이학석사학위논문

A comparison study of various SVR-GARCH models with applications to monitoring conditional volatility of stock markets

주식 시장의 조건부 변동성 모니터링 응용에 관한 SVR-GARCH 모델들의 비교 연구

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윤민혁의 이학석사 학위논문을 인준함

2022 년 8 월

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Abstract

In this paper, we compare the performance of the support vector regression-(SVR) and asymmetric Huber SVR (AHSVR) based-monitoring methods using the monitoring scheme regarding the change of conditional volatilities of the generalized autoregressive conditional heteroskedastic (GARCH) model. Specifically, we obtain the residuals via respectively fitting SVR- and AHSVR-GARCH models to a given time series, and seek for the optimal set of tuning parameters through a grid search. We confirm that AHSVR-GARCH has a better performance than SVR-GARCH by conducting simulation experiments, and conclude that utilizing robust methods when computing residuals indeed strengthen the detection ability in general. Moreover, the data analysis of log returns of S&P500 and KOSPI is conducted to further showcase its applicability.

Keywords: Monitoring, SVR-GARCH, Robust SVR, Volatility, CUSUM, financial market

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Chapter 1

Introduction

The primary interest regarding the participants of the financial market is the volatility, a measure that accounts for uncertainty, rather than changes in the average of financial returns.(조신섭, 2016) As the market volatility is generally regarded to be time-dependent, the generalized autoregressive conditional heteroskedastic model (GARCH) proposed by Engle (1982) and Bollerslev (1986) has gained enormous popularity among researchers and practitioners in the field of finance. Henceforth, the GARCH model is still one of the most widely used model that measures the volatility of financial returns data, and exploring the properties of the GARCH model and its variants, as well as their applications, continuously remains to be an enticing research topic. In general, the maximum likelihood method has been mainly used when estimating the parameters of the model, but recent studies attempt to estimate conditional variance with machine learning (ML) and deep learning (DL) techniques such as the support vector regression (SVR) and neural network.

In particular, SVR is a nonparametric method that is primarily used in

cases where the underlying structure of the dataset is considered to be nonlinear. As such, it liberates from the necessity of assuming the innovation distribution of the time series a priori when estimating the conditional variance of the GARCH model. This hybrid model of SVR and GARCH is generally referred as the SVR-GARCH model, and has been applied in several studies, including Lee et al. (2020), Pèrez-Cruz et al. (2003), Sun and Yu (2020), Chen et al. (2010), Santamarìa-Bonfil et al. (2015) . However, traditional SVR-GARCH models is prone to overfitting issues, and thus can be susceptible to outliers. To mitigate this issue, Lee et al. (2022) proposed a hybrid model that is based on the asymmetric Huber loss SVR (AHSVR) model (Balasundaram and Meena, 2019), which significantly reduced the effects of the outliers while successfully captured the underlying structure of the time-heteroskedastic volatility. . Therefore, in this paper, the primary focus is set towards investigating the necessity of using AHSVR compared to the traditional SVR models when estimating and monitoring a structural change of the conditional volatility.

Another interest of the financial market is to prospectively monitor the structural change regarding the volatility, because a larger volatility signifies a greater financial risk. Among the typical monitoring schemes, the methods based on CUSUM statistics, initially proposed by Page (1954), is being used in various fields. In addition, many studies on modified CUSUM-based monitoring methods based on model residuals have been conducted since then. e.g. Faisal et al. (2018), Oh and Lee (2017). Especially, Lee et al. (2020) devised a statistical monitoring method that both well detects the increase and decrease of the volatility. In this paper, the hybrid monitoring method of Lee et al. (2020) was adopted as a monitoring method of choice, and we here compare the SVR-GARCH and AHSVR-GARCH models when they are respectively utilized in obtaining the residuals required to formulate the monitoring scheme.

The remainder of this paper is structured as follows. Section 2 describes SVR and AHSVR models in general, and explain how they are incorporated when estimating the conditional volatility. Also, we briefly introduce the procedure of monitoring. Section 3 provides the simulation results, comparing the performance of the monitoring method via SVR-GARCH and AHSVR-GARCH models, respectively. Section 4 compares both models in monitoring real-world financial time series, namely, the S&P500 index and KOSPI. Section 5 describes the conclusions.

Chapter 2

Model Description

2.1 Support Vector Regression(SVR)

2.1.1 ϵ-SVR

Support vector regression, specifically the ϵ-SVR, is a nonlinear function estimation method that incorporates the ϵ-insensitive loss function, which neglects the error up to at most ϵ from y_i, where $\{(\mathbf{x_i}, \mathbf{y_i}) : i = 1, 2, \cdots, n\}$ denotes a training data. In addition, SVR is known to be highly flexible, as we can freely map the input data to a high-dimensional feature space by using the kernel trick.

In SVR, the function f to be estimated has the following form:

$$f(\mathbf{x}) = \langle \mathbf{w}, \phi(\mathbf{x}) \rangle + b, \tag{2.1}$$

where $\mathbf{x}$ denotes a input vector and $\mathbf{w}, b$ are the regression coefficient of the model to be estimated. $\phi(\cdot)$ is a Mercer kernel that satisfies $K(\mathbf{x}, \mathbf{z}) = \langle \phi(\mathbf{x}), \phi(\mathbf{z}) \rangle$, where ϕ is an implicit kernel operator.

To obtain the estimates $\hat{\mathbf{w}}$ and $\hat{b}$, we construct a constrained convex optimization problem, which is formulated as follows:

$$\text{minimize } \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) \tag{2.2}$$

$$\text{subject to } \begin{cases} y_i - \langle \mathbf{w}, \mathbf{x_i} \rangle & \leq \epsilon + \xi_i \\ \langle \mathbf{w}, \mathbf{x_i} \rangle + b - y_i & \leq \epsilon + \xi_i^* \\ \xi_i \geq 0, \xi_i^* \geq 0, \end{cases}$$

where the constant C is a penalty term that regulates between the flatness of function f and the training error, ξ_i, ξ_i^* are slack variables with nonnegative values, and C and ϵ are tuning parameters, the optimal values of which are obtained through a grid search method.

To solve the above constrained optimization problem, we transform the problem into the unconstrained one by exploiting the KKT conditions, and the formulate the Lagrangian dual problem. Then, we can utilize quadratic programming (QP) to find the lagrangian multiplier solution, and finally, obtain the estimates of the SVR parameters, see Smola (2004) for more details.

2.1.2 ϵ-AHSVR

When applying machine learning techniques, the overfitting issue is unavoidable. To deal with this phenomenon, we can employ a more robust loss function, which make function to be flat, and Balasundaram and Meena (2019) proposes a generalized SVR model by replacing the loss function in (2.2) with the ϵ-

insensitive asymmetric Huber loss, which is given as follows:

$$H_{\epsilon,\tau_L,\tau_R}(x) = \begin{cases} -\tau_L(2(x+\epsilon)+\tau_L) & , -\infty < x < -(\tau_L+\epsilon) \\ (x+\epsilon)^2 & , -(\tau_L+\epsilon) \leq x < -\epsilon \\ 0 & , -\epsilon \leq x < \epsilon \\ (x-\epsilon)^2 & , \epsilon \leq x < (\epsilon+\tau_R) \\ \tau_R(2(x-\epsilon)-\tau_R) & , (\epsilon+\tau_R) \leq x < \infty, \end{cases} \tag{2.3}$$

where ϵ and τ_L, τ_R are all nonnegative user-set parameters. The function $H_{\epsilon,\tau_L,\tau_R}$ returns zero if the difference between the estimated value and the actual observed value is less than ϵ. Also, $H_{\epsilon,\tau_L,\tau_R}$ converts the quadratic function into a linear function starting at points $(\epsilon+\tau_R)$ and $(\epsilon+\tau_L)$. As shown in Figure 2.1, the smaller τ_L or τ_R of function H, the smaller the loss. This behavior result the model to be less sensitive to noise present in the training data.

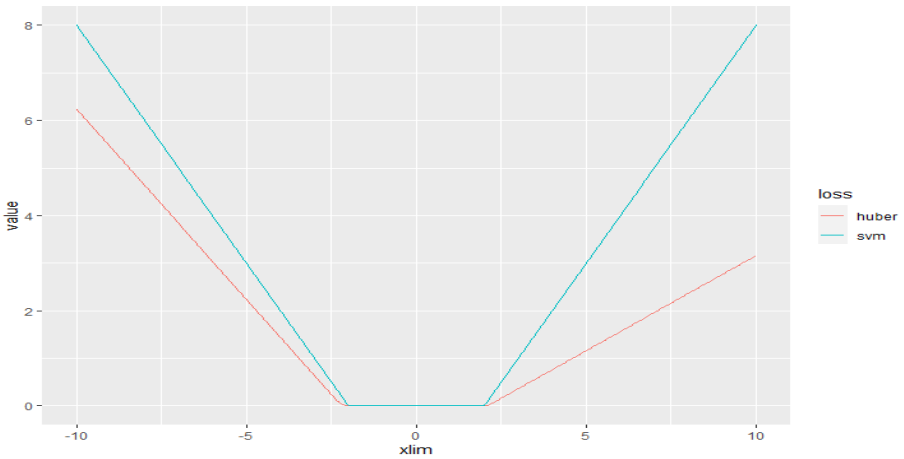

Figure 2.1 The loss functions of SVR with $\epsilon = 2$ (blue) and AHSVR with $(\epsilon, \tau_L, \tau_R) = (2, 0.4, 0.2)$ (red), respectively.

Next, we introduce the function $H_{\epsilon,\tau_L,\tau_R}$ to the penalty term of the equation

(2.2) to obtain the parameter of (2.1) (Balasundaram and Meena, 2019):

$$\underset{(\mathbf{w},b)}{\operatorname{argmin}} \frac{1}{2}(\mathbf{w}^T\mathbf{w} + b^2) + \frac{C}{2}\sum_{i=1}^{n} H_{\epsilon,\tau_L,\tau_R}(y_i - f(\mathbf{x_i})). \tag{2.4}$$

For convenience, we can rewrite (2.1) using matrix notations. Specifically, we write the matrix of explanatory variables as $X = [\mathbf{x}_1, \cdots, \mathbf{x}_n]^T$ and the vector of response variables as $y_i = f(\mathbf{x_i})$. Then, we write

$$\mathbf{y} = Q\mathbf{z}, \tag{2.5}$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{w} \\ b \end{bmatrix}$ and $Q = \begin{bmatrix} A(X, X^T) & \mathbb{1} \end{bmatrix}$ with the kernel matrix $A_{n\times n}$ such that $\left[A(X, X^T)\right]_{i,j} = K(\mathbf{x_i}, \mathbf{x_j})$ and $\mathbb{1}$ is ones vector of length n.

Using the notations presented in (2.5), we formulate the unconstrained variant of the problem in (2.4):

$$\min_{\mathbf{z}} S(\mathbf{z}) = \frac{1}{2}\mathbf{z}^T\mathbf{z} + \frac{C}{2}\left[\|\mathbf{y} - Q\mathbf{z}\|^2 - \|(\mathbf{y} - Q\mathbf{z} - \tau_R\mathbb{1})_+\|^2 - \|(Q\mathbf{z} - \mathbf{y} - \tau_L\mathbb{1})_+\|^2\right], \tag{2.6}$$

where $\mathbf{a}_+$ is the operator where its i-th element is defined as $\max(a_i, 0)$ ($\mathbf{a} = (a_1, \ldots, a_p)$). Balasundaram and Meena (2019) verified that $S(\mathbf{z})$ has a unique solution because of its strong convexity. Therefore, we can find an approximation of the optimal solution using some iterative solvers, and the desired solution is given by

$$\mathbf{z}^{new} = \left(\frac{I}{C} + Q^TQ\right)^{-1} Q^T\left[\mathbf{y} + (Q\mathbf{z}^{old} - \mathbf{y} - \tau_L\mathbb{1})_+ - (\mathbf{y} - Q\mathbf{z}^{old} - \tau_R\mathbb{1})_+)\right]. \tag{2.7}$$

2.2 Monitoring via Robust SVR based GARCH

GARCH(1,1) model is defined as follows:

$$\begin{aligned} y_t &= \sigma_t \eta_t \\ \sigma_t &= \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned} \tag{2.8}$$

where σ_t is the time-varying conditional volatility, and η_t is an iid process with mean zero and unit variance. The parameter ω, α, β satisfy the condition $\omega > 0$, $\alpha, \beta \geq 0$. In particular, $\alpha + \beta < 1$ is necessary for y_t to be stationary.

Lee et al. (2022) used the AHSVR-GARCH to estimate the conditional variance of various linear and nonlinear GARCH models. Specifically, AHSVR-GARCH model is expressed by the following formula:

$$\begin{aligned} \sigma_t^2 &= h(y_{t-1}^2, \sigma_{t-1}^2) \\ \Rightarrow \log \sigma_t^2 &= \log h(y_{t-1}^2, \sigma_{t-1}^2) =: f(y_{t-1}^2, \sigma_{t-1}^2), \end{aligned} \tag{2.9}$$

where f takes the form of (2.1). By taking the logarithm on both sides of Equation (2.9), it is ensured that the σ_t^2 estimated via AHSVR is all positive. Since the conditional variance is not observable in real-world circumstances, for practicality, we use the proxy $\tilde{\sigma}_t^2$ of σ_t^2 by taking the moving average of squared observations using the prescribed window size s, namely,

$$\tilde{\sigma}_t^2 = \frac{1}{s} \sum_{\ell=t-s+1}^{t} (y_\ell - \bar{y}_{s,\ell})^2, \tag{2.10}$$

where $\bar{y}_{s,\ell} = \frac{1}{s} \sum_{\ell=t-s+1}^{t} y_t$. $\tilde{\sigma}_t^2$ replaces the unknown quantity σ_t^2 in (2.9) when estimating f, and it is adopted in many studies in the literature, including Lee et al. (2022).

The training sample is divided into two subsets, one for model fitting and the other utilized as a validation set to select the optimal set of tuning parameters

through a grid search. For the validation process, we additionally assume that the error process $\{\eta_t\}$ is a standard normal distribution, and employ a different loss function that resembles the negative quasi log-likelihood function (Hwang and Shin, 2010) [Estimating GARCH models using kernel machine learning]:

$$L(\boldsymbol{\omega}, b) = \sum_{t=1}^{n} \left[y_t^2 e^{-f(\mathbf{x}_t)} + f(\mathbf{x}_t) \right] + \frac{\lambda}{2} \|\boldsymbol{w}\|^2, \tag{2.11}$$

where w is the parameter in (2.1), and $\lambda \geq 0$ is the regularization parameter.

Subsequently after obtaining the estimate $\hat{f}$ of f, we obtain the residual

$$\hat{\eta}_t = \frac{y_t}{\hat{\sigma}_t}. \tag{2.12}$$

This residual will be used when formulating the hybrid CUSUM test proposed by Lee et al. (2020) purposed for monitoring. The proposed CUSUM Test has the test statistic of

$$\hat{M}_n = \max \left\{ \hat{M}_n^{(1)}, \hat{M}_n^{(2)} \right\} \tag{2.13}$$

with

$$\begin{cases} \hat{M}_n^{(1)} & = \max\limits_{1 \leq k \leq n} \max\limits_{m \leq k} \frac{1}{\sqrt{n}} (\hat{W}_m - \hat{W}_k) \\ \hat{M}_n^{(2)} & = \max\limits_{1 \leq k \leq n} |\min\limits_{m \leq k} \frac{1}{\sqrt{n}} (\hat{W}_m - \hat{W}_k)|, \end{cases} \tag{2.14}$$

where $\hat{W}_k$ is defined as

$$\hat{W}_k = \sum_{t=1}^{k} (\hat{\eta}_t^2 - 1)/\hat{\delta},$$

and $\hat{\delta}^2$ is a sample variance of $\hat{\eta}_t^2$'s. Oh and Lee (2019) and Lee and Kim (2022) established that the limiting distribution of $\hat{M}_n$ under the null hypothesis of no

structural change is given as

$$\hat{M}_n \xrightarrow{d} M_{11}^* \vee M_{12}^*,$$

$$M_{11}^* = \sup_{0 \le t \le 1} \left| \sup_{0 \le s \le t} \mathcal{W}(s) - \mathcal{W}(t) \right|,$$

$$M_{12}^* = \sup_{0 \le t \le 1} \left| \inf_{0 \le s \le t} \mathcal{W}(s) - \mathcal{W}(t) \right|,$$

where $\mathcal{W}$ denotes the one-dimensional standard Brownian motion.

Notice that $\hat{M}_n$ is a combination of $\hat{M}_n^{(1)}$ and $\hat{M}_2^{(2)}$, and has the advantage of being able to detect not only increasing processes but also decreasing processes. However, in this paper, as promptly detecting an increase in volatility may be prioritized by market participants, we focus on such cases in empirical studies of the subsequent sections. Moreover, using the limiting null distribution, Lee et al. (2020) confirmed that the critical value $c = 2.465$ at the significance level of 0.05 via Monte Carlo simulation.

Chapter 3

Simulation Study

This section presents a comparison study of monitoring schemes respectively based on AHSVR-GARCH and SVR-GARCH models via simulation experiments. Specifically, the simulation is performed by using GARCH(1,1) as the base model, and assessing the average stopping time, namely, the average run length (ARL), to check whether the performance of AHSVR-GARCH model is relatively superior compared to the existing SVR-GARCH model.

We adopt the Gaussian kernel as a kernel function of choice to fit both models above:

$$K(\mathbf{x}, \mathbf{z}) = \exp(-\gamma \|\mathbf{x} - \mathbf{z}\|^2), \tag{3.1}$$

where $\gamma > 0$ is a user-defined parameter, and an optimal value is obtained by performing a grid search, similar to the searching method of SVR tuning parameters.

3.1 Finding optimal tuning parameters

We first find the optimal set of tuning parameters (γ, C, ϵ, τ_L, τ_R) of AHSVR that can be well describe the conditional volatility of the given dataset without overfitting. For the procedure, we generate 1,000 instances of training time series of length 1,000 from GARCH(1,1) model. In addition, we divide each generated time series into two chunks, the first 400 observations are reserved for training the model, and the remaining 600 observations are utilized when selecting the optimal tuning parameters.

Next, the optimal tuning parameter is selected as the pair of elements that minimizes the negative log-likelihood of (2.11) using the grid search method among the values presented in Table 3.1. As fitting SVR models are computationally less demanding than fitting AHSVR models, we consider a larger space of tuning parameters for SVR-GARCH models. The results of the grid search are shown in Table 3.2.

Tuning Parameter	AHSVR	SVR
γ	(1, 0.5, 0.05)	(1, 0.5, 0.1, 0.05, 0.01)
C	(100, 1, 0.01)	(1000, 100, 10, 1, 0.1, 0.01, 0.001)
ϵ	(1, 0.5)	(2, 1, 0.5, 0.1, 0.05)
τ_L, τ_R	(0.5, 0.2, 0.1, 0.05, 0.01)	-

Table 3.1 Set of tuning parameter for grid search

3.2 Simulation Results

In this section, we use the tuning parameters obtained from the preceding section, and obtain ARL_1 for each cases, and performs monitoring simulations on

Model	(γ, C, ϵ, τ_L, τ_R)
AHSVR	(0.05, 1, 0.5, 0.01, 0.5)
SVR	(0.1, 0.01, 0.05, $\cdot$, $\cdot$)

Table 3.2 The optimal tuning parameter

the changed model based on the estimated regression. Here, we fix the ARL_0 via Monte Carlo simulations based on the limiting distribution, and use $c = 1.016$ as our control limit.And then, for practical task, we independently generate 1,200 additional observations for evaluating ARL_1, specifically 400, 800 allocated as the training set and the changed set, respectively. The samples generated according to the prescribed parameters are plotted in Figures 3.1 to 3.4. In addition, we consider the following three cases of change:

- Case 1 : (ω, α, β) : $(0.3, 0.3, 0.3) \rightarrow (1, 0.3, 0.3)$
- Case 2 : (ω, α, β) : $(0.3, 0.3, 0.3) \rightarrow (0.3, 0.6, 0.3)$
- Case 3 : (ω, α, β) : $(0.3, 0.3, 0.3) \rightarrow (0.3, 0.3, 0.6)$

We input the test data into the fitted model to obtain residual (2.12), and perform the proposed CUSUM monitoring procedure in Section 2.2 to output the point at which the change is occurred. Note that due to the definition of ARL_1, we report the ARL of both methods after adjusting the starting point of the run lengths to precisely be located at the start of the structural change. This process was repeated 1,000 times, and it is summarized in Table 3.3.

As a result, monitoring process using AHSVR generally yield better results. In both models, performance of AHSVR-based monitoring scheme is observed to be superior in all cases. For AHSVR, when α increases, the performance is

(ω, α, β)	AHSVR	SVR
$(1.0, 0.3, 0.3)$	46.076	189.5221
$(0.3, 0.6, 0.3)$	86.385	99.328
$(0.3, 0.3, 0.6)$	52.436	129.176

Table 3.3 Results of Simulation

significantly affected by outliers that occasionally occur than in other cases. We plot the estimated values as examples in Figures 3.5-3.13, to illustrate the estimated σ_t^2 and their residuals based on AHSVR. In Figures 3.5, 3.7, 3.9, 3.11, and 3.13 for conditional variances, the red line represents the $\hat{\sigma}_t^2$, and the black line represents the true σ_t^2. Also, in Figures 3.6, 3.8, 3.10 and 3.12 regarding the residuals, the yellow section means training set, and the red line indicates the point where the change was detected.

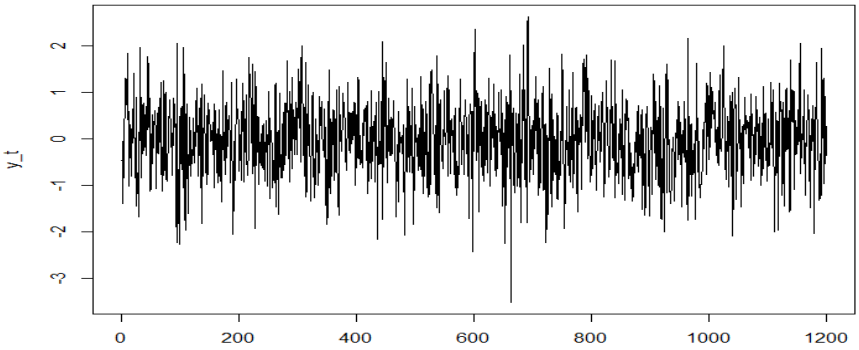

Figure 3.1 Sample : $\sigma_t^2 = 0.3 + 0.3y_t^2 + 0.3\eta_t^2$

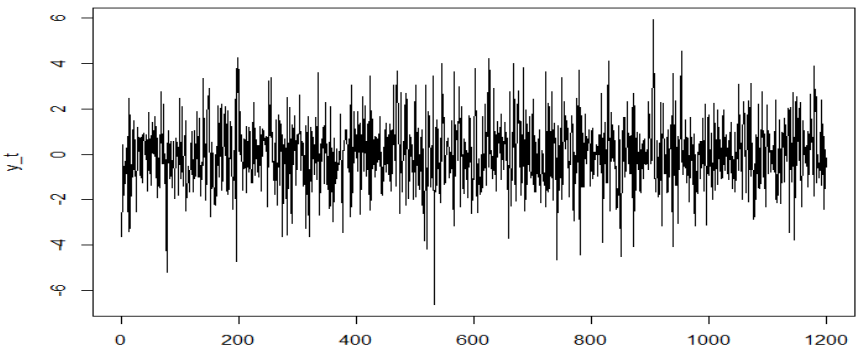

Figure 3.2 Sample : $\sigma_t^2 = 1 + 0.3y_t^2 + 0.3\eta_t^2$

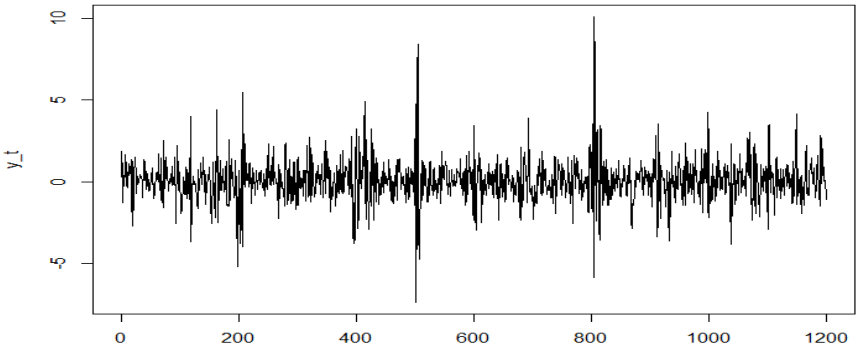

Figure 3.3 Sample : $\sigma_t^2 = 0.3 + 0.6y_t^2 + 0.3\eta_t^2$

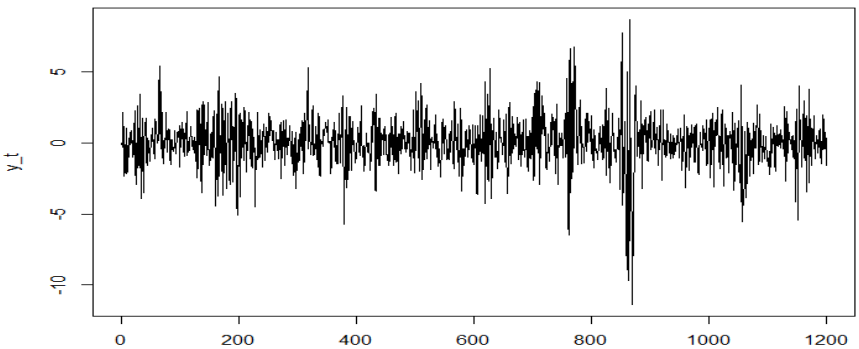

Figure 3.4 Sample : $\sigma_t^2 = 0.3 + 0.3y_t^2 + 0.6\eta_t^2$

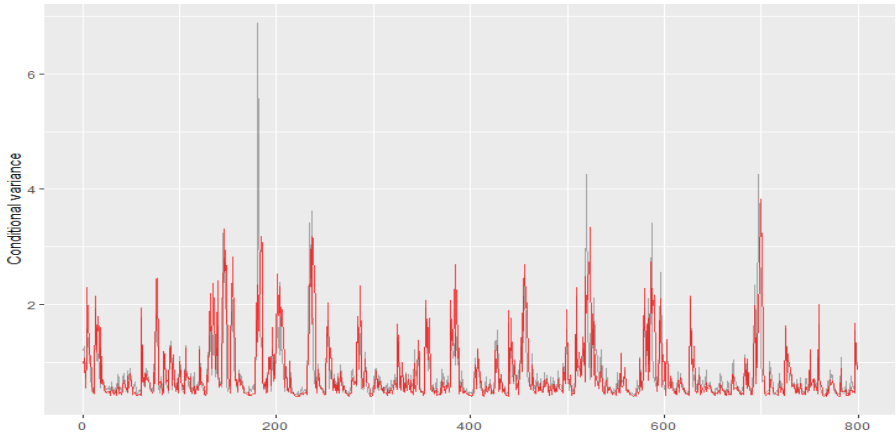

Figure 3.5 Plot $\hat{\sigma}_t^2$: $(\omega, \alpha, \beta) = (0.3, 0.3, 0.3)$

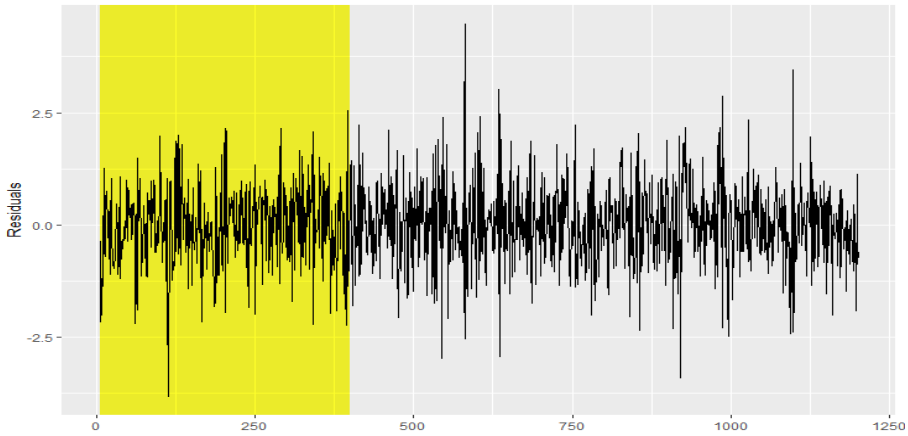

Figure 3.6 Plot $\hat{\eta}_t$: $(\omega, \alpha, \beta) = (0.3, 0.3, 0.3)$

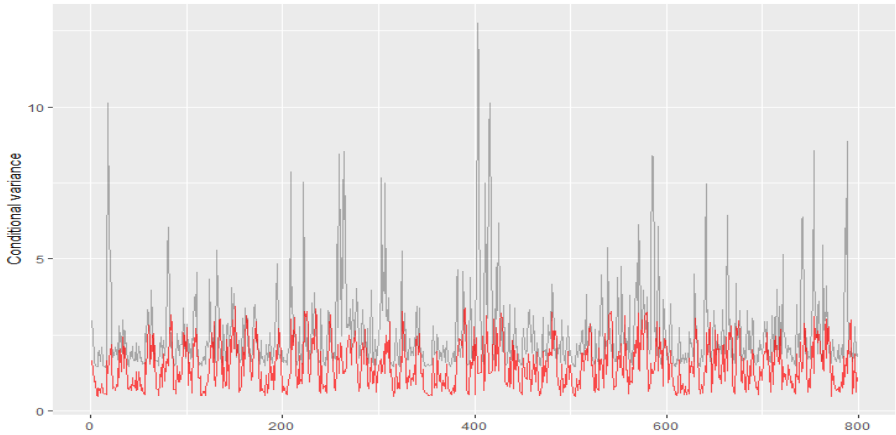

Figure 3.7 Plot $\hat{\sigma}_t^2 : (\omega, \alpha, \beta) = (1, 0.3, 0.3)$

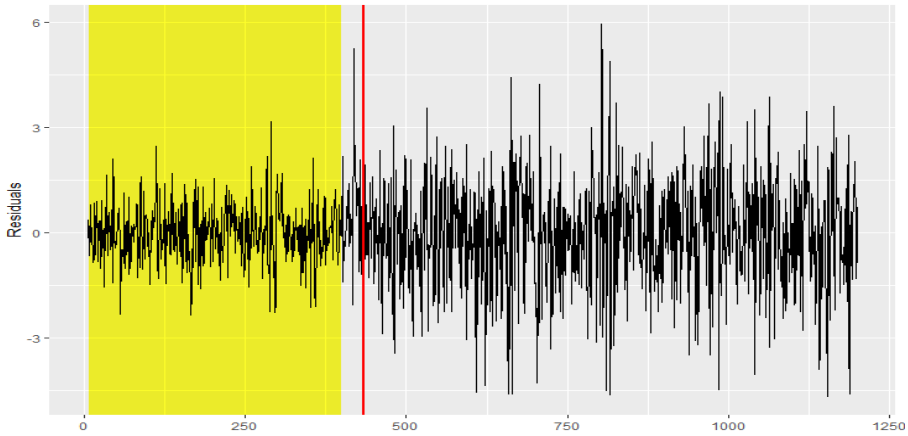

Figure 3.8 Plot $\hat{\eta}_t : (\omega, \alpha, \beta) = (1, 0.3, 0.3)$

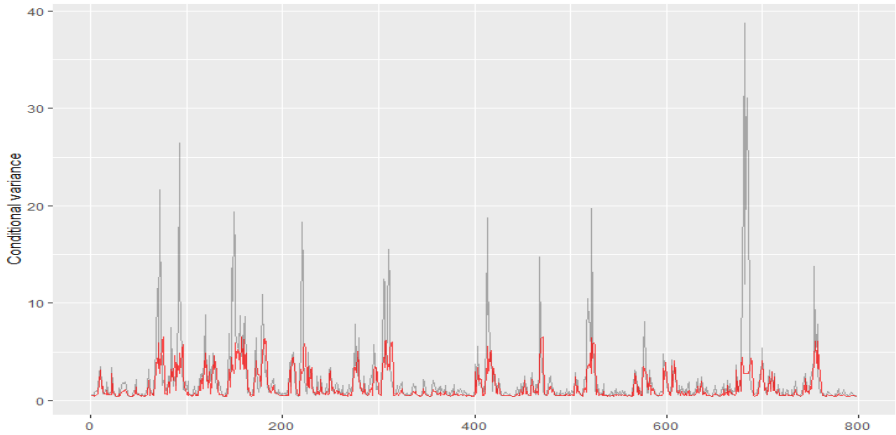

Figure 3.9 Plot $\hat{\sigma}_t^2$: $(\omega, \alpha, \beta) = (0.3, 0.6, 0.3)$

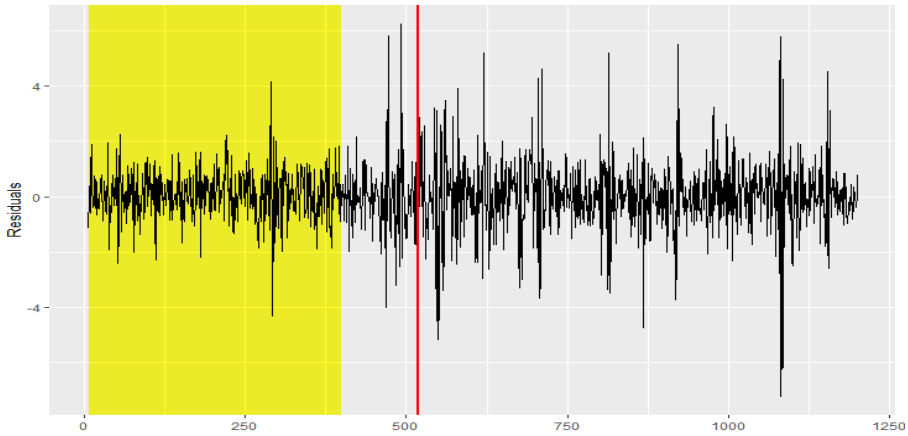

Figure 3.10 Plot $\hat{\eta}_t$: $(\omega, \alpha, \beta) = (0.3, 0.6, 0.3)$

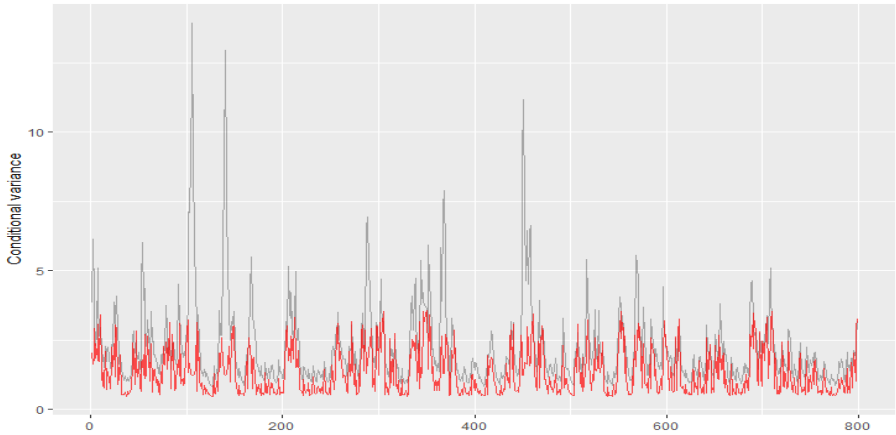

Figure 3.11 Plot $\hat{\sigma}_t^2$: $(\omega, \alpha, \beta) = (0.3, 0.3, 0.6)$

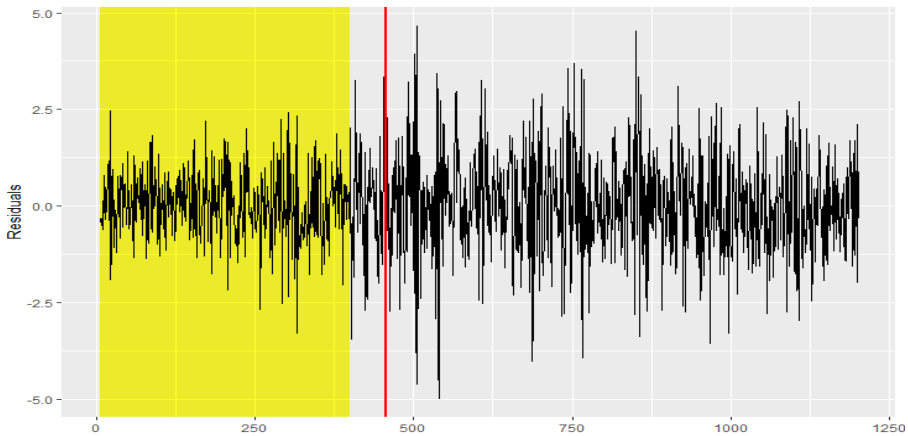

Figure 3.12 Plot $\hat{\eta}_t$: $(\omega, \alpha, \beta) = (0.3, 0.3, 0.6)$

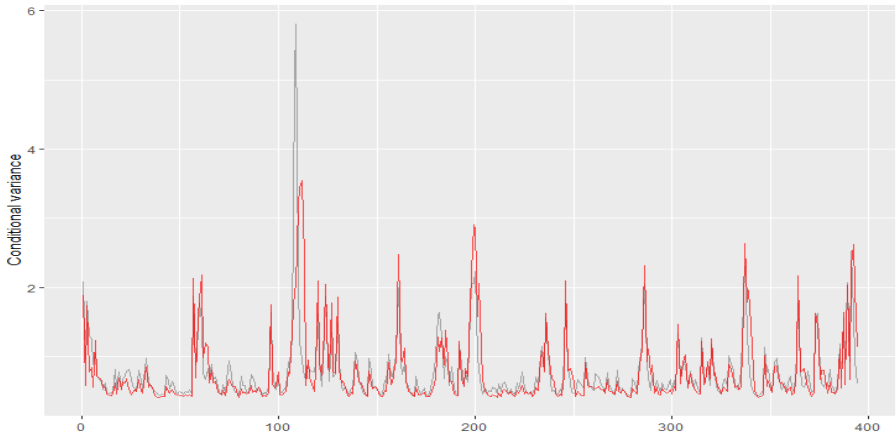

Figure 3.13 Plot of estimated AHSVR from training sample. $(\omega, \alpha, \beta) = (0.3, 0.3, 0.3)$

Chapter 4

Real Data Analysis

In this section, we use AHSVR-GARCH to monitor the volatility of returns of real-world financial time series, specifically, the daily log-returns of S&P 500 index and Korea Stock Price Composite Index (KOSPI). Throughout the section, we denote p_t to be the daily closing price, and y_t the log-returns, namely,

$$y_t = 100 \times (\log p_t - \log p_{t-1}) \quad (t \geq 1). \tag{4.1}$$

This is the most widely used transformation in the method for catching variability changes in real stock price data, and is known to be structurally similar to that of the GARCH model.

We use S&P 500 stock prices from January 1, 2012 to June 28, 2022, and KOSPI stock prices from April 1, 2020 to June 28, 2022, which can be obtained from the website "Investing.com". For KOSPI, as the volatility of financial returns before April 2020 was large and appeared to be nonstationary, we therefore set the starting date of the training set on April 1, 2020. This process is critical as nonstationarity of the training time series violates the assumption

of the CUSUM monitoring scheme, see Lee et al. (2020). Moreover, we use $c = 2.465$ in this analysis.

For S&P500 index, we set the training time series from January 1, 2012 to December 31, 2015, and then sequentially monitor the remaining dataset daily until a change is present. Moreover, Figures 4.1 and 4.2 are visualizations of S&P500's raw price and log returns, the yellow shaded area of which is the training set, and the red line depicts the date on which the change was detected. The monitoring scheme using AHSVR-GARCH model detected a change in volatility on March 13, 2020. The outcome of the monitoring may be primarily due to the stock market's sharp drop resulting from the WHO's declaration of the COVID-19 pandemic on March 12, 2020.

When analyzing KOSPI index, we assigned the dataset from April 1, 2020, to December 31, 2020 as the training data and performed the aforementioned procedure of monitoring a volatility change. Then, the nominated date of a change appeared to be on September 24, 2021. On such date, there was a widespread opinion that the inflation may occur throughout the global economy, and the U.S. Consumer Price Index (CPI) for November was scheduled to be announced. In the end, the CPI rose 6.8% to its highest level in 39 years, and referring to Figure 4.3, it can be seen that the stock market has since shifted towards a downwards trend.

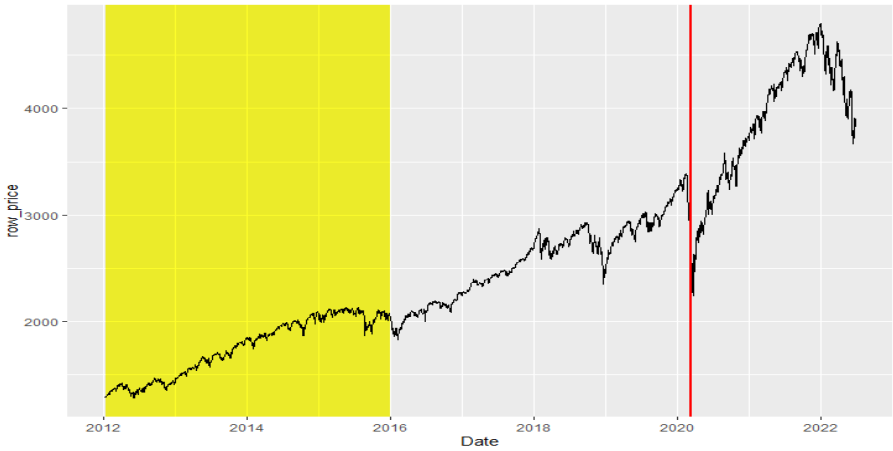

Figure 4.1 Raw Price of S&P500

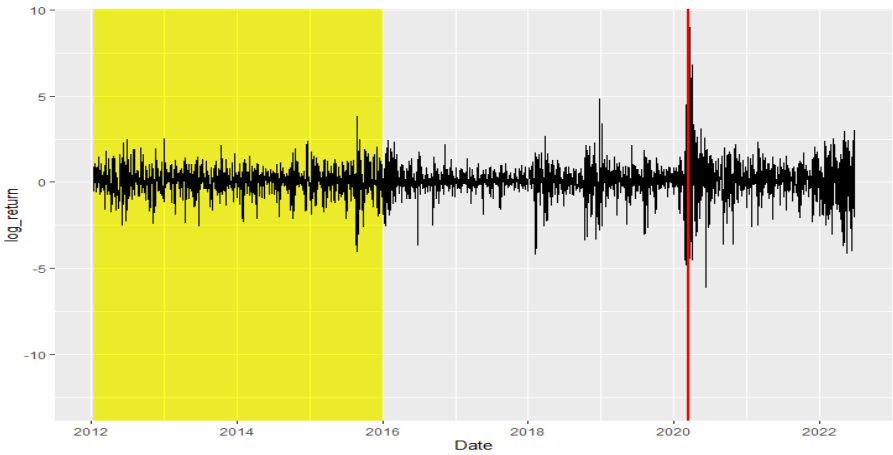

Figure 4.2 Log Return of S&P500

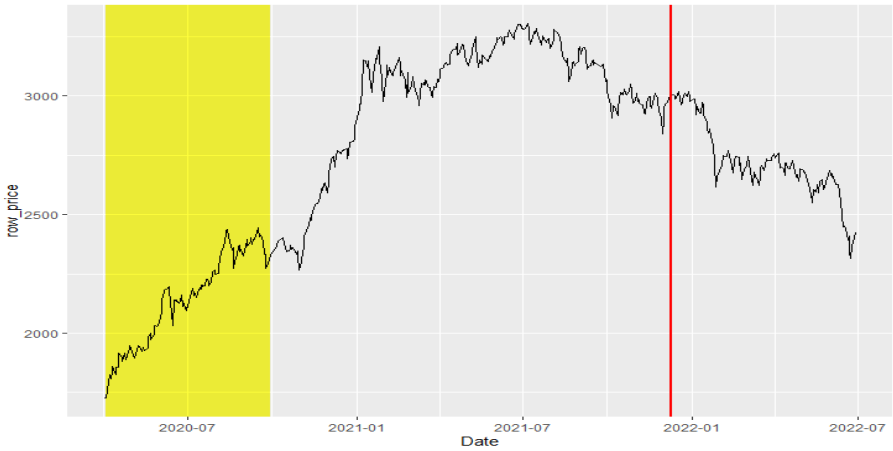

Figure 4.3 Raw Price of Kospi

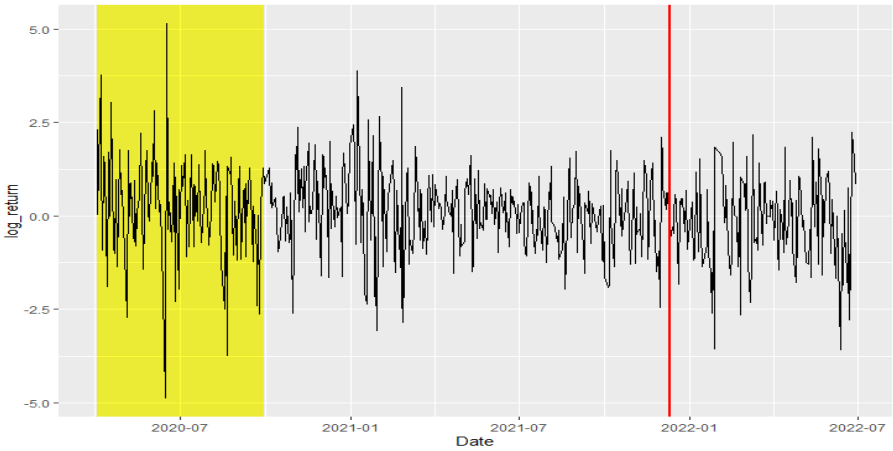

Figure 4.4 Log Return of Kospi

Chapter 5

Conclusion and Discussion

In this paper, we compared the results of monitoring scheme of Lee et al. (2020) based on the estimated volatility using AHSVR-GARCH and SVR-GARCH, respectively. The tuning parameters required for both models are respectively obtained by performing a grid search, and the monitoring procedure was conducted by using residual-based CUSUM method. Our simulation study revealed that the GARCH model with AHSVR proved to have a superior results in terms of the early-detection ability.

However, considering that ML techniques are heavily influenced by tuning parameters, the search space of the tuning parameters being relatively small may somewhat have affected the overall quality of the residuals, thus affecting the whole monitoring procedure in general. In addition, it is unclear whether the estimation of AHSVR is well applied not only to linear GARCH but also to non linear GARCH models. As the scope of this study was limited to the standard GARCH model, we leave these tasks as our future research topic.

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국문초록

본 연구에서는 일반화 자기회귀이분산성(GARCH) 시계열 모형의 조건부 분산 변화에 관하여 모니터링 체계를 사용하여 SVR(support vector regression)과 비대칭 Huber SVR(AHSVR) 기반 모니터링 방법의 성능을 비교한다. 구체적으로, 우리는 각각 SVR-GARCH와 AHSVR-GARCH로 적합시켜 잔차를 얻고, 그리드 서치를 통해서 튜닝 파라미터의 최적의 조합을 찾는다. 그 다음 시뮬레이션 실험을 수행하여 AHSVR-GARCH가 SVR-GARCH의 성능이 우수함을 보여주고, 일반적으로 잔차를 계산할 때 로버스트 방법을 사용하는 것이 실제로 탐지 성능을 강화한다는 결론을 내린다. 또한, 실제 적용 가능성을 보여주기 위해 S&P 500과 KOSPI의 로그 수익률에 대한 데이터 분석을 수행한다.

주요어: 모니터링, 서포트 벡터 회귀, 이분산성시계열모형, 로버스트, 변동성, 누적합관리도, 금융시장

학번: 2020-20706

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먼저, 저를 연구실의 일원으로 받아주시고 학업 뿐만 아니라 많은 가르침을 주신 이상열 교수님께 깊은 존경과 감사를 드립니다. 그리고 아낌없이 알려주시고 잘 해낼 수 있다고 격려해주신 조민영, 이상조, 김동원, 김창겸, 김동욱 선배님께, 힘든 시간 속에서도 항상 웃을 수 있게 해주고 잘 적응할 수 있도록 도와준 송원석, 김명동 동기들과 김세호, 엄규원 조항범 후배님들께도 감사한 마음을 전하며, 연구실의 모든 구성원이 앞으로 좋은 일만 있기를 기원합니다.

마지막으로, 항상 믿을 수 있고, 기댈 수 있는 부모님과 누나들께도 감사드립니다. 옆을 든든히 지켜주는 은지에게 감사한 마음을 표합니다.

석사과정의 끝이자, 박사과정의 시작인 이 시점에서 저를 지지해주고 응원해주신 모든 분들을 생각하며, 앞으로도 초심을 잃지 않고 노력하는 모습으로 좋은 연구자가 되어 보답하겠습니다.