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A Recursive Logit Model with Non-Link-Additive Attributes in a Multimodal Network

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Sedong Moon

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Dong-Kyu Kim

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Graduate School of Engineering
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Sedong Moon

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Chair _____(Seal)

Vice Chair _____(Seal)

Examiner _____(Seal)

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Abstract

Travelers' trip patterns are becoming more personalized and complex with the emergence of new mobility services, such as ride-hailing, demand-responsive transportation (DRT), and shared mobility. Also, with the emergence of the mobility-as-a-service (MaaS) concept, which provides various mobility services in an integrated manner to enable travelers to use multiple modes sequentially, the importance of intermodal trips is being more emphasized. For intermodal trips in multimodal networks, there are many combinations of modes and paths. Moreover, mode and path choices are strongly correlated with each other. Therefore, a model simultaneously predicting mode and path choices is needed, not a model predicting path choice after mode choice as in the conventional four-step travel demand forecasting model. Recursive logit models can predict mode and path choices at the same time by modeling mode and path choices as a sequence of link choices in a transportation network. However, recursive logit models can incorporate only link-additive attributes: the value of a path attribute must be the same as the sum of link attributes of links belonging to the path. This characteristic constrains the applicability of recursive logit models by restricting variables that can be included.

Therefore, this study proposes a methodology to include non-link-additive attributes to the recursive logit model to analyze and predict users'

intermodal path and mode choices on a multimodal network. To achieve this, this study developed a link-additive approximation method that approximates a non-link-additive path attribute into a corresponding link attribute that holds the link-additivity. The link-additive approximation is performed by the singular value decomposition and Moore-Penrose Pseudoinverse methods. The methodology is applied to the actual multimodal network and intermodal trip data in Seoul, Korea. The multimodal network consists of road, bus, and rail networks. The intermodal trip data is mainly the National Household Travel Survey data, supported by transit smartcard data for routing the transit trip stages. This study used two non-link-additive attributes: transit fare and transfer penalty. The link-additive approximation method was applied to these attributes for all observed paths and by O-D pairs.

To compare RL models with respect to the inclusion of link-additive approximated transit fare, this study specified four models: M0 without fare, M1 with the fare proportional to link length, M2 with the fare approximated for all observed paths, and M3 with the fare approximated by O-D pairs. Also, to compare RL models with respect to the inclusion of link-additive approximated transfer penalty, this study specified model M4. The models were estimated using 10% of the dataset (3,209 trips and 9,710 trip stages out of 32,094 trips and 97,175 trip stages). Among the models, model M4, which includes both transit fare and transfer penalty, shows the best goodness-of-fit in terms of log-likelihood and AIC. The models were tested using the rest of

the dataset (28,885 trips and 87,465 trip stages). The testing was performed by comparing the predicted choice probabilities of alternatives connecting a certain O-D pair to the actual choice probabilities. Because the actual trajectories were unknown, the orders, modes, and transfer points of trip stages were used instead. As a result, all of the RL models showed better accuracies compared to benchmark models, MNL and PSL. Among them, the model M4 showed the best accuracy. It was followed by M3, M2, and M1, with M0 showing the worst accuracy among RL models.

All of the results showed that the inclusion of link-additive approximated transit fare and transfer penalty in the RL model improves both goodness-of-fit and accuracy of the model. Especially, the link-additive approximation by O-D pairs showed better goodness-of-fit and accuracy compared to the link-additive approximation for all observed paths.

Keyword : Recursive logit model, multimodal network, intermodal trip, non-link-additive attribute, link-additive approximation

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Table of Contents

List of Tables

List of Figures

보존용 학위논문 정오표

페이지	정정 전	정정 후
p. 9 : 18	a sequence of two or mode modes	a sequence of two or more modes
p. 12 : 22	Van Eck et al.	van Eck et al.
p. 50 : 5	Korea Transport Database	Korea Transport Institute
p. 99 : 16	(누락)	Baek, J., & Sohn, K. (2016). Deep-learning architectures to forecast bus ridership at the stop and stop-to-stop levels for dense and crowded bus networks. *Applied Artificial Intelligence,* 30(9), 861-885.
p. 102 : 11	Lee, E. H., Lee, I., Cho, S.-H., Kho, S.-Y., & Kim, D.-K. (2019).	Lee, E. H., Lee, I., Cho, S.-H., Kho, S.-Y., & Kim, D.-K. (2019a).
p. 102 : 14	Lee, E. H., Lee, H., Kho, S.-Y., & Kim, D.-K. (2019).	Lee, E. H., Lee, H., Kho, S.-Y., & Kim, D.-K. (2019b).

Chapter 1. Introduction

1.1. Study Background

Most transportation networks are multimodal: they consist of various modes, such as private vehicles, buses, rails, taxis, and bicycles. In multimodal transportation networks, intermodal trips are common, consisting of two or more trip stages with distinct modes. It is essential to analyze intermodal trips in multimodal networks in terms of performance evaluation, planning, and operation. Especially, travelers' trip patterns are becoming more personalized and complex with the emergence of new mobility services, such as ride-hailing, demand-responsive transportation (DRT), and shared mobility (Spickermann et al., 2014; Meyer de Freitas et al., 2019). Also, with the emergence of the mobility-as-a-service (MaaS) concept, which provides various mobility services in an integrated manner to enable travelers to use multiple modes sequentially and seamlessly, the importance of intermodal trips is being more emphasized. Moreover, in policy, the overall efficiency of the transportation system can be improved by guiding travelers to make intermodal trips and be redistributed in near-saturated transportation networks (van Nes & Bovy, 2004; Schade et al., 2011; Spickermann et al., 2014; Rode & da Cruz, 2018; Meyer de Freitas et al., 2019).

To forecast travel demands, a conventional four-step travel demand

forecasting model consisting of trip generation, trip distribution, mode choice, and trip assignment is commonly used in practice. However, many combinations of modes and paths exist for intermodal trips. Moreover, mode and path choices are strongly correlated with each other. Therefore, mode choice and trip assignment (or path choice) cannot be divided when analyzing and predicting intermodal trips (Arentze & Molin, 2013; van Eck et al., 2014; Meyer de Freitas et al., 2019). Furthermore, intermodal trips accompany other choice behaviors, such as choices of transit routes, boarding stations, and alight stations, which cannot be classified into mode or path choices. (van Eck et al., 2014).

Therefore, to predict travelers' intermodal trip behaviors and the resultant trip flow by modes and paths, a model simultaneously predicting mode and path choices is needed, not a model predicting path choice after mode choice as in the conventional four-step model. Previous studies have developed several models to deal simultaneously with mode and path choices. Iterative mode and path choice models are some of those models. However, they require a long prediction time due to iteration between mode and path choice models. Formulating intermodal mode and path choices into a single discrete choice model, such as multimodal logit (MNL) or path-size logit (PSL) models, can predict users' behaviors within a relatively shorter time. However, it requires path sampling to generate a choice set among the infinite number of possible alternative paths. Machine learning models can predict

the results of mode and path choices directly and accurately. However, analyzing user behavior from machine learning models is not easy.

Recursive logit (RL) models can predict mode and path choices at the same time by modeling mode and path choices as a sequence of link choices in a transportation network. RL models have successfully predicted path and mode choices simultaneously in multimodal networks (Zimmermann et al., 2018; Meyer de Freitas, 2018; Meyer de Freitas et al., 2019). However, RL models can incorporate only link-additive attributes: the value of a path attribute must be the same as the sum of link attributes of links belonging to the path. For example, travel time and the number of transfers can be used in RL models because they are link-additive. However, transit fares are not link-additive under some fare structures and a transfer discount. In this case, transit fares cannot be incorporated into RL models. This characteristic constrains the applicability of RL models by restricting variables that can be included. Therefore, a methodology to incorporate non-link-additive attributes in RL models is needed.

1.2. Study Purpose

The purpose of this study can be divided into three-fold (Figure 1). First, this study develops RL models with non-link-additive attributes. This study considers two non-link-additive attributes: transit fare and transfer penalty.

Next, this study develops a methodology to incorporate non-link-additive attributes into RL models, which is called “link-additive approximation” in this study. This study develops a methodology to approximate non-link-additive path attributes to their corresponding link-additive link attributes. Also, this study compares the goodness-of-fit and accuracies of RL models according to the incorporation of non-link-additive attributes using the existing multimodal network and intermodal trip data.

This study analyzes the mode and path choice behaviors of single-purpose intermodal trips in urban areas using the proposed RL model with non-link-additive attributes. The multimodal network and intermodal trip data of Seoul, Korea, are used. This study constructs a network of road, bus, and rail (subways and metropolitan rails) and analyzes intermodal data resulting from the National Household Travel Survey (NHTS) and transit smartcard data.

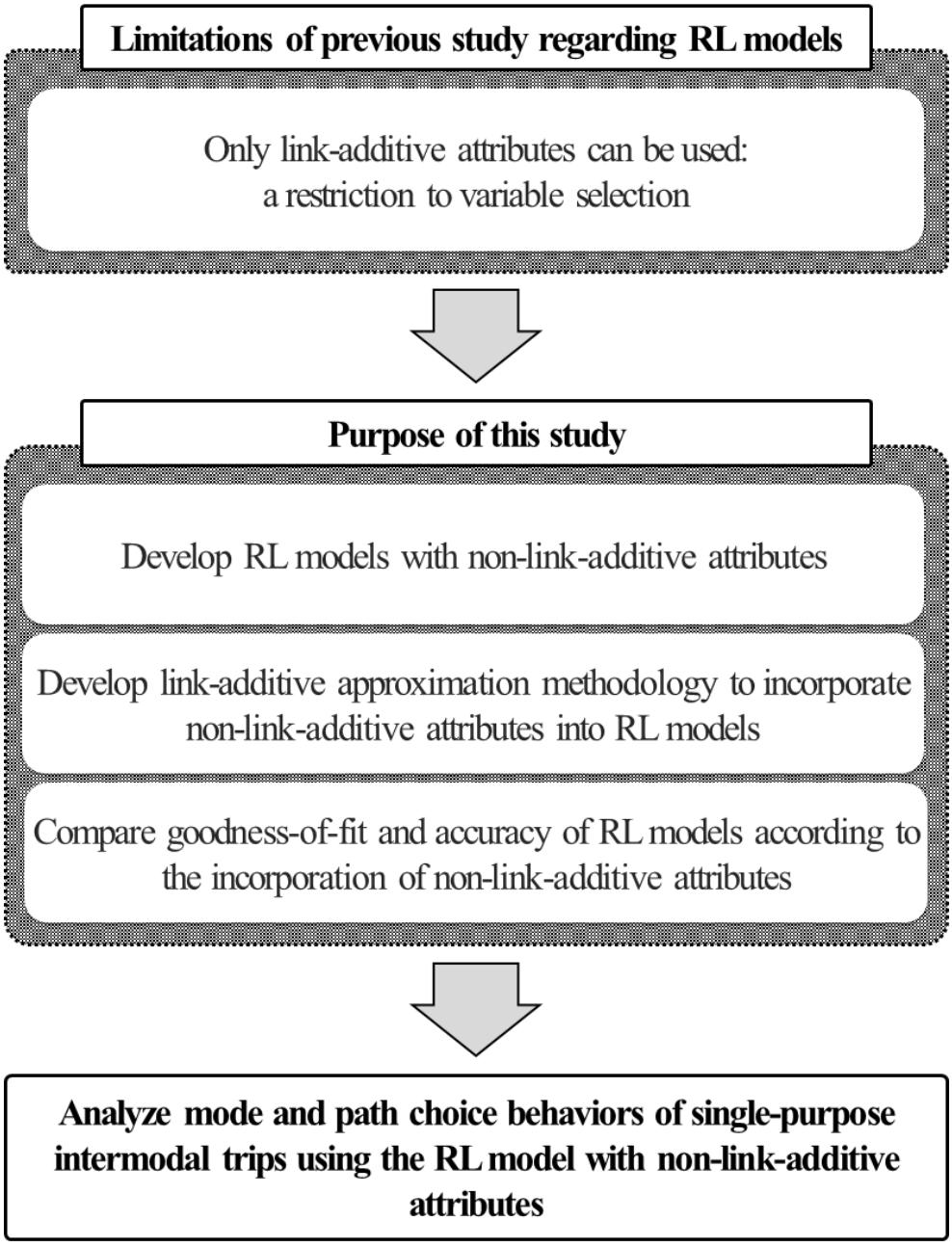


Figure 1. Purpose of this study

1.3. Terminologies

In this study, the terminologies "trip" and "trip stage" have distinct meanings. As Axhausen (2007) defines, a trip stage, or simply a "stage," means a "continuous movement with one mode of transport, respectively one vehicle." Meanwhile, a trip means "a continuous sequence of stages between two activities." An activity can be defined as a purpose of a travel demand at a certain location and time period. This concept of activity is primarily applied to activity-based models, which suggests that travel demands are generated by the need of people to participate in activities at different locations and times (Ben-Akiva et al., 1996; Castiglione et al., 2014; Kim, 2021; Min, 2021; Kim et al., 2022). Home-stay, working, schooling, shopping, and leisure are common activities that generate trips between them. Each continuous travel from home to workplace, workplace to shopping, or shop to home is an example of a trip. Also, a trip can consist of either a single or multiple stages. A transit trip from an origin to a destination involving multiple transfers is a typical example of a trip consisting of multiple stages. Each riding of a vehicle, from boarding to alighting, is a stage.

An intermodal trip consists of two or more stages with different modes, for example, bus and rail. A multimodal network consists of multiple modes, and intermodal trips can occur on the multimodal network.

In the literature dealing with path choice modeling (also called route

choice modeling), the word “path” and “route” are used interchangeably. However, this study uses the word “path” to refer to a sequence of links comprising a trip or trip stage because the word “route” can also refer to a specific transit service. In this study, specific bus services are called “bus routes,” whereas specific rail services are called “rail lines.”

The remainder of this thesis is structured as follows. Chapter 2 provides the literature review regarding intermodal mode and path choice models and recursive logit models. The methodology of this study is given in Chapter 3, including the theoretical backgrounds of the recursive logit model and the methodology of the proposed link-additive approximation of non-link-additive attributes. Chapter 4 explains the application of this study’s methodology on the existing multimodal network and intermodal trip data. The results are provided in Chapter 5. Finally, Chapter 6 states this study’s conclusion.

Chapter 2. Literature Review

2.1. Intermodal Mode and Path Choice Models

There have been various modeling approaches regarding intermodal mode and path choices. Though it is more reasonable to integrate mode and path choices when modeling intermodal trip behaviors, as some studies mentioned (Arentze & Molin, 2013; van Eck et al., 2014; Meyer de Freitas et al., 2019), some studies did separate mode and path choice models, while others iterated or integrated them.

2.1.1. Separation of Mode and Path Choices

The conventional four-step trip demand forecasting model, widely used in practice, is a typical example of the separation of mode and path choices. When intermodal trip behaviors are modeled with separate mode and path choices, trip assignment is performed after applying a mode choice model. Prespecified mode chains (a sequence of two or mode modes) are considered alternative modes in the mode choice model. Because those mode chains are correlated, a multinomial logit model cannot be directly used. A nested logit model can be used to model a choice among alternative modes or mode chains (Ben-Akiva and Bierlaire, 1999). In the nested logit model, two nests

corresponding to private cars and mode chains with transit are constructed. After choosing between those two nests, a choice among mode chain alternatives is performed. Private car trips are assigned using road traffic assignment models, such as user equilibrium models, and intermodal trips using mode chains are assigned using transit assignment models, such as stochastic assignment.

The separation of mode choice and path choice cannot consider a strong correlation between mode choice and trip assignment in intermodal trips. From a behavioral point of view, mode and path choices are integrated into a single-choice stage (van Eck et al., 2014). Furthermore, intermodal trips accompany choice behaviors beyond mode and path choices, such as choices of transit routes, boarding stations, and alight stations. The underlying behavioral choice processes are reasonable to be integrated into a single model describing the overall intermodal trip-making process because they are challenging to be described in a mathematically tractable way (van Eck et al., 2014).

2.1.2. Iterative Mode and Path Choice Models

As one of the methodologies to integrate mode and path choices in intermodal trip behavior modeling, mode and path choice models can be executed iteratively to consider a correlation between them (Abdulaal &

LeBlanc, 1979; Hou et al., 2020; Moon et al., 2021). Path choice is performed under the given modal split by the mode choice model, and mode choice is performed considering the resultant travel costs given by the path choice model. Abdulaal & LeBlanc (1979) is an early study dealing with the multimodal mode and path choices, although dealing only with auto and transit. The study proposed three modal split-assignment models. The first model only performs assignment by Wardrop's equilibrium principle while not considering a modal split. The second model performs a modal split by the logit modal split function and assignment by the equilibrium principle. The third model performs a modal split by the all-or-nothing function and assignment by the equilibrium principle. The study proved that there is no mathematical programming model for the multimodal equilibrium problems, then proposed an iterative solution method for the second and third models. Hou et al. (2020) used a combined modal split and traffic assignment model, in which the nested logit model performs the modal split among automobile, rail, and bus, and the equilibrium principle model performs the assignment. The modal split and assignment were also performed iteratively, and the study applied the model to optimize the locations of park-and-ride facilities. Moon et al. (2021) used a multinomial logit model to perform a modal split among private cars, conventional public transportation, and a new transit type called zonal express which connects users' aggregated origins and destinations directly. The study then optimized the route of the zonal express under the

assumption that zonal express users' paths are the same as the route, using the parallel cheapest insertion and tabu search methods. The modal split and zonal express route optimization (equivalent to zonal express path choice model) were performed iteratively.

Iterative mode and path choice models can consider the correlation between mode choice and trip assignment as well as other underlying choice behaviors. However, those models require a long computational time to repeat mode and path choice processes.

2.1.3. Integration of Mode and Path Choices into a Single-Choice Model

Several studies have simultaneously formulated the integrated mode and path choice problem into a single discrete choice model, such as logit-based models. Hoogendoorn-Lanser (2005) formulated nested logit and generalized nested logit models of multimodal inter-urban trips. The study focused on multimodal trips consisting of rail as a primary mode and urban public or private transportation modes as access/egress modes. The study defined choice sets consisting of boarding and alighting stations, train alternatives, access/egress modes, and access/egress paths. The choice models are then applied to the choice sets to determine the modes and paths of the overall trips. Van Eck et al. (2014) proposed a paired combinatorial logit model of mode

and path choices. The study also predefined choice sets, where each alternative is defined as a combination of mode (or mode chain) and path. The model is evaluated iteratively to reflect the resultant multimodal network load to mode and path alternatives. Montini et al. (2017) used a path size logit model as a combined mode and path choice model in a multimodal urban transportation network. The study used GPS trajectory data to identify trip paths for multimodal trips by one or more modes among private cars, public transportation, bicycles, and walking. Anderson et al. (2017) applied a path size logit model to estimate public transportation passengers' mode and path choice behaviors. The study incorporated revealed preference (RP) data collected by a survey to identify multimodal trip paths by one or more modes among metro, buses, regional trains, IC-trains, S-trains, and local trains. Nielsen et al. (2021) also applied a path size logit model to analyze intermodal public transportation mode and path choices, including specific transfer attributes: walking time, waiting time, and the number of transfers.

Those logit-based models need predefined choice sets of multimodal trips consisting of one or more modes and paths. Fiorenzo-Catalano (2007) proposed a methodology to generate path choice sets in a multimodal network. The study focused on interregional trips consisting of the main part (intercity train, interregional bus, car, metro), access from home (car, local train, taxi, metro, urban bus, urban tram, bicycle, walking), and egress to activity (taxi, local train, metro, urban bus, urban tram, bicycle, walking).

Those integrated choice models can predict users' mode and path choices quickly. However, there is a disadvantage in requiring choice set generation, which can cause biases (Meyer de Freitas, 2018; Meyer de Freitas et al., 2019). It is relatively easy to specify a mode choice set. However, a set of paths or mode-path combinations is essentially an infinite set that must be sampled to formulate a choice model. Biases can occur in a path sampling process, and a criterion to sample path is often arbitrary.

2.1.4. Intermodal Trip Models Based on Machine Learning

Machine learning models can also be used for trip prediction for unimodal and intermodal trips. Machine learning can predict the results of mode and path choices directly and accurately. Baek & Sohn (2016) directly predicted bus ridership (boarding and alighting of bus stops and ridership between stops) using a deep neural network model based on activity-related variables and variables related to bus routes. Yu et al. (2016) predicted bus passenger trip flow between origin and destination by an artificial neural network model based on land use, bus accessibility, and distance between the zones. Toqué et al. (2017) conducted short- and long-term temporal forecasting using machine learning models (random forests and long short-term memory neural networks) to predict multimodal transportation passenger flows (the number of boarding passengers at train stations, bus

stops, and tram stops). Sifringer et al. (2018) proposed a hybrid model of multinomial logit and dense neural network models to enhance conventional discrete choice models. The study added the neural network result into the utility function of the logit model. The study applied the model to a stated preference survey on mode choices between cars, trains, and the Swissmetro (a proposed inter-city express transit system project).

The major disadvantage of machine learning models is that they are challenging to interpret. To overcome this limitation, interpretable or explainable machine learning approaches, such as the Shapley additive explanations (SHAP) method, are being proposed. Lee et al. (2021) predicted users' choice behaviors between express and all-stop metro trains using the extreme gradient boosting (XGBoost) model. The study compared the effects and importance of features that can affect the choice behaviors (total travel times, in-vehicle times, waiting times, crowding, and the number of transfers) using their SHAP values. However, it is still challenging to explain users' specific behaviors using interpretable or explainable machine learning methods. For example, those methods are difficult to estimate trade-offs affecting user behaviors, such as the trade-off between travel time and the number of transfers or the trade-off between time and cost. Also, the SHAP value is affected by correlations between variables, which can result in wrong interpretations.

2.2. Recursive Logit Models

2.2.1. Recursive Logit Models with a Single Mode

RL model was first proposed by Fosgerau et al. (2013). The study focused on a private car path choice by formulating the route choice as a sequence of link choices. Link choices are formulated as a multimodal logit model, and a recursive value function is proposed to formulate the overall utility of a path. The study also proposed a concept of link size, which corresponds to path size in path size logit models, to consider overlapping paths sharing a single link. Mai et al. (2015) proposed a nested recursive logit (NRL) model to compute the value function more efficiently. The study concluded that NRL is better in terms of goodness of fit, based on log-likelihood and test error of RL and NRL models. The study also focused on a private car path choice. Zimmermann et al. (2017) applied the RL model to bicycle path choice. Mai et al. (2021) developed a recursive logit model in a stochastic time-dependent network to model car users' routing policy choices. The study also proposed an efficient algorithm to solve the model. Those studies focused on the path choice of a single mode while not considering mode choices. Although, unlike previous logit-based models, RL model does not require path choice set generation and path sampling.

2.2.2. Recursive Logit Models in a Multimodal Network

The first study to apply the RL model to a multimodal network and to consider a mode choice is conducted by Zimmermann et al. (2018). The study formulated public transportation users' choices of combined modes and paths. The study used in-vehicle time, waiting time, transfer dummy (1 if the link is a transfer link; 0 otherwise), and link constant as link attributes. Tram and bus dummies (1 if the link is a tram or bus link; 0 otherwise) multiplied by the in-vehicle time were also included in link attributes. The study is the first to integrate mode and path choices into a single RL model. Meyer de Freitas (2018) and Meyer de Freitas et al. (2019) applied the RL model not only to public transportation but also to a multimodal network consisting of public transportation, private car, bicycle, and walking. The study formulated travelers' choices of combined modes and paths. The study used survey data to identify travelers' intermodal trip modes, and then the trips were routed based on modes, origins, and destinations of each trip stage. The study used link constant, transit transfer dummy (1 if the link is a transfer link between transit lines), multimodal transfer dummy (1 if the link is a transfer link between transit line and street network), bike dummy, car dummy, bus dummy, tram dummy, heavy rail dummy, travel time, and headway dummies (H11 and H16; H11 is 1 when the headway is between 11 and 16 minutes, H16 is 1 when the headway is longer than 16 minutes). Among them, bike dummy, car

dummy, bus dummy, tram dummy, and heavy rail dummy were multiplied to travel time to make those attributes link-additive. The study is the first to use the RL model to simultaneously perform the mode and path choices in a multimodal network. Nassir et al. (2019) proposed a combined model of a recursive logit-based path choice model and a strategy-based transit route choice model. The recursive logit model was used to model the path choice on the network, including choices of boarding routes, alighting or transferring stops, and transfer routes. The strategy-based transit route choice model is similar to the previous optimal strategy model (Spiess & Florian, 1989). However, the study proposed a stochastic measure of attractiveness to model the choice of a transit route at a given stop. The study applied the combined model to a network consisting of bus and rail services.

Previous studies dealing with recursive logit models could not consider non-link-additive attributes, such as transit fares. In actual trip processes, the fare or cost of the trip is an important factor in choice-making. Travelers often choose paths with longer travel times or more transfers if their fares or costs are cheaper than other alternatives, or they often choose more expensive paths if they have shorter travel times or fewer transfers. Other non-link-additive attributes, such as varying transfer penalty, also could not be considered in previous RL models.

2.3. Comparison of Recursive Logit Model and Other Models

Table 1 provides a summary of the literature review. Compared to the RL model, other intermodal mode and path choice models have the following limitations. First, the separate mode and path choice models cannot consider a strong correlation between mode choice and trip assignment in intermodal trips. Next, the iterative mode and path choice models can consider this strong correlation, but they require a long time for estimation and prediction. This can be a significant obstacle to practical applications, often requiring time-efficient methodologies with short prediction times. The logit-based simultaneous mode and path choice models can consider the correlation between mode and path choices and have a relatively short prediction time. However, the choice set of modes and paths must be prespecified by choice set generation. The mode choice set is relatively easy to generate because the number of possible alternatives is small: only individual modes and their combinations are to be considered. However, this is not the case in path choice set generation. Because there are practically an infinite number of alternative paths, path sampling is needed to make a finite choice set. As mentioned by previous studies, the choice generation and path sampling of alternative paths can cause biases (Meyer de Freitas, 2018; Meyer de Freitas et al., 2019). Also, rule-based methods, such as finding paths with the least transfers, or assuming that users use the first vehicle arriving at a certain stop, are often used in path

sampling. These rule-based methods are one of the factors causing biases in path sampling: they can generate different choice sets when the rule is changed. Also, some alternative paths with significant choice probabilities can be omitted in the choice set if the path sampling is not carefully conducted. Finally, machine learning models can predict the results of mode and path choices directly and accurately within a shorter prediction time and without any choice set generation. However, they are difficult to interpret, especially in terms of users' behaviors.

Compared to those models, RL models have the following advantages. First, RL models can consider a strong correlation between mode and path choices. Next, though RL models require a relatively long computational time during estimation, once estimated, the prediction time of mode and path choices using the RL model is short. Also, RL models do not require any choice set generation of alternative paths, which can cause biases. With those advantages, The RL model is promising to predict intermodal trip demands on a multimodal network. Significantly, the model can simultaneously predict the sequence of trip stages and their paths and modes.

Table 1. Summary of literature review

Study	Modes	Mode Choice Model	Path Choice Model	Characteristics of the Study
Separate mode and path choice models				
Ben-Akiva & Bierlaire (1999)	• Private car • Mode chain including transit	Nested logit	• User equilibrium (car) • Stochastic assignment (transit)	Cannot consider a strong correlation between mode choice and trip assignment in intermodal trips
Iterative mode and path choice models				
Abdulaal & LeBlanc (1979)	• Private car • Transit	• Logit • All-or-nothing	User equilibrium	An early study to deal with multimodal mode and path choices
Hou et al. (2020)	• Private car • Rail • Bus	Nested logit	User equilibrium	Applied the model to locate park-and-ride facilities
Moon et al. (2021)	• Private car • Transit • Zonal express	Logit	• Parallel cheapest insertion • Tabu search	Applied the model to optimize zonal express routes
Integrated choice models				
Hoogendoorn -Lanser (2005)	• Interregional rail • Access/egress modes: urban transit & private car	Nested logit		Choice set generation is needed by path sampling
Fiorenzo-Catalano (2007)	• Interregional modes • Access/egress modes	-		The study dealt with the methodology of choice set generation by path sampling
van Eck et al. (2014)	• Train • Bus-tram-metro • Park & Ride	Paired combinatorial logit model of mode and path choices		Predefined choice sets, where each alternative is defined as a combination of mode (or mode chain) and path
Montini et al. (2017)	• Private cars • Transit • Bicycle • Walking	Path size logit		Choice set generation is needed by path sampling

Anderson et al. (2017)	• Metro • Bus • Regional train • IC-Train • S-Train • Local train	Path size logit		Choice set generation is needed by path sampling
Nielsen et al. (2021)	• Metro • Bus • Regional train • S-Train • Local train	Path size logit		Included transfer attributes (walking & waiting time, number of transfers) Choice set generation is needed by path sampling
Machine learning models				
Baek & Sohn (2016)	• Bus	-	Deep neural network	Predicted boarding and alighting of bus stops and ridership between stops
Yu et al. (2016)	• Bus	-	Artificial neural network	Predicted bus passenger flows by O-D pairs
Toqué et al. (2017)	• Train • Bus • Tram	• Random forest • Long short-term memory		Predicted multimodal transportation passenger flows temporally
Sifringer et al. (2018)	• Private car • Train • Swissmetro (proposed transit project)	A hybrid model of multinomial logit and dense neural network models	-	The neural network result was incorporated into the utility function of the logit model
Lee et al. (2021)	• Metro (express and all-stop)	XGBoost		Interpreted the result using SHAP values
Recursive logit models				
Fosgerau et al. (2013)	• Private car	-	Recursive logit	The first study to propose the recursive logit model
Mai et al. (2015)	• Private car	-	Nested recursive logit	Proposed the nested recursive logit model to calculate value function more easier
Zimmermann et al. (2017)	• Bicycle	-	Recursive logit	Applied recursive logit model to bicycle path choice

Zimmermann et al. (2018)	• Transit	Recursive logit		Applied recursive logit model to transit mode and path choices The first study to integrate mode and path choices into a single recursive logit model
Meyer de Freitas (2018) Meyer de Freitas et al. (2019)	• Transit • Private car • Bicycle • Walk	Recursive logit		Applied recursive logit model to multimodal mode and path choices The first study to use the recursive logit model to perform the mode and path choices simultaneously in a multimodal network
Nassir et al. (2019)	• Bus • Rail	A combined model of recursive logit and strategy-based models		The recursive logit model was used to model choices of routes and stops The strategy-based model was used to model the choice of a transit route at a given stop
Mai et al. (2021)	• Private car	-	Recursive logit	Applied recursive logit model to a stochastic time-dependent network

Chapter 3. Methodology

3.1. Recursive Logit Model

The recursive logit (RL) model was first proposed by Fosgerau et al. (2013). As the study proposed, the path choice problem is expressed as a sequence of link choice problems in multimodal logit models. Also, it is assumed that link attributes are link-additive and deterministic.

The notation of the RL model used in this study follows the original notations by Fosgerau et al. (2013), as shown in Figure 2. k is a link on which a traveler is currently located. $A(k)$ is a set of possible next links, or actions, chosen sequentially after the current link k. The chosen next link is denoted as a $(a \in A(k))$. The destination node is denoted as D, and a virtual link d is added after the node to formulate the recursive logit model as a sequence of link choices.

On every link k, a traveler chooses the next link a that maximizes the total utility, which is decomposed into two terms: instantaneous utility and expected downstream utility. The instantaneous utility is a utility of choosing the next link a conditional to the current link k. The expected downstream utility is the expectation of utility of the downstream path from a to d.

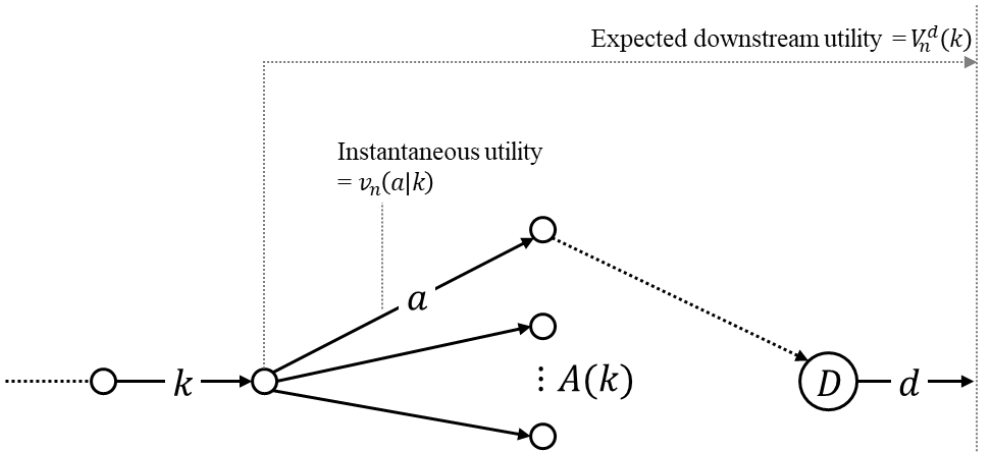


Figure 2. Notation of recursive logit model. Adapted from Fosgerau et al. (2013)

The instantaneous utility of a traveler n choosing link a conditional to current link k is expressed as Equation 1:

$$u_n(a|k) = v_n(a|k) + \mu\varepsilon_n(a) \tag{1}$$

where $v_n(a|k)$ is the deterministic term, $\varepsilon_n(a)$ is a random error term that is assumed identically and independently distributed (i.i.d.) extreme value type 1 with zero mean, and μ is the scale factor. The expected downstream utility of a traveler n choosing link a conditional to current link k is expressed as the Bellman equation (Bellman, 1957):

$$V_n^d(k) = E\left[\max_{a\in A(k)}\left(v_n(a|k) + V_n^d(a) + \mu\varepsilon_n(a)\right)\right] \tag{2}$$

The probability of choosing a next link a conditionally to current link k and destination link d is expressed as the multinomial logit model (Equation 3):

$$P_n^d(a|k) = \frac{\exp\left(\frac{v_n(a|k) + V_n^d(a)}{\mu}\right)}{\sum_{a' \in A(k)} \exp\left(\frac{v_n(a'|k) + V_n^d(a')}{\mu}\right)} \tag{3}$$

The solution of the value function $V_n^d(k)$ can be obtained by expressing the value function as a logsum function (Equation 4), which is derived from Equation 3:

$$V_n^d(k) = \begin{cases} \mu \ln \sum_{a \in A} \delta(a|k) \exp\left(\frac{v_n(a|k) + V_n^d(a)}{\mu}\right) & \forall k \in A \\ 0 & k = d \end{cases} \tag{4}$$

where A is the set of links, and $\delta(a|k) = 1$ if $a \in A(k)$ and 0 otherwise. Equation 4 can be transformed into Equation 5:

$$\exp\left(\frac{V_n^d(k)}{\mu}\right) = \begin{cases} \sum_{a \in A} \delta(a|k) \exp\left(\frac{v_n(a|k) + V_n^d(a)}{\mu}\right) & \forall k \in A \\ 1 & k = d \end{cases} \tag{5}$$

To express Equation 5 in a matrix form, let $\mathbf{M}$ $(|\tilde{A}| \times |\tilde{A}|)$, where $\tilde{A} = A \cup d$, be the incidence matrix with instantaneous utilities. Then entries of $\mathbf{M}$ can be expressed as Equation 6:

$$M_{ka} = \begin{cases} \delta(a|k) \exp\left(\frac{v_n(a|k)}{\mu}\right) & a \in A(k) \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

Also, let $\mathbf{z}$ $(|\tilde{A}| \times 1)$ and $\mathbf{b}$ $(|\tilde{A}| \times 1)$ be vectors with elements defined as Equations 7 and 8:

$$z_k = e^{\frac{V(k)}{\mu}} \tag{7}$$

$$b_k = \begin{cases} 0 & k \neq d \\ 1 & k = d \end{cases} \tag{8}$$

Then Equation 5 can be written in a matrix form as a system of linear equations (Equation 9):

$$\mathbf{z} = \mathbf{M}\mathbf{z} + \mathbf{b} \tag{9}$$

The solution for $\mathbf{z}$, or the solution for the value function $V_n^d(k)$, can be obtained from Equation 10, which is derived from Equation 9:

$$\mathbf{b} = (\mathbf{I} - \mathbf{M})\mathbf{z} \tag{10}$$

where $\mathbf{I}$ is an $|\tilde{A}| \times |\tilde{A}|$ identity matrix. Equation 10 has a solution if $(\mathbf{I} - \mathbf{M})$ is invertible.

The probability of choosing a path can be expressed using the Markov property of the model. Let a path σ be a sequence of links $(k_0, \dots, k_I)$ with $k_{i+1} \in A(k_i)$. Then the probability of choosing path σ is given by Equation 11:

$$\begin{aligned} P_n^d(\sigma) &= \prod_{i=0}^{I-1} \exp\left(\frac{v_n(k_{i+1}|k_i) + V_n^d(k_{i+1}) - V_n^d(k_i)}{\mu}\right) \\ &= \exp\left(-\frac{V_n^d(k_0)}{\mu}\right) \prod_{i=0}^{I-1} \exp\left(\frac{v_n(k_{i+1}|k_i)}{\mu}\right) \\ &= \frac{\exp\left(\frac{v_n(\sigma)}{\mu}\right)}{\exp\left(-\frac{V_n^d(k_0)}{\mu}\right)} = \frac{\exp\left(\frac{v_n(\sigma)}{\mu}\right)}{\sum_{\sigma' \in \Omega} \exp\left(\frac{v_n(\sigma')}{\mu}\right)} \end{aligned} \tag{11}$$

where $v_n(\sigma) = \sum_{i=0}^{I-1} v_n(k_{i+1}|k_i)$, and Ω is the set of all possible paths, which is an infinite and discrete set. Note that Equation 11 has a form of a path-based multinomial logit model.

3.2. Link-Additive Approximation of Non-Link-Additive Attributes

This study develops a methodology to approximate non-link-additive path attributes to their corresponding link-additive link attributes to enable those attributes to be incorporated into RL models. In this study, the methodology is called "link-additive approximation." Let σ_p $(p = 1, 2, \ldots, P)$ be a path, where P is the number of observed paths. The path attribute of path σ_p, which is non-link-additive, is denoted a_{σ_p}. Also, let k_l $(l = 1, 2, \ldots, L)$ be a link, where L is the number of links in the network. The link attribute of link k_l, which is link-additive and corresponds to the path attribute, is denoted a_{k_l}. Also, let $\delta_{\sigma k}$ be a binary variable that equals one if $k \in \sigma$ and zero otherwise. Then, the link-additive approximation of the path attribute into its corresponding link-additive link attribute can be expressed as Equation 12:

$$a_{\sigma_p} \approx \widehat{a_{\sigma_p}} = \sum_{l=1}^{L} \delta_{\sigma_p k_l} a_{k_l} \qquad \forall p \in \{1, 2, \dots, P\} \tag{12}$$

where $\widehat{a_{\sigma_p}}$ is the approximated path attribute corresponding to a_{σ_p}.

Equation 12 can be expressed as a matrix form. Let $\mathbf{A}_{\boldsymbol{\sigma}}, \widehat{\mathbf{A}_{\boldsymbol{\sigma}}}, \boldsymbol{\Delta}, \mathbf{A}_{\mathbf{k}}$ be matrices of path attributes, approximated path attributes, the relationship between paths and links, and link attributes, as shown in Equations 13-16:

$$\mathbf{A}_{\boldsymbol{\sigma}} = \begin{bmatrix} a_{\sigma_1} \\ a_{\sigma_2} \\ \vdots \end{bmatrix} \tag{13}$$

$$\widehat{\mathbf{A}_{\boldsymbol{\sigma}}} = \begin{bmatrix} \widehat{a_{\sigma_1}} \\ \widehat{a_{\sigma_2}} \\ \vdots \end{bmatrix} \tag{14}$$

$$\boldsymbol{\Delta} = \begin{bmatrix} \delta_{\sigma_1 k_1} & \delta_{\sigma_1 k_2} & \cdots \\ \delta_{\sigma_2 k_1} & \delta_{\sigma_2 k_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{15}$$

$$\mathbf{A}_{\mathbf{k}} = \begin{bmatrix} a_{k_1} \\ a_{k_2} \\ \vdots \end{bmatrix} \tag{16}$$

Then Equation 12 can be expressed as a matrix form (Equation 17):

$$\mathbf{A}_{\boldsymbol{\sigma}} \approx \widehat{\mathbf{A}_{\boldsymbol{\sigma}}} = \boldsymbol{\Delta}\mathbf{A}_{\mathbf{k}} \tag{17}$$

Because $\boldsymbol{\Delta}$ is not necessarily a square matrix and not necessarily invertible even if it is a square matrix, the exact solution for $\mathbf{A}_{\mathbf{k}}$ cannot be usually obtained. Instead, the approximate solution for $\mathbf{A}_{\mathbf{k}}$, which is denoted $\mathbf{A}_{\mathbf{k}}^{*}$ such that Equation 18 can be obtained by the singular value decomposition (Equation 19) and the Moore-Penrose Pseudoinverse method (Moore, 1920; Bjerhammar, 1951; Penrose, 1955) (Equations 20-22).

$$\mathbf{A}_{\mathbf{k}}^{*} = \underset{\mathbf{A}_{\mathbf{k}}}{\operatorname{argmin}} \parallel \widehat{\mathbf{A}_{\boldsymbol{\sigma}}} - \mathbf{A}_{\boldsymbol{\sigma}} \parallel = \underset{\mathbf{A}_{\mathbf{k}}}{\operatorname{argmin}} \parallel \boldsymbol{\Delta}\mathbf{A}_{\mathbf{k}} - \mathbf{A}_{\boldsymbol{\sigma}} \parallel \tag{18}$$

Equation 18 means that $\mathbf{A}_{\mathbf{k}}^{*}$ is a solution for $\mathbf{A}_{\mathbf{k}}$ which minimizes the norm of the difference between $\mathbf{A}_{\boldsymbol{\sigma}}$ and $\boldsymbol{\Delta}\mathbf{A}_{\mathbf{k}}$. To obtain $\mathbf{A}_{\mathbf{k}}^{*}$, the singular value decomposition of $\boldsymbol{\Delta}$ is first obtained by Equation 19:

$$\boldsymbol{\Delta} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{T}} \tag{19}$$

where $\mathbf{U}$ is an $m \times m$ orthogonal matrix, $\mathbf{V}$ is an $n \times n$ orthogonal matrix, and $\boldsymbol{\Sigma}$ is an $m \times n$ diagonal matrix when $\boldsymbol{\Delta}$ is an $m \times n$ matrix. Then, the Moore-Penrose Pseudoinverse of $\boldsymbol{\Delta}$ is obtained by Equation 20:

$$\mathbf{\Delta}^{+} = \mathbf{V}\mathbf{\Sigma}^{+}\mathbf{U}^{\mathbf{T}} \tag{20}$$

where $\mathbf{\Delta}^{+}$ is the Moore-Penrose Pseudoinverse of $\mathbf{\Delta}$, and $\mathbf{\Sigma}^{+}$ is an $n \times m$ diagonal matrix, as shown in Equation 21, when $\mathbf{\Sigma}$ is an $m \times n$ diagonal matrix, as shown in Equation 22:

$$\mathbf{\Sigma}^{+} = \begin{bmatrix} \frac{1}{s_1} & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & \frac{1}{s_n} & 0 \end{bmatrix} \tag{21}$$

$$\mathbf{\Sigma} = \begin{bmatrix} s_1 & & 0 \\ & \ddots & \\ 0 & & s_n \\ 0 & \cdots & 0 \end{bmatrix} \tag{22}$$

Finally, $\mathbf{A}_{\mathbf{k}}^{*}$ is obtained by multiplying the Moore-Penrose Pseudoinverse of $\mathbf{\Delta}$ and $\mathbf{A}_{\boldsymbol{\sigma}}$, as shown in Equation 23:

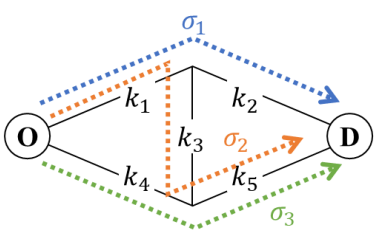
$$\mathbf{A}_{\mathbf{k}}^{*} = \mathbf{\Delta}^{+}\mathbf{A}_{\boldsymbol{\sigma}} \tag{23}$$

In this study, $\mathbf{A}_{\mathbf{k}}^{*}$ is called the link-additive approximation of $\mathbf{A}_{\mathbf{k}}$.

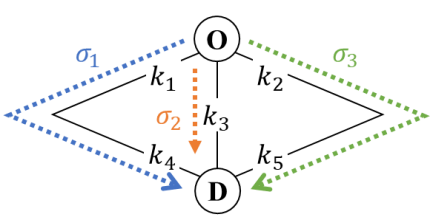
3.3. Link-Additive Approximation by O-D Pairs

3.3.1. The Necessity of Link-Additive Approximation by O-D Pairs

The link-additive approximation method explained in Section 3.2 conducts the approximation for all observed paths simultaneously. However, the link-additive approximation results can be significantly different according to the origins and destinations of those observed paths. For example, on a network shown in Figure 3, there are three paths with different origin-destination (O-D) pairs on each of Figure 3(a) and Figure 3(b).



(a)



(b)

Figure 3. Example of a network on which link-additive approximation results differ by O-D pairs

Values of the path attribute are the same both in Figure 3(a) and 3(b), which are $\mathbf{A}_{\boldsymbol{\sigma}} = [1000\ \ 1000\ \ 1000]^T$. However, $\mathbf{A}_{\boldsymbol{k}}^*$ in Figure 3(a) is calculated as Equations 24-26:

$$\boldsymbol{\Delta} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

$$\boldsymbol{\Delta}^+ = \begin{bmatrix} 3/8 & 1/4 & -1/8 \\ 1/8 & -1/4 & 5/8 \\ -1/4 & 1/2 & -1/4 \\ 5/8 & -1/4 & 1/8 \\ -1/8 & 1/4 & 3/8 \end{bmatrix} \tag{25}$$

$$\mathbf{A}_{\mathbf{k}}^* = \boldsymbol{\Delta}^+ \mathbf{A}_{\boldsymbol{\sigma}} = \begin{bmatrix} 3/8 & 1/4 & -1/8 \\ 1/8 & -1/4 & 5/8 \\ -1/4 & 1/2 & -1/4 \\ 5/8 & -1/4 & 1/8 \\ -1/8 & 1/4 & 3/8 \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 500 \\ 500 \\ 0 \\ 500 \\ 500 \end{bmatrix} \tag{26}$$

Meanwhile, $\mathbf{A}_{\boldsymbol{k}}^*$ in Figure 3(b) is calculated as Equations 27-29:

$$\boldsymbol{\Delta} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{27}$$

$$\mathbf{\Delta}^{+} = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \\ 0 & 0 & 1/2 \end{bmatrix} \tag{28}$$

$$\mathbf{A}_{\mathbf{k}}^{*} = \mathbf{\Delta}^{+}\mathbf{A}_{\boldsymbol{\sigma}} = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 500 \\ 500 \\ 1000 \\ 500 \\ 500 \end{bmatrix} \tag{29}$$

Comparing Equations 26 and 29, the third entries of $\mathbf{A}_{\mathbf{k}}^{*}$ are different: 0 in Figure 3(a) and 1000 in Figure 3(b).

This example shows the necessity of conducting the link-additive approximation by O-D pairs. In this process, the observed paths should be grouped by their origins and destinations. Then the link-additive approximation should be conducted for each group.

3.3.2. Link-Additive Approximation by O-D Pairs

Let $\delta_{o\sigma}$ be a binary variable that equals one if the path σ originates from the origin o and zero otherwise. Also, let $\delta_{d\sigma}$ be a binary variable that equals one if the path σ is destined to the destination d and zero otherwise. Then, for observed paths that originate from o and are destined to d, the link-additive approximation in Equation 17 is rewritten as Equation 30-32:

$$\begin{bmatrix} \delta_{o\sigma_1}\delta_{d\sigma_1}a_{\sigma_1} \\ \delta_{o\sigma_2}\delta_{d\sigma_2}a_{\sigma_2} \\ \vdots \end{bmatrix} \approx \begin{bmatrix} \delta_{o\sigma_1}\delta_{d\sigma_1}\delta_{\sigma_1 k_1} & \delta_{o\sigma_1}\delta_{d\sigma_1}\delta_{\sigma_1 k_2} & \cdots \\ \delta_{o\sigma_2}\delta_{d\sigma_2}\delta_{\sigma_2 k_1} & \delta_{o\sigma_2}\delta_{d\sigma_2}\delta_{\sigma_2 k_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} a_{k_1}^{od} \\ a_{k_2}^{od} \\ \vdots \end{bmatrix} \tag{30}$$

$$\begin{aligned} &\begin{bmatrix} \delta_{o\sigma_1} & & 0 \\ & \delta_{o\sigma_2} & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} \delta_{d\sigma_1} & & 0 \\ & \delta_{d\sigma_2} & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} a_{\sigma_1} \\ a_{\sigma_2} \\ \vdots \end{bmatrix} \\ &\approx \begin{bmatrix} \delta_{o\sigma_1} & & 0 \\ & \delta_{o\sigma_2} & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} \delta_{d\sigma_1} & & 0 \\ & \delta_{d\sigma_2} & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} \delta_{\sigma_1 k_1} & \delta_{\sigma_1 k_2} & \cdots \\ \delta_{\sigma_2 k_1} & \delta_{\sigma_2 k_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} a_{k_1}^{od} \\ a_{k_2}^{od} \\ \vdots \end{bmatrix} \end{aligned} \tag{31}$$

$$\mathbf{\Delta_o \Delta_d A_\sigma} \approx (\mathbf{\Delta_o \Delta_d \Delta})\mathbf{A_k^{od}} \tag{32}$$

where $\mathbf{\Delta_o}$, $\mathbf{\Delta_d}$, and $\mathbf{A_k^{od}}$ are matrices defined as Equations 33-35:

$$\mathbf{\Delta_o} = \begin{bmatrix} \delta_{o\sigma_1} & & 0 \\ & \delta_{o\sigma_2} & \\ 0 & & \ddots \end{bmatrix} \tag{33}$$

$$\mathbf{\Delta_d} = \begin{bmatrix} \delta_{d\sigma_1} & & 0 \\ & \delta_{d\sigma_2} & \\ 0 & & \ddots \end{bmatrix} \tag{34}$$

$$\mathbf{A_k^{od}} = \begin{bmatrix} a_{k_1}^{od} \\ a_{k_2}^{od} \\ \vdots \end{bmatrix} \tag{35}$$

Then the link-additive approximation of $\mathbf{A_k^{od}}$, defined as $\left(\mathbf{A_k^{od}}\right)^*$, is calculated as Equation 36:

$$(\mathbf{A_k^{od}})^* = (\mathbf{\Delta_o \Delta_d \Delta})^+ \mathbf{\Delta_o \Delta_d A_\sigma} \tag{36}$$

where $(\mathbf{\Delta_o \Delta_d \Delta})^+$ is the Moore-Penrose Pseudoinverse of $\mathbf{\Delta_o \Delta_d \Delta}$. When the link-additive approximation is conducted by O-D pairs, Equation 36 is evaluated for every group of observed paths with the same origins and destinations.

Chapter 4. Application

4.1. Overview of the Application

The methodology of this study is applied to a multimodal network and intermodal trip data in Seoul, Korea. The multimodal network consists of road, bus, and rail (subway and metropolitan railroad) networks. The intermodal trip data consist of two data. The first data are the National Household Travel Survey (NHTS) data which describe the information of individual trip stages comprising a traveler's trip: departure points, arrival points, modes, departure times, arrival times, and the order of the trip stages. Since the NHTS data do not include each traveler's specific path, each trip stage of NHTS data must be routed on a network to estimate the RL model. The second data, transit smartcard data, are used for routing transit (bus and rail) trip stages. Smartcard data describe transit users' boarding station and time, alighting station and time, mode type, route number (in case of bus trip stage), and fare of each transit trip stage. While transit trip stages are routed based on smartcard data, road trip stages are routed on a network by the shortest path. The paths of trip stages are concatenated into each single-purpose trip.

This study incorporates two non-link-additive attributes: transit fare and transfer penalty. The link-additive approximation was performed on the fares of transit trip stages so that the approximated fare of each transit link could

be obtained. The link-additive approximation of the transfer penalty is performed by approximating the number of transfers and calculating approximated transfer orders of links.

The application process can be divided into multimodal network construction, preprocessing of intermodal trip data, link-additive approximation, and multimodal RL model estimation, as shown in Figure 4.

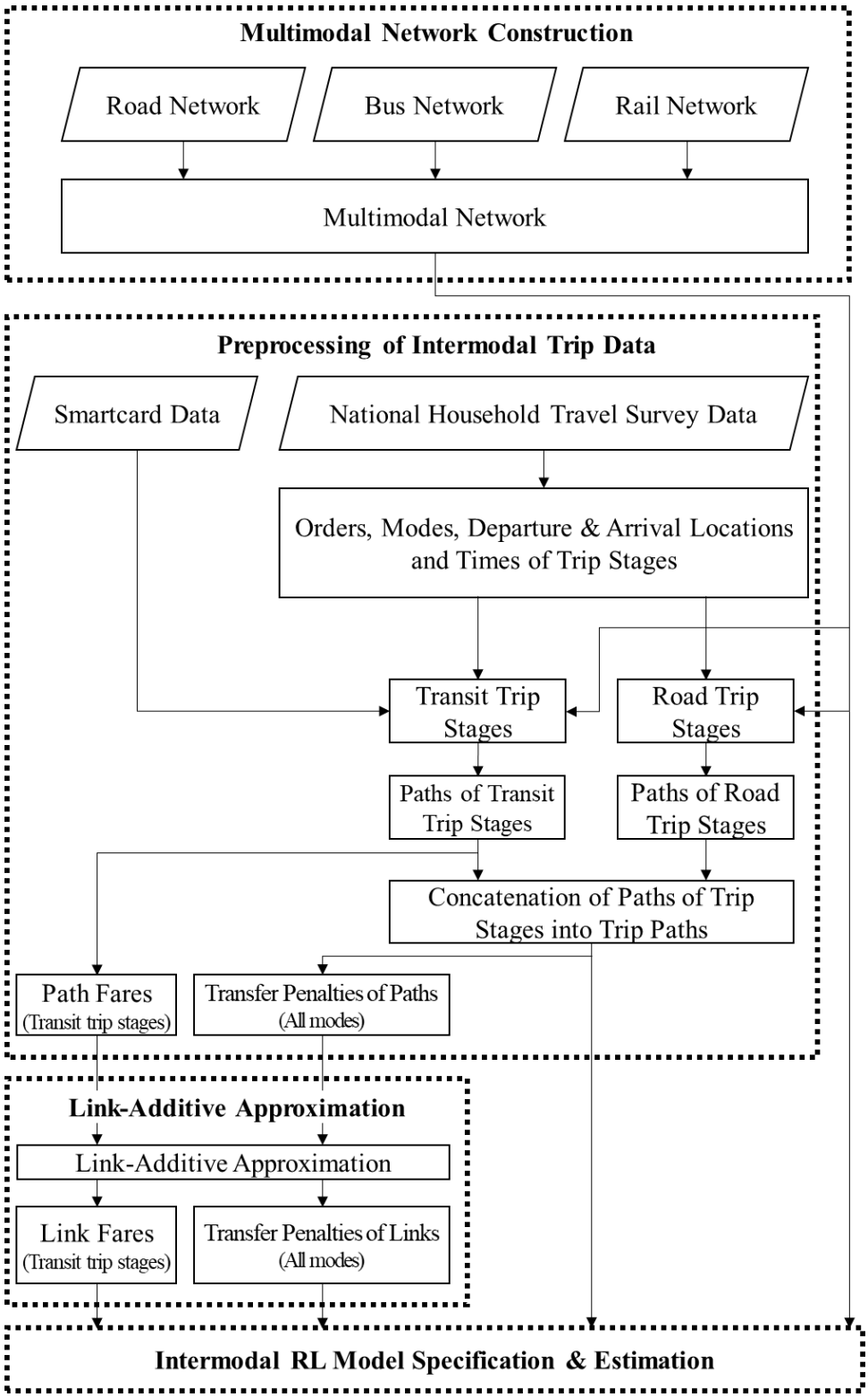


Figure 4. The overview of the application process

4.2. Multimodal Network Construction

4.2.1. Description of Network Data

This study uses the National Standard Node-Link data provided by the National Transport Information Center of Korea (National Transport Information Center, 2016) as the road network. The network contains spatial information of nodes and links of the road network and their corresponding information: e.g., road name, road type, number of lanes, and maximum speed. 47,868 nodes and 127,326 directional links in Seoul and its vicinities were used in this study.

For the bus network, the base information of bus routes and stops and their spatial information were used. The travel time of each link between stops was calculated based on users' boarding and alighting stops and corresponding times which are recorded in smartcard data. 14,588 bus stops and 446,461 directional links of 628 bus routes in Seoul were used in this study.

For the rail network, the base information of subway and metropolitan rail lines and stations and their spatial information were used. The travel time of each link between stations was calculated based on train log data, which record each train's approach, arrival, and departure times by station. Train log data are similar to train schedules, but the former are actual records of train

approach, arrival, and departure times, while the latter are planned timetables in which operational delays are not considered. 701 rail stations and 1,624 directional links of 20 rail lines in Seoul were used in this study.

4.2.2. Integration of Networks into the Multimodal Network

In the National Household Travel Survey data, each trip stage's departure and arrival points are expressed in units of administrative neighborhoods ("Dong" in Korean). Therefore, to complete a trip routing, connectors connecting each neighborhood centroid and the nearest road nodes, bus stops, and rail stations were constructed.

To consider transfers between bus routes, transfer links between adjacent bus stops were constructed. This study uses the density-based spatial clustering of applications with noise (DBSCAN) algorithm (Ester et al., 1996) to identify and cluster adjacent bus stops to construct transfer links between them. Let us assume that $S_i = (x_i, y_i)$ is the spatial coordinate of bus stop i. With a given S_i, its spatial neighborhood set $N_\varepsilon(S_i)$ defined by Equation 37 is classified as the same cluster with S_i by the DBSCAN algorithm

$$N_\varepsilon(S_i) = \{(x_j, y_j) \in S \| \|(x_i, y_i), (x_j, y_j)\| \leq \varepsilon\} \tag{37}$$

where S is the set of bus stops, $\|A, B\|$ is the distance between points A

and B, and ε is the predefined maximum distance (radius) between two points for one to be considered as in the neighborhood of the other. In this study, the radius was set as 100 meters: bus stops within 100 meters of each other were considered transferable. If two or more bus routes share the same bus stop, the bus stop was divided for each bus route to prevent a direct connection between different bus routes. Then transfer links were constructed between those divided stops.

To consider transfers between rail lines, transfer links between transferable rail stations were constructed. If two or more rail lines share the same rail station, the rail station was divided for each rail line to prevent a direct connection between different rail lines. Then transfer links were constructed between those divided stations.

Figures 5-14 show the multimodal network constructed in this study. Figures 5 and 6 show the neighborhood centroids in Seoul, Figures 7 and 8 show the road network used in this study, Figures 9 and 10 show the bus network used in this study, Figures 11 and 12 show the rail network used in this study, and Figures 13 and 14 show the multimodal network consisting of road, bus, and rail networks used in this study.

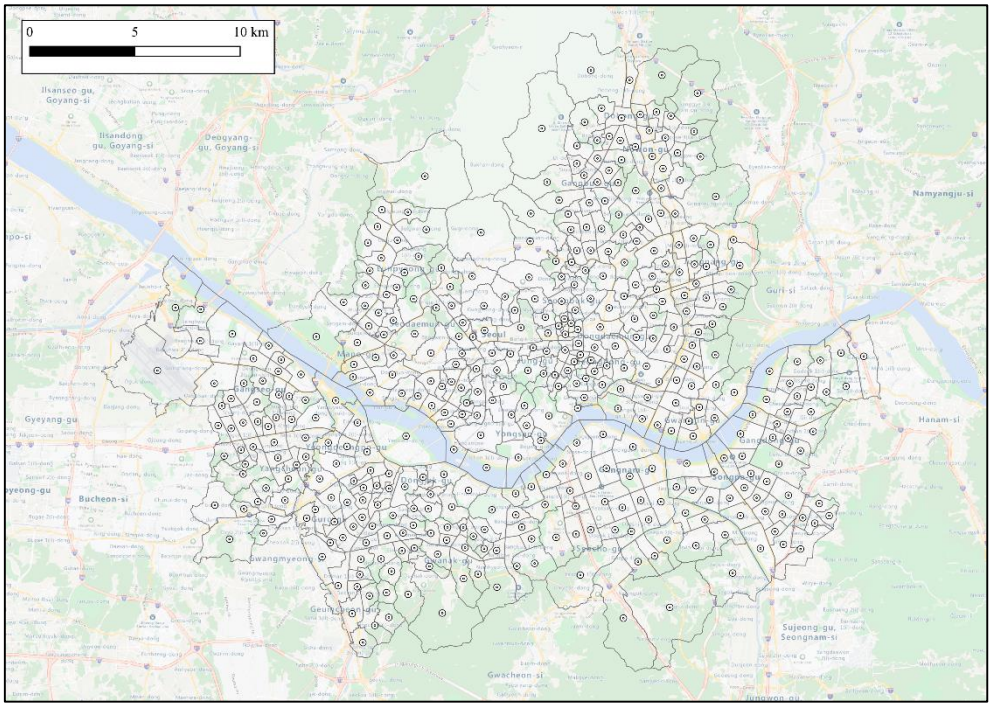


Figure 5. Neighborhood centroids in Seoul

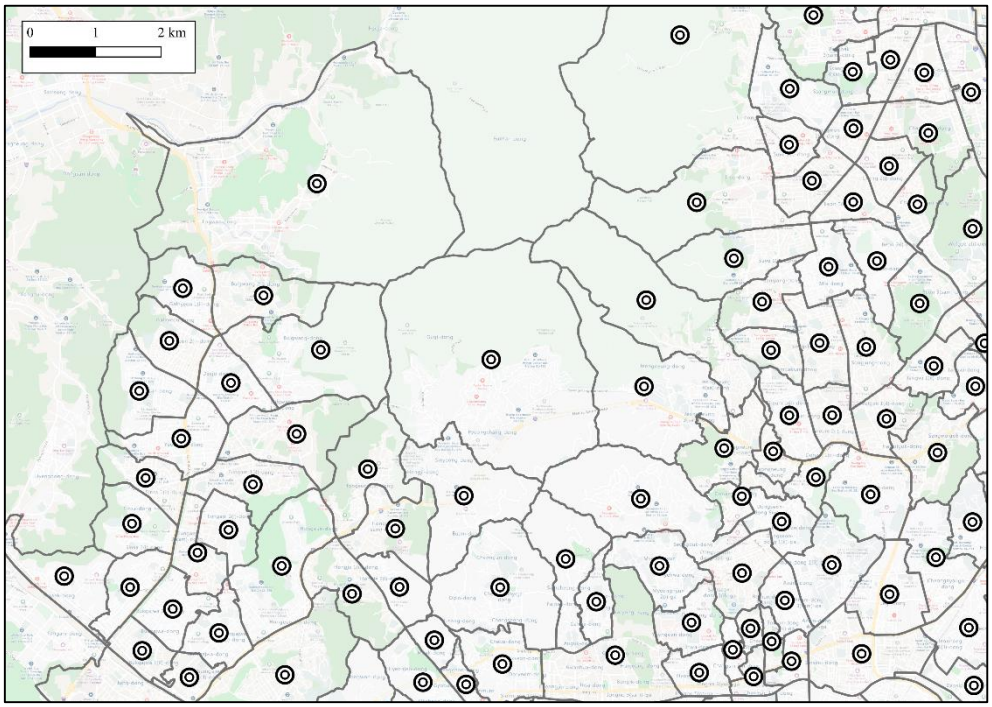


Figure 6. Neighborhood centroids in Seoul (enlarged)

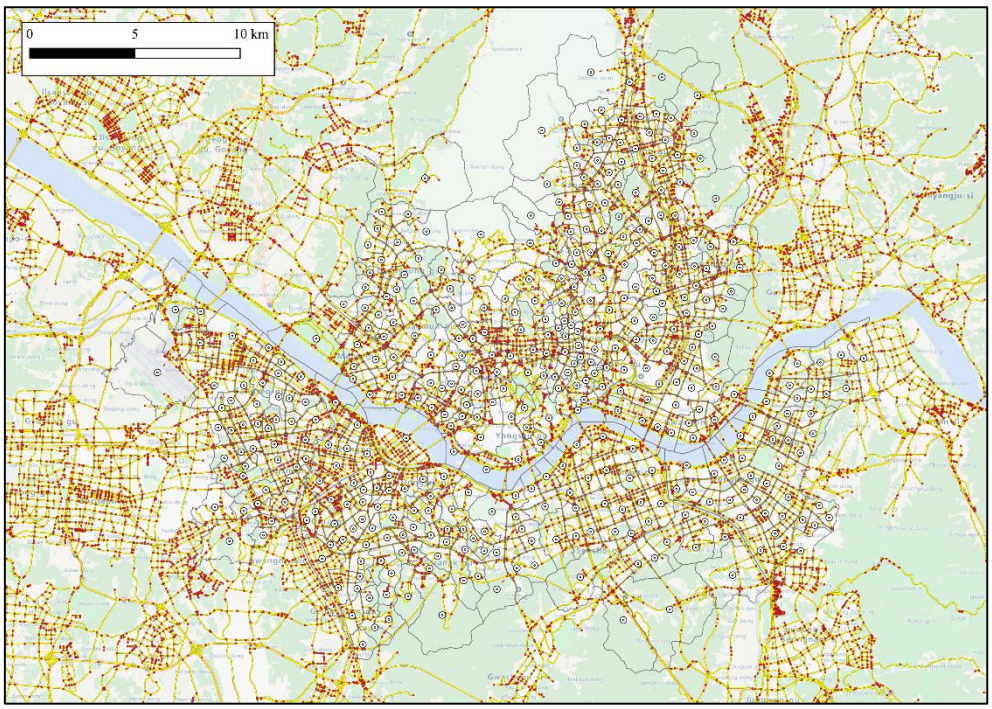
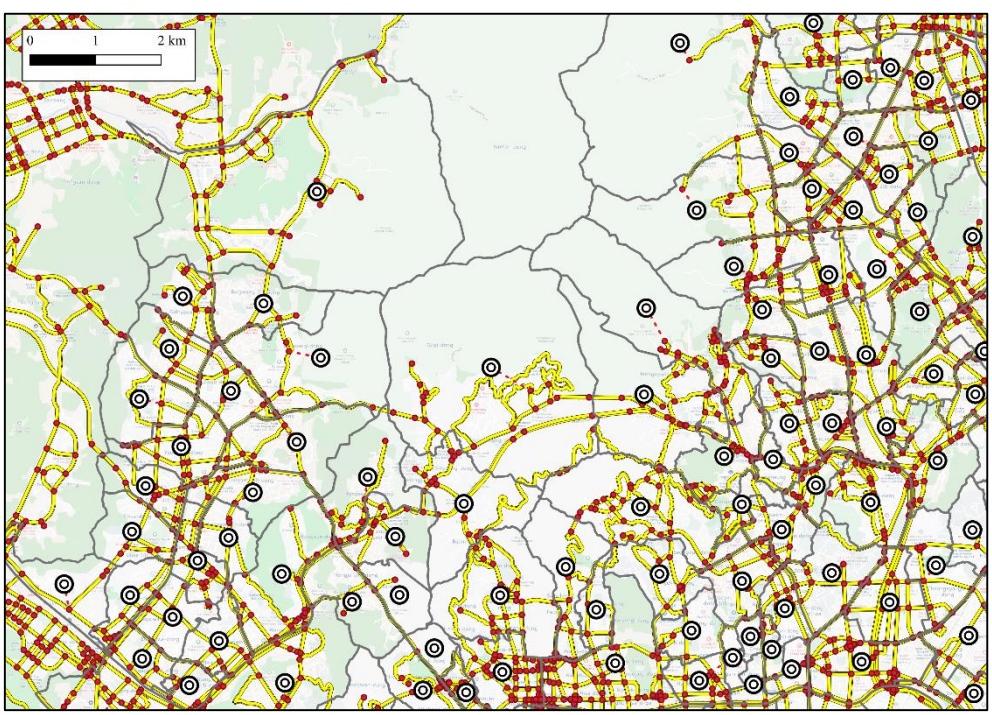


Figure 7. Road network in this study



** Solid lines are road links; dashed lines are connectors.*

Figure 8. Road network in this study (enlarged)

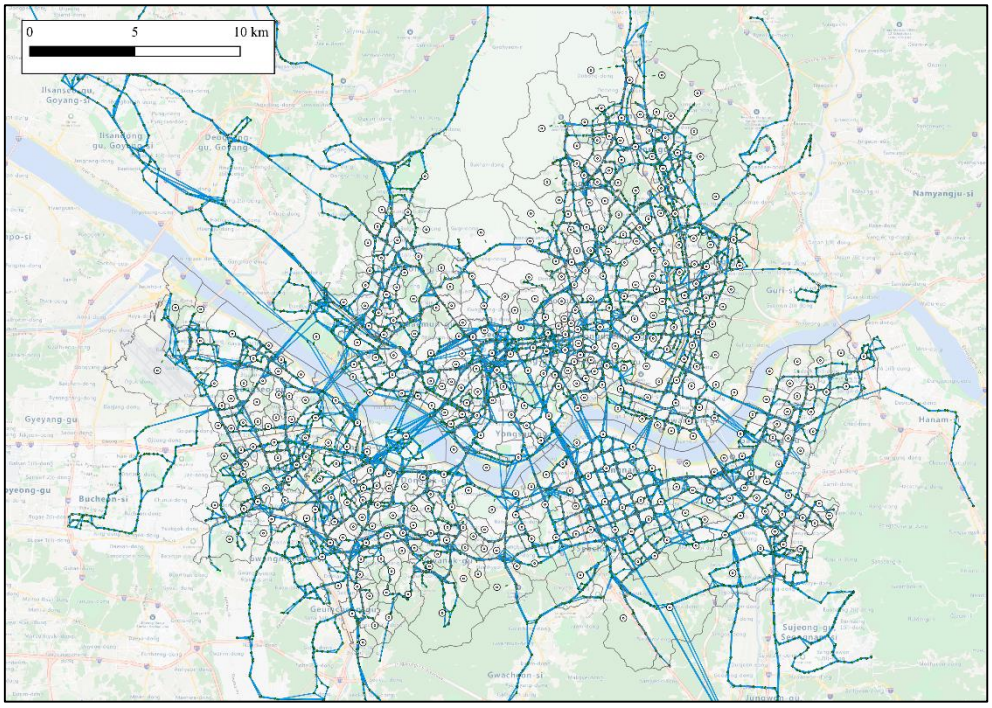
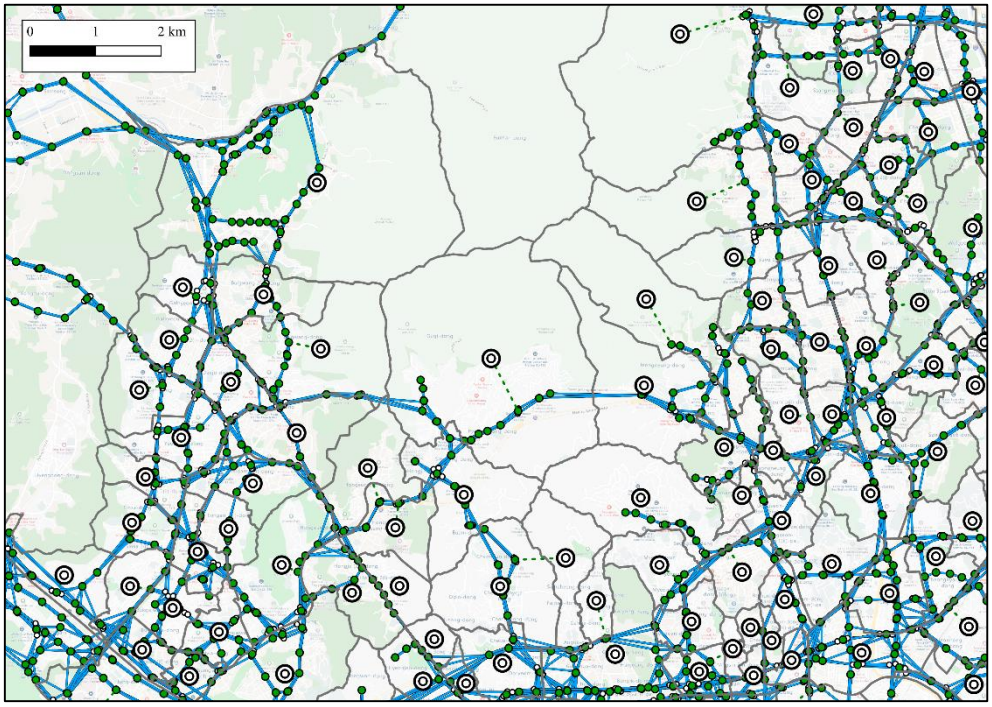


Figure 9. Bus network in this study



** Solid lines are bus links; dashed lines are connectors.*

Figure 10. Bus network in this study (enlarged)

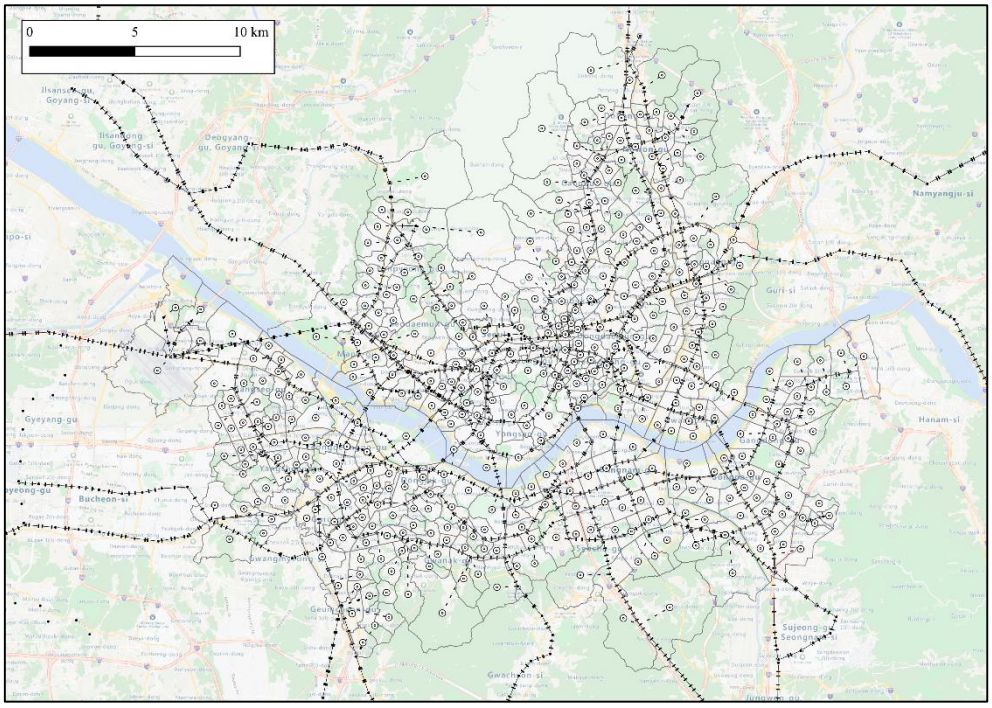
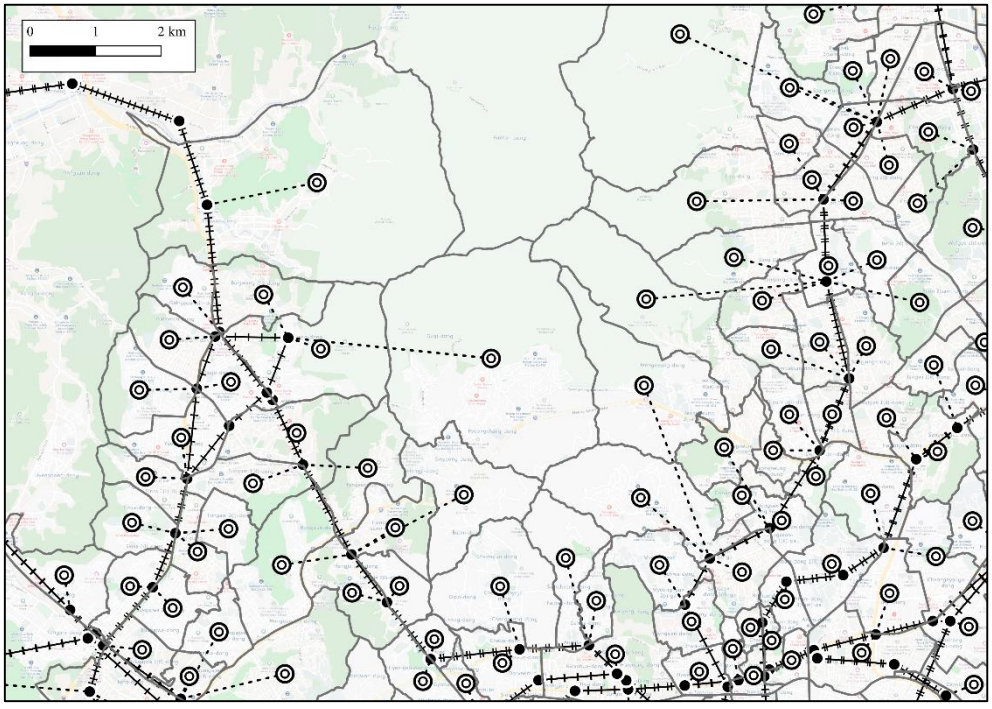


Figure 11. Rail network in this study



** Solid lines are rail links; dashed lines are connectors.*

Figure 12. Rail network in this study (enlarged)

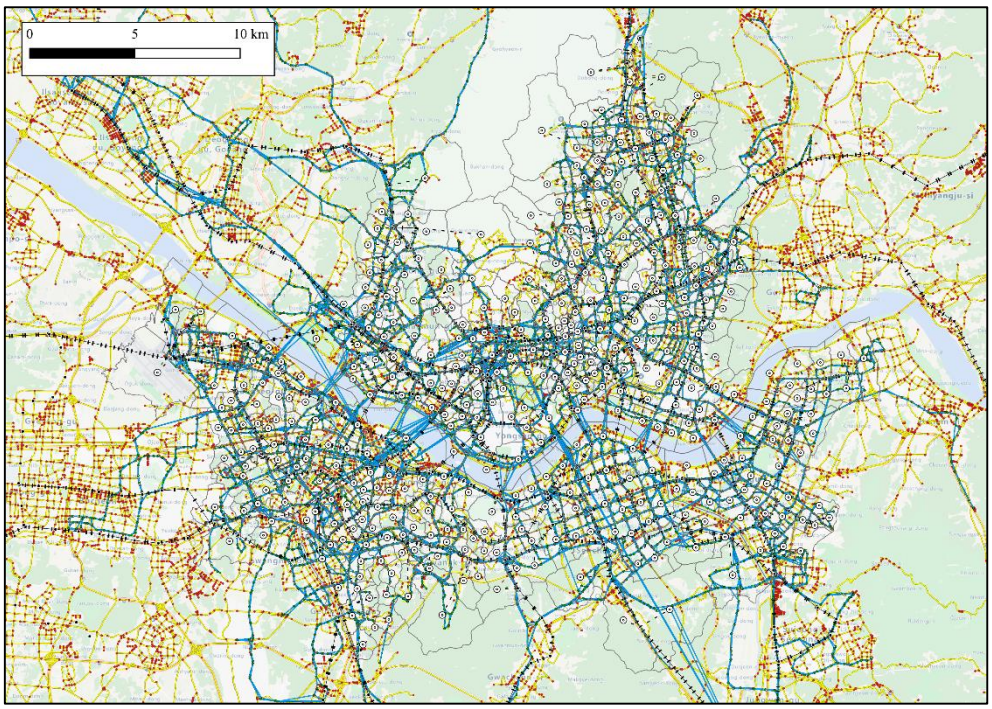
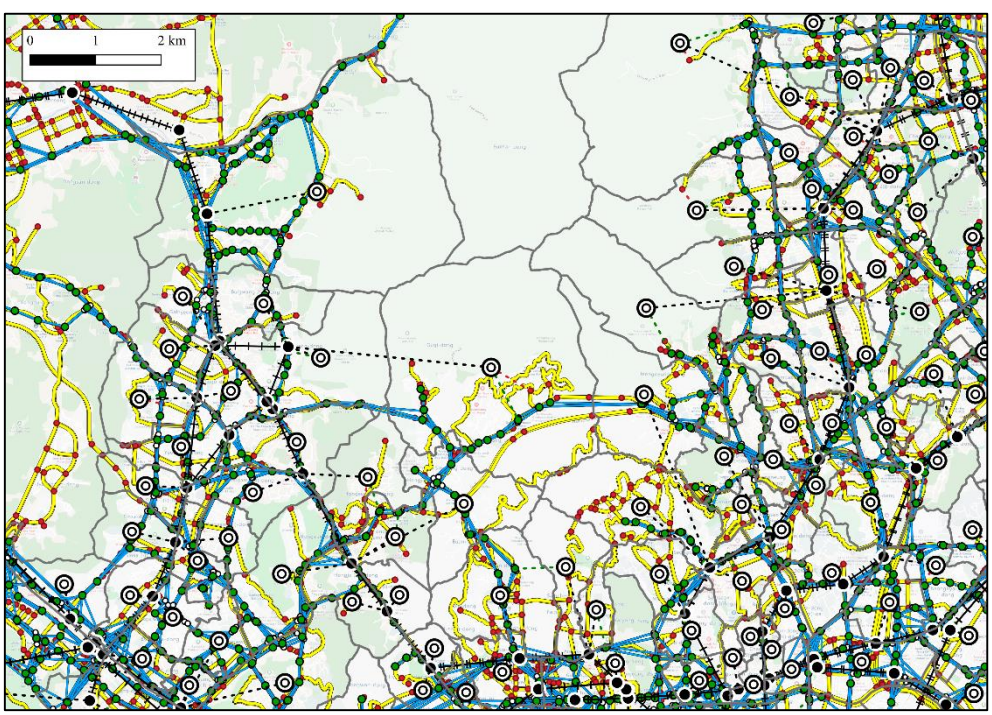


Figure 13. Multimodal network in this study



** Solid lines are links; dashed lines are connectors.*

Figure 14. Multimodal network in this study (enlarged)

4.3. Preprocessing of Intermodal Trip Data

4.3.1 NHTS Data Description

This study used NHTS data collected in 2016 (Korea Transport Database, 2016). Trips having both origins and destinations in Seoul were used. The parameter estimation results of the RL model can differ according to trip purposes. Among several trip purposes of the data, only trips to work were used because the proportion of trips with this purpose is the largest. There are several transportation modes in the NHTS data. This study uses walking, private car, taxi, bicycle, and motorcycle as road modes; urban transit buses, neighborhood buses, and metropolitan buses as bus modes; subway or metropolitan railroad and light rail as rail modes. Because this study focuses on urban commute trips, intercity buses, express buses, other buses, express trains, intercity trains, small trucks, mid-sized trucks, large trucks, airplanes, ships, and others were not used.

Among 32,094 trips and 97,175 trip stages satisfying those conditions, 10 percent of trips (3,209 trips and 9,710 trip stages) were used for model estimation, and the rest were used to test the model. Table 2 describes the information of each column in the NHTS data.

Table 2. Description of the NHTS data (Korea Transport Institute, 2016)

Column	Information	
1	Household information	Household ID
2		Administration district ID
3		Region
4		Total members in the household
5		Total members in the household (≥5 years old)
6		Administrative neighborhood code
7		Type of housing
8		Mean monthly income
9		Ownership of cars
10-21		Car type and manufacture year
22-25		Number of motorcycles
26		Number of motorized or electric bicycles
27		Number of normal bicycles
28		Ownership and type of other vehicles
29		Number of other vehicles
30	Household member information	Relationship to householder
31		Household member type
32		Year of birth
33		Sex
34		Ownership of driver's license
35		Education
36		Administrative neighborhood code of the school
37-38		Occupation
39		Whether the member is a telecommuter

40		Administrative neighborhood code of workplace
41		Number of working days per week
42		Full-time/part-time job
43		Whether the member serves transportation or door-sales businesses
44-47		Year, month, date, and day of the trip
48		Whether the member made any trip
49		The reason why the member did not make any trip
50	Trip information	The order of the trip on the day
51-62		Trip purpose *
63-65		The departure time of the trip
66		Type of origin of the trip
67		Administrative neighborhood code of origin of the trip
68-70		Arrival time of the trip
71		Type of destination of the trip
72		Administrative neighborhood code of destination of the trip
73	Trip stage information	The order of the trip stage during the trip
74-94		Transportation mode **
95-97		The departure time of the trip stage
98		Type of origin of the trip stage
99		Administrative neighborhood code of origin of the trip stage
100-102		Arrival time of the trip stage
103		Type of destination of the trip stage
104		Administrative neighborhood code of destination of the trip stage
105		Occupancy during the trip stage (for private vehicles or taxis)

** Trip purposes are:*

- *To pick up or drop off others*
- *To return to the workplace after an outside work*
- *To return home*
- *To work*
- *To school*
- *To academy*
- *Trips related to work*
- *To shopping*
- *Leisure / Sightseeing*
- *To eat outside*
- *To visit friends or relatives*
- *Others (e.g., religious activities and personal affairs)*

*** Transportation modes are:*

- *Walking (except transfers)*
- *Private cars/private vans (driving self)*
- *Private cars/private vans (driven by another)*
- *Urban/rural transit buses*
- *Neighborhood buses*
- *Metropolitan buses*
- *Intercity buses*
- *Express buses*
- *Other buses (e.g., academy, charter, and tour buses)*
- *Subway or metropolitan railroad*
- *Light rail*
- *Express train*
- *Intercity train*
- *Taxi*
- *Small trucks (<2.5 tons)*
- *Mid-sized or large trucks (≥2.5 tons)*
- *Bicycles*
- *Motorcycles*
- *Airplanes*
- *Ships*
- *Others*

4.3.2. Routing of Trip Stages

Since the NHTS data do not include each traveler's specific path, each trip stage of NHTS data must be routed on a network to estimate the RL model. Road trip stages are routed on the road network by the shortest path. The travel speed of each road link was considered to find the path with the shortest time. For bus trip stages, smartcard data were used to route the trip stages. The bus smartcard data contain each traveler's used bus route, boarding and alighting stops, and boarding and alighting times for every bus boarding and alighting. Therefore, it is relatively easy to route bus trip stages on a bus network. For rail trip stages, most rail smartcard data in Seoul do not contain information regarding transfers between rail lines because users tag their transit cards only at first boarding and last alighting stations, while no tag is required during transfers between most rail lines. Therefore, this study uses the method developed by Lee et al. (2019a, 2019b, 2021) to estimate rail users' paths using the rail smartcard data and the train log data. Rail passengers' boarding and alighting stations and times are matched to the approach, arrival, and departure times of trains. Then the most likely combination of each passenger's boarded rail lines and trains among possible alternative combinations is identified. The combination of rail lines and trains provides the passenger's traveled path on a rail network.

This study used smartcard data collected in 2017. It is assumed that trip

patterns are the same in 2016 (the time scope of NHTS data) and 2017 (the time scope of smartcard and train log data). Table 3 describes the information of each column in the smartcard data. Each row in the smartcard data represents a single trip stage traveled by bus or rail. To identify the overall transit trip of each user, the rows were aggregated using virtual card ID and the number of transfers.

Table 3. Description of smartcard data

Column	Information
1	Row ID
2	Virtual card ID
3	Region code
4	Card classification code
5-6	Vehicle ID
7	Vehicle registration code
8	Vehicle departure time from its depot
9	Vehicle arrival time to its depot
10	Mode ID
11-12	Route ID
13	Transit operator ID
14	Boarding date & time
15	Ticketing date & time
16-17	Boarding station ID
18	Alighting date & time
19-20	Alighting station ID
21	Transaction ID
22	Number of transfers
23	User classification ID
24	Number of users
25	Fare paid at boarding
26	Fare paid at alighting
27	Total travel distance
28	Total travel time

The resultant paths of trip stages are concatenated by trips to construct the overall intermodal path of a trip. The intermodal path is then used for estimating the RL model.

4.4. Incorporation of Non-Link-Additive Attributes

4.4.1. Transit Fare

This study regarded the transit fare as the first non-link-additive attribute and conducted a link-additive approximation to derive approximated fares of transit links. Transit fares are not link-additive if they are not linearly proportional to travel distances, especially under flat or distance-based fare systems. A flat fare system charges a fixed price regardless of the distance traveled between boarding and alighting stations. Distance-based fare system charges a fare based on traveled distance. Most distance-based fare systems are divided into base fares and additional fares increasing by distance. Also, the additional fares of most distance-based fare systems increase stepwise to simplify the fare system. Figure 15 compares three transit fare structures: link-additive, flat, and distance-based fares. Note that the link-additive fare is linearly proportional to travel distance, which is a hypothetical fare system and unlikely to exist in actual transit systems. Also, the distance-based fare shown in Figure 15 consists of base fare and additional fare increasing in a

stepped manner.

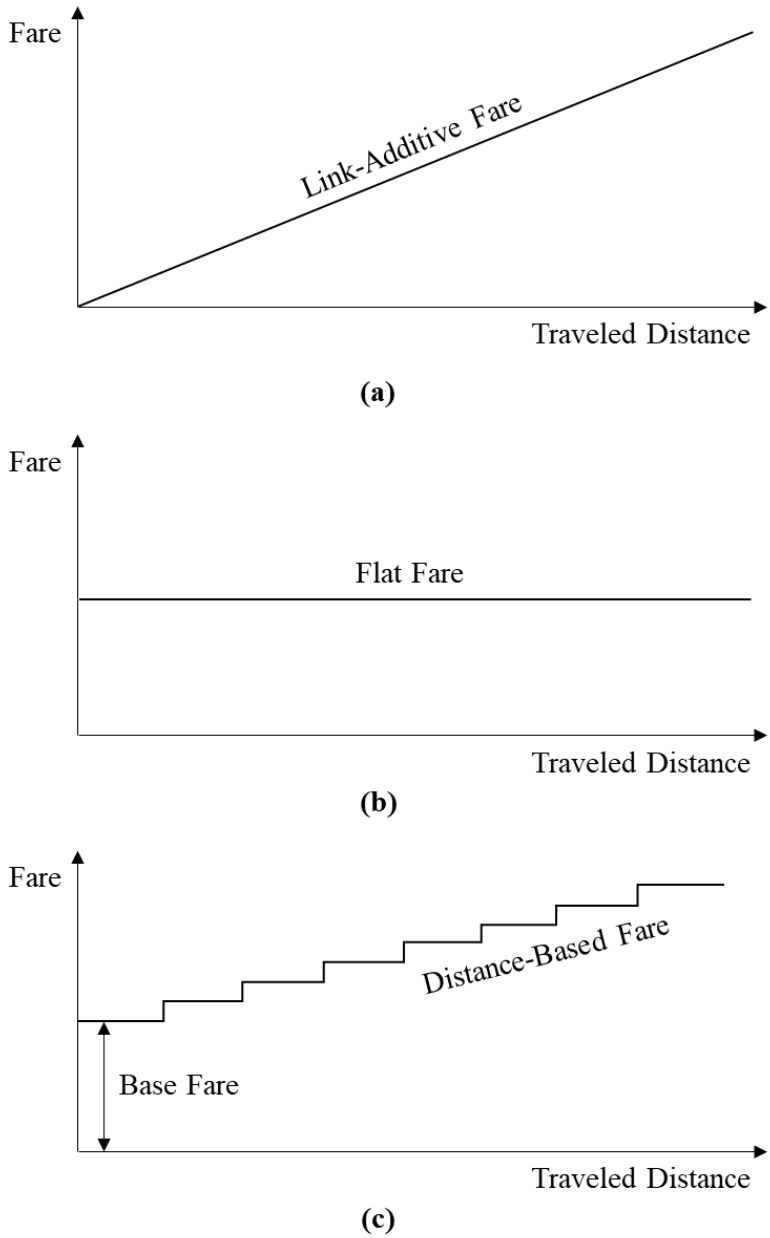


Figure 15. Comparison of transit fare structures: (a) link-additive fare, (b) flat fare, and (c) distance-based fare.

Also, some transit systems provide transfer discounts by reducing fares of consecutive transit trip stages to encourage people to use transit. Under the transfer discount, the fare of a transit trip is cheaper than the sum of individual trip stages forming the overall trip. The existence of transfer discounts is another reason why transit fares are not link-additive. The fare of a trip must equal the sum of fares of its constituent trip stages under a link-additive fare system.

In the transit systems of Seoul, both factors make their fares not link-additive: flat/distance-based fare system and transfer discounts. In Seoul, the bus fare system without transfers is flat regardless of traveled distance, and the rail fare follows a distance-based system with a base fare. If there is any transfer, the fare of the overall trip follows a distance-based system with a transfer discount: the base fare of the trip is the highest base fare of constituent trip stages, not the sum of their base fares. For example, when a traveler uses a neighborhood bus (base fare at 900 won) and subway (base fare at 1,250 won) for 9 kilometers (no additional fare charged), the total fare is 1,250 won (same as the subway's base fare), not 2,150 won (the sum of base fares of the neighborhood bus and the subway). Moreover, since it is impossible to know rail passengers' actual trip path, the rail fare is charged based on the shortest distance between the first boarding and final alighting stations, regardless of the actual path. This is also a factor making rail fares not link-additive. Table 4 shows the summary of the transit fare system in Seoul.

Table 4. Transit fare system in Seoul (Seoul Metropolitan Government, 2022a; Seoul Metropolitan Government, 2022b)

Mode		Fare structure	Base fare*	Additional fare*
Bus (without transfer)	Trunk line & most branch line buses	Flat	1,200 won	-
	Circular line & some branch line buses		1,100 won	
	Metropolitan buses		2,300 won	
	Night buses		2,150 won	
	Neighborhood buses		900 won	
Bus (with transfers)	Without metropolitan buses	Distance-based	The highest base fare of trip stages (≤10 km)	100 won per 5 km
	With metropolitan buses		The highest base fare of trip stages (≤30 km)	100 won per 5 km
Rail (subway and metropolitan rail)		Distance-based	1,250 won** (≤10 km)	10-50 km: 100 won per 5 km >50 km: 100 won per 8 km
Multimodal (bus and rail)	Without metropolitan buses	Distance-based	The highest base fare of trip stages (≤10 km)	
	With metropolitan buses		The highest base fare of trip stages (≤30 km)	

** The fares are as of the temporal scope of this study (2016-2017)*
*** Some rail lines (Uijeongbu Light Rail, Yongin Everline, Shinbundang Line, and Airport Railroad) have surcharges*

4.4.2. Transfer Penalty

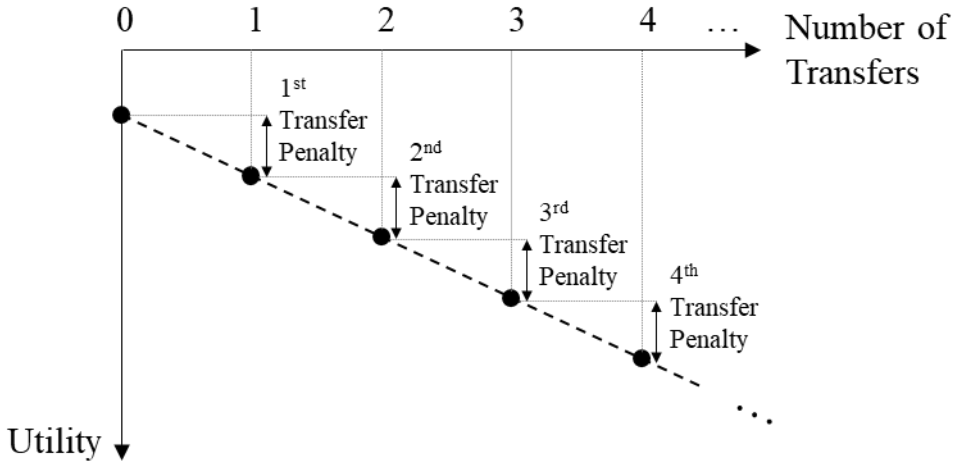
This study also regarded the transfer penalty as the second non-link-additive attribute. A transfer penalty can be defined as the disutility of a transfer, which is usually considered a fixed value (Nielsen et al., 2021). If the transfer penalty per one transfer is fixed, the cumulative transfer penalty is proportional to the number of transfers (Figure 16a). This is the case in most previous studies. However, this study assumes that the transfer penalty per one transfer can change according to how many a traveler has encountered transfers so far, i.e., the order of the transfer. In this case, the cumulative transfer penalty is not proportional to the number of transfers (Figure 16b).

This study considers the cumulative transfer penalty as nonlinear and non-link-additive. Therefore, it is necessary to conduct the link-additive approximation to the cumulative transfer penalty of a given path. However, the exact value of the transfer penalty per one transfer is unknown and must be estimated. Assuming the transfer penalty per one transfer as the constant, which changes according to the transfer order only, the link-additive approximation of cumulative transfer penalty can be calculated as Equations 38-39.

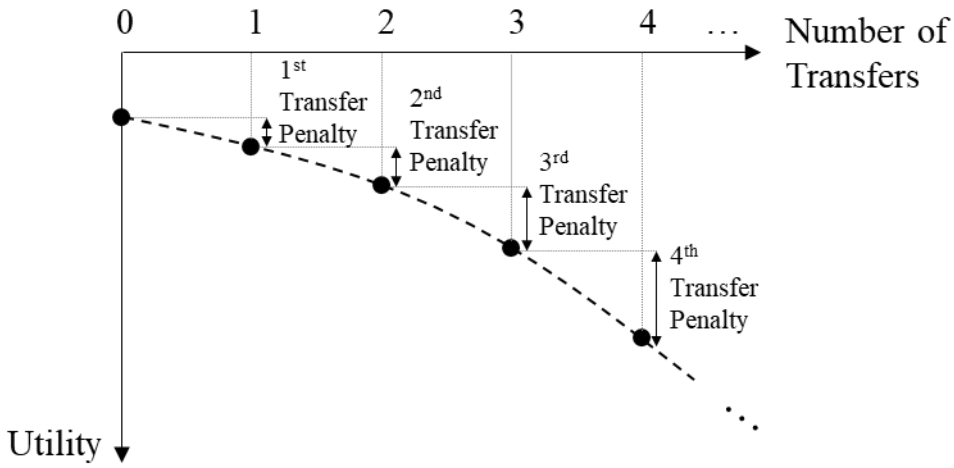
$$TP = \sum_{j} \beta_{Tr,\,j} \delta_{Tr,j} \tag{38}$$

$$\widehat{TP} = \sum_{j} \beta_{Tr,\,j} \widehat{\delta_{Tr,J}} \tag{39}$$

where TP is the cumulative transfer penalty of a path, $\beta_{Tr,\,j}$ is the transfer penalty of the j-th transfer, $\delta_{Tr,j}$ is a binary variable which equals one if the path includes the j-th transfer and zero otherwise. $\widehat{TP}$ is the link-additive approximated transfer penalty and $\widehat{\delta_{Tr,J}}$ is the link-additive approximation of $\delta_{Tr,j}$. As shown in Equations 38-39, to conduct the link-additive approximation of cumulative transfer penalty, it is enough to conduct the approximation of $\delta_{Tr,j}$.



(a)



(b)

Figure 16. Fixed transfer penalty (a) and varying transfer penalty (b)

4.5. Recursive Logit Model Specification

4.5.1. Models According to the Incorporation of Transit Fare

Four models are specified to compare RL models according to whether link-additive approximated fare is included: M0, M1, M2, and M3. Model M0 does not include any fare. Model M1 includes fare proportional to link length. Specifically, the base fare is applied to boarding links for buses, whereas in-vehicle links are assumed free because buses have flat fare structures. For rail links, the base fare of 1,250 won is applied to boarding links, and the additional fare of 20 won/km is applied to in-vehicle links. In both modes, alighting links are assumed free.

Models M2 and M3 include link-additive approximated fares. The link-additive approximation was performed for all observed paths simultaneously in Model M2, while it was performed by O-D pairs in Model M3. Equations 40-43 are specifications of RL models M0, M1, M2, and M3, respectively, which are expressed as instantaneous utility functions in terms of link-additive attributes.

$$\text{M0:}\ v_n(a|k) = \beta_{Const} + \beta_{TT}TT + \beta_{Tr}\delta_{Tr} + \sum_{i \in M} \beta_i \delta_i \tag{40}$$

$$\text{M1: } v_n(a|k) = \beta_{Const} + \beta_{TT}TT + \beta_{Tr}\delta_{Tr} + \sum_{i \in M} \beta_i \delta_i + \delta_{road}\beta_F F_{road} + \delta_{transit}\beta_F F_{length} \tag{41}$$

$$\text{M2: } v_n(a|k) = \beta_{Const} + \beta_{TT}TT + \beta_{Tr}\delta_{Tr} + \sum_{i \in M} \beta_i \delta_i + \delta_{road}\beta_F F_{road} + \delta_{transit}\beta_F \widehat{F_{link}} \tag{42}$$

$$\text{M3: } v_n(a|k) = \beta_{Const} + \beta_{TT}TT + \beta_{Tr}\delta_{Tr} + \sum_{i \in M} \beta_i \delta_i + \delta_{road}\beta_F F_{road} + \delta_{transit}\beta_F \widehat{F_{link}^{od}} \tag{43}$$

where $v_n(a|k)$ is the instantaneous utility of a link, β_{Const} is a link constant, and β_{TT}, β_{Tr}, β_F, and β_i are coefficients. TT is the link travel time, δ_{Tr} is a binary variable which equals one if the link is a transfer link, and zero otherwise. δ_i is a binary variable that equals one if the link is a link of mode i, and zero otherwise. M is a set of possible modes consisting of road modes (private car, taxi, bike, or motorcycle), bus, and rail in this study. F_{road}, which is included only in M1-3, is the out-of-pocket cost of road vehicles: the sum of fuel cost, parking fee, and toll. The fuel cost is assumed to be 140.68 won per kilometer, and the parking fee is assumed to be 2,520 won (Korea Transport Institute, 2021). The toll was applied only for toll roads, and their actual toll prices were applied. F_{length}, which is included in M1, is

the transit link fare which is assumed as proportional to link length. $\widehat{F_{link}}$ and $\widehat{F_{link}^{od}}$, which are included in M2 and M3, are the link-additive approximated fares of transit links or the out-of-pocket costs of road links. For transit links, $\widehat{F_{link}}$ in M2 is the result of link-additive approximation performed for all observed paths at the same time; $\widehat{F_{link}^{od}}$ in M3 is the result of link-additive approximation performed by O-D pairs.

4.5.2. Models According to the Incorporation of Transfer Penalty

To compare RL models according to whether link-additive approximated transfer penalty is included, model M4 was also specified. Model M4 includes the approximation of $\delta_{Tr,j}$ (a binary variable that equals one if the path includes the j-th transfer and zero otherwise), which is essentially the link-additive approximated transfer penalty. The transfer penalty was approximated by O-D pairs only. Also, the approximation was conducted only for transfer links. Equation 44 is the specification of M4.

$$\begin{aligned} \text{M4: } v_n(a|k) &= \beta_{Const} + \beta_{TT}TT + \sum_j \beta_{Tr,j}\widehat{\delta_{Tr,j}^{od}} \\ &+ \sum_{i \in M} \beta_i \delta_i + \delta_{road}\beta_F F_{road} + \delta_{transit}\beta_F \widehat{F_{link}^{od}} \end{aligned} \tag{44}$$

where $\widehat{\delta_{Tr,\,j}^{od}}$ is the link-additive approximation of $\delta_{Tr,j}$ by O-D pairs.

Chapter 5. Results

5.1. Link-Additive Approximation Result

5.1.1. Link-Additive Approximation for All Observed Paths

Figure 17 shows the out-of-pocket cost of road links. Though the out-of-pocket cost is not the result of the link-additive approximation, it is shown here for comparison. Figures 18 and 19 show the link-additive approximation results of transit fares performed for all observed paths simultaneously. Figure 18 shows link-additive approximated bus fares, and Figure 19 shows link-additive approximated rail fares.

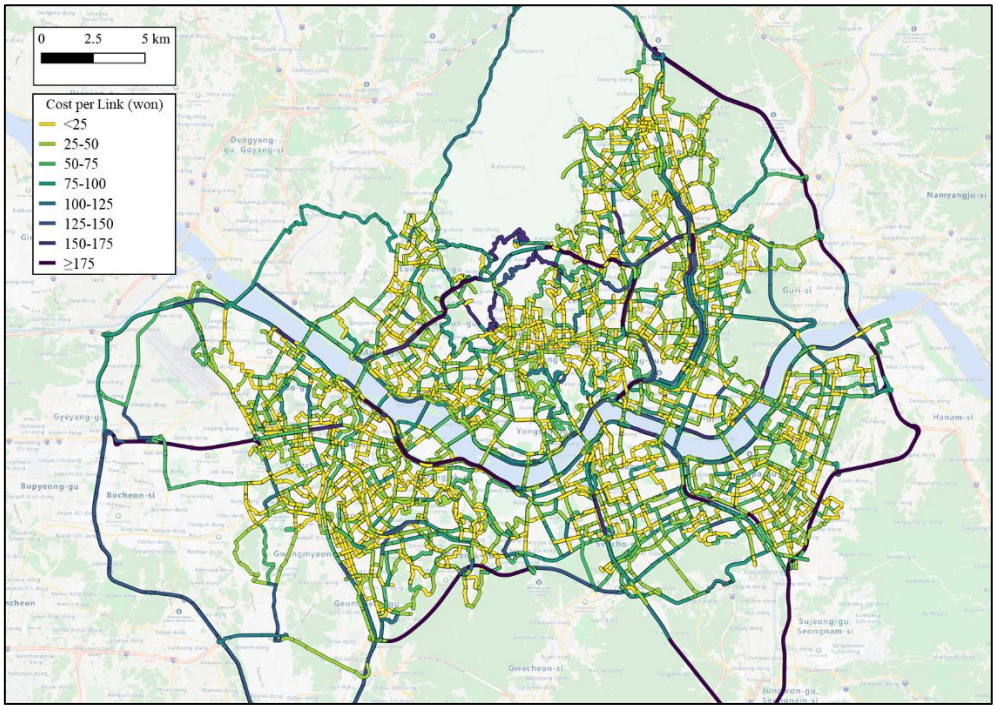


Figure 17. The out-of-pocket cost of road links

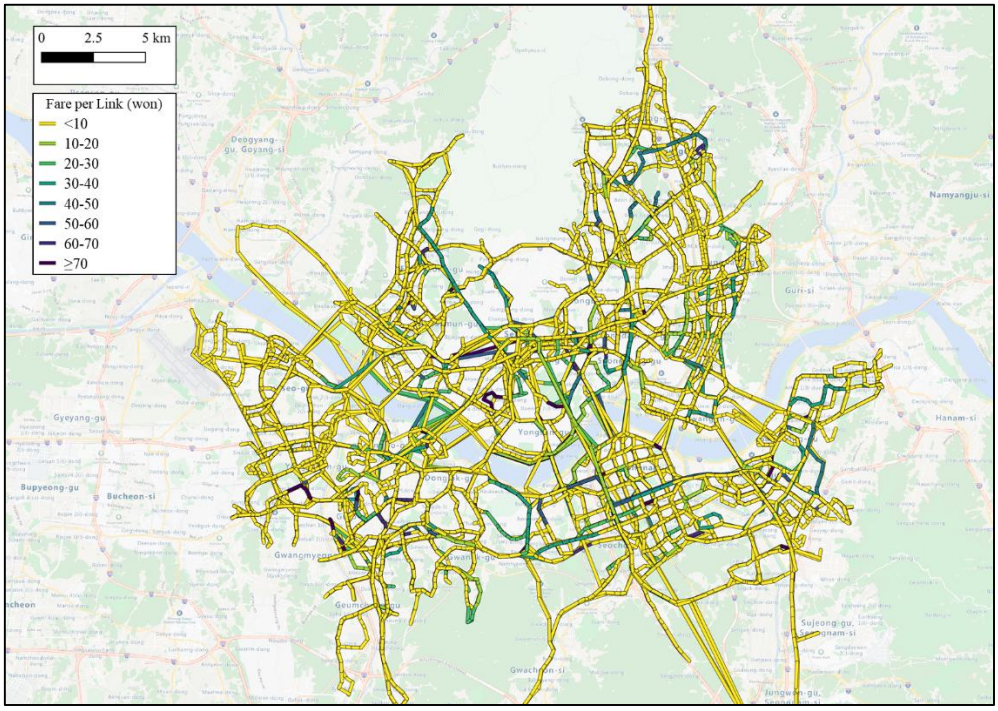


Figure 18. Link-additive approximation result of bus fares (approximation performed for all observed paths)

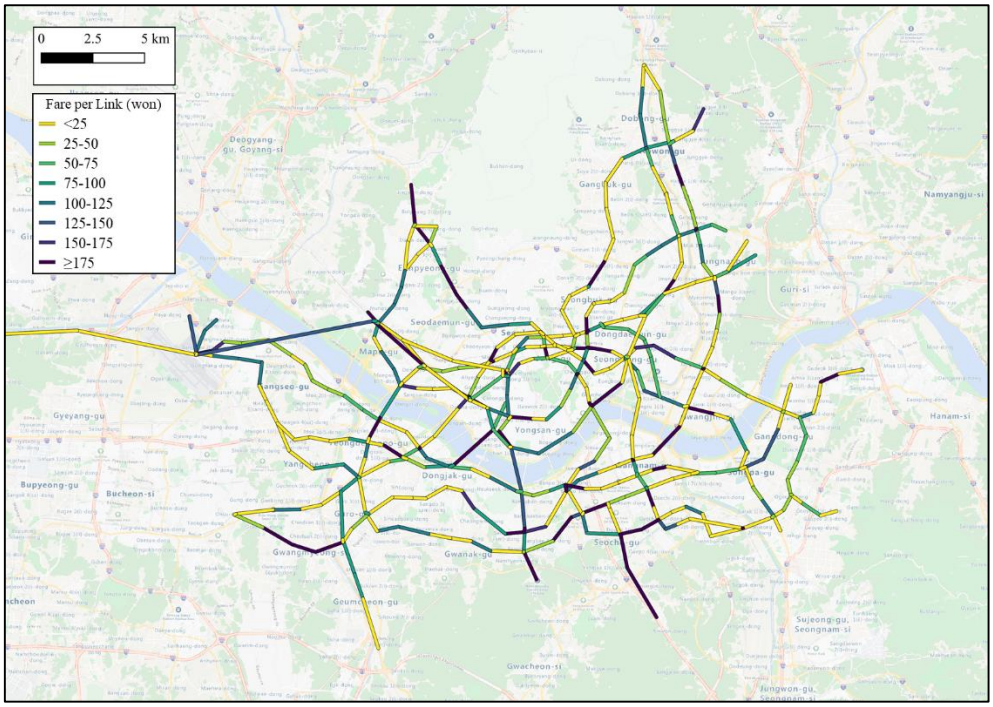


Figure 19. Link-additive approximation result of rail fares (approximation performed for all observed paths)

5.1.2. Link-Additive Approximation by O-D Pairs

The link-additive approximated results performed by O-D pairs are shown by some example O-D pairs in Figures 20-27. Figures 20-23 show link-additive approximated bus fares, and Figures 24-27 show link-additive approximated rail fares. Note that the link-additive approximation is performed only for links included in any path connecting those O-D pairs. Therefore, links shown in Figures 20-27 also show alternative paths between the O-D pairs.

As shown in Figures 20-27, multiple paths can exist between those O-D

pairs. Some paths have significant detours compared to the shortest paths. For example, in Figure 20, one of the paths is stretched southeastward of the map and retraces its path backward to reach its destination. Because transit trips are routed based on the smartcard data of actual users, those paths with unreasonably long detours are caused by users who are detoured in such a manner. Most of those detours are actually two different trips connected within a short time. The smartcard system in Seoul regards two consecutive transit trip stages as a transfer if the time difference between first alighting and second boarding is less than 30 minutes. In this case, two different transit trips can be connected by a transfer and show a significant detour. In the actual RL model estimation, those transit trips with significant detours have a negligible effect because the proportion of those trips is very low.

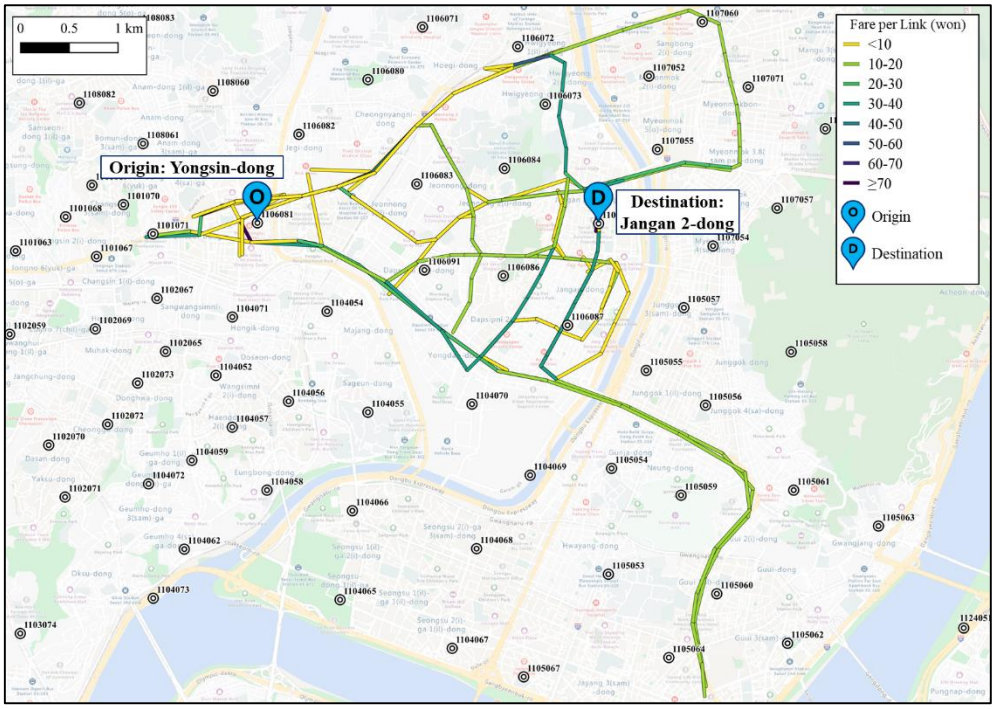


Figure 20. Link-additive approximated bus fare from Yongsin-dong to Jangan 2-dong

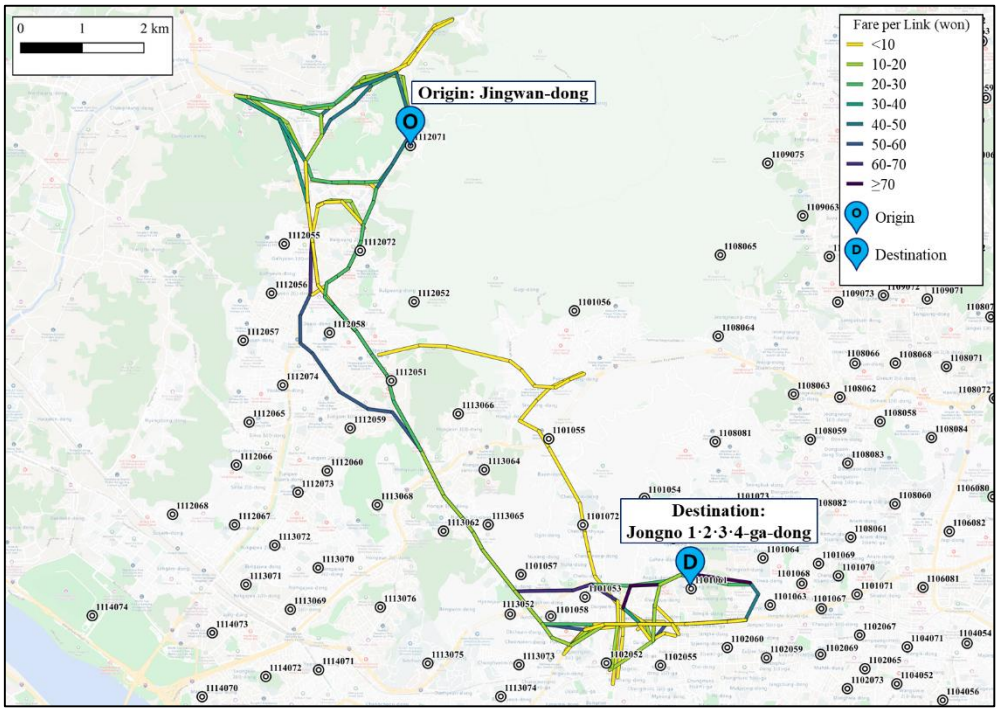


Figure 21. Link-additive approximated bus fare from Jingwan-dong to Jongno 1 · 2 · 3 · 4-ga-dong

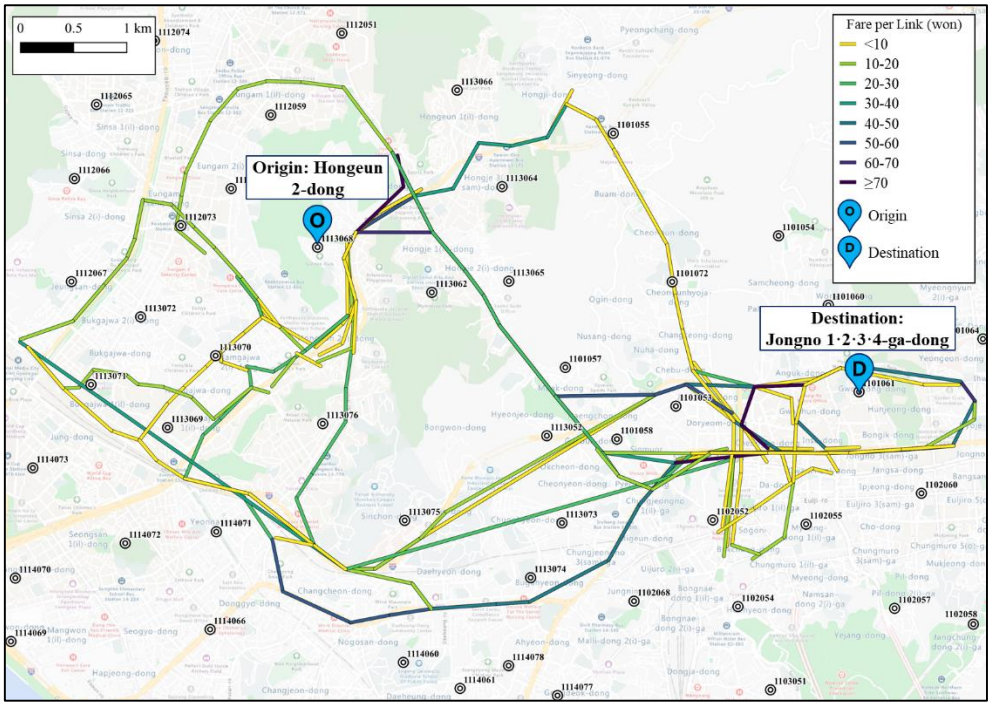


Figure 22. Link-additive approximated bus fare from Hongeun 2-dong to Jongno 1 · 2 · 3 · 4-ga-dong

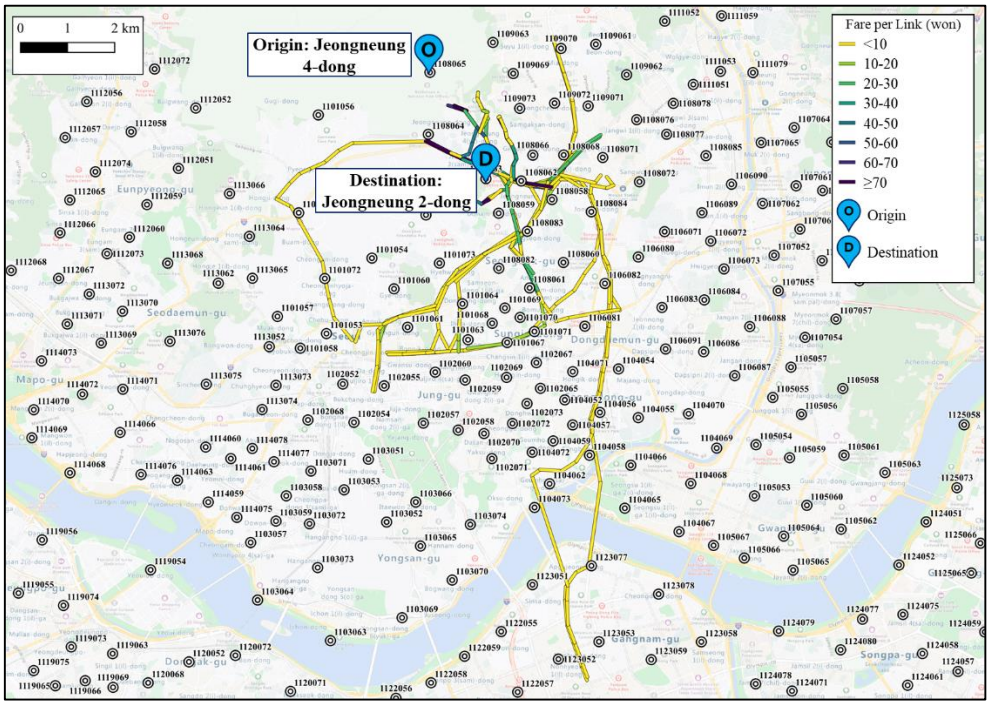


Figure 23. Link-additive approximated bus fare from Jeongneung 4-dong to Jeongneung 2-dong

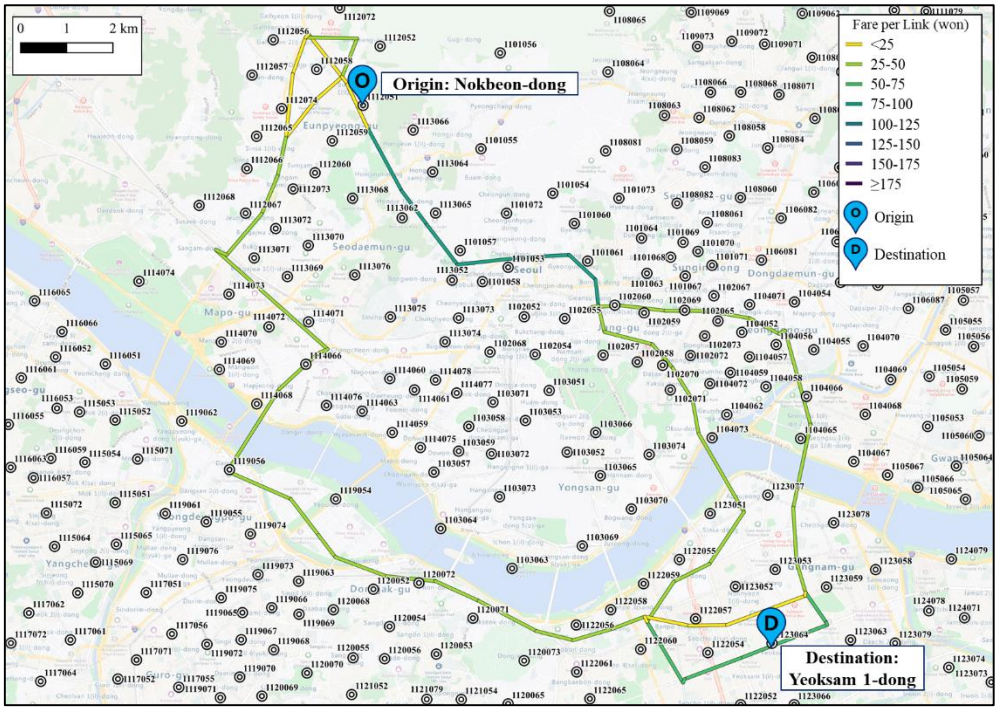


Figure 24. Link-additive approximated rail fare from Nokbeon-dong to Yeoksam 1-dong

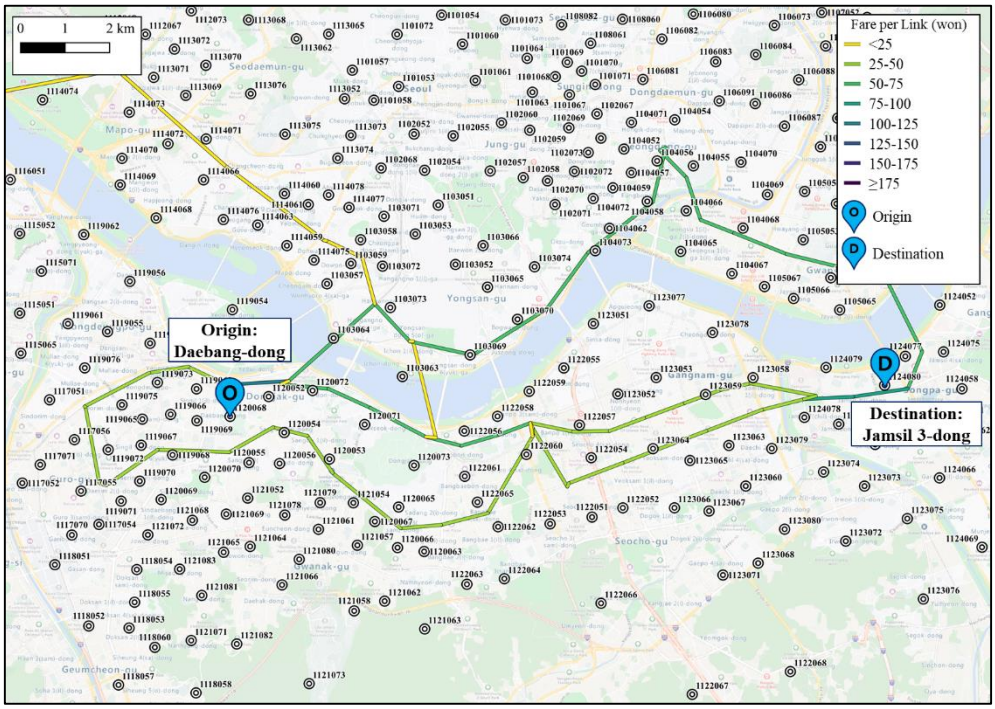


Figure 25. Link-additive approximated rail fare from Daebang-dong to Jamsil 3-dong

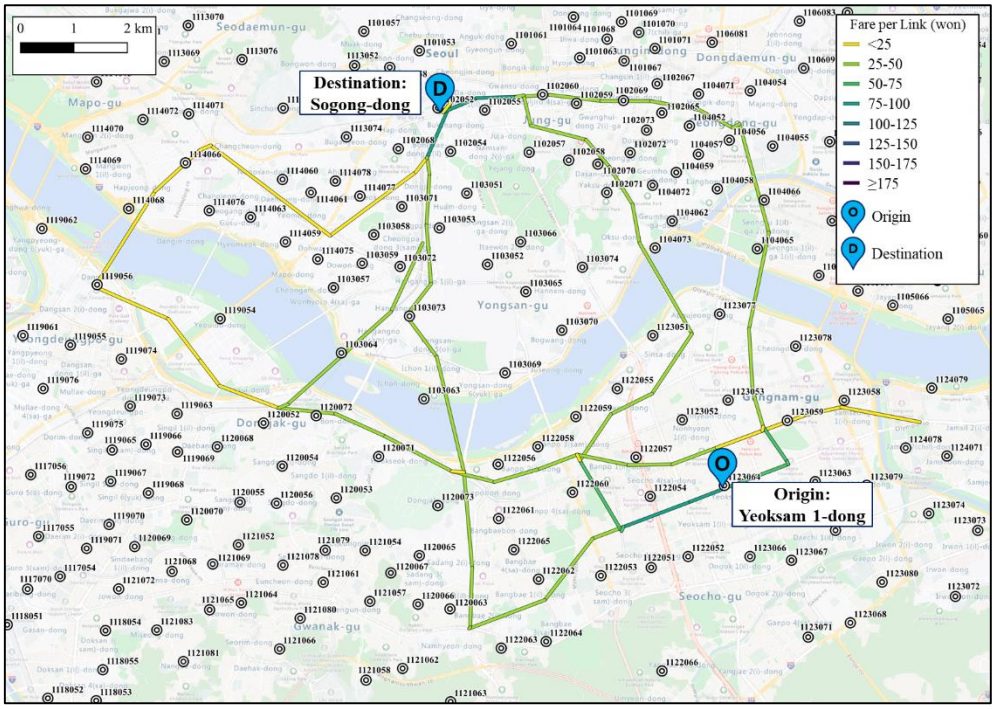


Figure 26. Link-additive approximated rail fare from Yeoksam 1-dong to Sogong-dong

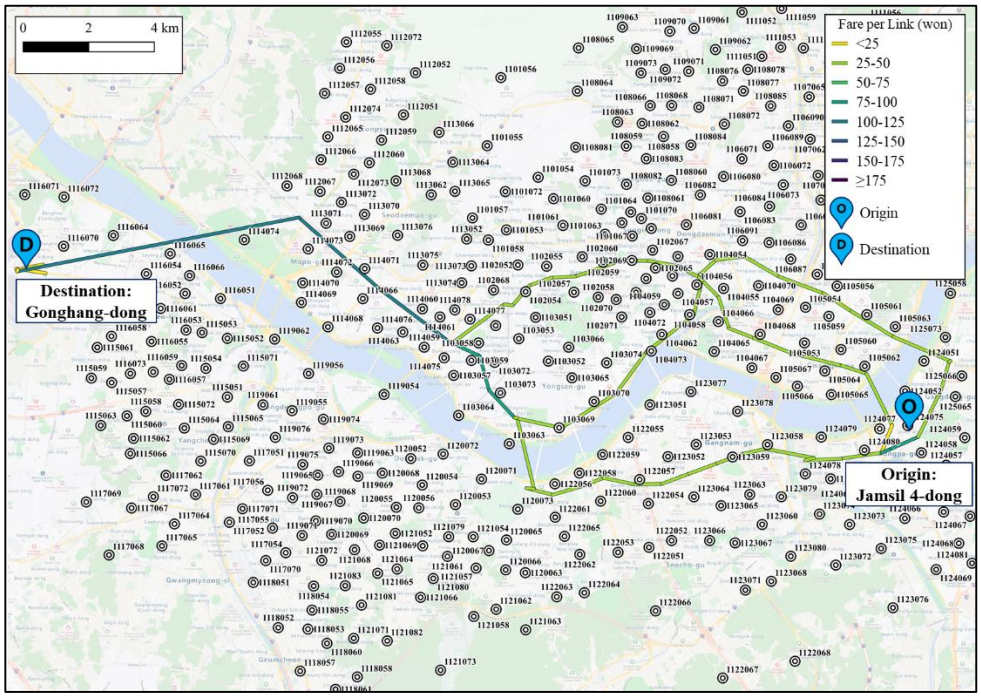


Figure 27. Link-additive approximated rail fare from Jamsil 4-dong to Gonghang-dong

5.1.3. Accuracy of Link-Additive Approximations

The approximated link fares are not actual fares charged to travelers passing a link. Therefore, it must be verified whether the link-additive approximation accurately reflects actual fares on individual links. To evaluate the accuracies of two link-additive approximation methods proposed here, path fares of transit trip stages are reconstructed based on approximated link fares and compared to actual path fares. Table 5 shows the result of accuracies of two link-additive approximation methods: link-additive approximation for all observed paths and link-additive approximation by O-D pairs. There were 3,333 trip stages of which path fares were reconstructed and compared, and their mean path fare was 1,508 won, which is the same for actual and reconstructed ones.

This study used three indices to evaluate the accuracies: root-mean-squared error (RMSE, Equation 45), mean absolute error (MAE, Equation 46), and mean absolute percentage error (MAPE, Equation 47).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \hat{f}(x_i)\right)^2}{n}} \tag{45}$$

$$MAE = \frac{\sum_{i=1}^{n}\left|y_i - \hat{f}(x_i)\right|}{n} \tag{46}$$

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{y_i - \hat{f}(x_i)}{y_i} \right|}{n} \times 100 \ \ (\%) \tag{47}$$

where n is the number of data (the number of alternative intermodal paths), y_i is the actual dependent variable (the actual probability of choosing a certain intermodal path), and $\hat{f}(x_i)$ is the predicted dependent variable (the predicted probability of choosing a certain intermodal path).

Table 5. Accuracies of link-additive approximation methods

		Approximation for all observed paths	Approximation by O-D pairs
Number of transit trip stages		3,333	
Actual mean path fare		1,508 won	
Reconstructed mean path fare		1,508 won	1,508 won
Error	RMSE	143.9 won	120.4 won
	MAE	96.2 won	53.9 won
	MAPE	6.78%	4.23%

As shown in Table 5, the approximation by O-D pairs is more accurate compared to the approximation for all observed paths. The error decreases by 44.0% in the approximation by O-D pairs compared to the approximation for all observed paths in terms of MAE.

5.2. RL Model Estimation Result

5.2.1. Parameter Estimation Results

The parameter estimation result of M0, M1, M2, and M3 is given in Table 6. As shown in Table 6, the coefficients of link travel time, transfer dummy, and link fare (or link cost) are negative. This is reasonable because travelers prefer paths with shorter travel time, fewer transfers, and cheaper costs. Also, a model with a higher log-likelihood value or lower AIC value has a better goodness-of-fit. In terms of goodness-of-fit, M3 has the best goodness-of-fit, followed by M2, M1, and M0.

It is shown that by including the transit fare, a non-link-additive attribute, the goodness-of-fit of the RL model can be improved. Especially, model M3, which has the link-additive approximated fare by O-D pairs, has better goodness-of-fit than model M2, which has the fare approximated for all observed paths.

Table 6. Parameter estimation result of RL models (M0-M3)

Parameters / Variables		M0	M1	M2	M3
β_{Const}	Constant	-2.4352	-2.4703	-2.4791	-2.3356
β_{TT}	Link travel time (minutes)	-0.1438	-0.2079	-0.2239	-0.2110
β_{Tr}	Transfer dummy	-4.7638	-4.7674	-4.7683	-4.6974
β_{F}	Link fare or cost (100 won)	-	-0.1100	-0.0996	-0.0579
β_{bus}	Bus link dummy	1.0541	1.0411	1.0379	1.0212
β_{rail}	Rail link dummy	0.4130	0.4091	0.4081	0.3994
β_{road}	Road link dummy	1.2972	1.2686	1.2614	1.2311
Log-likelihood		-20.036	-18.120	-17.641	-13.957
AIC*		52.072	49.840	49.282	41.913

** AIC: Akaike information criterion; AIC= −2 ×(log-likelihood)+2 × (number of variables).*

The parameter estimation result of M3 and M4 is given in Table 7. As shown in Table 7, the goodness-of-fit of M4 is better than M3 in terms of log-likelihood and AIC. Also, the absolute values of coefficients $\beta_{Tr,1}$, $\beta_{Tr,2}$, $\beta_{Tr,3}$, $\beta_{Tr,4}$ increases by the order of transfer, which means that the transfer penalty increases by the number of transfers. In other words, a traveler regards the transfer more uncomfortable if the number of transfer increases.

Table 7. Parameter estimation result of RL models (M3 and M4)

Parameters / Variables		M3	M4
β_{Const}	Constant	-2.3356	-2.3352
β_{TT}	Link travel time (minutes)	-0.2110	-0.2112
β_{Tr}	Transfer dummy	-4.6974	-
$\beta_{Tr,1}$	1st transfer dummy	-	-3.0211
$\beta_{Tr,2}$	2nd transfer dummy	-	-4.2434
$\beta_{Tr,3}$	3rd transfer dummy	-	-5.2595
$\beta_{Tr,4}$	4th transfer dummy	-	-6.1155
β_{F}	Link fare or cost (100 won)	-0.0579	-0.0579
β_{bus}	Bus link dummy	1.0212	1.0203
β_{rail}	Rail link dummy	0.3994	0.3996
β_{road}	Road link dummy	1.2311	1.2319
Log-likelihood		-13.958	-10.904
AIC*		41.913	41.808

** AIC: Akaike information criterion; AIC= −2 ×(log-likelihood)+2 × (number of variables).*

5.2.2. Discussion of Parameter Estimation Results

From the parameter estimation results, useful indices and policy implications can be derived. Especially, the trade-off between trip-related attributes, such as the value of time (trade-off between time and cost) and the value of transfer (trade-off between time and transfer), can be easily derived from the estimated parameters of the RL model. The value of time (VOT), in terms of trip cost, can be derived as shown in Equation 48:

$$VOT(\text{won/hour}) = \frac{\beta_{TT}(\text{min}^{-1})}{\beta_F((100\ \ \text{won})^{-1})} \times 100 \times 60 \tag{48}$$

This study's value of transfer (VOTR) is defined as the equivalent travel time per transfer. VOTR can be derived as shown in Equation 49:

$$VOTR(\text{min/transfer}) = \frac{\beta_{Tr}(\text{transfer}^{-1})}{\beta_{TT}(\text{min}^{-1})} \tag{49}$$

The results of VOT and VOTR are shown in Table 8.

Table 8. The results of values of time and transfer

Parameters / Variables		M0	M1	M2	M3	M4
β_{TT}	Link travel time (minutes)	-0.1438	-0.2079	-0.2239	-0.2110	-0.2112
β_{Tr}	Transfer dummy	-4.7638	-4.7674	-4.7683	-4.6974	-3.0211 (1st) -4.2434 (2nd) -5.2595 (3rd) -6.1155 (4th)
β_F	Link fare or cost (100 won)	-	-0.1100	-0.0996	-0.0579	-0.0579
VOT	Value of time (won/hour)	-	11,340	13,488	21,865	21,886
VOTR	Value of transfer (min/transfer)	33.1	22.9	21.3	22.3	14.3 (1st) 20.1 (2nd) 24.9 (3rd) 29.0 (4th)

The VOT and VOTR in previous studies are shown in Table 9. The difference between the previous and this studies' VOT and VOTR can be due to the difference in model structures, variable selection, regions, and trip purposes.

Table 9. The values of time and transfer in previous and this studies

Value of Time (VOT)

Study	Region	VOT	Remark
Ministry of Land, Infrastructure, and Transport (2022)	Seoul, Korea	6,355-12,322 won/hour (value in 2020)	All purposes
This Study	Seoul, Korea	11,340-21,886 won/hour (value in 2017)	Trips to work

Value of Transfer (VOTR)

Study	Region	VOTR (equivalent in-vehicle time per transfer, minutes)	Remark
Yoo (2015)	Seoul, Korea	11.24×(transfer time)	
Garcia-Martinez et al. (2018)	Madrid, Spain	15.2 + 1.14×(waiting time)+0.79×(walking time)	
Nielsen et al. (2021)	Copenhagen, Denmark	7.92+0.15×(waiting time)+0.69×(walking time)	Commute trips
Jara-Diaz et al. (2022)	Vitoria, Spain	18.4	
This Study	Seoul, Korea	14.3-33.1	Trips to work

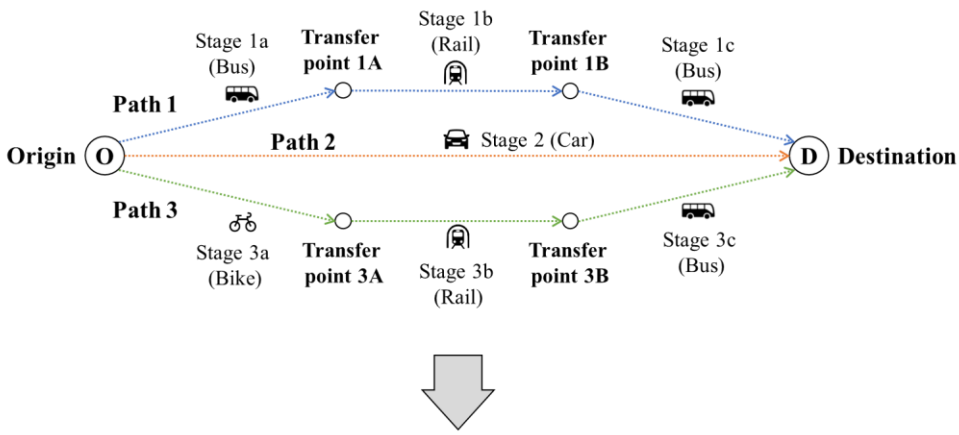
5.3. RL Model Test Result

5.3.1. Test of RL Models

To prove that including transit fare to the RL model improves the model's applicability, not only its goodness-of-fit, the test of RL models was also conducted. To test the accuracy of RL models, this study used the rest of NHTS data (28,885 trips and 87,465 trip stages) which were not used for model estimation. Because intermodal paths constructed in this study are not actual paths due to the lack of spatial trajectory information in the NHTS data, the orders, modes, and transfer points of trip stages comprising a trip are used for the prediction instead. The RL model is stochastic rather than deterministic, which predicts the probability of using a certain path among possible alternative paths. Therefore, this study compared the predicted and actual probabilities of choosing alternative paths of intermodal trips according to O-D pairs. The test set of trips was classified into 7,145 groups according to origins and destinations, and 15,313 alternative intermodal paths according to orders, modes, and transfer points. The prediction accuracy was evaluated by three indices: root-mean-squared error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

Figure 28 shows an example of alternative paths between a certain O-D pair and compares their actual and predicted path choice probabilities. In

Figure 28, there are three alternative paths. Path 1 consists of three stages: Stage 1a by bus, Stage 1b by rail, and Stage 1c by bus. The transfer point between Stage 1a and Stage 1b is Transfer Point 1A, and the transfer point between Stage 1b and Stage 1c is Transfer Point 1B. Path 2 consists of a single stage: Stage 2 by car. Path 3 consists of three stages: Stage 3a by bike, Stage 3b by rail, and Stage 3c by bus. The transfer point between Stage 3a and Stage 3b is Transfer Point 3A, and the transfer point between Stage 3b and Stage 3c is Transfer Point 3B. Each path can be characterized by a vector of the orders, modes, and transfer points of constituent trip stages. In this manner, Path 1 is characterized by a vector $P1 = (O, Bus, 1A, Rail, 1B, Bus, D)$. Also, Paths 2 and 3 are characterized by vectors $P2 = (O, Car, D)$ and $P3 = (O, Bike, 3A, Rail, 3B, Bus, D)$, respectively. If there were 10 trips between O and D, in which 5 trips used Path 1, 3 trips used Path 2, and 2 trips used Path 3, then their actual path choice probabilities are 0.500, 0.300, and 0.200, respectively. As Figure 28 shows, if their predicted choice probabilities are 0.530, 0.307, and 0.163, respectively, then MAE, MAPE, and MAE are calculated as 0.0278, 0.0245, and 8.944%, respectively.



Origin	Destination	Alternative path		Actual path choice probability	Predicted path choice probability
O	D	Path 1	$P1 = (O, Bus, 1A, Rail, 1B, Bus, D)$	0.500	0.530
		Path 2	$P2 = (O, Car, D)$	0.300	0.307
		Path 3	$P3 = (O, Bike, 3A, Rail, 3B, Bus, D)$	0.200	0.163

$$\text{RMSE} = \sqrt{\frac{(0.500-0.530)^2 + (0.300-0.307)^2 + (0.200-0.163)^2}{3}} = 0.0278$$

$$\text{MAE} = \frac{|0.500-0.530| + |0.300-0.307| + |0.200-0.163|}{3} = 0.0245$$

$$\text{MAPE} = \frac{\left|\frac{0.500-0.530}{0.500}\right| + \left|\frac{0.300-0.307}{0.300}\right| + \left|\frac{0.200-0.163}{0.200}\right|}{3} \times 100 = 8.944\%$$

Figure 28. Description of the RL model testing

5.3.2. Test Result of RL Models

Table 10 shows the test result of M0-M4. As shown in Table 10, M4 has the least error in terms of RMSE, MAE, and MAPE, followed by M3, M2, M1, and M0.

Table 10. Test result of RL models

Accuracy index		M0	M1	M2	M3	M4
RMSE	Root-mean-square error	0.12204	0.08952	0.07686	0.06941	0.06559
MAE	Mean absolute error	0.10755	0.07570	0.06774	0.06117	0.05956
MAPE	Mean absolute percentage error	28.70%	19.44%	17.13%	15.47%	15.06%

Note: n=15,313

It is shown that by including the transit fare, a non-link-additive attribute, the prediction accuracy of the RL model can be improved. Especially, model M3, which has the link-additive approximated fare by O-D pairs, is more accurate than model M2, which has the fare approximated for all observed paths.

5.3.3. Comparison with Benchmark Models

To compare the accuracy of this study's RL models to other models, two widely used logit-based path choice models, the multinomial logit model (MNL) and path-size logit model (PSL), were used as benchmark models. Both models calculate the path choice probability as shown in Equation 50, whereas the utility functions of MNL and PSL are shown in Equations 51 and 52, respectively.

$$P_{ni} = \frac{e^{V_{ni}}}{\sum_{l \in C_n} e^{V_{nl}}} \tag{50}$$

$$\text{MNL: } V_{ni} = \beta_{Const} + \beta_{TT} TT_i + \sum_j \beta_{Tr,\,j} \left(\delta_{Tr,j}\right)_i + \beta_F F_i \tag{51}$$

$$\begin{aligned} \text{PSL: } V_{ni} = {} & \beta_{Const} + \beta_{TT} TT_i + \sum_j \beta_{Tr,\,j} \left(\delta_{Tr,j}\right)_i + \beta_F F_i \\ & + \beta_{PSC} PSC_i \end{aligned} \tag{52}$$

where P_{ni} is the probability of traveler n to choose path i, C_n is the choice set of traveler n, and V_{ni} is the deterministic term of the utility function of traveler n to choose path i. β_{Const} is a constant, β_{TT}, $\beta_{Tr,\,j}$, β_F, and β_{PSC} are coefficients, TT_i is the travel time of path i, $\left(\delta_{Tr,j}\right)_i$ is a binary variable that equals one if path i includes the j-th transfer and zero otherwise, and F_i is the cost of path i. PSC_i (Equation 53) is the path-size correction factor of path i.

$$PSC_i = -\sum_{a \in \Gamma_i} \left(\frac{L_a}{L_i} \ln \sum_{l \in C_n} \delta_{al} \right) \tag{53}$$

where Γ_i is the set of links included in path i, L_a is the length of link a, and δ_{al} is a binary variable that equals one if link a is included in another

path l and zero otherwise.

The variables used for the benchmark models are travel time, transfer order, and trip cost, similar to this study's RL model. The path choice set of the benchmark models consists of actual paths in the smartcard data for transit trip stages and the shortest path for road trip stages.

The test results of the benchmark models are shown in Table 11. Though PSL showed a smaller error than MNL, both models showed higher errors compared to this study's RL models.

Table 11. The test result of benchmark models

Accuracy index		MNL	PSL
RMSE	Root-mean-square error	0.45473	0.21507
MAE	Mean absolute error	0.20761	0.14278
MAPE	Mean absolute percentage error	55.4%	38.1%

Note: n=15,313

Chapter 6. Conclusion

6.1. Conclusion

This study proposes a methodology to include non-link-additive attributes to the RL model for predicting users' intermodal path and mode choices on a multimodal network. To achieve this, this study developed a link-additive approximation method that approximates a non-link-additive path attribute into a corresponding link attribute that holds the link-additivity. The link-additive approximation is performed by the singular value decomposition and Moore-Penrose Pseudoinverse methods. The methodology is applied to the actual multimodal network and intermodal trip data in Seoul. The multimodal network consists of road, bus, and rail networks. The intermodal trip data is mainly the National Household Travel Survey data, supported by transit smartcard data for routing the transit trip stages. Transit fare and transfer penalty were used as non-link-additive attributes in this study, and the link-additive approximation method was applied to them, both for all observed paths and by O-D pairs.

To compare RL models with respect to the inclusion of link-additive approximated transit fare, this study specified four models: M0 without fare, M1 with the fare proportional to link length, M2 with the fare approximated for all observed paths, and M3 with the fare approximated by O-D pairs. Also,

to compare RL models with respect to the inclusion of link-additive approximated transfer penalty, this study specified model M4.

The models were estimated using 10% of the dataset (3,209 trips and 9,710 trip stages out of 32,094 trips and 97,175 trip stages). Among the models, model M4, which includes both transit fare and transfer penalty, shows the best goodness-of-fit in terms of log-likelihood and AIC. It was followed by M3, M2, and M1, with M0 showing the worst goodness-of-fit. As a remarkable result, the transfer penalty per one transfer increased by the number of transfers, indicating that a traveler regards a transfer more uncomfortable with the increasing number of transfers. Also, the values of time and transfers could be derived from the estimation results.

The models were tested using the rest of the dataset (28,885 trips and 87,465 trip stages). The testing was performed by comparing the predicted choice probabilities of alternatives connecting a certain O-D pair to the actual choice probabilities. Because the actual trajectories were unknown, the orders, modes, and transfer points of trip stages were used instead. As a result, all of the RL models showed better accuracies compared to benchmark models, MNL and PSL. Among them, the model M4 showed the best accuracy. It was followed by M3, M2, and M1, with M0 showing the worst accuracy among RL models.

All of the results showed that the inclusion of link-additive approximated transit fare and transfer penalty in the RL model improves both goodness-of-

fit and accuracy of the model. Especially, the link-additive approximation by O-D pairs showed better goodness-of-fit and accuracy compared to the link-additive approximation for all observed paths.

6.2. Practical Implications

The RL model is promising to predict intermodal trip demands on a multimodal network. Especially, the model can predict the sequence of trip stages and their paths and modes simultaneously. However, several problems have obstructed the application of the RL model in practice.

First, the model takes a long time to estimate, especially when the dataset is large. However, this is not a significant problem because the estimation is seldom needed in practice, considering that the estimated parameter values for mode choice models are already given in guidelines or handbooks. For example, in Korea, the Transportation Infrastructure Investment Evaluation Manual provides the estimated parameter values for logit mode choice models among private cars, buses, and rail (Ministry of Land, Infrastructure, and Transport, 2017; Ministry of Land, Infrastructure, and Transport, 2022).

Next, the RL models could not incorporate non-link-additive variables, especially transit fares. In actual trip-making, travelers regard the fare or cost of a mode or path alternative as very important, as well as travel time or the number of transfers. However, previous studies dealing with RL models did

not consider fares of transit trip stages or out-of-pocket costs of road trip stages. So far, this was one of the significant problems which disabled the application of the RL model in practice. This study's methodology would be one of the solutions to this problem by enabling the fares or costs to be incorporated into the RL model.

For the application of the RL model in practice, the collection of data and information related to travelers' intermodal trips should also be more specific than now. The National Household Travel Survey data used in this study do contain the orders, modes, departure, and arrival times/locations of trip stages, but they do not contain specific trajectories of the trip stages. Because of this problem, this study had to estimate and assume the trajectories of trip stages, which are not guaranteed that the estimated trajectories are the same as the actual ones. With the future adoption of the RL model to the practice of travel demand prediction, it is recommended that the dataset contain specific information regarding travelers' spatial trajectories as well as already being collected information of trip stages: orders, modes, departure and arrival times/locations. Foreign studies conducted by Montini et al. (2017) and Zimmermann et al. (2018), which collected GPS trajectories of travelers and applied them to the path and mode choice models, could be examples.

Suppose the RL model replaces current deterministic models of travel demand predictions in the practice, e.g., optimal strategy model (Spiess & Florian, 1989) widely used in transit demand assignment. In that case, it is

expected that transit users' actual behaviors of path and mode choices can be reflected much more realistic. Also, in the increasing number of intermodal trip-making and the increasing need for modeling intermodal trip behaviors, the RL model would be much more appropriate than the conventional path and mode choice models to predict intermodal trip behaviors.

The estimation results of parameters can provide some important indices in policy, such as VOT and VOTR derived in this study. Other indices can also be derived using the estimated parameters. Those indices also can be applied to other purposes, especially in policy, such as feasibility analysis.

6.3. Limitations of This Study and Future Research

In this study, two non-link-additive attributes, transit fare and transfer penalty, were considered. Other non-link-additive attributes, such as waiting time reliability, in-vehicle time reliability, or crowding, also should be considered in future studies. Some aspects that can potentially affect travelers' trip-making, such as safety, comfort, or path circuity (the total path length divided by Euclidean distance between origin and destination), would also be worth considering.

The estimation of the RL model needs a long computational time, which can be an obstacle to its application in practice. A computationally efficient methodology or algorithm to shorten its estimation time also should be

proposed in the future. The estimation of the RL model in this study only used 3,209 trips, which is less than 0.1% of the total daily trips in Seoul. Because RL models are computationally expensive, other studies dealing with the RL model also used only hundreds or thousands of trips. With the development of a computationally efficient algorithm to estimate the RL model, a larger dataset should be used to estimate the model in the future.

Also, with the limited modes in the National Household Travel Survey data, this study only considered some of the road modes (walking, private car, taxi, bicycle, and motorcycle), bus modes (urban transit buses, neighborhood buses, and metropolitan buses), and rail modes (subway, metropolitan railroad, and light rail). With the emergence of new mobility services, such as ride-hailing, demand-responsive transportation (DRT), and shared mobility, the future NHTS survey or dataset regarding intermodal trips should also include such modes.

In addition, as mentioned above, this study could not use the actual trajectories of the intermodal trips. In the future, data collection and information related to travelers' intermodal trips, especially their trajectories, should also be more specific than now. Using the dataset, intermodal trip trajectories predicted by RL models also should be compared to actual trajectories.

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국문초록

라이드헤일링, 수요응답형 교통체계, 공유교통 등 새로운 모빌리티 서비스의 등장으로 사람들의 통행패턴은 점점 개인화되고 복잡해지고 있다. 또한, 다양한 모빌리티 서비스를 통합하여 제공하는 Mobility-as-a-Service (MaaS) 개념의 등장으로 사람들이 여러 가지 교통수단을 연속적으로 이용할 수 있게 되면서 수단간(intermodal) 통행의 중요성은 더욱 커지고 있다. 다수단(multimodal) 교통망에서의 수단간(intermodal) 통행은 수단과 경로의 조합이 다양하며, 수단선택과 경로선택 사이에 서로 큰 상관성이 존재한다. 따라서 기존 4단계 교통수요추정 모형처럼 수단선택 후 경로선택을 예측하는 것이 아닌, 수단과 경로의 선택을 동시에 예측하는 모형이 필요하다. 재귀 로짓(recursive logit, RL) 모형은 수단 및 경로의 선택을 교통망에서의 개별 링크의 연속된 선택으로 모델링하므로, 수단과 경로의 선택을 동시에 예측할 수 있다. 그러나, 재귀 로짓 모형은 어떤 경로에 속한 개별 링크들의 속성값(attribute)들을 더했을 때 그 경로 전체의 속성값과 동일한 '링크 가산(link-additive)' 성질을 만족하는 속성값만을 사용할 수 있다. 이로 인해 재귀 로짓 모형은 사용할 수 있는 변수의 종류에

제약이 발생하며, 이는 재귀 로짓 모형의 활용가능성에도 제약을 가한다.

따라서 본 연구에서는 다수단 네트워크에서의 이용자들의 수단간 경로 및 수단 선택을 분석 및 예측하기 위하여, 재귀 로짓 모형에 비 링크 가산(non-link-additive) 속성값을 포함하기 위한 방법론을 제시한다. 이를 위해, 본 연구는 먼저 경로 단위의 비 링크 가산 속성값을 링크 단위의 링크 가산 속성값으로 근사시키는 링크 가산 근사(link-additive approximation) 방법론을 개발하였다. 링크 가산 근사법은 특이값 분해(singular value decomposition) 및 Moore-Penrose 유사역행렬(Moore-Penrose Pseudoinverse)을 통해 수행된다. 이 방법론은 서울특별시의 실제 다수단 교통망 및 수단간 통행 데이터에 대해 적용되었다. 다수단 교통망은 도로, 버스, 철도 네트워크로 구성하였다. 수단간 통행 데이터는 가구통행실태조사 데이터를 위주로 사용하였으며, 대중교통 수단통행의 경로를 산정하기 위해 대중교통 스마트카드 데이터를 추가로 사용하였다. 본 연구에서는 대중교통 요금 및 환승 페널티 등 2종류의 비 링크 가산 속성값을 사용하였다. 이들에 대해 링크 가산 근사법은 전체 경로에 대해 동시에 수행하고, 또한 O-D쌍별로도 수행하였다.

링크 가산 근사를 수행한 대중교통 요금의 포함 여부에 따

라 재귀 로짓 모형을 비교하기 위해서, 본 연구는 4개의 모형을 구축하였다. 모형 M0은 요금을 포함하지 않으며, 모형 M1은 링크 길이에 비례하는 요금을 포함하였고, 모형 M2는 전체 경로에 대해 링크 가산 근사가 수행된 요금을 포함하였고, 모형 M3은 O-D 쌍별로 링크 가산 근사가 수행된 요금을 포함하였다. 또한, 링크 가산 근사를 수행한 환승 페널티의 포함 여부에 따라 재귀 로짓 모형을 비교하기 위해서 모형 M4를 추가하였다.

구축된 모형들에 대해서는 전체 데이터셋의 10%(32,094건의 목적통행 및 97,175건의 수단통행 중 3,209건의 목적통행 및 9,710건의 수단통행)를 이용하여 추정을 수행하였다. 모형 중에서 대중교통 요금과 환승 페널티를 모두 포함한 모형 M4가 log-likelihood 및 AIC 측면에서 가장 우수한 적합도를 보였다. 또한, 나머지 90% 데이터셋(28,885건의 목적통행 및 87,465건의 수단통행)을 이용하여 모형 검증을 수행하였다. 모형 검증은 특정 O-D쌍을 연결하는 대안 수단 및 경로들에 대해 모형으로 예측된 선택 확률과 실제 선택 확률을 비교함으로써 수행되었다. 실제 이동 궤적은 알 수 없으므로 이를 대신하여 각 수단통행의 순서, 수단, 환승지점을 검증에 사용하였다. 검증 결과, 모든 재귀 로짓 모형이 벤치마크 모형에 비해 우수한 정확도를 나타내었다. 모형 중에서 모형 M4가 가장 우수한 정확도를 나타내었고, 그 다음이 M3, M2,

M1 및 M0 순서로 나타났다.

본 연구의 결과는 링크 가산 근사가 수행된 대중교통 요금 및 환승 페널티를 재귀 로짓 모형에 포함함으로써 모형의 적합도와 정확도를 모두 제고할 수 있음을 의미한다. 특히, 링크 가산 근사를 전체 경로에 대해 수행하는 것보다 O-D쌍별로 수행하는 것이 더욱 우수한 적합도 및 정확도를 보였다.