruct the true resistivity and boundaries of the long structure along the x-axis. In contrast to Z_{xy} , Z_{yx} dominantly inverts the long structure along the x-axis. The inverted model of Z_{yy} has similar tendencies to that of Z_{xx} . The inversion results of the components of the impedance tensor are in good agreement with the features of their sensitivity patterns described in Section 4.2.

Fig. 4.9 shows inversion results obtained with only the low frequencies (0.1~1.778 Hz). Because the low-frequency data contain both the responses of the shallow high-resistivity cross-shaped and lower conductive cube-shaped anomalous bodies in Fig. 4.7, the inversion models are constructed up to 4 km along the z-axis. In Fig. 4.9, the inversion results are represented at depths of 0.75 and 3 km where the two structures exist. In Fig. 4.9, the diagonal components of the impedance tensor (i.e., Z_{xx} and Z_{yy}) similarly invert the lower structures well, while the off-diagonal components of the impedance (i.e., Z_{xy} and Z_{yx}) do not recover both the structures. These inversion results are consistent with the features of the sensitivity patterns for each impedance component shown in Fig. 4.6.



Fig. 4.8. Inversion results of the components of the impedance tensor with high frequencies (10~100 Hz) for the model of Fig. 4.7 at

z=0.75 km. The white dotted rectangles represent the boundaries of 3D structures at the depth in Fig. 4.7.





In Fig. 4.10, the inversion results obtained using all, xx-, and xy-impedance components (i.e., Z_{all} , Z_{xx} , and Z_{xy}) with all frequencies (0.1~100 Hz) are presented. In addition, Fig. 4.10 shows inversion results obtained selectively using the off-diagonal components (i.e., Z_{xy} and Z_{yx}) for the high frequencies and the diagonal components (i.e., Z_{xx} and Z_{yy}) for the low frequencies, which is indicated by Z_{sel} . Inverted models of Z_{yx} and Z_{yy} are omitted in Fig. 4.10, because the inversion results of Z_{yx} are rotated versions of Z_{xy} , and the inverted model of Z_{yy} is similar to that of Z_{xx} . The updated models with only Z_{xx} and Z_{xy} have similar tendencies to those of the components shown in Figs. 4.8 and 4.9, respectively. Although the model inverted with all impedance components allows to infer the true subsurface structures to some extent, the adverse effects of both the diagonal and off-diagonal components represented in Figs. 4.8 and 4.9 are also observed. In other words, the diagonal components lead the high resistivity of the upper structure to spread beyond the actual boundaries, and the underestimation of the lower structure is attributed to the off-diagonal components. Compared with the inverted model of Z_{all} , the updated model denoted by Z_{sel} shows that the deviation of the upper cross-shaped structure is suppressed, and the resistivity values of the lower cube-shaped structure are closer to the true value.

Fig. 4.11 shows the inversion results obtained using all, yx-, and yy-components of the phase tensor, which are represented by Φ_{all} , Φ_{yx} , and Φ_{yy} , respectively, with all frequencies. In Figs. 4.10 and 4.11, the inverted models of Φ_{yx} and Z_{xx} , which belong to the same Group 1, similarly present that they recover the lower conductive structure well, but the upper cross-shaped structure is inverted slightly wider than the true model. The inversion results of Φ_{yy} and Z_{xy} belonging to the same Group 2 are analogous in that the upper long anomalous body along the y-axis is updated close to the true one, whereas the low resistivity of the lower cube-shaped structure is not well retrieved. Because the sensitivity patterns of Φ_{yy} and Z_{xy} are not exactly the same, there exist differences in the inverted models of the upper anomaly extended along the x-axis. The updated models for Φ_{all} and Z_{all} have different values of the resistivity, but show similar structures.

In summary, the different sensitivity patterns in Figs. 2.1, 2.2, and 4.6 imply that it is generally recommended to use all the impedance components together for 3D MT inversion, but Z_{xx} and Z_{yy} , which are suitable for imaging of vertically placed structures, may have disadvantages in imaging of structures close to 2D, and Z_{xy} and Z_{yx} , which are optimal for imaging of structures close to 2D in y- and x-axes, respectively, may adversely affect inversion of vertically existing structures. In Figs. 4.8 and 4.9, the pros and cons of the different sensitivity patterns for each component are represented in the inversion results for the model of Fig. 4.7, and Fig. 4.10 shows the selective use of the components according to the frequencies in consideration of their sensitivity patterns can improve the inversion results for the 3D model. Additionally, Figs. 4.10 and 4.11 demonstrate that the phase tensor can yield similar subsurface structures even with the halved number of data compared to the impedance tensor, because each component of the same group can give.

Finally, the synthetic examples of inversion for different MT exploration in Chapter 4 are summarized in Table 4.2. Table 4.2 can be used as a guideline for the selection of MT response functions in inversion based on their sensitivity patterns.



Fig. 4.10. Inversion results with the whole range of frequencies (0.1~100 Hz) for the model of Fig. 4.7: the cross-section at y=0 km (top) and the plan views at z=0.75 km (middle) and 3 km (bottom). Z_{all} , Z_{xx} , and Z_{yy} mean the inversion results obtained using all, xx-, and xy-components of the impedance tensor, respectively, and Z_{xel} indicates the inversion results obtained selectively using the impedance components. The white dotted rectangles represent the boundaries of 3D structures on the planes in Fig. 4.7.



Fig. 4.11. Inversion results with the whole range of frequencies (0.1~100 Hz) for the model of Fig. 4.7: the cross-section at y=0 km (top) and the plan views at z=0.75 km (middle) and 3 km (bottom). Φ_{all} , Φ_{yx} , and Φ_{yy} mean the inversion results obtained using all, yx-, and yy-components of the phase tensor, respectively. The white dotted rectangles represent the boundaries of 3D structures on the planes in Fig. 4.7.

Imaging 3D structure (receivers on the surface)	Sub-major (good for multi-structures vertically placed)	Major (optimal for nearly 2D structures with y-axis strike)	Major (optimal for nearly 2D structures with x-axis strike)	Sub-major	Sub-major	Not recommended
Imaging 2D structure (strike: x-axis, receiver line: y-axis)	N/A	Not recommended (TE)	Major (TM) (optimal for ideal 2D cases)	N/A	Sub-major (optimal for receivers with no data right above a target)	Sub-major
Imaging 2D structure (strike: y-axis, receiver line: x-axis)	N/A	Major (TM) (optimal for ideal 2D cases)	Not recommended (TE)	Sub-major (optimal for receivers with no data right above a target)	N/A	Sub-major
Imaging 1D structure (one receiver)	N/A	Available but 2 nd place (dimensionality error)	Available but 2 nd place (dimensionality error)	N/A	N/A	Optimal (dimensionality error)
MT response functions	$Z_{_{XYR}} \frac{ ext{Group I}}{ ext{Z}_{_{XYI}}} \frac{ ext{Z}_{_{YYR}}}{ ext{Z}_{_{YY}}} \Phi_{_{YX}}$	$Z_{xyt} \begin{array}{c} { m Group} \ 2 \\ Z_{xyt} \ Z_{xyt} \ ho_{a,xy} \\ arphi_{xy} \ \Phi_{yy} \end{array}$	$\begin{array}{c} \operatorname{Group} 3\\ Z_{\operatorname{jark}} Z_{\operatorname{jarl}} \rho_{a, \operatorname{jar}}\\ \varphi_{\operatorname{jar}} \Phi_{\operatorname{ar}} \end{array}$	Group 4 $T_{xR}^{}$	Group 5 $T_{_{\mathcal{Y}\!R}}$ $T_{_{\mathcal{Y}\!I}}$	${ m Group}~6$ $Z_{ m effr}~Z_{ m effr}$

Table. 4.2. Recommendation of input MT response functions in inversion for several MT exploration cases.

Chapter 5. Case study: Utah FORGE field data

5.1. Utah FORGE site and MT field data

The Utah Frontier Observatory for Research in Geothermal Energy (FORGE) is a field laboratory located about 30 km northeast of Milford, Utah. The production area of the Roosevelt Hot Springs geothermal system is located about 5 km east of the FORGE site. To explore the technical feasibility of characterizing, creating, and maintaining enhanced geothermal system (EGS) reservoirs, the U.S. Department of Energy (U.S. DOE) selected the Utah FORGE in 2018 (Moore et al. 2020; Wannamaker et al. 2021). For these purposes, a variety of geological and geophysical methods have been applied. As one of them, MT data were acquired around the Utah FORGE and Roosevelt Hot Springs geothermal system sites to clarify resistivity structures involving potential heat sources.

Wannamaker et al. (2020) performed 3D MT inversion using the newly measured FORGE MT data along with the existing Subsurface Science, Technology and Engineering Research, and Development (SubTER) and Cove Fort MT data measured in other projects. Fig. 5.1 modified from Figure 16 of Wannamaker et al. (2020) shows the cross-section of the 3D MT inversion model, and the white dotted lines indicate a lower resistivity body (about 100 Ω m). They described that the depth range of the lower resistivity body is appropriate for the possibly cooled magma storage zone, and the lower resistivity body may represent the fracture zone of residual hot fluids probably related to the Roosevelt Hot Springs. The black lines in Fig. 5.1 are used later in comparison with inversion results in Chapter 5.3.



Fig. 5.1. The cross-section of the 3D MT inversion model. A clockwise rotation of 20 degrees is required to align with true north. This figure is modified from Figure 16 of Wannamaker et al. (2020) by adding the white dotted lines and black lines.

Among the three MT data sets used in Wannamaker et al. (2020), publicly accessible FORGE (https://gdr.openei.org/submissions/1255, Energy and Geoscience Institute at the University of Utah. 2020. Utah FORGE: Phase 3 Magnetotelluric Data [data set]. Retrieved from https://dx.doi.org/10.15121/ 1776598) and SubTER (https://gdr.openei.org/submissions/1331, Energy and Geoscience Institute at the University of Utah. 2021. SubTER Final Magnetotelluric Data: Mineral Mountains, Retrieved Utah [data set]. from https://dx.doi.org/10.15121/1822377) MT data can be downloaded in the website for the U.S. DOE Geothermal Data Repository (GDR). In Fig. 5.2, the MT survey area including both the FORGE and SubTER data in the Utah is represented using the Google Earth Pro. Fig. 5.3 shows the locations of the FORGE and SubTER MT data with the MT survey area, origin, axes, and surface-range of the aforementioned lower resistivity body in the cross-section of Fig. 5.1 (indicated by the white line) on a map from the Google Earth Pro.

The total number of MT data is 181, and the majority of the data are concentrated around the origin. The MT data are rotated according to the coordinates in Fig. 5.3, and post-processing for data is performed in the frequency domain. The total number of frequencies is 48, and the frequency range is from 0.0122 to 230.47 Hz. To comparably image the main structures of Fig. 5.1 using less MT data than the data used by Wannamaker et al. (2020), it is necessary to optimally select input MT response functions for inversion in consideration of the characteristics of the sensitivity patterns and the surroundings of the MT survey. Because many receivers are distributed three-dimensionally away from the white line in the negative y-axis direction near the center in Fig. 5.3, it can be expected that the MT response function

of T_y whose surface-sensitivity patterns are wide and strong in the y-axis will make a great contribution to the imaging of the target structure. In this thesis, inversion results are compared according to the four strategies of selecting MT response functions of Z_{xy} and Z_{yx} ; Z_{xy} , Z_{yx} , and T_y ; Z_{xy} , Z_{yx} , and T_x ; and Z_{xx} , Z_{xy} , Z_{yx} , and Z_{yy} , as input data types. They are denoted as $(Z_{xy}, Z_{yx}), (Z_{xy}, Z_{yx}, Z_{yx}, T_y), (Z_{xy}, Z_{yx}, T_x)$, and $(Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy})$, respectively.



Fig. 5.2. MT survey area for the FORGE and SubTER MT data in a Utah map from the Google Earth Pro.



Fig. 5.3. The survey area and locations of the FORGE and SubTER MT data on a map from the Google Earth Pro. The origin is (38°30'12.96"N, 112°53'47.80"W) and the axes are rotated eastward by 20 degrees in reference to the North. The surface-range of the target lower resistivity body in the cross-section of Fig. 5.1 is indicated by the white line.

5.2. Settings of 3D MT inversion for the field data

A modeling mesh of the Utah area is constructed by the tetrahedral elements using the 'Gmsh' mesh generator (Geuzaine & Remacle 2009). The modeling mesh should be finely divided near the locations of the receivers, and reflect the elevations of the Utah area. In Fig. 5.4, the procedures for creating the modeling mesh are briefly represented. The modeling mesh consists of a total of 901,557 tetrahedral elements. Fig. 5.5(a) shows that the modeling mesh has a small mesh size near the receivers. In Fig. 5.5(b), the modeling mesh considering the heights at the surface correctly depicts the topography of Fig. 5.3.

An inversion mesh is composed of mixed elements: hexahedron, tetrahedron, and prism. In Fig. 5.6, the region of interest inside the $(-15\sim15 \text{ km}) \times (-20\sim20 \text{ km}) \times$ $(-4\sim20 \text{ km})$ box (indicated by red lines) is discretized into small 9, 720 (20 x 27 x 18) hexahedrons, whereas the remaining region is filled with large 2,136 tetrahedrons and 3,096 prisms for computational efficiency. The inversion mesh consists of a total of 14,952 elements.

For 3D MT inversion of the field data, a homogeneous model of 100 Ω m is used as an initial model, and other settings are demonstrated in Table 5.1.

- Receivers (Lat, Lon, 0) → (x, y, 0)
 <u>geo file 1</u>: nodes for surface boundary (±300, ±300, 0) and receivers
- Gmsh, geo file 1 → msh file 1: surface mesh composed of small triangular elements near the receivers
- 3) <u>Msh file 1</u> nodes (x, y, 0) → (Lat, Lon, 0)
 → GPS Visualizer (Schneider, 2019), (Lat, Lon, z) → (x, y, z)
 <u>msh file 2</u>: msh file 1 + consideration of elevations
- 4) Receivers (x, y, 0) \rightarrow (x, y, z) interpolated by <u>msh file 2</u>
- 5) Gmsh, nodes for surface lines (±300, ±300, z) and outer boundary (±300, ±300, -90 or 300) with receivers (x, y, z)
 → msh file 3: outer boundary mesh
- 6) Gmsh, msh file 2 + msh file 3 \rightarrow msh file 4: 3D MT modeling mesh with tetrahedral elements
- Fig. 5.4. Simple workflow of constructing the modeling mesh for the field data.



Fig. 5.5. Modeling mesh at the surface of the survey area (a) with and (b) without edges.



Fig. 5.6. Inversion mesh composed of hexahedral, tetrahedral, and prism elements. The red lines represent the region of interest in 3D MT inversion.

3D MT modeling with edge-based FEM					
Modeling mesh (Chapter A.2)	Tetrahedral elements (Figs. 5.4 and 5.5)				
3D MT inversion					
Objective function (Chapter 2.2.1)	Creeping method in equation (2-63)				
Jacobian calculation (Chapter 2.2.2)	Sensitivity equation approach in equation (2-73)				
Blocky parameterization (Chapter 2.2.3.1)	Hexahedral, tetrahedral, and prism inversion mesh (Fig. 5.6)				
Model parameterization (Chapter 2.2.3.2)	Equations (2-77) and (2-84)				
Data weighting matrix (Chapter 2.2.3.3)	Equations (2-90), (2-91), and (2-92)				
Roughness matrix (Chapter 2.2.3.4)	Equation (2-97)				
Lagrange multiplier (Chapter 2.2.3.5)	Equations (2-98), (2-100), and (2-102)				
Line search (Chapter 2.2.3.6)	Ο				

Table. 5.1. Settings of 3D MT modeling and inversion for the field data.

5.3. 3D MT inversion results for the field data

In this section, inversion results of the MT field data for the Utah FORGE site obtained by the aforementioned strategies of selecting MT response functions as input data types denoted by $(Z_{xy}, Z_{yx}), (Z_{xy}, Z_{yx}, T_y), (Z_{xy}, Z_{yx}, T_x)$, and $(Z_{xx}, T_y), (Z_{xy}, Z_{yx}, T_y)$ Z_{xy} , Z_{yx} , Z_{yy}) are represented. Fig. 5.7 shows the updated models of the yz crosssection at x=0 km obtained from the four strategies with the ParaView (Ahrens et al. 2005). In Fig. 5.7, the white dotted lines represent the boundaries of the target body with lower resistivity (marked with the white dotted lines in Fig. 5.1), whereas the black lines indicate the overlapping part with the cross-section of Wannamaker et al. (2020) (marked with the black lines in Fig. 5.1). In Fig. 5.7(a), the strategy of (Z_{xy} , $Z_{\rm vx}$) images the low resistivity structure near the surface in the negative y-axis and the underlying high resistivity structure similar to Fig. 5.1, but the inversion result in the white dotted rectangle does not show low resistivity body. The inversion result for the selection strategy of (Z_{xy}, Z_{yx}, T_y) shown in Fig. 5.7(b) reconstructs the subsurface structures including the target body comparable to those of Fig. 5.1. Comparing the inversion results of Figs. 5.7(a) and 5.7(c) with each other, the T_x data are added, but the recovered structures for the strategy of (Z_{xy}, Z_{yx}, T_x) are not significantly different from those for the (Z_{xy}, Z_{yx}) strategy. In Fig. 5.7(d), the model inverted by the strategy of $(Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy})$ shows the low resistivity body near the target structure, but it exceeds the boundaries of the target structure in the y-axis. Consequently, as expected in Chapter 5.1, the strategy of (Z_{xy}, Z_{yx}, T_y) containing T_{y} data visualizes the subsurface structures including the target body most similar to the result of Wannamaker et al. (2020) inverted by more MT data.



Fig. 5.7. Inversion results of the yz cross-section at x=0 km for the four strategies of selecting MT response functions as input data types denoted by (a) (Z_{xy}, Z_{yx}) , (b) (Z_{xy}, Z_{yx}, T_y) , (c) (Z_{xy}, Z_{yx}, T_x) , and (d) $(Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy})$. The white dotted and black lines represent the boundaries of the lower resistivity target body and overlapping part in the cross-section of Fig. 5.1, respectively.



Fig. 5.7. (Continued)

Fig. 5.8 shows the data misfits of the objective functions over the iteration numbers in the inverse procedures for the four strategies of selecting MT response functions. The total number of iterations for the strategy of (Z_{xy}, Z_{yx}) is 5, whereas that for the other strategies is 7. In the inverse problems of the four strategies, the data misfits decrease and finally converge to the data misfits smaller than 1% of the initial values. In other words, all four inversion processes are reasonable and adequately reflect their observed data.



Fig. 5.8. Data misfits over the iteration numbers in the inverse procedures for the four strategies of selecting MT response functions as input data types denoted by $(Z_{xy}, Z_{yx}), (Z_{xy}, Z_{yx}, T_y), (Z_{xy}, Z_{yx}, T_x), \text{ and } (Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy}).$

In Fig. 5.9, the ratios of root-mean-square (RMS) errors for the inverted models of the (Z_{xy}, Z_{yx}) and (Z_{xy}, Z_{yx}, T_y) strategies are presented according to the receiver locations. Receiver positions with the ratio greater than 1 (indicated in blue) mean that the RMS errors for the strategy of (Z_{xy}, Z_{yx}, T_y) are smaller than those for the strategy of (Z_{xy}, Z_{yx}) , whereas the red points with a ratio less than 1 mean that the RMS errors for the strategy of (Z_{xy}, Z_{yx}) are smaller than those for the strategy of (Z_{xy}, Z_{yx}, T_y) . Fig. 5.9(a) shows the ratios of RMS errors for the MT response functions of Z_{xy} and Z_{yx} are close to 1 in most receivers (i.e., the RMS errors for the MT response functions of Z_{xy} and Z_{yx} in the two inverted models are similar). In Fig. 5.9(b), many receivers have a dark blue color on the map of the ratios of RMS errors for the MT response function of T_y , and it implies the responses of T_y for the inverted model of the (Z_{xy}, Z_{yx}, T_y) strategy are in better agreement with the observed data.

Fig. 5.10 shows the observed data and MT responses obtained from the two inverted models for the (Z_{xy}, Z_{yx}) and (Z_{xy}, Z_{yx}, T_y) strategies in the receiver located at (-0.76, 0.75, -1.68 km) away from the target structure (Fig. 5.3) in the negative y-axis. For the MT response functions of $\rho_{a, xy}$, $\rho_{a, yx}$, φ_{xy} , and φ_{yx} (i.e., Z_{xy} and Z_{yx}), both strategies result in the updated models with the MT modeling results close to the observed data. On the other hand, the inverted model by the strategy of (Z_{xy}, Z_{yx}, T_y) reproduces the responses of T_y more similar to the observed data than that for the strategy of (Z_{xy}, Z_{yx}) . Like Fig. 5.10, the MT responses in the receiver located at (1.69, 17.2, -2.24 km) away from the lower resistivity body (Fig. 5.3) in the positive y-axis are represented in Fig. 5.11. Compared to the results in Fig. 5.10, the responses of $\rho_{a, xy}$, $\rho_{a, yx}$, φ_{xy} , and φ_{yx} have similar tendencies, and the responses of T_y show more distinct differences according to the two strategies in Fig. 5.11.

In summary, Fig. 5.7 demonstrates the strategy of selecting MT response functions as input data types denoted by (Z_{xy}, Z_{yx}, T_y) restores the subsurface structures including the main lower resistivity zone most comparable to the inverted model in Fig. 5.1. Figs. 5.9, 5.10, and 5.11 show that the different inversion results are attributed to the MT response function of T_y . These results are consistent with the presumption based on the sensitivity patterns, which is described at the end of Chapter 5.1. As in the case study explained in Chapter 5, inversion results in various MT case studies will be improved because optimal MT response functions can be selected by considering the sensitivity patterns and the environment of MT exploration (distribution of receivers, location of main structures, frequencies, etc.).



Fig. 5.9. Maps of the ratios of RMS errors between the models inverted by the strategies of selecting MT response functions as input data types denoted by (Z_{xy}, Z_{yx}) and (Z_{xy}, Z_{yx}, T_y) for the MT response functions of (a) Z_{xy} and Z_{yx} and (b) T_y .



Fig. 5.10. MT responses of $\rho_{a, xy}$, $\rho_{a, yx}$, φ_{xy} , φ_{yx} , T_{yR} , and T_{yI} at the receiver located at (-0.76, 0.75, -1.68 km) obtained from the observed data (black circles) and the inverted models for the strategies of selecting MT response functions as input data types denoted by (Z_{xy}, Z_{yx}) (red circles) and (Z_{xy}, Z_{yx}, T_y) (blue circles).



Fig. 5.11. MT responses of $\rho_{a, xy}$, $\rho_{a, yx}$, φ_{xy} , φ_{yx} , T_{yR} , and T_{yI} at the receiver located at (1.69, 17.2, -2.24 km) obtained from the observed data (black circles) and the inverted models for the strategies of selecting MT response functions as input data types denoted by (Z_{xy}, Z_{yx}) (red circles) and (Z_{xy}, Z_{yx}, T_y) (blue circles).

Chapter 6. Conclusions

The 3D sensitivity patterns and surface-sensitivity patterns were investigated to describe the characteristics of the 22 major MT response functions: 8 components of the impedance tensor (Z_{xxR} , Z_{xxI} , Z_{xyR} , Z_{xyI} , Z_{yxR} , Z_{yxI} , Z_{yyR} , and Z_{yyI}); 2 components of the apparent resistivity ($\rho_{a, xy}$ and $\rho_{a, yx}$); 2 components of the phase (φ_{xy} and φ_{yx}); 4 components of the tipper (T_{xR} , T_{xI} , T_{yR} , and T_{yI}); 2 components of the effective impedance (Z_{effR} and Z_{effI}); and 4 components of the phase tensor (Φ_{xx} , Φ_{xy} , Φ_{yx} , and Φ_{yy}) for inversion.

First, the various MT response functions were classified into 6 groups according to their surface-sensitivity patterns so that MT response functions with similar roles in the inverse process belong to the same group. Group 1 contains Z_{xxR} , Z_{xxl} , Z_{yyR} , Z_{yyl} , Φ_{xy} , and Φ_{yx} that have the diagonal four petals-shaped surfacesensitivity patterns. Because their surface-sensitivity patterns are strong in all four quadrants, the MT response functions of Group 1 are useful when receivers are evenly installed on the surface to image 3D structures. Z_{xyR} , Z_{xyl} , $\rho_{a, xy}$, φ_{xy} , and Φ_{yy} with the linear surface-sensitivity patterns along the y-axis belong to Group 2, and the MT response functions of Z_{yxR} , Z_{yxl} , $\rho_{a, yx}$, φ_{yx} , and Φ_{xx} whose surface-sensitivity patterns are linear in the x-axis are classified into Group 3. The intensive surface-sensitivity patterns along the y- and x-axes indicate that the MT response functions of Groups 2 and 3 contribute significantly to inversion of nearly 2D structures extending along the y- and x-axes, respectively. The x- and ycomponents of the tipper possess the linear two petals-shaped surface-sensitivity patterns along the x- and y-axes, and belong to Groups 4 and 5, respectively. Because the part with strong surface-sensitivity patterns of the tipper appears widely along one axis, the tipper can be helpful for retrieving both 2D and 3D structures. The surface-sensitivity patterns for the effective impedance (Group 6) appear as the small circle. The characteristic of surface-sensitivity patterns, which appear strongly only around the center, makes the effective impedance an advantage in interpretation of 1D strucutres.

Next, the synthetic examples for 1D, 2D, and 3D interpretation of MT data were represented to show how the sensitivity patterns are reflected in observed data and inversion results. The synthetic examples were summarized in one table that recommends appropriate input MT response functions for inversion according to the environment of MT exploration. The examples for 1D interpretation of MT data showed that Groups 2, 3, and 6 can be applied for 1D inversion, and the effective impedance is an optimal MT response function for the cases with an anomalous body causing the dimensionality error. In the examples for 2D interpretation of MT data, Groups 2, 3, 4, and 6 respond to 2D structures with the strike along the y-axis. If geological structures are nearly 2D and the sufficient data can be obtained crossing the main structure, the MT response functions having strong surface-sensitivity patterns along the strike direction (i.e., TM mode) are recommended. The tipper is of great help in the cases where data acquired right above the target structure cannot be used. The tipper and effective impedance are good substitutes for the TM mode MT response functions when the off-plane structures exist. In the examples for 3D interpretation of MT data, all data types of MT response functions can be applied to inversion, but the features of the components of the impedance tensor that are mainly used in 3D inversion were mostly described. It is generally helpful to use all the impedance components together because they have different sensitivity patterns. However, in imaging of some specific structures, selectively using the impedance components can be a better strategy than using all the components, and such specific structures can be guessed through the sensitivity patterns. As previously mentioned, the responses to structures close to 2D are mainly contained in off-diagonal components having strong sensitivity patterns in the same direction as the strike, while responses to nearly 2D structures hardly appear in the diagonal components. In the case of vertically existing structures, the responses to upper and lower structures at limited frequencies are recorded separately in different receivers for the diagonal components, whereas they are superposed on the same receiver for the offdiagonal components. Corresponding to the features of the sensitivity pattern for each component, the 3D models inverted by the diagonal impedance components reflected the vertically existing structures well but did not properly represent the structures close to 2D, whereas the inversion results of the off-diagonal impedance components had opposite tendencies. The synthetic examples for 3D inversion also showed that inversion results may be improved by selectively using the impedance components according to frequencies in imaging of structures close to 2D or two structures separated in the vertical direction. Additionally, it was demonstrated that the phase tensor can yield similar subsurface structures to those of the impedance tensor even with a smaller number of data.

Finally, the sensitivity patterns for MT response functions were applied to inversion of real 3D MT field data. The field data were acquired near the Utah FORGE and Roosevelt Hot Springs geothermal system sites. Most of the field data are located widely away from the target structure in the y-axis direction. Considering this MT exploration situations, it could be inferred that many receivers contain the responses to the target structure in the data type of MT response function of T_y whose surface-sensitivity pattern is spread in the shape of two petals in the y-axis direction. As expected, the model most compatible to the inversion result of Wannamaker et al. (2020) imaged from more MT field data was inverted when T_y is adopted with Z_{xy} and Z_{yx} . Furthermore, some error analyses for the inverted models demonstrated that the MT response function of T_y has a large contribution to imaging the target body with low resistivity.

Consequently, the sensitivity patterns presented in this study give insight into the characteristics of the MT response functions. The analyzed sensitivity patterns of the MT response functions can be used in various MT studies such as postprocessing of data and survey design. In this thesis, the synthetic examples for several specific situations and the field data example at the Utah FORGE site describe approximate structures suitable for inversion according to the sensitivity patterns of the MT response functions and show how to apply their sensitivity patterns to MT inversion. Therefore, this thesis can be used as a guideline for selecting optimal input data types of MT response functions in different case studies for MT inversion.

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Appendix A. Tetrahedral and hexahedral elements

In this appendix, information on tetrahedral and hexahedral elements used for 3D MT modeling with the edge-based FEM is described. Because the tetrahedral or hexahedral elements have various shapes and sizes, their basic elements are required to explain in general. The coordinate system used in the basic element is called the local coordinate system, whereas the global coordinate system is for the elements of the entire 3D model. In Chapter A.1, the contents for the tetrahedral element: the basic element; the transformation between local and global coordinates; the edge shape function; the curl of edge shape function; the elementary stiffness matrix; and the elementary mass matrix are represented. In Chapter A.2, the same contents for the hexahedral element are described.

A.1. Tetrahedral elements

For the basic tetrahedral element in the FEM, the local coordinate system, number of nodes, edge directions, and number of edges are shown in Fig. A.1. To handle all tetrahedral elements in general, a volume coordinate system can be used instead of considering a tetrahedron of a specific shape and size. For example, let four nodes of a tetrahedron (nodes 1, 2, 3, and 4) and an arbitrary point P inside the tetrahedron be (x_1, y_1, z_1) , (x_2, y_2, z_2) , (x_3, y_3, z_3) , (x_4, y_4, z_4) , and (x, y, z) in the global coordinate system, respectively. The volume coordinates of the point P $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ are defined as follows:

$$\begin{split} \lambda_{1} &= \frac{V(\text{point P, node 2, node 3, node 4})}{V(\text{node 1, node 2, node 3, node 4})} \\ \lambda_{2} &= \frac{V(\text{node 1, point P, node 3, node 4})}{V(\text{node 1, node 2, node 3, node 4})} \\ \lambda_{3} &= \frac{V(\text{node 1, node 2, point P, node 4})}{V(\text{node 1, node 2, node 3, node 4})} \\ \lambda_{4} &= \frac{V(\text{node 1, node 2, node 3, node 4})}{V(\text{node 1, node 2, node 3, node 4})} \end{split}$$
(A-1)

where V(a, b, c, d) is a volume of the tetrahedron composed of four points, a, b, c, and d. From equation (A-1), $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$.



Fig. A.1. Local coordinate system, number of nodes, edge directions, and number of edges for the basic tetrahedral element in the FEM.

The transformations between the global coordinates (x, y, z) and the local coordinates $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ (i.e., the volume coordinates) of the arbitrary point P are defined as follows (Jin 2002):

$$\begin{pmatrix} \lambda_{1} \\ \lambda_{2} \\ \lambda_{3} \\ \lambda_{4} \end{pmatrix} = \frac{1}{6V} \begin{pmatrix} a_{1} & b_{1} & c_{1} & d_{1} \\ a_{2} & b_{2} & c_{2} & d_{2} \\ a_{3} & b_{3} & c_{3} & d_{3} \\ a_{4} & b_{4} & c_{4} & d_{4} \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} ,$$
 (A-2)

$$\begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} ,$$
 (A-3)

where

$$\begin{aligned} a_{1} &= \begin{vmatrix} x_{2} & x_{3} & x_{4} \\ y_{2} & y_{3} & y_{4} \\ z_{2} & z_{3} & z_{4} \end{vmatrix}; b_{1} &= -\begin{vmatrix} x_{1} & x_{3} & x_{4} \\ y_{1} & y_{3} & y_{4} \\ z_{1} & z_{3} & z_{4} \end{vmatrix}; c_{1} &= \begin{vmatrix} x_{1} & x_{2} & x_{4} \\ y_{1} & y_{2} & y_{4} \\ z_{1} & z_{2} & z_{4} \end{vmatrix}; d_{1} &= -\begin{vmatrix} x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \\ z_{1} & z_{2} & z_{3} \end{vmatrix}; \\ a_{2} &= -\begin{vmatrix} 1 & 1 & 1 \\ y_{2} & y_{3} & y_{4} \\ z_{2} & z_{3} & z_{4} \end{vmatrix}; b_{2} &= \begin{vmatrix} 1 & 1 & 1 \\ y_{1} & y_{3} & y_{4} \\ z_{1} & z_{3} & z_{4} \end{vmatrix}; c_{2} &= -\begin{vmatrix} 1 & 1 & 1 \\ y_{1} & y_{2} & y_{4} \\ z_{1} & z_{2} & z_{4} \end{vmatrix}; d_{2} &= \begin{vmatrix} 1 & 1 & 1 \\ y_{1} & y_{2} & y_{3} \\ z_{1} & z_{2} & z_{3} \end{vmatrix}; \\ a_{3} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{2} & x_{3} & x_{4} \\ z_{2} & z_{3} & z_{4} \end{vmatrix}; b_{3} &= -\begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{3} & x_{4} \\ z_{1} & z_{3} & z_{4} \end{vmatrix}; c_{3} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{4} \\ z_{1} & z_{2} & z_{4} \end{vmatrix}; d_{3} &= -\begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ z_{1} & z_{2} & z_{3} \end{vmatrix}; \\ a_{4} &= -\begin{vmatrix} 1 & 1 & 1 \\ x_{2} & x_{3} & x_{4} \\ y_{2} & y_{3} & y_{4} \end{vmatrix}; b_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{3} & x_{4} \\ y_{1} & y_{3} & y_{4} \end{vmatrix}; c_{4} &= -\begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{4} \\ y_{1} & y_{2} & y_{4} \end{vmatrix}; d_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= -\begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{4} \\ y_{1} & y_{2} & y_{4} \end{vmatrix}; d_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{3} \\ y_{1} & y_{2} & y_{3} \end{vmatrix}; \\ a_{4} &= \begin{vmatrix} 1$$

$$V = \text{ volume of tetrahedron} = \frac{1}{6} \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & y_4 \end{vmatrix}$$

For the tetrahedral elements, the edge shape functions and the curl of the edge shape functions are defined as follows using the volume coordinates (Jin 2002):

$$\boldsymbol{\Phi}_{i} = \left(\lambda_{N1(i)} \nabla \lambda_{N2(i)} - \lambda_{N2(i)} \nabla \lambda_{N1(i)}\right) l_{i}; \ i = 1 \sim 6 \quad , \tag{A-4}$$

$$\nabla \times \mathbf{\Phi}_i = 2l_i \nabla \lambda_{N1(i)} \times \nabla \lambda_{N2(i)}; \ i = 1 \sim 6 \quad , \tag{A-5}$$

where l_i is the length of the *i*-th edge. N1(i) and N2(i) are the start and end nodes of the *i*-th edge along its direction in Fig. A.1, respectively. For example, N1(1)=1 and N2(1)=2 for the edge number 1.

For arbitrary tetrahedral elements, the elementary stiffness and mass matrices have analytic solutions (Jin 2002). The components of the elementary stiffness matrix for the tetrahedral element are defined as follows:

$$K_{ij} = \frac{4l_i l_j V}{\left(6V\right)^4} \left[\mathbf{v}_i \cdot \mathbf{v}_j \right]; \ i, j = 1 \sim 6 \quad , \tag{A-6}$$

where

$$\mathbf{v}_{i} = \left(c_{N1(i)}d_{N2(i)} - d_{N1(i)}c_{N2(i)}, \ d_{N1(i)}b_{N2(i)} - b_{N1(i)}d_{N2(i)}, \ b_{N1(i)}c_{N2(i)} - c_{N1(i)}b_{N2(i)}\right).$$

The components of the elementary mass matrix for the tetrahedral element are defined as follows:

$$\begin{split} \mathcal{M}_{11} &= \frac{(l_1)^2}{360V} \left(f_{22} - f_{12} + f_{11} \right) , \\ \mathcal{M}_{12} &= \frac{l_1 l_2}{720V} \left(2f_{23} - f_{21} - f_{13} + f_{11} \right) , \\ \mathcal{M}_{13} &= \frac{l_1 l_3}{720V} \left(2f_{24} - f_{21} - f_{14} + f_{11} \right) , \\ \mathcal{M}_{14} &= \frac{l_1 l_4}{720V} \left(f_{23} - f_{22} - 2f_{13} + f_{12} \right) , \\ \mathcal{M}_{15} &= \frac{l_1 l_5}{720V} \left(f_{22} - f_{24} - f_{12} + 2f_{14} \right) , \\ \mathcal{M}_{16} &= \frac{l_1 l_6}{720V} \left(f_{24} - f_{23} - f_{14} + f_{13} \right) , \\ \mathcal{M}_{22} &= \frac{(l_2)^2}{360V} \left(f_{33} - f_{13} + f_{11} \right) , \\ \mathcal{M}_{23} &= \frac{l_2 l_3}{720V} \left(2f_{34} - f_{13} - f_{14} + f_{11} \right) , \\ \mathcal{M}_{24} &= \frac{l_2 l_4}{720V} \left(f_{33} - f_{23} - f_{13} + 2f_{12} \right) , \\ \mathcal{M}_{25} &= \frac{l_2 l_5}{720V} \left(f_{13} - f_{33} - 2f_{14} + f_{34} \right) , \\ \mathcal{M}_{26} &= \frac{l_2 l_6}{720V} \left(f_{13} - f_{33} - 2f_{14} + f_{34} \right) , \\ \mathcal{M}_{33} &= \frac{(l_3)^2}{360V} \left(f_{44} - f_{14} + f_{11} \right) , \\ \mathcal{M}_{35} &= \frac{l_3 l_4}{720V} \left(f_{34} - f_{24} - f_{13} + f_{12} \right) , \\ \mathcal{M}_{35} &= \frac{l_3 l_5}{720V} \left(f_{24} - f_{44} - 2f_{12} + f_{14} \right) , \\ \mathcal{M}_{36} &= \frac{l_3 l_6}{720V} \left(f_{43} - f_{14} + 2f_{13} \right) , \\ \mathcal{M}_{44} &= \frac{(l_4)^2}{360V} \left(f_{33} - f_{23} + f_{22} \right) , \\ \mathcal{M}_{44} &= \frac{(l_4)^2}{360V} \left(f_{33} - f_{23} + f_{22} \right) , \\ \mathcal{M}_{44} &= \frac{(l_4)^2}{360V} \left(f_{33} - f_{23} + f_{22} \right) , \\ \end{array}$$

$$\begin{split} M_{45} &= \frac{l_4 l_5}{720V} \left(f_{23} - 2f_{34} - f_{22} + f_{24} \right) , \\ M_{46} &= \frac{l_4 l_6}{720V} \left(f_{34} - f_{33} - 2f_{24} + f_{23} \right) , \\ M_{55} &= \frac{\left(l_5 \right)^2}{360V} \left(f_{22} - f_{24} + f_{44} \right) , \\ M_{56} &= \frac{l_5 l_6}{720V} \left(f_{24} - 2f_{23} - f_{44} + f_{34} \right) , \\ M_{66} &= \frac{\left(l_6 \right)^2}{360V} \left(f_{44} - f_{34} + f_{33} \right) , \end{split}$$

where $f_{ij} = b_i b_j + c_i c_j + d_i d_j$ $(i, j = 1 \sim 4)$. Because the elementary mass matrix is symmetric, $M_{ij} = M_{ji}$ where $i, j = 1 \sim 6$.

A.2. Hexahedral elements

For the basic hexahedral element in the FEM, the local coordinate system, number of nodes, edge directions, and number of edges are shown in Fig. A.2. The basic hexahedral element in this thesis is a regular hexahedron composed of eight nodes, which are (-1 or 1, -1 or 1, -1 or 1). In the local coordinate system, the origin is the center of the basic hexahedral element and the local coordinates are based on the ξ , η , and ζ axes.



Fig. A.2. Local coordinate system, number of nodes, edge directions, and number of edges for the basic hexahedral element in the FEM.

Let the eight nodes of the structured hexahedral element used in this study be $(x_1 \text{ or } x_2, y_1 \text{ or } y_2, z_1 \text{ or } z_2)$ in the global coordinate system. The transformations between the global coordinates (x, y, z) and the local coordinates (ξ, η, ζ) of an arbitrary point P inside the structured hexahedron are defined as follows:

$$\xi = 2 \left(\frac{x - x_1}{x_2 - x_1} \right) - 1$$

$$\eta = 2 \left(\frac{y - y_1}{y_2 - y_1} \right) - 1 , \qquad (A-8)$$

$$\zeta = 2 \left(\frac{z - z_1}{z_2 - z_1} \right) - 1$$

$$x = \frac{(x_2 - x_1)(\xi + 1)}{2} + x_1$$

$$y = \frac{(y_2 - y_1)(\eta + 1)}{2} + y_1 \quad .$$
 (A-9)

$$z = \frac{(z_2 - z_1)(\zeta + 1)}{2} + z_1$$

For the hexahedral elements, the edge shape functions and the curl of the edge shape functions are defined as follows (Miyata 2006):

$$\mathbf{\Phi}_{i} = \frac{l_{i}}{8} (1 + \xi_{i} \xi) (1 + \eta_{i} \eta) (1 + \zeta_{i} \zeta) \mathbf{G}_{3D} \begin{bmatrix} \delta_{\xi}^{i} \\ \delta_{\eta}^{i} \\ \delta_{\zeta}^{i} \end{bmatrix}; \ i = 1 \sim 12 \quad , \quad (A-10)$$

$$\nabla \times \mathbf{\Phi}_{i} = l_{i} \cdot \frac{\mathbf{J}_{3D}}{8 \cdot \det(\mathbf{J}_{3D})} \begin{bmatrix} (1 + \xi_{i}\xi) (\eta_{i}\delta_{\zeta}^{i} - \zeta_{i}\delta_{\eta}^{i}) \\ (1 + \eta_{i}\eta) (\zeta_{i}\delta_{\xi}^{i} - \xi_{i}\delta_{\zeta}^{i}) \\ (1 + \zeta_{i}\zeta) (\xi_{i}\delta_{\eta}^{i} - \eta_{i}\delta_{\xi}^{i}) \end{bmatrix}; \ i = 1 \sim 12 \quad , \quad (A-11)$$

where
$$\mathbf{G}_{3D} = \begin{pmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} & \frac{\partial \zeta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} & \frac{\partial \zeta}{\partial y} \\ \frac{\partial \xi}{\partial z} & \frac{\partial \eta}{\partial z} & \frac{\partial \zeta}{\partial z} \end{pmatrix} = (\mathbf{J}_{3D}^{-1})^{\mathrm{T}}; \ \mathbf{J}_{3D} = \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial \zeta}{\partial \zeta} \end{pmatrix};$$

 $\delta_{\xi}^{i} = |\eta_{i}\zeta_{i}|; \ \delta_{\eta}^{i} = |\zeta_{i}\xi_{i}|; \text{ and } \ \delta_{\zeta}^{i} = |\xi_{i}\eta_{i}|. \ \xi_{i}, \ \eta_{i}, \text{ and } \ \zeta_{i} \text{ are in Table A.1.}$

Table. A.1. ξ_i , η_i , and ζ_i for hexahedral element.

Edge No.	ξ_i	η_i	ζ_i
1	0	-1	-1
2	0	1	-1
3	0	1	1
4	0	-1	1
5	-1	0	-1
6	1	0	-1
7	1	0	1
8	-1	0	1
9	-1	-1	0
10	1	-1	0
11	1	1	0
12	-1	1	0

The elementary stiffness and mass matrices of the arbitrary hexahedral element should be calculated with a numerical integration. For example, the components of the elementary stiffness matrix in the local coordinate system are calculated as follows:

$$K_{ij} = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} (\nabla \times \mathbf{\Phi}_{i}) \cdot (\nabla \times \mathbf{\Phi}_{j}) |\mathbf{J}_{3D}| d\xi d\eta d\zeta$$

= $\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} k(\xi, \eta, \zeta) d\xi d\eta d\zeta; \ i, j = 1 \sim 12$ (A-12)

The Gauss-Legendre quadrature is a form of Gaussian quadrature to approximate a definite integral of a function with a weighted sum of the function values at specified points. The Gauss-Legendre quadrature is represented as follows:

$$\int_{-1}^{1} f(x) dx \approx \sum_{i=1}^{n} w_i f(x_i) , \qquad (A-13)$$

where *n* is the number of sample points; w_i is the quadrature weight; and x_i is the quadrature point between -1 and 1. Using the Gauss-Legendre quadrature of equation (A-13), the components of the elementary stiffness matrix of equation (A-12) can be computed as follows:

$$K_{ij} = \sum_{m=1}^{n_3} \sum_{l=1}^{n_2} \sum_{k=1}^{n_1} w_m w_l w_k k(\xi_k, \eta_l, \zeta_m); \ i, j = 1 \sim 12 \quad .$$
(A-14)

In Table A.2, the points and weights for the Gauss-Legendre quadrature are described. In the same way, the components of the elementary mass matrix of the hexahedral element can also be calculated.

Order	Points	Weights
1	0	2
2	-0.57735026918962576	1
	0.57735026918962576	1
3	-0.77459666924148338	0.8888888888888888888
	0	0.55555555555555555
	-0.77459666924148338	0.88888888888888888888
4	-0.86113631159405258	0.34785484513745386
	-0.33998104358485626	0.65214515486254614
	0.33998104358485626	0.65214515486254614
	0.86113631159405258	0.34785484513745386
5	-0.90617984593866399	0.23692688505618909
	-0.53846931010568309	0.47862867049936647
	0	0.5688888888888888
	0.53846931010568309	0.47862867049936647
	0.90617984593866399	0.23692688505618909

Table. A.2. 1D Gauss-Legendre quadrature points and weights.

초 록

MT 반응함수들의 민감도 패턴 분석 및 역산에의 적용

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자기지전류(Magnetotelluric; MT) 탐사를 통해 자연 송신원에 의해 발생하는 전자기장을 시간에 따라 지표에서 측정하고, 이를 주파수 영역의 다양한 MT 반응함수로 변환시켜 지하구조를 영상화하는 역산 등에 활용한다. 역산 기술은 모델 변수 및 입력자료 형태에 따라 그 결과가 달라지고, 민감도 행렬을 활용해 지구물리탐사 역산에서 여러 모델 변수의 특성을 분석하는 연구가 진행되었다. 또한, MT 탐사 분야에서는 MT 반응함수에 따라 다른 역산 결과를 제시하는 연구들이 진행되었다. 하지만, MT 반응함수들의 어떤 특성이 역산 결과의 차이를 야기하는지에 대한 연구는 부족하다.

이 논문에서는 주요한 MT 반응함수들(임피던스 텐서, 겉보기 비저항, 위상, 티퍼, 유효 임피던스, phase 텐서)의 민감도 패턴을 분석하고, 그 민감도 패턴의 특징에 따라 역산 결과가 어떻게 달라지는지 확인한다. 민감도 패턴은 특정 위치의 모델 변수의 작은 변화에 자료가 변하는 정도를 공간에 나타낸 것으로, 모델 변수 벡터의 모델링 자료를

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관측자료와 맞추는 역산 과정에서 MT 반응함수 형태별 특성을 나타내는 지표가 된다. 이 논문에서는 MT 반응함수 형태별 3차원 민감도 패턴과 MT 탐사가 이루어지는 지표에서의 2차원 민감도 패턴을 모두 제시한다. 그리고, 총 22개의 MT 반응함수 형태들은 6개의 그룹으로 분류하는데, 지표에서의 민감도 패턴이 비슷한, 즉 역산에서 비슷한 역할을 하는 MT 반응함수들끼리 같은 그룹으로 분류된다. 서로 다른 그룹에 포함되는 MT 반응함수들끼리는 역산에서 상호보완적인 역할을 할 수 있다.

민감도 패턴의 특징이 역산에 미치는 영향을 알아보고자 MT 자료의 1차원, 2차원, 3차원 해석 상황을 가정해 MT 반응함수별 관측자료와 역산 결과를 수치 예시로 제시한다. 이를 통해 MT 자료의 1차원 해석 상황에서 dimensionality 오차를 고려했을 때, 유효 임피던스가 최적의 MT 반응함수 형태임을 확인한다. 일반적인 MT 자료의 2차원 해석 상황에서는 transverse magnetic (TM) 모드의 MT 반응함수 형태들을 활용하는 것이 추천되지만, 목표 구조 위의 수신기를 사용하지 못하는 경우 티퍼가 더 적합하다. 두 이상체가 수직적으로 존재하는 특정 구조를 영상화 할 때는 임피던스 텐서의 성분들을 선택적으로 활용해 개선된 3차원 MT 역산 결과를 얻을 수 있다. 수치 예시들은 하나의 표로 정리되어, MT 탐사 상황에 따라 역산의 입력자료로 적합한 MT 반응함수 형태들을 추천한다.

민감도 패턴의 MT 현장자료 역산 적용성을 알아보기 위해, Utah Frontier Observatory for Research in Geothermal Energy (FORGE) 근처에서 획득된 현장자료에 다른 MT 반응함수들의 조합을 입력자료 형태로 활용하는 4가지 역산 전략을 고려한다. 이 사례 연구는 민감도 패턴으로부터 선정된 MT 반응함수들을 입력자료 형태로 활용해 역산

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결과를 개선시킬 수 있다는 사실을 보여준다. 현장자료가 목표 구조로부터 y축 방향으로 떨어진 곳에 넓게 분포하는 상황을 고려하면, y축 방향으로 두 개의 꽃잎 모양 민감도 패턴을 갖는 티퍼의 y성분에 그 구조의 반응이 주로 포함되어 있을 것이라 추론할 수 있다. 실제로, 4가지 전략 중 티퍼의 y성분을 포함하는 자료선택 전략으로부터 역산된 지하 모델은 목표 구조를 뚜렷하게 나타낸다. 이후 오차 분석을 통해 티퍼의 y성분이 목표 지하구조를 더 정확히 영상화하는데 크게 기여하고 있다는 사실을 확인할 수 있다.

이 논문은 MT 탐사의 환경과 MT 반응함수들의 민감도 패턴에 따라 역산의 최적 입력자료 형태를 선택하는 데 큰 도움이 될 것이다.

주요어: 자기지전류(Magnetotelluric; MT), MT 반응함수, 민감도 패턴, 역산, Utah FORGE

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