



공학석사 학위논문

Experimental Investigation of Turbulent Effect on Settling Velocity of Inertial Particles

관성입자의 침강속도에 난류가 미치는 영향에 대한 실험연구

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서울대학교 대학원

건설환경공학부

백 승 준

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ABSTRACT

Experimental Investigation of Turbulent Effect on Settling Velocity of Inertial Particles

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Existing particle tracking models predict the vertical velocity of particles using the linear summation of carrier fluid velocity, the settling velocity in still fluid, and the random value following normal distribution to represent the effect of diffusion and dispersion. However, it has been reported that the terminal settling velocity of inertial particles changed in a turbulent flow. Therefore, it is necessary to investigate the interactions between advection by carrier fluid, settling velocity in stagnant water, and changes of settling velocity in a turbulent flow to improve the performance of predicting particle transport in particle tracking models. To this end, numerical simulations and laboratory experiments were conducted in the present study. First of all, the numerical simulations for the particle settlement in a steady uniform flow have been carried out to evaluate the effect of a parallel advection on the settling velocity. The resultant settling velocity was the same as the velocity calculated by superposing the advection by carrier fluid and the settling velocity in still fluid because the particles' relative velocity has to be consistent according to the particle and fluid characteristics. To investigate the turbulence effect on the settling velocity, two kinds of experiments, namely, the open-channel flow experiments and experiments using the Vertical Recirculation Tube (VeRT), were conducted. In both of the experiments, the velocity of inertial particles was measured using particle tracking velocimetry (PTV), and fluid velocity and turbulence were measured using particle image velocimetry (PIV). In the present study, the PTV algorithm, which can track multiple settling particles, has been constructed. The experimental results showed that the settling velocity of the particles was generally larger in turbulent flow than in stagnant water. Then, several parameters representing particle and turbulence characteristics, such as Stokes number (St) and Rouse number (Sv) were investigated to determine which parameter depends on settling velocity change. As a result, the combination of Stokes and Rouse number, SvSt, which can be seen as a length scale parameter, appears to show a more evident correlation with the settling velocity change than other parameters. Thus, it is maintained that SvSt can be used as a defining parameter to describe the turbulence effect on the settling velocity change of inertial particles in a turbulent flow.

In conclusion, through the experiments conducted in the present study and preceding studies, it was evident that the settling velocity generally increases with increasing level of turbulence. Hence, the existing particle tracking model could overestimate the transport distance, which is mainly determined by the ratio of the settling distance to vertical velocity of particles. Thus, it is important to take into account the turbulence effect on the settling velocity of inertial particles in order to improve the performance of particle transport.

Keywords: inertial particles, settling velocity, turbulence, particle tracking model

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TABLE OF CONTENTS

LIST	OF FIGURES	vi
LIST	OF TABLES	X
LIST	OF SYMBOLS	xi
LIST	OF ABBREVIATIONS	xvi
1. Int	roduction	1
1.1	Background and necessities of study	1
1.2	Research objectives	3
2. Th	eoretical background	7
2.1 I	nertial particles in a viscous fluid	7
	2.1.1 Equation of motion	7
	2.1.2 Numerical integration scheme for the MRE	12
2.2 \$	Settling velocity of inertial particles	16
	2.2.1 Terminal settling velocities in Stokes regime	16
	2.2.2 Settling velocity changes in turbulence	17
2.3 I	Estimating turbulent kinetic energy (TKE) dissipation rate for	
turbi	ulence analysis	22
	2.3.1 Particle Image Velocimetry (PIV)	22
	2.3.2 TKE dissipation rate	22
	2.3.3 The method estimating TKE dissipation rate from PIV data	

suggested by Sheng et al. (2000)	25
2.4 Empirical Mode Decomposition (EMD)	32
3. Experimental setup and instrumentations	34
3.1 Experimental setup	34
3.1.1 Experiment 1: Open-channel flume	34
3.1.2 Experiment 2: Vertical Recirculation Tube (VeRT)	44
3.2 Particle Tracking Velocimetry (PTV)	57
4. Results and discussion	63
4.1 Effect of parallel advection on the settling velocity	63
4.1.1 Modified drag force in MRE	63
4.1.2 Validation of the numerical scheme	65
4.1.3 Numerical simulation in a steady uniform flow	70
4.2 Experimental results	73
4.2.1 Experiment 1: Open-channel flume	73
4.2.2 Experiment 2: VeRT	88
4.3 Effect of turbulence on settling velocity change	101
5. Conclusion	113
REFERENCES	116
APPENDIX	121
국문초록	131

LIST OF FIGURES

Fig.	1. 1. Flowchart of the research
Fig.	2. 1. Graphical description of forces exerted on a spherical particle in fluid9
Fig.	2. 2. Summary of the preceding experimental studies about the settling
	velocity change of inertial particles in turbulent water; w_t / w_s is the settling
	velocity change and <i>St</i> is the Stokes number
Fig.	2. 3. A schematic diagram of the energy cascade with lengthscale demarcations
	and following subranges
Fig.	3. 1. The schematic diagram of the open-channel flume; (a) side view; (b) top
	view
Fig.	3. 2. Photograph of the open-channel flume and image acquisition system. $\dots 36$
Fig.	3. 3. The reflection mirror installed on the optic table
Fig.	3. 4. Cross-sectional view of the test section of the open-channel flume41
Fig.	3. 5.CMOS camera with 60 mm lens42
Fig.	3. 6. DPSS laser and supply42
Fig.	3. 7. PIV particle; silver coated hollow glass spheres with a mean particle
	diameter of 10 µm43
Fig.	3. 8. Schematic diagram of the Vertical Recirculation Tube (VeRT); (a) Top
	view (b) Front view (c) Side view46
Fig.	3. 9. (a) Photograph of the VeRT (b) The experimental section when the laser
	supply is activated
Fig.	3. 10. The flow direction according to the configuration of valves; (a) upward
	flow (b) downward flow
Fig.	3. 11. (a) Photograph of the particle injection pipe (b) Schematic diagram for
	mechanism of the particle injection
Fig.	3. 12. Instruments for the experimental channel improvement; (left) stainless
	steel honeycomb (right) mesh grid
Fig.	3. 13. Lateral flow velocity profiles at the center line in the FOV (a) before and
	(b) after the improvement

Fig. 3. 14. High-speed camera with 24-70 mm lens55
Fig. 3. 15. Camera and laser traversing system; (a) camera traverse (b) laser
traverse and CMOS camera used as side camera on the top of the frame55
Fig. 3. 16. Halogen lamp installed to provide a sufficient light source for the PTV
algorithm
Fig. 3. 17. Snapshots of an image pre-process procedure; (a) original image (b)
gamma calibration (c) median subtraction (d) binarization through adaptive
thresholding60
Fig. 3. 18. Overall scheme of PTV algorithm; Step 1: The first location of particle
is selected manually; Step 2: Moving window is set based on the locations of
Step 1; Step 3: Find the longest continuous bundles of I^+ pixels and
determine an approximate location of center point (red x mark); Step 4:
Calculating the area containing the approximate location and using weighted
average, a precise location of center point (blue x mark) is determined61
Fig. 4. 1. Calculated settling velocity when Stokes or Cheng drag was applied;
example case with a diameter of 390 μ m and specific gravity of 1.3567
Fig. 4. 2. The magnitude of accelerations in the Maxey-Riley equation with
calculating the drag force term (a) as the Stokes drag (b) with the drag
coefficient suggested by Cheng (2009)
Fig. 4. 3. Numerical calculation results with various particle diameters and fixed
specific gravity of 1.35. The dashed lines indicate the terminal velocity in a
Stokes regime with a large (blue) and small (red) density ratio, β . The dash-
dot and dotted black line represent the results of calculating the drag force as
the Stokes drag with the drag coefficient suggested by Cheng (2009),
respectively
Fig. 4. 4. Comparison of the results of the numerical simulations of settling
velocities according to particle sizes with or without advection and the linear
summation of advection and terminal settling velocity72
Fig. 4. 5. Settling trajectories of inertial particles for the open-channel experiment:

examples of case (a) OC-Q1D1 (b) OC-Q2D4 (c) OC-Q2D6 (d) OC-Q3D2..75

Fig. 4. 6. Histogram of measured settling velocities for each particle for the open	-
channel experiment: examples of case (a) OC-Q1D1 (b) OC-Q2D4 (c) OC-	
Q2D6 (d) OC-Q3D2	76
Fig. 4. 7. Measured settling velocities according to the particle diameter; the	
errorbars were calculated by the standard deviation of measured velocities	78
Fig. 4. 8. Relative velocities of particles to fluid velocity; OC-Q2D1, OC-Q1D3,	
OC-Q1D5, and OC-Q1D6 showed the relative velocities smaller than the	
terminal settling velocity in stagnant water	78
Fig. 4. 9. Example of decomposed time series signal by the EMD method; The	
signal at the top is the original signal, the bottom is the residual, and the rest	
are the IMFs	80
Fig. 4. 10. Comparison of the fluctuating velocity component between the origina	ıl
signal and the reconstructed signal	81
Fig. 4. 11. Comparison of the streamwise turbulence intensity between the	
experimental data and the empirical equation of Nezu & Nakagawa, (1993) t	Ö
verify the appropriateness of measured turbulence	86
Fig. 4. 12. Turbulent energy spectrum of the experimental cases; (a) OC-Q1 (b)	
OC-Q2 (c) OC-Q3	87
Fig. 4. 13. Description image of divided ROI and acquired mean flow field; (a)	
upward flow (b) downward flow	89
Fig. 4. 14. Settling trajectories of inertial particles for the VeRT experiment:	
examples of a single trial of case (a) V-Q1D1 (b) V-Q2D1 (c) V-Q1D4 (d) V	-
Q2D4	91
Fig. 4. 15. Histogram of measured settling velocities for entire trials of the VeRT	1
experimental case: examples of case (a) V-Q1D1 (b) V-Q2D1	92
Fig. 4. 16. Measured settling velocities according to the particle diameters for the	;
VeRT experiments and stagnant water; the errorbar indicates the standard	
deviation of velocities	95
Fig. 4. 17. Comparison of settling velocities in stagnant water and the measured	
relative velocities for the VeRT experiments according to the particle diamet	er
	95

Fig. 4. 18. Snapshots of the calibration procedure for the turbulence measurement;
Left figure shows the trajectory of the calibration particle with the full FOV,
and the rest are presented to show the detailed view of the detected area of a
particle
Fig. 4. 19. Turbulent energy spectrum of the experimental cases; (a) V-Q1 (b) V-Q2
Fig. 4. 20. Summary of the settling velocities for entire experimental cases
according to the particle diameter104
Fig. 4. 21. Comparison of experimental results with preceding studies about the
relation between the settling velocity change and Stokes number; The
errorbars were calculated with the error propagation according to statistical
properties of particle sizes and TKE dissipation rate105
Fig. 4. 22. Comparison of experimental results with preceding studies about the
relation between the settling velocity change and the ratio of the particle
diameter to the Kolmogorov lengthscale106
Fig. 4. 23. Settling velocity change according to velocity scale parameter using
Kolmogorov velocity scale, $Sv_{\eta} = w_s / u_{\eta}$
Fig. 4. 24. Settling velocity change according to velocity scale parameter using
turbulence intensity, $Sv_i = w_s / \tilde{u}'_{rms}$
Fig. 4. 25. Settling velocity change according to the grouping parameters of
velocity scale parameter using Kolmogorov velocity scale, $Sv_{\eta} = w_s / u_{\eta}$ and
time scale parameter, i.e., $Sv_{\eta}St = w_s\tau_p / \eta$
Fig. 4. 26. Settling velocity change according to the grouping parameters of
velocity scale parameter using Kolmogorov velocity scale, $Sv_l = w_s / u'$ and
time scale parameter, i.e., $Sv_{\eta}St = w_s \tau_p / \tilde{u}'_{rms} \tau_k$

LIST OF TABLES

Table 2. 1. Preceding studies which investigated the turbulence effect on settling
velocities of particles
Table 2. 2. The ratio of estimated to actual TKE dissipation rate and effective
Smagorinsky constants for different combinations of parameters (Bertens et
al., 2015)
Table 3. 1. The particle information used in the experiments
Table 3. 2. Experimental cases for the open-channel experiments
Table 3. 3. Experimental cases for the Vertical Recirculation Tube
Table 4. 1. Comparison of experimental results of the settling velocities in stagnant
water and open-channel flow77
Table 4. 2. Results of turbulence analysis of the open-channel experiment
Table 4. 3. Comparison of experimental results of the settling velocities in stagnant
water and vertical flow; Relative settling velocity was calculated by
subtracting average of flow velocity along the particle trajectory94
Table 4. 4. Results of turbulence analysis of the VeRT experiment

LIST OF SYMBOLS

Latin Uppercase

A_p	Area of particle
C_s	Smagorinsky constant
C_s^{eff}	Effective Smagorinsky constant
E(k)	Turbulent energy spectrum
F _{AM}	Added mass force
F_B	Buoyancy force
F_D	Drag force
F_H	Boussinesq-Basset history force
F_P	Pressure gradient force
G	Abbreviation for several forces in calculation scheme of MRE
Н	Abbreviation for a history force in calculation scheme of MRE
I_1	Brightness intensity of pixel after gamma correction
I_0	Brightness intensity of pixel before gamma correction
I _{max}	Maximum possible brightness intensity
I_t	Turbulent intensity
I^+	Binarized light pixel with brightness intensity of 255
I^-	Binarized dark pixel with brightness intensity of 0
K	Integration kernel function of history force
\overline{P}	Resolved-scale pressure for PIV method

Q	Flowrate
Re _p	Particle Reynolds number
\overline{S}	Characteristic rate of strain
$\overline{S_{\iota J}}$	Resolved-scale strain rate tensor for PIV method
U	Cross-section averaged velocity
\overline{U}	Resolved-scale velocity for PIV method
Ũ	Magnitude of mean velocity
V	Particle velocity
W_0	Terminal settling velocity in a Stokes regime with a large density ratio
W_0^*	Terminal settling velocity in a Stokes regime with a similar density ratio

Latin Lowercase

d	Diameter of particle
d_{50}	Median particle diameter
$e_{\max}(t)$	Envelope passing through local maxima for the EMD algorithm
$e_{\min}(t)$	Envelope passing through local minima for the EMD algorithm
f	Frequency
g	Gravitational acceleration
h	Flow depth
$h_i(t)$	Difference between the original signal and local mean for the i-th iteration in the EMD algorithm
k	Wavenumber
l	Lengthscale of turbulence in the inertial subrange

Lengthscale of the largest eddies l_0 Lengthscale as the demarcation between energy and inertial l_{EI} subrange Lengthscale as the demarcation between dissipation and inertial l_{DI} subrange Local mean for the i-th iteration in the EMD algorithm $m_i(t)$ Mass of the fluid excluded by a particle m_f Mass of a particle m_n The number of intervals for the calculation of Maxey-Riley п equation Pressure р Unresolved-scale pressure for PIV method р Radius of a particle r Residual for the i-th iteration in the EMD algorithm $r_i(t)$ S Fitted spline function of a particle trajectory Time variable t Initial time for the particles to start settling motion t_0 Fluid velocity u \mathbf{u}_{rms}' Root-mean-square streamwise fluid velocity fluctuation ũ'_{rms} Root-mean-squared magnitude of velocity fluctuation ū Time average streamwise fluid velocity Unresolved-scale velocity for PIV method и w Relative velocity between particle and fluid Settling velocity at stagnant water W_{S}

Wt	Settling velocity at turbulent water
<i>w′</i>	Root-mean-square vertical fluid velocity fluctuation
\overline{W}	Time average vertical fluid velocity
x(t)	Time series signal for the EMD algorithm
x _p	Particle displacement

Greek Uppercase

- Δ Interrogation window size for the PIV measurement
- Δt Time-step for the numerical study

Greek Lowercase

Window overlap rate for PIV
Coefficients for the quadrature scheme to calculate the history force
Density parameter
Gamma correction parameter
Turbulent kinetic energy dissipation rate
Actual TKE dissipation rate
Kolmogorov length scale
von Karman constant
Dynamic viscosity of fluid
Kinematic viscosity of fluid

- ρ Density ratio of particle to fluid
- ρ_f Density of fluid
- ρ_p Density of particle
- σ Standard deviation
- τ_{ij}^{r} Residual stress tensor
- τ_k Kolmogorov time scale
- au_p Particle relaxation time
- au_p^* Particle relaxation time with a density ratio
- ϕ Smoothing parameter for spline fitting

LIST OF ABBREVIATIONS

- CFD Computational Fluid Dynamics
- DNS Direct Numerical Simulation
- EMD Empirical Mode Decomposition
- FOV Field of View
- IMF Intrinsic Mode Function
- LES Large-Eddy Simulation
- PIV Particle Image Velocimetry
- PTM Particle Tracking Model
- PTV Particle Tracking Velocimetry
- TKE Turbulent Kinetic Energy
- MRE Maxey-Riley Equation
- MPs Micro-Plastics
- VeRT Vertical Recirculation Tube

1. Introduction

1.1 Background and necessities of study

Predicting the behaviors of inertial particles in a water system, such as sediment, suspended soil and even microplastics (MPs), is essential to manage hydraulic structures constructed at rivers or coastal areas and preserve water resources. Especially since microplastics (MPs) have been treated as one of the most significant issues for the environment nowadays, the behavior of the MPs in a water system, such as where MPs are accumulated and where they are transferred to, also has been considered as crucial as the sediment behavior. These interests are mainly determined by the transport distance of heavy particles. Consequently, the settling velocity of particles that dominates vertical displacements during transportation must be concerned. Many experimental and theoretical investigations on determining settling velocity according to the particle characteristics, such as sizes, shapes, and densities, have been conducted. However, most of them were focused on the terminal settling velocity, which is the velocity of particles in stagnant water or vertically steady flow (Brown and Lawler, 2003; Cheng, 2009, 1997; Dey et al., 2019; Turton and Clark, 1987).

Various particle tracking models (PTM) based on the random walk method have been developed to predict those particles' movements in a natural water system. Most PTMs adopt the particles' settling velocity as a model parameter using the empirical equation derived from the experimental results conducted in stagnant water. Then, the model considers the turbulence effect using the random walk term, reflecting the effect of vertical diffusivity following a Gaussian distribution. For instance, the particle tracking module in Delft3D-PART predicts the particles' lateral and streamwise velocities (and corresponding displacements) following the flow velocity of a carrier fluid, considering the dispersion effect with the random walk method. But, since the CFD model is based on the shallow water equation, it is hard to calculate accurate vertical flow velocity because particles' vertical displacement is predicted only with their settling velocity, not with a flow velocity. Therefore, the settling velocity used in the module of Delft3D-PART is calculated with user-defined parameters (Deltares, 2022). For another example, the MATLAB-based open-source model, TrackMPD, is a 3D model that calculates the particles' vertical displacement through the linear summation of vertical flow velocity and settling velocity of particles calculated with the empirical equation using the input particle characteristics (Jalón-Rojas et al., 2019). On the other hand, the particle tracking module in EFDC doesn't consider particle characteristics and only predicts the vertical displacement of particles with vertical diffusion and flow velocity. Like these examples, most PTMs use the random walk method to reflect the turbulent effect as the random amount having the zero mean.

However, suppose the settling velocities of inertial particles in a natural water body were affected by the turbulence and changed either increasingly or decreasingly. In that case, it is inappropriate that the turbulent effect is considered only a random value. Still, the random effect should be considered to reflect diffusion and dispersion into the PTM, but the settling velocity input in the model should be changed. If not, the predicted transport distance of particles using the settling velocity in stagnant water could be under- or overestimated. So, it is necessary to analyze the relationship between turbulence and the settling velocity of inertial particles in nature-like flow. Also, whether the consistent or noticeable tendency of the relationship exists needs to be investigated.

1.2 Research objectives

The overall objective of the present study was to investigate the relationship between turbulence and settling velocity changes of inertial particles. To this end, the governing equation used in PTMs for particle velocity calculation has to be reviewed. At the 3D-PTM like TrackMPD, the particle velocity is estimated by the equation below (Jalón-Rojas et al., 2019):

$$w_t = w_s + w + R \tag{1.1}$$

where w_t represents the settling velocity of a particle in turbulent flow, w_s the terminal settling velocity in stagnant water, w the vertical component of the flow velocity of a carrier fluid, and R is a random walk term which can be calculated with vertical diffusivity. In the present study, two kinds of laboratory experiments and a numerical simulation have been conducted to investigate the practicality of Eq. (1.1). By dividing the settling velocity change into advective effect and turbulent

effect, the relation between settling velocities in a turbulent flow and stagnant water can be expressed as:

$$w_t = w_s + w + f(St, Sv) \tag{1.2}$$

where f(St,Sv) is used instead of *R* in Eq. (1.1) to represent the velocity change caused by the turbulent effect. *St* is Stokes number, a ratio between the particle response time to Kolmogorov time scale. *Sv* indicates Rouse number, a velocity ratio between turbulence fluctuation velocity to the Kolmogorov velocity scale. In order to validate Eq. (1.2), three series of investigations were carried out.

First, to inspect the effect of the vertical flow velocity of a carrier fluid as an advection to inertial particles, the numerical simulation of the settling behavior of particles was carried out by solving the equation of motion numerically. This numerical study is used to prove whether the settling velocity of inertial particles in an advective flow (a steady uniform flow without any turbulence) can be explained as a linear summation of a flow velocity and a particle's settling velocity at stagnant water ($w_t = w_s + w$). In the second place, the laboratory experiment using an open-channel flume that can reflect a nature-like flow condition on a laboratory scale has been conducted. Under the environment where the vertical component of flow velocity is small enough, and the advection effect perpendicular to the streamwise direction can be negligible, the settling velocity of inertial particles was measured. Lastly, the second laboratory experiment was carried out using a vertical

recirculating tube (VeRT) which can generate bi-directional flows, both upward and downward. In this experiment, the vertical flow can be controlled, and only the turbulent effect on settling velocity can be investigated by eliminating the relation between the advection parallel to the settling motion and the particles' settling velocity that has been inspected in numerical simulation. In the two experiments, the flow velocity and turbulence were measured by the particle image velocimetry (PIV), and the particles' settling velocities were measured by the particle tracking velocimetry (PTV). Fig. 1. 1 shows the flowchart of this study, and it summarizes the research objectives and the overall procedure.



Fig. 1. 1. Flowchart of the research

2. Theoretical background

2.1 Inertial particles in a viscous fluid

2.1.1 Equation of motion

The governing equation of motion for a small spherical particle with radius r and mass m_p in a viscous fluid is given by the Maxey-Riley equation (MRE) (Maxey and Riley, 1983):

$$m_{p} \frac{d\mathbf{V}}{dt} = \sum F = F_{p} + F_{B} + F_{D} + F_{AM} + F_{H}$$

$$= m_{f} \frac{D\mathbf{u}}{Dt}\Big|_{\mathbf{x}_{p}(t)} + (m_{p} - m_{f}) g\vec{e_{z}}$$

$$- 6\pi r \rho_{f} v \left(\mathbf{V} - \mathbf{u} - \frac{r^{2}}{6} \Delta \mathbf{u}\right)\Big|_{\mathbf{x}_{p}(t)}$$

$$- \frac{m_{f}}{2} \left[\frac{d\mathbf{V}}{dt} - \frac{D}{Dt} \left(\mathbf{u} + \frac{r^{2}}{10} \Delta \mathbf{u}\right)\Big|_{\mathbf{x}_{p}(t)}\right]$$

$$- 6r^{2} \rho_{f} \sqrt{\pi v} \int_{t_{0}}^{t} \frac{1}{\sqrt{t - \tau}} \left[\frac{d\mathbf{V}}{d\tau} - \frac{d}{d\tau} \left(\mathbf{u} + \frac{r^{2}}{6} \Delta \mathbf{u}\right)\Big|_{\mathbf{x}_{p}(t)}\right] d\tau$$
(2.1)

where $\mathbf{V}(t) = d\mathbf{x}_{\mathbf{p}}/dt$ is the particle velocity, m_f the mass of that fluid excluded by the particle, $\mathbf{u}(\mathbf{x}_{\mathbf{p}}, t)$ the fluid velocity at the particle position, ρ_f the density of the fluid, g the gravitational acceleration, ν the kinematic viscosity of the fluid, and t_0 the initial time for the particles to start settling motion. The terms involving $r^2\Delta(\mathbf{u})$ are usually referred to as the Fauxén correction. Assuming that the particle is small enough, i.e. $r \ll 1$, these terms can be negligible. The two types of derivatives were used in Eq. (2.1) according to the carrying particle of the convective term. The derivative d/dt is used to denote a total derivative along the trajectory of the moving inertial particle:

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{u}$$
(2.2)

The derivative D/Dt, in contrast, is the material derivative of the fluid element occupied by the particle:

$$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}$$
(2.3)

The terms on the right-hand side of Eq. (2.1) correspond to the force exerted by the fluid on the fluid element, the buoyancy force, the Stokes drag, the added mass term, and the Boussinesq-Basset history force. Fig. 2. 1 shows the graphical description of these terms. Before further discussions about MRE, brief descriptions of the unacquainted last three forces are followed.



Fig. 2. 1. Graphical description of forces exerted on a spherical particle in fluid

First, in terms of the Stokes drag (drag force) term, preceding research has been conducted under the assumption that the disturbance flow produced by the motion of the sphere was at a sufficiently low particle Reynolds number $\left(Re_p = \frac{r|\mathbf{V}-\mathbf{u}|}{v} << 1\right)$ so that the drag force on the sphere could be calculated at the regime of Stokes flow, which can be expressed as $F_D = 6\pi \mu_f r(\mathbf{u}-\mathbf{V})$ (Maxey and Riley, 1983; Armenio and Fiorotto, 2001; Candelier et al., 2004; Haller and Sapsis, 2008; Sapsis et al., 2011; Monroy et al., 2016).

Second, the force by added mass is generated when a particle moves in a fluid. The moving particle makes some amount of fluid surrounding the particle move, which means when the particle gets accelerated due to external forces, so does the fluid. Thus, more force is required to accelerate the particle in the viscous fluid than in a vacuum. The difference between the force required in fluid and a vacuum is called an added mass force, and the corresponding imaginary mass of fluid is called added mass. The added mass force F_{AM} for a spherical particle can be calculated as:

$$F_{AM} = -\frac{2}{3}\rho_f \pi r^3 \left(\frac{d\mathbf{V}}{dt} - \frac{D\mathbf{u}}{Dt}\right)$$

$$= -\frac{m_f}{2} \left(\frac{d\mathbf{V}}{dt} - \frac{D\mathbf{u}}{Dt}\right)$$
(2.4)

where a mass of fluid occupied by the particle is $m_f = \rho_f \frac{4}{3}\pi r^3$ for a spherical

particle.

Lastly, the history force has been first proposed by Boussinesq and then by Basset (Boussinesq, 1885; Basset, 1888). This term describes the force due to the lagging boundary layer development with changing the relative velocity of bodies moving through a fluid (Crowe, 2012). The viscous effects of fluid are exerted with the temporal delay on the particle along its trajectory. Basset (1888) suggested that the history force F_H on an accelerating spherical particle in a viscous fluid is

$$F_{H} = 6r^{2}\sqrt{\pi\rho_{f}\mu_{f}}\int_{t_{0}}^{t}\frac{1}{\sqrt{t-\tau}}\left(\frac{\partial\mathbf{u}}{\partial\tau}+\mathbf{V}\cdot\nabla\mathbf{u}-\frac{d\mathbf{V}}{d\tau}\right)d\tau$$
$$= -6r^{2}\rho_{f}\sqrt{\pi\nu}\int_{t_{0}}^{t}\frac{1}{\sqrt{t-\tau}}\left(\frac{d\mathbf{V}}{d\tau}-\frac{d\mathbf{u}}{d\tau}\right)d\tau$$
(2.5)

Dividing both sides by the mass of the particle $m_p = \rho_p \frac{4}{3} \pi r^3$ and

rearranging it, MRE can be expressed as a general form:

$$\frac{d\mathbf{V}}{dt} = \beta \frac{D\mathbf{u}}{Dt} + (1-\beta)g\vec{e_z} - \frac{1}{\tau_p^*}(\mathbf{V}-\mathbf{u}) - \sqrt{\frac{3\beta}{\tau_p^*\pi}} \int_{t_0}^t \frac{1}{\sqrt{t-\tau}} \left(\frac{d\mathbf{V}}{dt} - \frac{d\mathbf{u}}{dt}\right) d\tau \qquad (2.6)$$

with

$$\beta = \frac{3\rho_f}{2\rho_p + \rho_f}, \quad \tau_p^* = \frac{r^2}{3\beta v}$$

where β is the nondimensional density parameter, and τ_p^* is particle relaxation time. This is the time scale parameter that describes the properties of a particle to approach a steady motion.

From the expression of the history force, Eq.(2.5), the term that attenuates the history force in time is generally referred to as the history force kernel $K(t-\tau) = (t-\tau)^{-1/2}$. Mei and Adrian (1992) proposed a modified version of this kernel which can describe their analytical and numerical results of the decaying history force as t^{-2} , instead of $t^{-1/2}$, at large elapsed time. Candelier et al. (2004) investigated the effect of the history force on the trajectory of particles falling in a rotating flow with solid-body fluid. They observed that the history forces calculated by the original Basset's kernel and the empirical kernel proposed by Mei and Adrian, (1992) gave similar results for the time range considered in their experiments. Considering that the laboratory experiment results in the present study showed short elapsed time ranges for settlement, the original kernel, $K(t-\tau) = (t-\tau)^{-1/2}$, was used.

2.1.2 Numerical integration scheme for the MRE

Some numerical studies using MRE and DNS data to predict particle behaviors in turbulent fluid have been conducted. (Bec et al., 2014; Wang and Maxey, 1993; Yang and Lei, 1998a). However, those studies focused on the settling motion of particles with a large density ratio, $\rho = \rho_p / \rho_f \approx 1,000$, which corresponds to the ratio, such as water droplets in a cloud. In that density ratio, β in MRE, Eq. (2.6), approximates as 0. Thus, their research could be carried out with a simplified MRE that ignores several terms including β , and used it as a governing equation of particles' motion. In this study, however, the particles in water have been treated with a relatively small density ratio. So, the full MRE, including terms ignored in preceding studies, should be applied to this study.

Since the history force included in full MRE accounts for the dissipative viscous effects along the time and the trajectory of a particle, this term is expressed in an integrodifferential equation. Generally, the solution of an integrodifferential equation can be obtained by applying the Laplace transform to the equation. Several studies were conducted to find the explicit solution for MRE using Laplace transform (Michaelides, 1992; Mei and Adrian, 1992; Candelier et al., 2004). However, all the proposed solutions still have a similar form of the "history integral". This integral term has several difficulties in the numerical calculation: the singularity of the kernel, and stems from the necessity to recompute the history force – an integral over all previous time steps – for every new time step (Daitche, 2015). In the present study, the numerical scheme proposed by Daitche (2015) was adopted to calculate the history force term effectively.

The MRE has been derived with the assumption of an inertial particle and the fluid having the same initial velocity, i.e. $\mathbf{u}(t_0) = \mathbf{V}(t_0)$. To be valid for initial conditions $\mathbf{u}(t_0) \neq \mathbf{V}(t_0)$, the history force term of Eq. (2.6) can be rewritten as:

$$\int_{t_0}^t K(t-\tau) \frac{d}{d\tau} f_H(\tau) d\tau + K(t-\tau) f_H(t_0) = \frac{d}{dt} \int_{t_0}^t K(t-\tau) f_H(\tau) d\tau \qquad (2.7)$$

where the integral kernel is $K(t-\tau) = (t-\tau)^{-1/2}$ and $f_H(t) = \sqrt{\frac{3\beta}{\tau_p^* \pi}} (\mathbf{V} - \mathbf{u})$.

Daitche (2015) introduced the quadrature scheme up to the third-order approximation to calculate the integral term of the history force. For the first order,

$$\int_{t_0}^{t} \frac{f_H(\tau)}{\sqrt{t-\tau}} d\tau = \sqrt{\Delta t} \sum_{k=0}^{n} \alpha_k^n f_H(\tau_{n-k}) + \mathcal{O}(\Delta t^2) \sqrt{t-t_0}$$
(2.8a)

$$\alpha_{k}^{n} = \frac{4}{3} \begin{cases} 1 & (k=0) \\ (k-1)^{3/2} + (k+1)^{3/2} - 2k^{3/2} & (0 < k < n) \\ (n-1)^{3/2} - n^{3/2} + \frac{3}{2}\sqrt{n} & (k=n) \end{cases}$$
(2.8b)

where Δt is the time-step, and the coefficients α_k^n depend on *n*, the number of intervals for the approximation of the integral.

The quadrature scheme for calculating history force is incorporated in the integration scheme for the full MRE. In order to reduce the number of terms to be calculated, Eq. (2.6) can be rearranged with the relative velocity between a particle and a fluid $\mathbf{w} = \mathbf{V} - \mathbf{u}$.

Subtracting $\frac{d\mathbf{u}}{dt}$ to both sides of Eq. (2.6),

$$\frac{d\mathbf{w}}{dt} = -\frac{d\mathbf{u}}{dt} + \beta \frac{D\mathbf{u}}{Dt} + (1-\beta)g\vec{e_z} - \frac{1}{\tau_p^*}\mathbf{w} - \sqrt{\frac{3\beta}{\tau_p\pi}}\frac{d}{dt}\int_0^t \frac{\mathbf{w}}{\sqrt{t-\tau}}d\tau$$

$$= (\beta-1)\frac{d\mathbf{u}}{dt} + (1-\beta)g\vec{e_z} - \beta \mathbf{w} \cdot \nabla \mathbf{u} - \frac{1}{\tau_p^*}\mathbf{w} - \sqrt{\frac{3\beta}{\tau_p\pi}}\frac{d}{dt}\int_0^t \frac{\mathbf{w}}{\sqrt{t-\tau}}d\tau$$
(2.9)

Integrating Eq. (2.9) from t to $t + \Delta t$ and using abbreviations,

$$\mathbf{G} = (\beta - 1)\frac{d\mathbf{u}}{dt} - \beta \,\mathbf{w} \cdot \nabla \mathbf{u} - \frac{1}{\tau_p^*} \,\mathbf{w}$$
$$\mathbf{H} = -\sqrt{\frac{3\beta}{\tau_p^* \pi}} \frac{d}{dt} \int_0^t \frac{\mathbf{w}}{\sqrt{t - \tau}} d\tau$$

the equation for the calculation scheme can be derived as follows:

$$\mathbf{w}(t+\Delta t) = \mathbf{w}(t) + \int_{t}^{t+\Delta t} \mathbf{G}(\tau) d\tau + \mathbf{H}(t+\Delta t) - \mathbf{H}(t)$$
(2.10)

The integral of the abbreviation term G is approximated using first-order polynomial interpolation.

$$\int_{t}^{t+\Delta t} \mathbf{G}(\tau) d\tau = \Delta t \, \mathbf{G}(t) + \mathcal{O}(h^2)$$

Then, denoting the time grid as $t_n = t_0 + n\Delta t$ and using the abbreviations for representing the time-steps $\mathbf{w}_n = \mathbf{w}(t_n)$, the complete integration scheme of the first order for the MRE can be written as:

$$\left(1+\sqrt{\frac{3\beta h}{\tau_p \pi}}\alpha_0^{n+1}\right)\mathbf{w}_{n+1} = \mathbf{w}_n + h\mathbf{G}_n - \sqrt{\frac{3\beta h}{\tau_p \pi}}\sum_{j=0}^n \left(\alpha_{j+1}^{n+1}\mathbf{w}_{n-j} - \alpha_j^n\mathbf{w}_{n-j}\right) + \mathcal{O}(h^2) \quad (2.11)$$

2.2 Settling velocity of inertial particles

2.2.1 Terminal settling velocities in Stokes regime

The terminal settling velocity can be calculated from the general form of MRE, Eq. (2.6). When settling particles in stagnant water approaches equilibrium, i.e., $d\mathbf{v}/dt = 0$, the buoyancy and the drag force become the same, neglecting the history force; indeed, the calculated history force can be neglected considering its relative magnitude to the other force terms (See Sec. 4.1.2). So, let the drag force,

$$F_D = 6\pi \mu_f r(\mathbf{u} - \mathbf{V})$$
, be equal to the buoyancy force, $F_B = \frac{4}{3}\pi r^3 (\rho_f - \rho_p) g \vec{e_z}$, it is

arranged as $V = \frac{d_p^2(\rho_p - \rho_f)}{18\mu_f}g$. Herein, the widely used form of particle relaxation

time, $\tau_p = \frac{d_p^2 \rho_p}{18 \mu_f}$, can be derived by omitting the fluid density with an assumption

that the fluid density is much smaller than the particle density. Therefore, the terminal settling velocity in a Stokes regime is different according to the density ratio. When the density ratio is sufficiently large, i.e., $\rho = \rho_p / \rho_f \gg 1$ and $\beta \approx 0$, the terminal settling velocity is written as Eq. (2.12a), whereas the terminal settling velocity with a comparable density ratio can be derived from Eq. (2.6) as Eq. (2.12b)

$$W_0 = \tau_p g \tag{2.12a}$$

$$W_0^* = \tau_0^* (1 - \beta)g \tag{2.12b}$$

Then, this gives the relation between τ_p and τ_p^* as:

$$\tau_p^* = \frac{3}{3-\beta}\tau_p \tag{2.13}$$

2.2.2 Settling velocity changes in turbulence

Several studies have been conducted about the turbulence effect on the settling velocity of inertial particles. Table 2. 1 shows a summary including research methodology and contents. Most of the studies propose the settling velocity changes at the turbulent state of the fluid, and its change has the dependency on the Stokes
number, St, and two velocity ratios, Sv_{η} and Sv_{l} , which are defined as:

$$St = \frac{\tau_p}{\tau_k} \tag{2.14a}$$

$$Sv_{\eta} = \frac{W_0}{u_{\eta}} \tag{2.14b}$$

$$Sv_l = \frac{W_0}{\tilde{u}'_{rms}} \tag{2.14c}$$

where τ_k and u_η are the Kolmogorov time and velocity scales, and \tilde{u}_{rms} is the turbulence intensity, the root-mean-squared magnitude of velocity fluctuation, that is $\tilde{u}_{rms}' = \sqrt{u_{rms}'^2 + v_{rms}'^2 + w_{rms}'^2}$. Thus, the Stokes number is the parameter that represents both particle and turbulent characteristics, and velocity ratio, Sv, indicates the ratio of terminal settling velocity to the velocity in a turbulence scale.

Several studies have investigated the settling velocity change of inertial particles in a turbulent fluid using direct numerical simulation (DNS) (Bec et al., 2014; Wang and Maxey, 1993; Yang and Lei, 1998). However, they all dealt with particles and fluids with relatively large density ratios up to 1,000 ($\rho = 1,000$ and $\beta \approx 0$), such as droplets in clouds and aerosols in the atmosphere. At the same time, the sediments or MPs in water have a density ratio from nearly 1 to less than 10. Other studies have been conducted experimental research on the inertial particles in the turbulent water body (Good et al., 2014; Jacobs et al., 2016; Nielsen, 1993; Petersen et al., 2019; Wang et al., 2018; Yang and Shy, 2003, 2021). Kawanisi and

Shiozaki (2008) and Wang et al. (2018) investigated the settling velocity of inertial particles in the open-channel flow. Yang and Shy (2021) conducted experiments using glass particles in the airflow. So, the turbulence and particle interactions of Yang and Shy (2021) may show different characteristics from the other experiments using inertial particles in a water flow. The others used the turbulence tank to generate the turbulence in an essentially stationary water. The recent studies mainly proposed the grouping parameter of Stokes number and velocity ratio parameter, Sv_{η} and Sv_l , dominates in explaining the turbulence effects on settling velocity change. So, in the present study, various parameters were tested, and parameters showing a clear tendency were presented. Fig. 2. 2 shows a summary of the available experimental results of the preceding research, where w_t / w_s represents the settling velocity change. These data were used to be compared the experimental results conducted in the present study.

References	Method	Technique	Flow Condition	Settling velocity change (+: increase, -: decrease)	Contents
Nielsen (1993)	LE	Displacement in image	Stationary*	±	Proposed mechanisms of settling velocity changes
Wang & Maxey (1993)	NS	DNS	Stationary*	+	Maximum w_p change when $St = \tau_p / \tau_k \sim 1$
Yang & Lei (1998)	NS	DNS, LES	Stationary*	+	Maximum w_p change when $St = \tau_p / \tau_k \sim 1$
Yang & Shy (2003)	LE	PTV, PIV	Stationary*	±	Maximum w_p change when $St = \tau_p / \tau_k \sim 1$
Kawanisi & Shiozaki (2008)	LE & NS	ADV, KS	Flume	±	The loitering effect slows w_p in relatively weak turbulence.
Bec et al. (2014)	NS	DNS	Vertical Flow	+	w_p change dependent on $Fr = \epsilon^{3/4}/(gv^{1/4}), Re_{\lambda} = u_{rms}\sqrt{15/(\epsilon v)}$
Good et al. (2014)	LE & NS	PTV, DNS	Stationary*	±	w_p change depends on $Sv_\eta = \tau_p g/u_\eta, Sv_l = \tau_p g/u', St$
Jacobs et al. (2016)	LE	PTV, PIV	Stationary*	±	Maximum w_p change when $0.01 < St < 0.1$
Wang et al. (2018)	LE	PTV, PIV	Flume	±	Maximum w_p change when $d_p/\eta \sim 1$
Petersen et al. (2019)	LE	PTV, PIV	Stationary*	±	w_p change depends on $Sv_\eta\cdot St$
Yang & Shy (2021)	LE	PTV, PIV	Rotating cylinder	+	w_p change depends on $Sv_\eta\cdot St$

Table 2. 1. Preceding studies which investigated the turbulence effect on settling velocities of particles

*Stationary turbulence made by turbulence tank; LE: Laboratory Experiment, NS: Numerical Simulation, DNS: Direct Numerical Simulation, LES: Large-Eddy Simulation, PTV: Particle Tracking Velocimetry, PIV: Particle Image Velocimetry, ADV: Acoustic Doppler Velocimetry, KS: Kinematic Simulation



Fig. 2. 2. Summary of the preceding experimental studies about the settling velocity change of inertial particles in turbulent water; w_t / w_s is the settling velocity change and St is the Stokes number.

2.3 Estimating turbulent kinetic energy (TKE) dissipation rate for turbulence analysis

2.3.1 Particle Image Velocimetry (PIV)

Particle Image Velocimetry (PIV) is a flow-field measurement system for experimental fluid mechanics. PIV can measure the instantaneous velocity field by calculating a displacement vector for each interrogation window with a crosscorrelation between two consecutive frames. Since this method measures the flow field based on an image, it is known that the PIV is a non-intrusive measurement technique. Numerous studies have been conducted to improve the performance of the PIV technique. As a result, various kinds of commercial and open-source software for the PIV method are available. This study used the PIVlab software developed by Thielicke and Stamhuis (2014), capable of performing in MATLAB® (Mathwork Inc.).

2.3.2 TKE dissipation rate

The turbulent kinetic energy (TKE) dissipation rate ϵ is one of the essential turbulence characteristics to estimate turbulence parameters such as Kolmogorov microscales. However, since the PIV methods provide a single representative value over an interrogation window, the direct estimating of ϵ whose conventional calculation demands high spatial resolution, is not suitable for using PIV data without any processing. Thus, in the present study, the large-eddy PIV method developed by Sheng et al. (2000) and improved by Bertens et al. (2015) was used to

estimate the TKE dissipation rate using PIV data.

From Kolmogorov's second similarity hypothesis,

"In every turbulent flow at sufficiently high Reynolds number, the statistics of the motions of scale l in the range $l_0 \gg l \gg \eta$ have a universal form that is uniquely determined by ϵ , independent of ν ."

where l is turbulence lengthscale and l_0 the lengthscale of the largest eddies, the range can be expressed as inertial subrange $l_{EI} > l > l_{DI}$ by adopting the demarcations. The suffixes EI and DI indicate that l_{EI} is between the energy (E) and inertial (I) range, as l_{DI} is that of between the dissipation (D) and inertial (I) ranges. In the inertial subrange, the rate of energy transfer from the large scales determines the constant rate of energy transfer until the rate enters the dissipation range. That is, the rate of turbulent kinetic energy transfer is nearly equal to the rate of turbulent kinetic energy dissipation in the inertial subrange (Pope, 2000). Fig. 2. 3 shows the sketch of the lengthscales and following subranges with a schematic diagram of the energy cascade.



Fig. 2. 3. A schematic diagram of the energy cascade with lengthscale demarcations and following subranges

2.3.3 The method estimating TKE dissipation rate from PIV data suggested by Sheng et al. (2000)

Since only the lengthscales within the inertial subrange are needed to estimate the rate of TKE transfer (or TKE dissipation rate) under Kolmogorov's hypothesis, the measured velocity field is not required to be resolved down to the Kolmogorov scales. Therefore, a direct estimation of the dissipation rate over large flow regions can be provided from the PIV data by borrowing the concept of the large-eddy simulation (LES) (Sheng et al., 2000). Like the filtering operation of LES, the PIV method also filters out the turbulence of a lengthscale smaller than the interrogation window. The mathematical description based on LES modeling is followed below.

The velocity and pressure can be written as:

$$u_i = \overline{U_i} + u_i \tag{2.15a}$$

$$p = \overline{P} + p \tag{2.15b}$$

where $\overline{U_i}$, \overline{P} are the resolved-scale velocity and pressure, respectively, measured by PIV, and u_i , p are the unresolved-scale velocity and pressure. By borrowing filtered conservation equations for LES, those for the large-eddy PIV method are obtained:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{2.16a}$$

$$\frac{\partial \overline{U_j}}{\partial t} + \frac{\partial \left(\overline{U_i} \overline{U_j}\right)}{\partial x_i} = \nu \frac{\partial^2 \overline{U_j}}{\partial x_i \partial x_i} - \frac{\partial \tau_{ij}^r}{\partial x_i} - \frac{1}{\rho} \frac{\partial \overline{P}}{\partial x_j}$$
(2.16b)

where τ_{ij}^r is the residual (or SGS) stress tensor, which must be modeled with some assumptions. Multiplying Eq. (2.16b) by $\overline{U_j}$ to derive the conservation equation for the kinetic energy of the resolved-scale velocity field, the energy transport balance equation is obtained (Sheng et al., 2000):

$$\frac{\partial \overline{q}^{2}}{\partial t} + \overline{U_{i}} \frac{\partial \overline{q}^{2}}{\partial x_{i}} = \frac{\partial}{\partial x_{i}} \left(-\overline{P}\overline{U_{i}} + \nu \frac{\partial \overline{q}^{2}}{\partial x_{i}} - \tau_{ij}^{r}\overline{U_{j}} \right) - \nu \frac{\partial \overline{U_{j}}}{\partial x_{i}} \frac{\partial \overline{U_{j}}}{\partial x_{i}} + 2\tau_{ij}^{r}\overline{S_{ij}}$$
(2.17)

where $\overline{S_{ij}}$ is the resolved-scale strain rate tensor defined as

$$\overline{S_{ij}} = \frac{1}{2} \left(\frac{\partial \overline{U_j}}{\partial x_i} + \frac{\partial \overline{U_i}}{\partial x_j} \right)$$
(2.18)

The last term in Eq. (2.17) is the residual kinetic energy production rate (Pope, 2000).

Revisiting the aforementioned Kolmogorov's hypothesis, the kinetic energy production rate has a nearly equal quantity to the energy dissipation rate in the inertial subrange. Then, the energy dissipation rate can be obtained by adopting the Smagorinsky model to estimate the residual stress tensor. The Smagorinsky model is expressed as:

$$\tau_{ij}^{r} = -2(C_{s}\Delta)^{2} \left| \bar{S} \right| \overline{S_{ij}}$$
(2.19)

where $C_s = 0.17$ is the Smagorinsky constant, Δ is interrogation window size, and the characteristic rate of strain is defined as $\overline{S} = \left(2\overline{S_{ij}}\overline{S_{ij}}\right)^{1/2}$. Substituting Eq. (2.19) into the last term of Eq. (2.17), the energy dissipation rate can be written as:

$$\epsilon = -2\tau_{ij}^r \overline{S_{ij}} = \left(C_s \Delta\right)^2 \overline{S}^3 \tag{2.20}$$

Bertens et al. (2015) improved the large-eddy PIV method by suggesting effective Smagorinsky constants C_s^{eff} , which depend (1) on the degree of window overlap, (2) on how velocity derivatives are approximated, and (3) on which components of the strain tensor are used. In the present study, the overlap rate was set as 50%, and the velocity derivatives obtained by "least squared approximation".

$$\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right)_{i} = \frac{2\mathbf{u}_{i+2} + \mathbf{u}_{i+1} - \mathbf{u}_{i-1} - 2\mathbf{u}_{i-2}}{10\alpha\Delta}$$
(2.21)

The least-square approximation is the 5-point central difference method where α is the overlap rate and subscripts denote the spatial grid locations.

Bertens et al. (2015) rearranged Eq. (2.20) by defining

$$S = \bar{S} / 2^{1/2} = \left(\sum_{i,j} \overline{S_{ij}} \overline{S_{ij}} \right)^{1/2}$$
(2.22)

$$\left\langle S^2 \right\rangle = \frac{1}{2} \sum_{i,j} s_{i,j} \tag{2.23}$$

$$\epsilon = 2^{3/2} \left(C_s^{eff} \Delta \right)^2 S^3 \tag{2.24}$$

where $\langle \cdot \rangle$ means averaging over time and $s_{i,j} = \langle \left(\partial \overline{U}_i / \partial x_j\right)^2 \rangle$. Since the twodimensional PIV method can measure $s_{1,1}$, $s_{1,2}$, $s_{2,1}$, and $s_{2,2}$ but the third velocity component are unavailable. To this end, by using isotropy and incompressibility assumption, missing component for $\langle S^2 \rangle$ should be considered. The missing components for $\langle S^2 \rangle$ can be estimated by using only the diagonal velocity derivatives,

$$\langle S^2 \rangle = \frac{15}{4} (s_{1,1} + s_{2,2})$$
 (2.25)

or using both diagonal and off-diagonal ones Bertens et al., (2015).

$$\langle S^2 \rangle = \frac{3}{2} (s_{1,2} + s_{2,1}) + \frac{3}{4} (s_{1,1} + s_{2,2})$$
 (2.26)

In this study, the only diagonal velocity derivatives are considered to compensate the missing components of $\langle S^2 \rangle$. However, isotropy only applies to the ensemble-averaged squared gradients, so that only $\langle S^2 \rangle$ could be estimated, and the assumption $\langle S^3 \rangle = \langle S^2 \rangle^{3/2}$ is unavoidable (Meneveau and Lund, 1997).

In summary, the parameters to determine the effective Smagorinsky constants C_s^{eff} are chosen as (1) overlap rate α of 0.50, (2) velocity derivatives using the 5-point central difference method, and (3) the only diagonal velocity derivatives for missing components of $\langle S^2 \rangle$. Consequently, $C_s^{eff} = 0.225$ has been used for estimating TKE dissipation rate (See Table 2. 2).

Bertens et al. (2015) argued that the effective Smagorinsky constant is almost independent of the window size when the interrogation window size is $\Delta/\eta \gtrsim 20$. In other words, at this condition, the estimated TKE dissipation rate converges to a specific value that corresponds to the effective Smagorinsky constants. In Sec. 4.2, it was shown that the PIV data of each experiment were in the inertial subrange, and the energy dissipation rate was estimated by the large-eddy PIV method with setting appropriate interrogation window sizes for each case.

Table 2. 2. The ratio of estimated to actual TKE dissipation rate and the effective Smagorinsky constants for different combinations of parameters (Bertens et al., 2015)

	- / -	Ceff		
(1) overlap rate α	(2) method of velocity derivatives*	(3) used strain tensor components	ϵ / ϵ_o	L_s
0.50	3-point	Only diagonal	1.04	0.167
0.50	3-point	Both diagonal and off-diagonal	0.81	0.190
0.50	5-point	Only diagonal	0.57	0.225
0.50	5-point	Both diagonal and off-diagonal	0.41	0.264
0.25	3-point	Only diagonal	1.40	0.144
0.25	3-point	Both diagonal and off-diagonal	1.14	0.264
0.25	5-point	Only diagonal	1.10	0.162
0.25	5-point	Both diagonal and off-diagonal	0.86	0.183

* 3-point and 5-point indicate the 3-point and 5-point central difference method for calculating velocity derivatives, respectively. ϵ_o is the actual TKE dissipation rate.

2.4 Empirical Mode Decomposition (EMD)

The Empirical Mode Decomposition (EMD), suggested by Huang et al. (1998), is a method for decomposing nonlinear and unsteady time series signals into Intrinsic Mode Functions (IMFs) and residual, and extracting fluctuating components from original signals by removing higher-order IMFs, i.e., long - periodic trends. The flow velocity measured by the PIV method showed unsteady characteristics in the open-channel flume experiment. Thus, turbulence analysis was conducted by extracting the velocity fluctuation component from the original time series data of the flow velocity by applying EMD.

At each iteration of EMD procedure, the IMF is estimated by the trend corresponding to the residual from the previous iteration. Each IMF estimated by the EMD procedure have to follow two properties: (1) The IMF has a mean value equal to zero. (2) The IMF has only one extreme between the zero-crossings. This method has been used in several preceding studies about analyzing the turbulent characteristics of unsteady flow by separating the low frequency components, the trends, from the original flow velocity data (Chen et al., 2022; Foucher and Ravier, 2010; Sadeghi et al., 2019). Huang et al. (1998) proposed the algorithm of EMD as follows; the first iteration is given as an example:

- 1. Identify local extrema (maxima and minima) of a given signal x(t)
- Using a cubic spline interpolation, the local maxima are interpolated to obtain the upper envelope of the original signal passing through all maxima, e_{max}(t). Respectively, the lower envelope, e_{min}(t), is also obtained by interpolating the minima.

- 3. The local mean of two envelopes is defined as $m_1(t) = [e_{\max}(t) + e_{\min}(t)]/2$.
- 4. The local difference, $h_1(t)$, is obtained by subtracting the local mean from the given signal, i.e., $h_1(t) = x(t) - m_1(t)$.
- 5. Checking if the local difference $h_1(t)$ satisfies the two properties for being an IMF aforementioned above, then it is considered as the IMF and denoted $IMF_1(t) = h_1(t)$.
- 6. The residual is calculated by subtracting the denoted IMF from the given signal, r₁(t) = x(t) IMF₁(t), and it is taken as the new given signal in step 1 at the next iteration.

After several iterations, the EMD procedure can be stopped if the residual, $r_N(t)$, becomes negligible. Then, the original signal can be expressed as:

$$x(t) = \sum_{i=1}^{N} IMF_{i}(t) + r_{N}(t)$$
(2.27)

Finally, eliminating the IMFs with the dominant periodicity larger than the turbulence time scale, the flow velocity data were reconstructed by summing up the rest of the IMFs.

3. Experimental setup and instrumentations

3.1 Experimental setup

3.1.1 Experiment 1: Open-channel flume

3.1.1.1 Flume specification

This study conducted open-channel experiments in a rectangular laboratory flume of 3.6 m length, 0.3 m width, and 0.5 m height. The sidewalls of a flume were made of acrylic. The flume consists of the water supply system of a reservoir at the end of the flume and a recirculating pump, enabling discharges of up to 6.67×10^{-3} m³/s. A valve controls water discharge. The flowmeter was installed in the pipe connecting the reservoir and the head tank with a measuring capacity from 1.00×10^{-6} to 7.00×10^{-3} m³/s. The flow stabilizers were installed in the head tank to minimize the non-uniform flow condition. At the location of $0.9 \sim 1.8$ m down from the head tank, the bottom of the flume was replaced with glass to make a vertical laser sheet in the test section. Also, to this end, the reflection mirror has been installed under the glass bottom with a 45° angle. The optic table was used to position the camera and laser supply accurately. The schematic diagram and photographs of the flume are shown in Fig. 3. 1 to Fig. 3. 3.



Fig. 3. 1. The schematic diagram of the open-channel flume; (a) side view; (b) top view



Fig. 3. 2. Photograph of the open-channel flume and image acquisition system.



Fig. 3. 3. The reflection mirror installed on the optic table

3.1.1.2 Experimental cases

The experimental cases for the open-channel flume were determined through two factors, the flow condition and the size of the particles. The turbulence condition must be controlled to find out the turbulence effect on the settling velocity of particles. Thus, by varying the flowrate, the turbulence were changed. The particles used in this study are presented in Table 3. 1. Total 8 different sizes of plastic particles (manufactured by Cospheric LLC, Santa Barbara, California, USA) were used and they were made of polyethylene and polyoxymethylene with the size less than 5 mm, which is in the range of commonly defined size of micro-plastics (MPs). But, due to manufacturing limitations, the size of particles smaller than 2000 μ m is presented as a reliable range of diameters. Therefore, under the assumption that the particles are well-distributed in their size, the median particle size d_{50} is used for the representative size of particles. The results of settling velocity in stagnant water were conducted by Jung et al. (2022). Combining the flowrate and particle conditions, the experimental cases for open-channel flume are shown in Table 3. 2.

<i>d</i> [µm]	<i>d</i> ₅₀ [μm]	s.g	Туре	Settling velocity at stagnant water [mm/s]
180-212	196	1.35	PE	7.36
300-355	327.5	1.35	PE	9.77
355-425	390	1.35	PE	1.82×10^1
500-600	550	1.35	PE	2.45×10^{1}
710-850	780	1.35	PE	3.77×10^1
850-1000	925	1.35	PE	4.32×10^1
-	2000	1.41	POM	9.97×10^{1}
-	3000	1.41	POM	1.39×10^{2}

Table 3. 1. The particle information used in the experiments

PE is polyethylene, and POM is polyoxymethylene; s.g is the specific gravity.

Case	Q [L/min]	d [µm]	d ₅₀ [μm]	
OC-Q1D1		180-212	196	
OC-Q1D2	200	300-355	327.5	
OC-Q1D3		355-425	390	
OC-Q1D4		500-600	550	
OC-Q1D5		710-850	780	
OC-Q1D6		850-1000	925	
OC-Q1D7		-	2000	
OC-Q1D8		-	3000	
OC-Q2D1		180-212	196	
OC-Q2D2		300-355	327.5	
OC-Q2D3	250	355-425	390	
OC-Q2D4		500-600	550	
OC-Q2D5		710-850	780	
OC-Q2D6		850-1000	925	
OC-Q2D7		-	2000	
OC-Q2D8		-	3000	
OC-Q3D1		180-212	196	
OC-Q3D2		300-355	327.5	
OC-Q3D3	300	355-425	390	
OC-Q3D4		500-600	550	
OC-Q3D5		710-850	780	
OC-Q3D6		850-1000	925	
OC-Q3D7		-	2000	
OC-Q3D8		-	3000	

Table 3. 2. Experimental cases for the open-channel experiments

3.1.1.3 Image acquisition system

The PIV method was used to estimate the TKE dissipation rate for the turbulence analysis. The images were acquired by a CMOS (Complementary metal-oxide-semiconductor) camera sensor (pco.1200hs, PCO AG, Kelheim, Germany), which can take up to 200 frames per second with a resolution of 1280×1024 pixels. The 60 mm Nikon f/2.8D lens was mounted on the camera. Using the support jack, the camera's vertical position could be controlled precisely. All the frames were recorded in even time steps. The camera was about 15 cm far from the sidewall of the flume. The pixel size was 0.13 mm per pixel. Double-pulse diode pumped solid state (DPSS) laser with the frequency of 532nm was used (RayPower 2000, Dantec Dynamics, Skovlunde, Denmark). The position of the camera and laser supply can be fixed by using an optic table (See Fig. 3. 4 ~ Fig. 3. 6). As the tracer particle of PIV, Silver Coated Hollow Glass Spheres 10 μ m (S-HGS-10, Dantec Dynamics, Skovlunde, Denmark) were used (See Fig. 3. 7).



Fig. 3. 4. Cross-sectional view of the test section of the open-channel flume



Fig. 3. 5.CMOS camera with 60 mm lens



Fig. 3. 6. DPSS laser and supply



Fig. 3. 7. PIV particle; silver coated hollow glass spheres with a mean particle diameter of 10 μ m

3.1.2 Experiment 2: Vertical Recirculation Tube (VeRT)

3.1.2.1 Experimental channel specification

The second experiment was carried out using Vertical Recirculation Tube (VeRT). In order to control the vertical component of carrier fluid's velocity, the channel that can generate vertical directional flow has been constructed. (See Fig. 3. 8 and Fig. 3. 9) It consists of a head tank, PVC pipes with valves, a pump, an experimental section, and a particle injection pipe. The head tank was installed to prevent experimental particles from entering the pump and releasing the pressure inside the pipes. To generate bi-directional flow (upward and downward), two pairs of valves were installed at the PVC pipes. By adjusting the configuration of a pair of valves, the flow direction in the experimental section can be changed. (See Fig. 3. 10) The pump was chosen as having the flow velocity inside the experimental section being up to 1 m/s. The experimental section was 1.5 m height, 150 mm diameter, and made of acrylic. Because PVC pipes have a circular cross-section, the acrylic experimental section was also designed to have the same size of circular crosssection as PVC pipes, which can minimize the influences caused by contractions or expansions of the flow cross-section. However, the circular appearance of an acrylic pipe can cause errors due to the refraction during the optic measurement, such as PTV and PIV. To this end, a cuboid water chamber has been constructed outside the acrylic pipe to reduce the refraction effects. The particle injection pipe has been installed at the top of the VeRT so that the stabilization time for the settling motion of particles can be secured. The injection pipe was made of stainless steel 1.2 m long and 1 mm in diameter. At the top of the injection pipe, the valve is installed to control the injection of particles. The following process conducted the particle injections: (1) Open the valve. (2) Draw water into the silicon tube connected to the injection pipe. (3) Close the valve. (4) The experimental particles are stacked inside the silicon tube using a funnel. (5) Open the valve and close it when the settling particles appear at the end of the injection pipe. (See Fig. 3. 11)

Since the VeRT originally had a circulative structure and short length to stabilize the lateral profile of flow velocity, the improvement for flow stabilization was necessary. First, stainless steel honeycombs were inserted at both ends of the experimental section to make the flow more straightened. They are 75 mm height, and each cell has a diameter of 10 mm. Second, two types of mesh grids were installed at both ends of the experimental section. The sizes of the mesh grids were #24 and #50 with grid cell sizes of 1.06 mm and 0.85 mm, respectively. To make flow mixed well, the larger mesh was installed on the exterior and the smaller one on the interior of the experimental section. (See Fig. 3. 12) Fig. 3. 13 shows the lateral flow velocity profiles at the center line in the FOV measured using the PIV method before and after improvement.



Fig. 3. 8. Schematic diagram of the Vertical Recirculation Tube (VeRT); (a) Top view (b) Front view (c) Side view



Fig. 3. 9. (a) Photograph of the VeRT (b) The experimental section when the laser supply is activated.



Fig. 3. 10. The flow direction according to the configuration of valves; (a) upward flow (b) downward flow



Fig. 3. 11. (a) Photograph of the particle injection pipe (b) Schematic diagram for mechanism of the particle injection



Fig. 3. 12. Instruments for the experimental channel improvement; (left) stainless steel honeycomb (right) mesh grid



Fig. 3. 13. Lateral flow velocity profiles at the center line in the FOV (a) before and (b) after the improvement

3.1.2.2 Experimental cases

Like the open-channel flume experiment, experimental cases for the VeRT were also determined by particle sizes and flow conditions. Among the particles used in Experiment 1 (manufactured by Cospheric LLC, Santa Barbara, California, USA), particles bigger than 300 µm and smaller than 1 mm were selected and used. Because of the diameter of the injection pipe, particles larger than 1 mm cannot be tested. Thus, 5 sizes of particles were used, and their information is presented in Table 3. 1. As the flow conditions, two different directions, upward and downward, were generated in the experimental section. Although the flow directions were different, the experimental cases were selected as the cross-sectional flow velocity had similar absolute values of 16 mm/s. Combining the particle and flow conditions, the experimental cases for the VeRT are shown in Table 3. 3. Among them, the V-Q1D1 case couldn't be conducted because the particles with a diameter of 327.5 µm did not settle in the upward flow condition. Although it was possible to record the floating motions of particles, it was determined that the particle velocity data were unreliable considering the disturbed flow filed near the injection pipe because the particles were suspended as soon as they were ejected from the injection pipe.

	Flow condition				
Case	Direction	Cross-sectional mean velocity [mm/s]	<i>d</i> [μm]	d ₅₀ [μm]	
V-Q1D1		13.21	300-355	327.5	
V-Q1D2			355-425	390	
V-Q1D3	↑		500-600	550	
V-Q1D4			710-850	780	
V-Q1D5			850-1000	925	
V-Q2D1		12.54	300-355	327.5	
V-Q2D2			355-425	390	
V-Q2D3	\downarrow		500-600	550	
V-Q2D4			710-850	780	
V-Q2D5			850-1000	925	

Table 3. 3. Experimental cases for the Vertical Recirculation Tube

3.1.2.3 Image acquisition system

Like the open-channel flume experiment, the PIV method was adopted to measure the flow field and turbulence, and the PTV method was used to measure the settling velocity. In the VeRT experiment, the images were acquired by the highspeed camera (FASTCAM Mini UX50, Photron, Tokyo, Japan; Fig. 3. 14) at 500 frames per second for the PTV with a resolution of 1280×1024 pixels. The Tamron 24-70 mm f/2.8D lens was mounted on the camera. The laser supply was the same as the one used in the open-channel experiment. The camera and laser can be transported by the traverse bound with the frame of VeRT (See Fig. 3. 15). In each flow direction, the distance in the experimental section required for the flow stabilization was different. To this end, different FOVs were applied under the two flow directions. In the upward flow, the FOV was located where the distance between the center point of the FOV and the top of the experimental section was 870 mm. On the other hand, the distance was 1415 mm in the downward flow. That is, the FOV in downward flow was located at a relatively low position. The actual sizes of the FOVs in the two locations were the same, with a lateral width of 166.3 mm and a vertical height of 140.3 mm.

For the experiments of settling velocity measurement, the injected particles settled down as moving in the front-rear direction. Since the focus of the camera was calibrated on the laser sheet, it was necessary to judge if the captured particles were on the laser sheet for the accurate settling velocity measurement. Thus, the side camera, in addition to the main camera, was used to observe whether the particles passed through the laser sheet (See Fig. 3. 15). The side camera was the CMOS
camera used in the open-channel flume experiment. The specification can be found in Sec. 3.1.1.3. Additionally, in order to improve the performance of the PTV algorithm, the halogen lamp was used, as shown in Fig. 3. 16, to provide a sufficient light source that can make the particles reflect well.



Fig. 3. 14. High-speed camera with 24-70 mm lens



Fig. 3. 15. Camera and laser traversing system; (a) camera traverse (b) laser traverse and CMOS camera used as side camera on the top of the frame



Fig. 3. 16. Halogen lamp installed to provide a sufficient light source for the PTV algorithm

3.2 Particle Tracking Velocimetry (PTV)

Particle Tracking Velocimetry (PTV) is the technique that uses particles and images to measure their velocities by tracking the locations of particles at consecutive times. The images for PTV were acquired by the same image acquisition system introduced in Sec. 3.1. In this study, a PTV algorithm to track inertial particles settling in turbulent flow was constructed, and velocities were measured by their settling trajectories. Since the constructed PTV algorithm was designed to operate based on the pixel brightness of the captured image and the minimum pixel size recognized as a particle, several image pre-processing procedures are required to improve its performance and accuracy. The detailed process is as follows: (1) gamma correction of original images, (2) median subtraction, and (3) binarization through locally adaptive thresholding. First, a gamma correction is a method following Eq. (3.1) to increase the image contrast by stretching the interval between the specific brightness intensity bands, the lower brightness intensity band in this study.

$$I_1 = \left(\frac{I_0}{I_{\text{max}}}\right)^{\gamma} \times I_{\text{max}}$$
(3.1)

where I_{max} is the maximum possible brightness intensity value of a pixel, which is 255 (8-bit grayscale image) in this study, I_1 , I_0 are the brightness intensity values after and before applying gamma correction, respectively, and γ is a gamma value that decides the intensity of correction. When γ is smaller than 1, it can make dark

regions of the image lighter and distinguishable by increasing the image contrast of the lower brightness band. Second, the median brightness intensity in the time series of each pixel position was subtracted to remove the background image. Lastly, since the brightness of a laser sheet can vary depending on space, the locally adaptive thresholding method that applies different threshold values for each pixel position is used. Accordingly, better binarization results can be obtained by approximating the change in brightness intensity of the background to the primary plane and then removing it from the original image. The approximation to the primary plane was conducted by the least square method for each image of each time frame. The snapshots of each step of the image pre-processing process are presented in Fig. 3. 17. The final results of image pre-processing are binary images consisting of light pixels, I^+ , and dark pixels, I^- , which have a brightness intensity value of 255 and 0, respectively.

After the image pre-processing, the particles were tracked, and the PTV algorithm measured their velocities. The overall scheme is shown in Fig. 3. 18. First, the locations of the particles in the first frame are selected manually. Then, the moving window with an appropriate size is set based on the first particle locations. Next, each row of the moving window detects continuous bundles of I^+ pixels, and the length of each of them is measured to distinguish the noise on the image or remaining PIV particles from the target MP particles. Then, comparing the lengths of I^+ pixels, the approximate locations of the particles in an instantaneous frame are decided. Finally, to decide the more precise location, the area of a block of I^+ pixels containing the approximate location is calculated. Then, the particle's position is determined by a weighted average of the area.

The MATLAB code for the PTV algorithm is attached to APPENDIX.A. It is necessary that the described pre-processing should be conducted before running the algorithm.



Fig. 3. 17. Snapshots of an image pre-process procedure; (a) original image (b) gamma calibration (c) median subtraction (d) binarization through adaptive thresholding







Step 4



Fig. 3. 18. Overall scheme of PTV algorithm; Step 1: The first location of particle is selected manually; Step 2: Moving window is set based on the locations of Step 1; Step 3: Find the longest continuous bundles of I^+ pixels and determine an approximate location of center point (red x mark); Step 4: Calculating the area containing the approximate location and using weighted average, a precise location of center point (blue x mark) is determined.

The settling velocity can be calculated by the particle location acquired from the PTV method. But, the block of I^+ pixels recognized as a particle often didn't have a constant size during a settling motion. The reason is that the flow was not completely two-dimensional, so the phenomenon where the particles left the laser sheet and then came back made the size of the reflected area keep changing. As a result, if the complete shape of the particles was not reflected, the weight-averaged center point could be fluctuated and be distorted by the frame. To reduce this error, the trajectory tracked by the PTV algorithm was fitted with spline fitting by setting the smoothing parameter. The equation for fitting is introduced below:

$$\phi \sum_{i} (z_{i} - s(x_{i}))^{2} + (1 - \phi) \int \left(\frac{d^{2}s}{dx^{2}}\right)^{2} dx$$
(3.2)

where x_i , z_i are the horizontal and vertical location of tracked particles, and s is the fitted spline function, and ϕ is a smoothing parameter with a value between 0 and 1. $\phi = 0$ produces a least-squares straight line fit to the trajectory, and $\phi = 1$ produces a cubic spline interpolant which passes all points of the trajectory. The smoothing parameter for this study was set as $\phi = 0.999$ to avoid excessive data distortions.

4. Results and discussion

4.1 Effect of parallel advection on the settling velocity

As mentioned in Sec 1.2, the present study has been conducted to inspect whether the settling velocity in turbulent flow follows Eq. (1.2), $w_t = w_s + w + f(St, Sv)$. To this end, the effect of advection parallel to settling motion by a carrier fluid was investigated. In other words, the possibility of the linear summation of advection and the settling velocity from the governing equations of particle movements in PTMs, $w_s + w$, was concerned. However, except for the stagnant water, it is impossible to eliminate the turbulent characteristics of the carrier fluid. Therefore, instead of a laboratory experiment, the settling velocity of inertial particles in uniform steady flow was calculated by numerical simulation of the MRE.

4.1.1 Modified drag force in MRE

The MRE was established assuming that the inertial particles (sphere) have a sufficiently low particle Reynolds number $\left(Re_p = \frac{r|\mathbf{V} - \mathbf{u}|}{v} << 1\right)$. However, particles used in this study, introduced in Sec. 3.1, have the particle Reynolds number larger than unity. So, the drag force term in MRE should be reconsidered by adopting the drag coefficient, which can be used at large particle Reynolds numbers. Therefore, this study proposes the modified MRE by applying the drag coefficient suggested by Cheng (2009). The drag force can be expressed as:

$$F_D = \frac{1}{2} C_D \rho_f (\mathbf{V} - \mathbf{u})^2 A_p \tag{4.1a}$$

$$C_{D} = \frac{24}{\text{Re}_{p}} \left(1 + 0.27 \,\text{Re}_{p} \right)^{0.43} + 0.47 \left[1 - \exp\left(-0.04 \,\text{Re}_{p}^{0.38} \right) \right]$$
(4.1b)

where A_p is an area of particles and the applicable particle Reynolds number range for the suggested drag coefficient is $2 \times 10^{-3} < \text{Re}_p < 2 \times 10^5$. Then, applying this drag force, the general form of the MRE can be expressed as:

$$\frac{d\mathbf{V}}{dt} = \beta \frac{D\mathbf{u}}{Dt} + (1-\beta)g\vec{e_z} - \frac{\beta}{4r}C_D |\mathbf{V}-\mathbf{u}|(\mathbf{V}-\mathbf{u}) - \sqrt{\frac{3\beta}{\tau_p\pi}} \int_{t_0}^t \frac{1}{\sqrt{t-\tau}} \left(\frac{d\mathbf{V}}{dt} - \frac{d\mathbf{u}}{dt}\right) d\tau$$
(4.2)

Thus, the calculation scheme (Eq. 2.9) and the aforementioned abbreviation function **G** is rewritten as:

$$\frac{d\mathbf{w}}{dt} = (\beta - 1)\frac{d\mathbf{u}}{dt} + (1 - \beta)g\overline{e_z} - \beta \mathbf{w} \cdot \nabla \mathbf{u} - \frac{\beta}{4r}C_D |\mathbf{w}|\mathbf{w} - \sqrt{\frac{3\beta}{\tau_p^*\pi}}\frac{d}{dt}\int_0^t \frac{\mathbf{w}}{\sqrt{t - \tau}}d\tau \qquad (4.3)$$

$$\mathbf{G} = \left(\beta - 1\right) \frac{d\mathbf{u}}{dt} - \beta \,\mathbf{w} \cdot \nabla \mathbf{u} - \frac{\beta}{4r} C_D \left|\mathbf{w}\right| \mathbf{w}$$
(4.4)

4.1.2 Validation of the numerical scheme

First of all, the numerical model with "Cheng drag force", Eq. (4.3) and Eq. (4.4), has been validated by the experimental data of settling velocity at the stagnant water in Table 3. 1. So, the background flow field was set to be 0 in both horizontal and vertical directions. Fig. 4. 1 shows the simulated settling velocity of an example case with a particle diameter of 390 μ m and specific gravity of 1.35. The dashed and dotted lines in Fig. 4. 1 indicate the simulation results using Stokes and Cheng drag force, respectively. The velocity of a particle with Stokes drag force was deemed not converged enough compared to Cheng drag force. This is because, since the particle Reynolds number of the example case is $\operatorname{Re}_{n} = 7.02$, the particle is out of the Stokes regime, so the drag force estimated by Stokes drag should be underestimated. Thus, to reach the terminal settling velocity, the drag force must approximate the buoyancy force. Consequently, a higher settling velocity is required, and the convergence of settling velocity demands more time. Fig. 4. 2 illustrates the actual values of the acceleration terms using Stokes drag force and Cheng drag force. The magnitudes of acceleration terms were obtained by calculating each term in Eq. (2.6) and Eq. (4.2). Likewise, when Cheng drag force was applied, the settling velocity converged earlier than when Stokes drag force was applied. The acceleration of the drag has grown faster. Conducting the calculation with various diameters and fixed specific gravity

of 1.35, the dashed and dotted lines of Fig. 4. 3 can be drawn, which represent when the Stokes drag and Cheng drag are applied, respectively. As seen in Fig. 4. 3, it is confirmed that the calculation using the Cheng drag showed good agreement with the experimental results with the measured settling velocities at the stagnant water, even at the small particles with low particle Reynolds numbers. Thus, it can be concluded that the suggested numerical scheme with Cheng drag force, from Eq. (4.2) to Eq. (4.4), can simulate the settling motions of inertial particles in the fluid.



Fig. 4. 1. Calculated settling velocity when Stokes or Cheng drag was applied; example case with a diameter of 390 μ m and specific gravity of 1.35



Fig. 4. 2. The magnitude of accelerations in the Maxey-Riley equation with calculating the drag force term (a) as the Stokes drag (b) with the drag coefficient suggested by Cheng (2009).



Fig. 4. 3. Numerical calculation results with various particle diameters and fixed specific gravity of 1.35. The dashed lines indicate the terminal velocity in a Stokes regime with a large (blue) and small (red) density ratio, β . The dash-dot and dotted black line represent the results of calculating the drag force as the Stokes drag with the drag coefficient suggested by Cheng (2009), respectively.

4.1.3 Numerical simulation in a steady uniform flow

Before conducting the numerical simulation to investigate the effect of advection exerted in a parallel direction on the settling motion of particles, the implication of the motion equation was qualitatively interpreted. Eq. (4.3) can be rewritten in a steady uniform flow (i.e., $\frac{d\mathbf{u}}{dt} = 0$ and $\nabla \cdot \mathbf{u} = 0$) as below:

$$\frac{d\mathbf{w}}{dt} = (1-\beta)g\vec{e_z} - \frac{\beta}{4r}C_D |\mathbf{w}|\mathbf{w} - \sqrt{\frac{3\beta}{\tau_p\pi}}\frac{d}{dt}\int_0^t \frac{\mathbf{w}}{\sqrt{t-\tau}}d\tau$$
(4.5)

When a particle approaches the terminal settling velocity, $\frac{d\mathbf{w}}{dt} = 0$, the

buoyancy term and drag force term become the same, considering the history force is relatively negligible, as shown in Fig. 4. 2. Then, since the buoyancy is determined by the particle characteristics and the drag coefficient C_D is also the function of a relative velocity (See Eq. (4.1b)), the relative velocity \mathbf{w} is fixed as a specific value. Hence, the "terminal relative velocity" is decided according to the particle. In other words, if the flow velocity changed, the particle velocity should be changed with the same flow velocity change to maintain the magnitude of the relative velocity. Thus, the linear summation of the flow velocity of a carrier fluid and the particles' settling velocity is deemed reasonable.

Subsequently, the numerical simulations of a particle settlement in the steady uniform flow were conducted to verify the possibility of the linear summation

of advection and settling velocity. Fig. 4. 4 shows that the comparison of the results of numerical simulation with a linear summation of advection and settling velocity in stagnant water. The black dashed line indicates the numerical result of settling velocities without an advection (i.e., in stagnant water). Two dots (red and blue) represent the numerical results with the downward and upward advection. The other two dotted lines (magenta and cyan) were calculated by a simple linear summation of the advection and settling velocity in stagnant water. The two pairs of dots and dotted lines showed excellent agreement with each other, with the R-squared value of 1.0000 and 0.9999. In conclusion, it is proven that the effect of parallel advection on the settling velocity can be calculated as the linear summation in quantitative and qualitative ways.



Fig. 4. 4. Comparison of the results of the numerical simulations of settling velocities according to particle sizes with or without advection and the linear summation of advection and terminal settling velocity

4.2 Experimental results

4.2.1 Experiment 1: Open-channel flume

4.2.1.1 Results of the settling velocities in open-channel flow

This section describes a quantitative description of the settling velocity of inertial particles in open-channel flow. Fig. 4. 5 and Fig. 4. 6 show examples of the particle trajectories tracked by the PTV algorithm and the histograms of the measured velocities. Each experimental case was repeated to acquire the settling velocities of at least 7 particles. However, since the number of particles captured in each experimental case and the total time of PTV being operated was different, the resultant settling velocity was calculated with an ensemble average over the whole trial for the same experimental case.

The histograms of measured settling velocities showed that some particles tend to have a different mean settling velocity than others. This situation can be explained in two ways. One was that the instantaneous streamline had been generated, and the particle followed it. The other was the unordinary shape of particles. Due to the manufacturing error of the plastic particles, a few particles were fragmented or non-spherical. Also, the relatively small plastic particles were easy to flocculate because of their material properties. Since the floc of particles also has a non-spherical shape, the settling velocities might be varied during the settlement. Also, a non-spherical shape could cause a rotating motion, making the settling trajectory bend as Fig. 4. 5(c). On the other hand, although Fig. 4. 5(d) also showed a bent trajectory, relatively large particles could easily be controlled not to flocculate using a surfactant.

All experimental results are summarized in Table 4. 1 and compared with the results of the stagnant water. All the cases showed larger settling velocities in the turbulent flow, w_t , than in the stagnant water, w_s . Fig. 4. 8 illustrates the measurement results of settling velocity according to the particle diameter in which the errorbars were calculated by the standard deviation of measured velocities. As seen clearly in Fig. 4. 8, the smaller the particle, the larger the settling velocity increase was observed. As mentioned in the previous section, the advection can be superposed to settling velocity linearly, so the mean velocity of fluid velocity supposed to be exerted on the particle has been subtracted from the measured settling velocity to evaluate the turbulent effect. As a result, Fig. 4. 7 showed the particle's relative velocity to the fluid $(w_p - w)$. In some cases, OC-Q2D1, OC-Q1D3, OC-Q1D5, and OC-Q1D6, the relative velocity smaller than the terminal settling velocity in stagnant water has been observed.



Fig. 4. 5. Settling trajectories of inertial particles for the open-channel experiment: examples of case (a) OC-Q1D1 (b) OC-Q2D4 (c) OC-Q2D6 (d) OC-Q3D2



Fig. 4. 6. Histogram of measured settling velocities for each particle for the open-channel experiment: examples of case (a) OC-Q1D1 (b) OC-Q2D4 (c) OC-Q2D6 (d) OC-Q3D2

Case	Settling velocity [mm/s]		
	Stagnant water; w_s	Turbulent water; w_t	
OC-Q1D1	7.36	21.2	
OC-Q1D2	9.77	29.5	
OC-Q1D3	18.2	28.0	
OC-Q1D4	24.5	41.9	
OC-Q1D5	37.7	47.2	
OC-Q1D6	43.2	48.9	
OC-Q1D7	99.7	119.3	
OC-Q1D8	139.3	167.9	
OC-Q2D1	7.36	15.6	
OC-Q2D2	9.77	24.5	
OC-Q2D3	18.2	34.1	
OC-Q2D4	24.5	42.5	
OC-Q2D5	37.7	57.3	
OC-Q2D6	43.2	71.8	
OC-Q2D7	99.7	122.6	
OC-Q2D8	139.3	172.5	
OC-Q3D1	7.36	31.7	
OC-Q3D2	9.77	33.0	
OC-Q3D3	18.2	34.4	
OC-Q3D4	24.5	33.3	
OC-Q3D5	37.7	41.1	
OC-Q3D6	43.2	52.2	
OC-Q3D7	99.7	127.6	
OC-Q3D8	139.3	175.0	

Table 4. 1. Comparison of experimental results of the settling velocities in stagnant water and open-channel flow



Fig. 4. 7. Measured settling velocities according to the particle diameter; the errorbars were calculated by the standard deviation of measured velocities



Fig. 4. 8. Relative velocities of particles to fluid velocity; OC-Q2D1, OC-Q1D3, OC-Q1D5, and OC-Q1D6 showed the relative velocities smaller than the terminal settling velocity in stagnant water

4.2.1.2 Results of turbulence analysis in open-channel flow

The visualized results through the PIV data of the open-channel experiment have shown the periodic downward streamlines in the FOV and fluctuating flow velocity up to $\pm 50\%$ of time-averaged velocity at the specific point. This nonlinear time series signal of flow velocity causes an overestimation of the turbulence intensity because the deviation from the mean value increases due to the unordinary fluctuation. Therefore, the EMD process has been conducted for appropriate turbulence analysis by eliminating the long periodicity of time series data of the flow velocity. The EMD method was applied at each interrogation window of the PIV results. Fig. 4. 9 illustrates an example of the result of which the EMD method was performed. As the order of the IMFs increases, the signal with longer periodicity has been decomposed. Then, the IMFs with longer periods than the turbulence time scale, such as the integral time scale, were regarded as the trend, and those were eliminated with the residual together. As a result, the fluctuating part of flow velocity, the turbulence component, could be estimated by summing up the rest of the IMFs (from the first IMF to the certain order of remained IMF). Fig. 4. 10 shows the comparison of fluctuating part of flow velocity between the original signal and the signal reconstructed by the EMD method. It is noticeable that the trend is successfully eliminated, and meanwhile, the small scale of temporal variations still remains in the reconstructed signal.



Fig. 4. 9. Example of decomposed time series signal by the EMD method; The signal at the top is the original signal, the bottom is the residual, and the rest are the IMFs.



Fig. 4. 10. Comparison of the fluctuating velocity component between the original signal and the reconstructed signal

Using the turbulence signal obtained by applying the EMD method, the turbulence analysis has been conducted by the method introduced in Sec. 2.3.3. Through trial and error, the appropriate interrogation window size for each flowrate case to make the turbulent kinetic energy dissipation rate converge was determined. The applied interrogation window sizes of the PIV method were 96, 96, and 64 pixels for the OC-Q1, OC-Q2, and OC-Q3 cases, respectively. Using the estimated TKE dissipation rate, the Kolmogorov microscales were calculated. The results of turbulence analysis were shown in Table 4. 2 where $\overline{(\cdot)}$ means time average of velocity, the suffix "rms" means root-mean-square velocity fluctuation of fluid, and downward is represented as positive in a vertical direction. The streamwise relative turbulence intensity I_t represents the relative unsteadiness of fluctuation velocity components against the time-averaged velocity, which can be defined as:

$$I_t = \frac{u'_{rms}}{\overline{u}} \tag{4.6}$$

Nezu and Nakagawa (1993) has proposed the empirical equation for the streamwise relative turbulence intensity, combining the velocity-defect law and empirical equation of the turbulence intensity, normalized with the friction velocity, in open-channel flow.

$$\frac{u'}{\bar{u}} = \frac{2.3\exp(-z/h)}{\kappa^{-1}\ln(z/h) + w(z/h) + A'}$$
(4.7a)

$$A' = \kappa^{-1} \ln(\operatorname{Re}_*) + A \tag{4.7b}$$

where κ is von Karman constant, z/h is the depth ratio, w(z/h) is the wake function given by Coles (1956), Re. is Reynolds number calculated by a friction velocity, and A is the integration constant. The friction velocity was estimated by fitting the vertical profile from PIV results to the velocity-defect law. According to the estimated friction velocity, the following friction Reynolds number is 890. The solid line of Fig. 4. 11 is calculated by the empirical equation of Nezu and Nakagawa (1993) when the friction Reynolds number is 890. The experimental data were plotted as circle markers, and they have a similar streamwise relative turbulence intensity with the values of the empirical equation. This means that the turbulence intensity is in the reasonable range. Also, the turbulence signal obtained by the EMD method was reliable in considering the turbulence made in open-channel flow.

All results of turbulence intensity and Kolmogorov microscales in Table 4. 2 represent that the higher the flowrate, the stronger turbulence. Fig. 4. 12 illustrates the turbulent energy spectrum from PIV data. The wavenumber k was obtained by:

$$k = \frac{2\pi f}{\tilde{U}} \tag{4.8}$$

where f is a frequency and \tilde{U} is the magnitude of a mean velocity, i.e., $\tilde{U} = \sqrt{\overline{u}^2 + \overline{w}^2}$. The energy spectrum function E(k) follows the power-law behavior, $k^{-5/3}$, except for the high wavenumber range which came from a signal noise, where the energy spectrum piled up under the certain wave number corresponding to the interrogation window size. The turbulent energy within the window has been stacked and usually neglected in other studies.

The Taylor microscale introduced in Table 4. 2 was computed as follows:

$$\lambda = \sqrt{15\frac{\nu}{\varepsilon}} \cdot \tilde{u}'_{rms} \tag{4.9}$$

where \tilde{u}_{rms} is the root-mean-squared magnitude of velocity fluctuation. And following Taylor-scale Reynolds numbers, $\text{Re}_{\lambda} = \frac{\tilde{u}_{rms}^{'}\lambda}{v}$, are 42, 39, and 49.

Case	OC-Q1	OC-Q2	OC-Q3
Flow rate, Q [L/min]	200	250	300
Water depth, <i>h</i> [m]	0.235	0.240	0.245
Interrogation window size, Δ [pixel]	96	96	64
Interrogation window size, Δ [mm]	13.29	13.29	8.86
$ar{u}$ [mm/s]	39.94	56.33	62.45
\overline{w} [mm/s]	10.04	11.41	5.674
$u'_{ m rms}~[m mm/s]$	3.743	5.026	6.823
w' _{rms} [mm/s]	4.025	3.996	6.805
Streamwise relative turbulence intensity, I_t [%]	9.371	8.922	10.92
TKE dissipation rate, $\epsilon [mm^2/s^3]$	4.275	9.101	29.32
Kolmogorov length scale, η [µm]	751.7	622.3	464.5
Kolmogorov velocity scale, v_k [mm/s]	1.476	1.783	2.388
Kolmogorov time scale, τ_k [s]	0.5094	0.3491	0.1945
Taylor microscale, λ [mm]	7.668	6.139	5.133
Taylor-scale Reynolds number, $\operatorname{Re}_{\lambda}$	42	39	49

Table 4. 2. Results of turbulence analysis of the open-channel experiment



Fig. 4. 11. Comparison of the streamwise turbulence intensity between the experimental data and the empirical equation of Nezu & Nakagawa, (1993) to verify the appropriateness of measured turbulence



Fig. 4. 12. Turbulent energy spectrum of the experimental cases; (a) OC-Q1 (b) OC-Q2 (c) OC-Q3

4.2.2 Experiment 2: VeRT

4.2.2.1 Results of the settling velocities in vertical flow

Unlike the open-channel experiment, in the VeRT, the vertical flow existed, which could affect the settling velocity of particles. As mentioned in Sec. 4.1, when the particles reach a state of near-equilibrium, the advection exerted on the settling particles makes the settling velocity superpose linearly. Therefore, the relative velocity where the flow velocity has been subtracted from the measured particles' settling velocity has been used for analysis. To estimate the subtracted flow velocity, first, the whole flow field of the FOV was obtained by an ensemble average of the repeated PIV measurement. Since the laser supply could not cover the entire vertical length of the FOV, the PIV measurements were conducted by separating the FOV into three ROIs with upper, middle, and lower parts. The PIV measurements were repeated 20 times in each divided ROI, and the ensemble average was calculated. Then, the PIV results were connected, and the flow field for the entire FOV can be acquired as Fig. 4. 13. Fig. 4. 13 are for upward and downward flow, respectively. Then, the vertical velocities along the particle trajectory were obtained using the flow field, and their mean value was subtracted from the measured settling velocity.



Fig. 4. 13. Description image of divided ROI and acquired mean flow field; (a) upward flow (b) downward flow
For the reliability of the experimental results, particles' settling velocities for each experimental case were measured with repetitive trials at least 10 times. Fig. 4. 14 illustrates the settling trajectories of a single trial for specific experimental cases. Fig. 4. 14(a) and (c) are the cases for upward flow, and they showed more unvaried and straight trajectories than Fig. 4. 14(b) and (d). For the same experimental cases, the histograms of settling velocity for entire trials were represented in Fig. 4. 15. Likewise, (a) and (c) show the distributions more concentrated to the mean, which means the settling velocities did not vary during the settlement.

An ensemble average of entire trials for each experimental case was used for the experimental results. And relative velocities calculated by subtracting mean flow velocity along the particle's trajectory from the settling velocity were summarized in Table 4. 3. As mentioned in Sec. 3.1.2, the V-Q1D1 case could not be conducted because the particles with a diameter of $327.5 \mu m$ float immediately as being ejected from the injection pipe. Fig. 4. 17 shows the measured values of settling particles in vertical flows and stagnant water. Fig. 4. 16 illustrates the comparison between the settling velocity in stagnant water and the relative velocities of the VeRT experiment for both flow conditions to evaluate only the turbulent effect as done in the open-channel experiment. The figure shows that the relative velocities were faster than the settling velocities in stagnant water. Also, the smaller the particle, the larger the settling velocity increase was observed.



Fig. 4. 14. Settling trajectories of inertial particles for the VeRT experiment: examples of a single trial of case (a) V-Q1D1 (b) V-Q2D1 (c) V-Q1D4 (d) V-Q2D4



Fig. 4. 15. Histogram of measured settling velocities for entire trials of the VeRT experimental case: examples of case (a) V-Q1D1 (b) V-Q2D1



Fig. 4. 15. examples of case (c) V-Q1D4 (d) V-Q2D4

Table 4. 3. Comparison of experimental results of the settling velocities in stagnant water and vertical flow; Relative settling velocity was calculated by subtracting average of flow velocity along the particle trajectory.

	Settling velocity [mm/s]			
Case	Stagnant water; <i>W_s</i>	Turbulent water; W_t	Relative settling velocity; $w_t - w$	
V-Q1D1	9.77	-	-	
V-Q1D2	18.2	4.85	21.4	
V-Q1D3	24.5	14.0	30.5	
V-Q1D4	37.7	23.9	40.4	
V-Q1D5	43.2	33.6	50.2	
V-Q2D1	9.77	29.1	14.8	
V-Q2D2	18.2	35.0	20.7	
V-Q2D3	24.5	46.0	31.2	
V-Q2D4	37.7	56.6	38.6	
V-Q2D5	43.2	64.7	44.9	



Fig. 4. 16. Measured settling velocities according to the particle diameters for the VeRT experiments and stagnant water; the errorbar indicates the standard deviation of velocities



Fig. 4. 17. Comparison of settling velocities in stagnant water and the measured relative velocities for the VeRT experiments according to the particle diameter

4.2.2.2 Results of turbulence analysis in vertical flow

For the VeRT experiment, since the flow velocity was well controlled and showed steady characteristics, as described in Sec. 3.1.2, the pre-processing (EMD) for the measured velocity has not been carried out. Since the FOVs used in the experiments for the settling velocity measurement were too large to capture the small scales of turbulence, a more enlarged FOV should be applied. While the results of the VeRT experiment were calibrated using the width of the channel, 150 mm, the enlarged FOV could not cover the whole width of the channel. So, the PIV data for the turbulence measurement was calibrated by using the experimental particle with a diameter of 925 µm instead of the width of the channel. It is checked that the particle passes through the laser sheet precisely using the side camera for accurate calibration, considering the camera's focal length. And the particle size (vertical and lateral pixel lengths) for each frame was measured by the PTV algorithm. Fig. 4. 18 shows the snapshots for the calibration procedure. For each of the total 981 frames, all pixels occupied by a particle were detected. As a result, the mean values of lateral and vertical pixel length were calculated as 14.62 pixels and 14.43 pixels, respectively. The average of both measurements was applied to the calibration, whose value was 0.0637 mm/pixel.

The interrogation window sizes of the PIV method for both flow conditions were 64 pixels. Like the open-channel experiment, the Kolmogorov microscales are calculated using the estimated TKE dissipation rate. The results of turbulence analysis were shown in Table 4. 4, where downward is represented as positive in a vertical direction. The cross-sectional mean velocity is calculated at the vertical center line of each FOV. Naturally, the streamwise relative turbulence intensity was calculated by $I_t = w'/\bar{w}$, and the empirical equation used in the open-channel experiment was not considered in the VeRT experiment because it was derived for an open-channel flow. The results of turbulence analysis showed that the two flow conditions had similar turbulence intensity, not only in terms of the root-mean-square values but also the Kolmogorov microscales. Fig. 4. 19 illustrate the turbulent energy spectrum. The wavenumber range judged as a signal noise has been determined by the same portion of the wavenumber range for the open-channel experiment. Plus, Taylor-scale Reynolds numbers for the VeRT experiment are 14 and 15, respectively.



Fig. 4. 18. Snapshots of the calibration procedure for the turbulence measurement; Left figure shows the trajectory of the calibration particle with the full FOV, and the rest are presented to show the detailed view of the detected area of a particle.

Case	V-Q1	V-Q2
Cross-sectional mean velocity, U [mm/s]	-13.21	12.54
Interrogation window size, Δ [pixel]	32	32
Interrogation window size, Δ [mm]	1.984	8.241
$ar{u}$ [mm/s]	0.2995	-0.2402
\overline{w} [mm/s]	-13.28	12.57
$u'_{ m rms}~[m mm/s]$	2.972	3.017
w' _{rms} [mm/s]	3.526	3.585
Streamwise relative turbulence intensity, I_t [%]	26.56	28.52
TKE dissipation rate, $\epsilon [\text{mm}^2/\text{s}^3]$	17.72	16.02
Kolmogorov length scale, η [µm]	488.2	500.7
Kolmogorov velocity scale, v_k [mm/s]	2.224	2.002
Kolmogorov time scale, τ_k [s]	0.2027	0.2502
Taylor microscale, λ [mm]	3.004	3.210
Taylor-scale Reynolds number, $\operatorname{Re}_{\lambda}$	14	15

Table 4. 4. Results of turbulence analysis of the VeRT experiment



Fig. 4. 19. Turbulent energy spectrum of the experimental cases; (a) V-Q1 (b) V-Q2 $\,$

4.3 Effect of turbulence on settling velocity change

The results of the two experiments showed that the particles' relative velocity generally increases except for some cases in the open-channel experiment. So, it can be concluded that as an advection has been subtracted, the settling velocity in turbulent flow increased due to the turbulent effect. The detailed turbulence effect on the settling velocity change, i.e., w_t / w_s , was analyzed based on the turbulence measurement results in Sec 4.2. First of all, entire experimental results were represented in Fig. 4. 20. To evaluate the turbulent effect on settling velocity, the experimental data from previous studies were also plotted with results of the present study in Fig. 4. 21, which is the log-scale plot of settling velocity change according to Stokes number. Since Stokes number is a non-dimensional parameter which can represent both particle and turbulence characteristics, it is one of the parameters widely used in the preceding research. The errorbars in the figure were calculated with the law of error propagation, which is:

$$\sigma_f^2 = \sum_{i=1}^n \left(\frac{\partial f(X_1, X_2, \cdots, X_n)}{\partial X_i} \right)^2 \sigma_{X_i^2}$$
(4.10)

where σ is a standard deviation and X_i is the random variable. Thus, the error of settling velocity change and Stokes number were calculated by statistical properties of particle diameters (confidence level of manufacturing accuracy), measured

settling velocity, and estimated TKE dissipation rate.

In Fig. 4. 21, the experimental results have shown a generally increasing settling velocity change as the Stokes number has decreased. However, when the Stokes number is larger than 0.3, the diverged values of settling velocity change have been observed according to the experimental conditions. Especially the experimental results of the present study and Yang and Shy (2003) showed converged settling velocity changes with 10 to 20% increases and - 3% to + 10% changes, respectively. The experimental particles in that range of the present study had relatively large sizes with diameters of 2 and 3 mm and different densities from the other particles, whose specific gravity was 1.41 while the others' was 1.35. In addition, the results of Yang and Shy (2021) showed a relatively large settling velocity change compared to the corresponding Stokes numbers of other research. As mentioned in Sec. 2.2, Yang and Shy (2021) conducted experiments with glass particles in airflow, so it is possible that the different particle characteristics, such as specific gravity and density ratio of particles to the fluid, could result in a diverged values of settling velocity change despite of the similar Stokes number. Secondly, in order to inspect the effect of the relation between particles' size and turbulence length scale instead of times scale ratio, Stokes number, the settling velocity change according to the ratio of particles' diameter to Kolmogorov lengthscale was plotted in Fig. 4. 22, as introduced in Wang et al. (2018). The length ratio of the preceding research could be estimated by the given data of turbulence and particle information. The overall tendency was similar to Fig. 4. 21. Still, it was noticeable that the local minima of settling velocity change appeared in the experimental results of the present study when $d/\eta = 1 \sim 2$, which indicates that the particle size was

comparable to the Kolmogorov lengthscale. In addition, the experimental results of Yang and Shy (2021) moved to the more concentrated location compared to the figure with the Stokes number. Still, the settling velocity changes showed too various values in $0.5 < d/\eta < 10$, and this range covers most experimental results. Therefore, it can be concluded that the "direct" lengthscale ratio of particle diameter to Kolmogorov lengthscale seems inappropriate to describe the turbulence effect on settling velocity change.



Fig. 4. 20. Summary of the settling velocities for entire experimental cases according to the particle diameter



Fig. 4. 21. Comparison of experimental results with preceding studies about the relation between the settling velocity change and Stokes number; The errorbars were calculated with the error propagation according to statistical properties of particle sizes and TKE dissipation rate.



Fig. 4. 22. Comparison of experimental results with preceding studies about the relation between the settling velocity change and the ratio of the particle diameter to the Kolmogorov lengthscale

Using the Rouse number, the velocity scale parameters Sv_{η} and Sv_{l} , presented in prior studies (Good et al., 2014; Petersen et al., 2019; Yang and Shy, 2021), the additive investigation of the relation between Rouse numbers and the settling velocity change has been conducted. As mentioned in Sec. 2.2.1, however, W_{0} indicates the terminal settling velocity where the density ratio of a particle to fluid is sufficiently large, and a particle Reynolds number is small enough to be considered in a Stokes regime. Therefore, considering the experimental conditions where the density ratio of experimental particles was an order of O(1) and particle Reynolds numbers vary up to 400 which is out of a Stokes regime, the measured terminal settling velocity in stagnant water was used instead of W_{0} for velocity scale parameters. Consequently, the settling velocity change according to

$$Sv_{\eta} = \frac{w_s}{u_{\eta}}$$
 and $Sv_l = \frac{w_s}{\tilde{u}'_{rms}}$ have been plotted in Fig. 4. 23 and Fig. 4. 24. In both

figures, the experimental results of Yang and Shy (2021) have shown relatively small values in velocity scale parameters, compared to other experimental results. On the other hand, the grouping parameters of velocity and time scale parameters, $Sv_{\eta}St$ and $Sv_{l}St$ also have been introduced in preceding studies. Since they can be calculated by multiplying velocity and time scale parameters, these grouping parameters also can be interpreted as lengthscale parameters.

$$Sv_{\eta}St = \frac{w_s}{u_n} \cdot \frac{\tau_p}{\tau_k} = \frac{w_s \tau_p}{\eta}$$
(4.11a)

$$Sv_l St = \frac{w_s}{\tilde{u}'_{rms}} \cdot \frac{\tau_p}{\tau_k}$$
(4.11b)

Indeed, these grouping parameters can be compared with a simple lengthscale parameter mentioned above, d/η , used in Fig. 4. 22. Fig. 4. 25 and Fig. 4. 26 using the grouping parameters show more arranged experimental data including results of Yang and Shy (2021) and tungsten particles in Yang and Shy (2003) that had inconsistent values in Fig. 4. 22. Also, comparing with Fig. 4. 21 illustrated by using Stokes number, the graph seems to have a similar trend where the settling velocity increases as the grouping parameter decreases. But, it is noticeable that the 6 points at the large SvSt of Fig. 4. 25 and Fig. 4. 26 have been separated from the other results, compared to Fig. 4. 21. These 6 points were relatively large particles with a size of 2000 and 3000 µm experimented in the openchannel flow. In Fig. 4. 21, these experimental cases have Stokes numbers similar to results in prior studies, Yang and Shy (2003), so, overall, it seems that there is a tendency for the settling velocity change to converge into 1. Still, actually, the settling velocity changes of those cases were larger than the previous studies' results by two times. However, by adopting the grouping parameters, these cases were separated from the results of other studies, and it was observed that the envelope of graphs had clearer curve shapes with a reduced variation compared to Fig. 4. 21, which has local minima when $Sv_n \cdot St = 20$ or $Sv_l \cdot St = 10$. On the other hand, the experimental results of Yang and Shy (2021) showed slightly larger settling velocity changes than the other research near SvSt = 1. As aforementioned, it is considered as the different density ratio between particles and the fluid caused the differences in settling velocity changes.

In conclusion, both Fig. 4. 25 and Fig. 4. 26 have shown an increasing

tendency as the grouping parameter, $Sv_{\eta}St$ and $Sv_{l}St$, decreases. This means the ratio of length scale related particle settlement and turbulence characteristics can be considered as a dominant characteristic which reflects turbulence effect on settling velocity enhancement in a range of $O(10^{-2}) < SvSt < O(10)$. So, since the settling velocity increases, the predicted transport length can be overestimated in this range when an existing predicting method of vertical displacement in the particle tracking model has been applied. On the other hand, several studies suggested that the hindering of settling velocity can occurs at a large Stoke number ($O(10) < St < O(10^2)$), (Good et al., 2014; Petersen et al., 2019; Wang et al., 2018; Yang and Lei, 1998). When the conditions where heavier particles than microplastics are in the water are considered, *St* can exceed O(10). Hence, it is necessary that more experiments with various turbulence intensity and particle conditions should be conducted to find out the turbulent effect on settling velocity including a relatively large Stokes number.



Fig. 4. 23. Settling velocity change according to velocity scale parameter using Kolmogorov velocity scale, $Sv_{\eta} = w_s / u_{\eta}$



Fig. 4. 24. Settling velocity change according to velocity scale parameter using turbulence intensity, $Sv_i = w_s / \tilde{u}'_{rms}$



Fig. 4. 25. Settling velocity change according to the grouping parameters of velocity scale parameter using Kolmogorov velocity scale, $Sv_{\eta} = w_s / u_{\eta}$ and time scale parameter, i.e., $Sv_{\eta}St = w_s\tau_p / \eta$



Fig. 4. 26. Settling velocity change according to the grouping parameters of velocity scale parameter using Kolmogorov velocity scale, $Sv_l = w_s / u'$ and time scale parameter, i.e., $Sv_\eta St = w_s \tau_p / \tilde{u}'_{rms} \tau_k$

5. Conclusion

Three kinds of inspection were conducted to investigate the interactions of terms that comprise the governing equation of particle velocity calculation in PTMs. First, the numerical simulation of settling particles in a steady uniform flow was carried out to find the effect of the parallel direction of advection on settling velocity. Using the steady uniform conditions, the equation of motion has been rearranged with a particle's relative velocity. From the rearranged equation, it can be concluded that the particle characteristics determine the particle's relative velocity, and the particle's settling velocity can be superposed linearly with the fluid velocity and a settling velocity in a still fluid. Secondly, the open-channel experiment has been conducted to investigate the turbulence effect on settling velocity in a horizontal flowing condition. The settling velocities and flow field were measured using the PTV and PIV method. To eliminate long periodicity in the results of flow velocity measurements, the EMD process was performed, and higher orders of IMFs were subtracted from the original velocity signal. After removing long-periodicity, the TKE dissipation rate for computing Kolmogorov microscales has been estimated by the method suggested by Sheng et al. (2000) and developed by Bertens et al. (2015). Thirdly, Vertical Recirculation Tube experiments have been carried out to investigate the turbulence effect on settling velocity in a vertical flow. Likewise, the PTV and the PIV method were used to measure particles' settling velocities and flow field. From the results of the numerical simulation, the background flow should be subtracted from the measured particles' settling velocities, so the particles' relative velocities were used for the analyses in both laboratory experiments.

The results of laboratory experiments have shown generally larger settling velocities in turbulent flow than in stagnant water. So, turbulence can affect the enhancement of settling velocity. Several parameters containing turbulence and particle characteristics have been considered by using the results of turbulence analyses to find out the dominant parameters that can explain the turbulence effect on settling velocity increase. The parameters were expressed as the ratio of characteristic particles to turbulence in time, velocity, and length scales. First, Stokes number, the time scale ratio, has been investigated among those parameters. The settling velocity changes showed a consistent decreasing tendency as the Stokes number increased. But, when the Stokes number is larger than 0.3, settling velocity changes of the present study and those in previous studies showed different values, although they have similar Stokes numbers. Secondly, the length scale ratio, d/η , has been inspected. Although it showed the similar tendency with the graph using Stokes number, still there were multiple values of settling velocity change according to corresponding d/η values. Also, the velocity scale ratio, Sv_n and Sv_l , didn't show dominant features that can describe the turbulent effects on settling velocity change. Lastly, the length scale ratio calculated by multiplying the time scale and the velocity scale ratio, $Sv_{\eta}St$ and $Sv_{l}St$, showed a clearer decreasing tendency than the other graphs. In particular, the experimental results when the Stokes number is larger than 0.3 have been separated from the results of other previous studies. As a result, the envelope of experimental results became a more obvious curve shape with a reduced variation. Thus, these grouping parameters, Sv_nSt and Sv_lSt , can be determined as more dominant parameters that can describe the turbulent effect on settling velocity change of inertial particles in

turbulent flow than other parameters.

Although some cases have shown decreased settling velocity in turbulent flow, the settling velocity generally increased in most cases. So, this means the existing particle tracking method based on the settling velocity in stagnant water can predict the overestimated transport distance of particles because the vertical displacement will decrease when the settling velocity increases in a turbulent flow. However, if the material of experimental particles is replaced with heavier particles, such as sediments and metals, the range of SvSt can be changed, and the different relationships between settling velocity in turbulent flow has to be more inspected by conducting additional experiments, and the general degree of change in settling velocity in a specific particle and flowing condition must be determined and adopted by the solvers about particle behaviors in PTMs for improving the exactness of prediction of particle transport.

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APPENDIX

APPENDIX.A

MATLAB code for the PTV algorithm

```
% Created by S. Baek
% PTV algorithm for tracking particles in water body
% last update: 2022.11.13
clear all; close all;
% Calibration
file org = 'Z:\Settling Turb\';
date = '221020 d390\main cam\';
trial num = 'trial1 ';
part size = 'd390 ';
fps = '250fps ';
case name = [part size trial num fps 'C001H001S0001'];
filepath = [file org date];
videopath = [file org date case name '\'];
savepath = [file org date 'result\'];
dt = 1/str2num(fps(1:3));
cd([file org date])
cali img = imread('cali.bmp');
imtool(cali img)
pause()
mmperpixel = 150/distance; % 150 mm : channel width
save([savepath
'cali base.mat'],'case name','filepath','savepath','mmperpi
xel','dt','videopath','trial num')
%% Trial savepath designation
clear all; close all;
load('cali base.mat')
file org = 'Z:\Settling Turb\';
date = '221020 d327\main cam\'; trial num = 'trial8 ';
part size = 'd327 '; fps = '250fps ';
mkdir(trial num(1:end-1))
case name = [part size trial num fps 'C001H001S0001'];
filepath = [file org date];
videopath = [file org date case name '\'];
savepath = [file org date 'result\' trial num(1:end-1)
'\'];
```

save([savepath

```
'cali.mat'],'case name','filepath','savepath','mmperpixel',
'dt', 'videopath', 'trial num')
%% Particle tracking - automatic
clear all; close all; clc;
load('cali.mat');
vid = VideoReader([filepath 'postprocess\' case name
'.avi']);
vid start = 4445; % analysis start frame num.
vid end = 5715; % analysis end frame num.
len = vid end - vid start + 1;
frame = uint8(zeros(vid.Height,vid.Width,len));
for i=1:len
   frame rgb = read(vid,vid start+i-1);
   frame(:,:,i) = im2gray(frame rgb);
end
figure(1), imshow(frame(:,:,1))
pause()
[x start, y start] = getpts();
ea = length(x start);
save([savepath '\tracking start pts.mat'],'-v7.3')
close(figure(1))
%% auto-tracking start
clear all; close all;
load('tracking start pts.mat');
x before = x start; y before = y start;
x detect = cell(1, ea);
for i=1:ea
   x detect{i} = zeros(1,len);
end
y detect = x detect; x weighted = x detect; y weighted =
y detect;
frame num = 0;
pass ea = [];
for i = 1:len
   frame num = frame num + 1;
   img proto = frame(:,:,i);
   img = img proto;
   img(img <= 30) = 0;
   img(img > 30) = 255;
   for j=1:ea
      if isempty(find(pass ea==j))
          pass ea = [];
      else
          continue
       end
       if y before(j) >= 1018
```

```
x weighted{j}(frame num) = x before(j);
          y weighted{j}(frame num) = 1024;
          continue
       elseif x before(j) >= 1275
          x_weighted{j}(frame_num) = 1280;
          y weighted{j}(frame num) = y before(j);
          continue
       elseif x before(j) == 0
          continue
       else
          win x1 = ceil(x before(j)) - 10;
          win x^2 = ceil(x before(j)) + 10;
          win y1 = ceil(y before(j));
          win y^2 = ceil(y before(j)) + 10;
          if win y^2 > 1024
              win y2 = 1024;
          elseif win x2 > 1280
              win x^2 = 1280;
          end
          x \text{ tmp} = \text{zeros}(\text{win y2-win y1+1,1}); y \text{ tmp} = x \text{ tmp};
len tmp = x tmp;
          cnt jj = 0;
          for jj = win y1:win y2
              cnt jj = cnt jj + 1;
              row start = []; row end = [];
              cnt line = img(jj,win x1:win x2);
              for jjj = 1:length(cnt line)-1
                 if cnt line(jjj)~=0 & jjj==1 &
cnt line(jjj+1)~=0
                     row start = [row start; 0];
                 elseif cnt line(jjj)~=0 & jjj==1 &
cnt line(jjj+1)==0
                     row start = [row start; 0]; row end =
[row end; 1];
                 elseif cnt line(jjj)==0 &
cnt line(jjj+1)~=0 & jjj~=length(cnt line)-1
                    row start = [row start; jjj+1];
                 elseif cnt line(jjj)==0 &
cnt line(jjj+1)~=0 & jjj==length(cnt line)-1
                     continue
                 elseif cnt line(jjj)~=0 &
cnt line(jjj+1)==0
                     row end = [row end; jjj];
                     continue
                 elseif cnt_line(jjj)~=0 &
cnt line(jjj+1)~=0 & jjj==length(cnt line)-1
                        row end = [row end; jjj];
                         continue
                 else
                     continue
                 end
              end
              row = row end - row start;
```

```
jj max = max(row);
              jj ind = find(row==max(row));
              if isempty(jj max)
                 continue
              elseif max(row) < 2
                 continue
             elseif max(row) >= 2 & length(jj ind)==1
                 x tmp(cnt jj) = row start(jj ind) +
ceil(jj max/2);
                 y tmp(cnt jj) = jj;
                 len_tmp(cnt_jj) = jj_max;
              elseif max(row) >= 2 & length(jj ind)>1
                 jj ind = jj ind(1);
                 x tmp(cnt jj) = row start(jj ind) +
ceil(jj max/2);
                 y tmp(cnt jj) = jj;
                 len tmp(cnt jj) = jj max;
              end
          end
          [j max, j ind] = max(len tmp);
          x detect{j}(frame num) = win x1+x tmp(j ind)-1;
          y detect{j}(frame num) = y tmp(j ind);
       end
       img = img;
       [particle, img ] =
findParticle(y detect{j}(frame num), x detect{j}(frame num),
img ,[]);
       if isempty(particle)
          pass ea = [pass ea; j];
       else
          particle =
reshape (particle, 2, length (particle) /2);
          x weighted{j}(frame num) =
sum(particle(2,:))/length(particle);
          y weighted{j}(frame num) =
sum(particle(1,:))/length(particle);
       end
   end
   for j=1:ea
      x before(j) = x weighted{j}(frame num);
       y before(j) = y weighted{j}(frame num);
   end
   figure(1), imshow(img proto)
   hold on
   for j=1:ea
plot(x weighted{j}(1:frame num), y weighted{j}(1:frame num),
'.', 'MarkerSize',14)
   end
   hold off
   drawnow
   gif frame = getframe(1);
```

```
gif img = frame2im(gif frame);
   [imind, cm] = rgb2ind(gif img,256);
   if i == 1
       imwrite (imind, cm, [savepath
'auto tracking.gif'],'gif','Loopcount', Inf, 'DelayTime', 1/25
);
   elseif rem(i,10)==0
      imwrite(imind, cm, [savepath
'auto tracking.gif'],'gif','WriteMode','append','DelayTime'
,1/25);
   else
      continue
   end
end
x cor = x weighted;
y_cor = y weighted;
save([savepath]
'tracking pixel.mat'], 'x cor', 'y cor', 'x weighted', 'y weigh
ted')
%% Trajectory fitting & Outlier detection
clear all; close all;
load('cali.mat');
load('tracking pixel.mat');
particle legend = cell(1,ea);
for i=1:ea
   particle legend{i} = ['particle ' num2str(i)];
end
x = cell(1, ea); y = x;
for j=1:ea
   x tmp = x weighted{j}; y tmp = y weighted{j};
   idx disappear = find(x tmp == 0);
   x tmp(idx disappear) = [];
   y tmp(idx disappear) = [];
   x{j} = x tmp*mmperpixel;
   y{j} = (1024-y \text{ tmp}) * mmperpixel;
end
t = 0:dt:dt*(len-1);
y vel = cell(1, ea);
fig1 = figure(1);
hold on
for j=1:ea
   yy = y\{j\};
   if length(yy) \leq 5
      continue
   end
   while round(yy(end),5) == round(yy(end-1),5)
       yy(end)=[];
   end
```
```
ind y = find(yy==0);
   yy(ind y) = 0;
   yy = rmoutliers(yy(1:end), 'mean');
   fit =
fit(t(1:length(yy))', yy', 'smoothingspline', 'SmoothingParam'
,0.999);
   y vel{j} = abs(differentiate(fit ,t(1:length(yy))));
   w hist(j) = mean(y_vel{j});
   plot(t(1:length(yy)),y vel{j})
   yline(mean(y vel{j}),'--',[sprintf('mean velocity\n')
num2str(mean(y vel{j}))], 'FontSize', 20)
end
hold off
grid on
legend(particle legend, 'Location', 'northeast')
ylabel('$$ w {sT} \,\, [mm/s]$$','Interpreter','latex')
xlabel('Time [sec]')
ylim([floor(mean(y vel{1}))-2.5 floor(mean(y vel{1}))+2.5])
set(gca, 'FontSize', 16)
fig1.Position(1:4) = [200 800 800 600];
save('settling velocity.mat', 'x', 'yy', 'y vel', 't')
%% Particle area detection in a binary image
function [particle, img] = findParticle(yi,xi,img,particle)
[M, N] = size(imq);
if yi == M
   return
elseif xi == 0 | yi == 0
   return
elseif xi == 1
   return
elseif xi == N
   return
end
if ~img(yi,xi)
   return
end
particle = [particle yi xi];
img(yi, xi)=0;
[particle,img] = findParticle(yi+1,xi,img,particle);
[particle,img] = findParticle(yi-1, xi, img, particle);
[particle,img] = findParticle(yi,xi+1,img,particle);
[particle,img] = findParticle(yi,xi-1,img,particle);
end
```

APPENDIX.B

Experimental data

Ref.	d [µm]	$ ho_p$ [g/cm ³]	μ _f [Pa·s]	τ _p [s]	w _s [mm/s]	η [μm]	τ _k [s]	<i>v_k</i> [mm/s]	ũ' <i>rms</i> [mm/s]	w _t [mm/s]	w _t / w _s [-]	St [-]	Svη [-]	Sv _l [-]	Sv _η St [-]	Sv _l St [-]
	360	2.50	0.001001	1.80E-02	70.2	300	9.00E-02	3.3	7.2	72.26	1.029	2.00E-01	2.13E+01	9.75E+00	4.25E+00	1.95E+00
	360	2.50	0.001001	1.80E-02	70.2	240	5.80E-02	4.1	9.6	73.45	1.046	3.10E-01	1.71E+01	7.31E+00	5.31E+00	2.27E+00
	360	2.50	0.001001	1.80E-02	70.2	210	4.20E-02	4.9	12	74.30	1.058	4.29E-01	1.43E+01	5.85E+00	6.14E+00	2.51E+00
	360	2.50	0.001001	1.80E-02	70.2	180	3.20E-02	5.6	14.4	75.06	1.069	5.63E-01	1.25E+01	4.88E+00	7.05E+00	2.74E+00
	360	2.50	0.001001	1.80E-02	70.2	160	2.50E-02	6.3	16.8	75.18	1.071	7.20E-01	1.11E+01	4.18E+00	8.02E+00	3.01E+00
	360	2.50	0.001001	1.80E-02	70.2	140	2.10E-02	6.9	19.2	74.12	1.056	8.57E-01	1.02E+01	3.66E+00	8.72E+00	3.13E+00
	505	2.50	0.001001	3.54E-02	77.6	300	9.00E-02	3.3	7.2	79.87	1.029	3.94E-01	2.35E+01	1.08E+01	9.25E+00	4.24E+00
Yang &	505	2.50	0.001001	3.54E-02	77.6	240	5.80E-02	4.1	9.6	80.92	1.043	6.11E-01	1.89E+01	8.08E+00	1.16E+01	4.94E+00
Shy	505	2.50	0.001001	3.54E-02	77.6	210	4.20E-02	4.9	12	80.89	1.042	8.43E-01	1.58E+01	6.47E+00	1.34E+01	5.45E+00
(2003)	505	2.50	0.001001	3.54E-02	77.6	180	3.20E-02	5.6	14.4	79.12	1.020	1.11E+00	1.39E+01	5.39E+00	1.53E+01	5.96E+00
	505	2.50	0.001001	3.54E-02	77.6	160	2.50E-02	6.3	16.8	76.90	0.991	1.42E+00	1.23E+01	4.62E+00	1.75E+01	6.54E+00
	505	2.50	0.001001	3.54E-02	77.6	140	2.10E-02	6.9	19.2	75.41	0.972	1.69E+00	1.12E+01	4.04E+00	1.90E+01	6.82E+00
	160	19.3	0.001001	2.74E-02	103.1	300	9.00E-02	3.3	7.2	104.66	1.015	3.05E-01	3.12E+01	1.43E+01	9.53E+00	4.37E+00
	160	19.3	0.001001	2.74E-02	103.1	240	5.80E-02	4.1	9.6	105.66	1.025	4.73E-01	2.51E+01	1.07E+01	1.19E+01	5.08E+00
	160	19.3	0.001001	2.74E-02	103.1	210	4.20E-02	4.9	12	106.80	1.036	6.54E-01	2.10E+01	8.59E+00	1.38E+01	5.62E+00
	160	19.3	0.001001	2.74E-02	103.1	180	3.20E-02	5.6	14.4	106.13	1.029	8.58E-01	1.84E+01	7.16E+00	1.58E+01	6.14E+00
	160	19.3	0.001001	2.74E-02	103.1	160	2.50E-02	6.3	16.8	105.15	1.020	1.10E+00	1.64E+01	6.14E+00	1.80E+01	6.74E+00

	160	19.3	0.001001	2.74E-02	103.1	140	2.10E-02	6.9	19.2	102.79	0.997	1.31E+00	1.49E+01	5.37E+00	1.95E+01	7.02E+00
	71	1.44	0.001001	4.03E-04	5.00	550	3.04E-01	1.8	7.4	2.89	0.577	1.33E-03	2.78E+00	6.76E-01	3.68E-03	8.96E-04
	71	1.44	0.001001	4.03E-04	5.00	460	2.17E-01	2.2	9.7	11.63	2.326	1.86E-03	2.27E+00	5.15E-01	4.22E-03	9.58E-04
	71	1.44	0.001001	4.03E-04	5.00	440	2.04E-01	2.4	10.6	26.51	5.303	1.98E-03	2.08E+00	4.72E-01	4.12E-03	9.32E-04
	71	1.44	0.001001	4.03E-04	5.00	290	8.30E-02	3.5	18.3	27.32	5.464	4.86E-03	1.43E+00	2.73E-01	6.94E-03	1.33E-03
	71	1.44	0.001001	4.03E-04	5.00	320	1.05E-01	3.2	16.2	25.35	5.070	3.84E-03	1.56E+00	3.09E-01	6.00E-03	1.19E-03
Jacobs	71	1.44	0.001001	4.03E-04	5.00	220	5.00E-02	4.5	25.1	18.51	3.701	8.07E-03	1.11E+00	1.99E-01	8.96E-03	1.61E-03
(2016)	97	1.35	0.001001	7.06E-04	8.00	550	3.04E-01	1.8	7.4	6.92	0.865	2.32E-03	4.44E+00	1.08E+00	1.03E-02	2.51E-03
	97	1.35	0.001001	7.06E-04	8.00	460	2.17E-01	2.2	9.7	23.65	2.956	3.25E-03	3.64E+00	8.25E-01	1.18E-02	2.68E-03
	97	1.35	0.001001	7.06E-04	8.00	440	2.04E-01	2.4	10.6	24.74	3.092	3.46E-03	3.33E+00	7.55E-01	1.15E-02	2.61E-03
	97	1.35	0.001001	7.06E-04	8.00	290	8.30E-02	3.5	18.3	18.89	2.361	8.50E-03	2.29E+00	4.37E-01	1.94E-02	3.72E-03
	97	1.35	0.001001	7.06E-04	8.00	320	1.05E-01	3.2	16.2	36.52	4.565	6.72E-03	2.50E+00	4.94E-01	1.68E-02	3.32E-03
	97	1.35	0.001001	7.06E-04	8.00	220	5.00E-02	4.5	25.1	46.41	5.802	1.41E-02	1.78E+00	3.19E-01	2.51E-02	4.50E-03
	17	2.50	0.001001	2.17E-03	159.96	240	3.79E-03	64.17	172.82	200.10	1.251	5.73E-01	2.49E+00	9.26E-01	1.43E+00	5.30E-01
	17	2.50	0.001001	2.17E-03	159.96	160	1.69E-03	96.11	296.18	292.37	1.828	1.28E+00	1.66E+00	5.40E-01	2.14E+00	6.94E-01
Yang &	17	2.50	0.001001	2.17E-03	159.96	140	1.18E-03	114.98	376.16	252.76	1.580	1.84E+00	1.39E+00	4.25E-01	2.56E+00	7.82E-01
(2021)	17	2.50	0.001001	2.17E-03	159.96	110	8.00E-04	139.62	487.28	263.75	1.649	2.71E+00	1.15E+00	3.28E-01	3.11E+00	8.90E-01
	17	2.50	0.001001	2.17E-03	159.96	100	6.64E-04	153.23	551.61	211.40	1.322	3.27E+00	1.04E+00	2.90E-01	3.41E+00	9.47E-01
	17	2.50	0.001001	2.17E-03	159.96	90	5.36E-04	170.65	636.79	196.36	1.228	4.05E+00	9.37E-01	2.51E-01	3.80E+00	1.02E+00
	3000	1.41	0.001108	6.36E-01	139.3	751.7	5.09E-01	1.476	3.887	157.81	1.133	1.25E+00	9.44E+01	3.58E+01	1.18E+02	4.48E+01
This Study	3000	1.41	0.001108	6.36E-01	139.3	622.3	3.49E-01	1.783	4.54	161.10	1.157	1.82E+00	7.81E+01	3.07E+01	1.42E+02	5.59E+01
Study	3000	1.41	0.001108	6.36E-01	139.3	464.5	1.95E-01	2.388	6.814	169.37	1.216	3.27E+00	5.83E+01	2.04E+01	1.91E+02	6.69E+01

2000	1.41	0.001108	2.83E-01	99.7	751.7	5.09E-01	1.476	3.887	109.29	1.096	5.55E-01	6.75E+01	2.56E+01	3.75E+01	1.42E+01
2000	1.41	0.001108	2.83E-01	99.7	622.3	3.49E-01	1.783	4.54	111.23	1.116	8.10E-01	5.59E+01	2.20E+01	4.53E+01	1.78E+01
2000	1.41	0.001108	2.83E-01	99.7	464.5	1.95E-01	2.388	6.814	121.96	1.223	1.45E+00	4.18E+01	1.46E+01	6.07E+01	2.13E+01
925	1.35	0.001108	5.79E-02	43.2	751.7	5.09E-01	1.476	3.887	38.83	0.899	1.14E-01	2.93E+01	1.11E+01	3.33E+00	1.26E+00
925	1.35	0.001108	5.79E-02	43.2	622.3	3.49E-01	1.783	4.54	60.42	1.399	1.66E-01	2.42E+01	9.52E+00	4.02E+00	1.58E+00
925	1.35	0.001108	5.79E-02	43.2	464.5	1.95E-01	2.388	6.814	46.50	1.076	2.98E-01	1.81E+01	6.34E+00	5.39E+00	1.89E+00
925	1.35	0.001001	6.41E-02	43.2	488.2	2.03E-01	2.224	3.261	50.11	1.160	3.16E-01	1.94E+01	1.32E+01	6.15E+00	4.19E+00
925	1.35	0.001001	6.41E-02	43.2	500.7	2.50E-01	2.002	3.313	44.90	1.039	2.56E-01	2.16E+01	1.30E+01	5.53E+00	3.34E+00
780	1.35	0.001108	4.12E-02	37.7	751.7	5.09E-01	1.476	3.887	37.13	0.985	8.08E-02	2.55E+01	9.70E+00	2.06E+00	7.84E-01
780	1.35	0.001108	4.12E-02	37.7	622.3	3.49E-01	1.783	4.54	45.94	1.218	1.18E-01	2.11E+01	8.30E+00	2.49E+00	9.80E-01
780	1.35	0.001108	4.12E-02	37.7	464.5	1.95E-01	2.388	6.814	35.44	0.940	2.12E-01	1.58E+01	5.53E+00	3.34E+00	1.17E+00
780	1.35	0.001001	4.56E-02	37.7	488.2	2.03E-01	2.224	3.261	40.43	1.072	2.25E-01	1.70E+01	1.16E+01	3.81E+00	2.60E+00
780	1.35	0.001001	4.56E-02	37.7	500.7	2.50E-01	2.002	3.313	38.63	1.025	1.82E-01	1.88E+01	1.14E+01	3.43E+00	2.07E+00
550	1.35	0.001108	2.05E-02	24.5	751.7	5.09E-01	1.476	3.887	31.88	1.301	4.02E-02	1.66E+01	6.30E+00	6.67E-01	2.53E-01
550	1.35	0.001108	2.05E-02	24.5	622.3	3.49E-01	1.783	4.54	31.13	1.271	5.87E-02	1.37E+01	5.40E+00	8.06E-01	3.17E-01
550	1.35	0.001108	2.05E-02	24.5	464.5	1.95E-01	2.388	6.814	27.62	1.127	1.05E-01	1.03E+01	3.60E+00	1.08E+00	3.79E-01
550	1.35	0.001001	2.27E-02	24.5	488.2	2.03E-01	2.224	3.261	30.52	1.246	1.12E-01	1.10E+01	7.51E+00	1.23E+00	8.40E-01
550	1.35	0.001001	2.27E-02	24.5	500.7	2.50E-01	2.002	3.313	31.17	1.272	9.06E-02	1.22E+01	7.40E+00	1.11E+00	6.70E-01
390	1.35	0.001108	1.03E-02	18.2	751.7	5.09E-01	1.476	3.887	17.92	0.985	2.02E-02	1.23E+01	4.68E+00	2.49E-01	9.46E-02
390	1.35	0.001108	1.03E-02	18.2	622.3	3.49E-01	1.783	4.54	22.68	1.246	2.95E-02	1.02E+01	4.01E+00	3.01E-01	1.18E-01
390	1.35	0.001108	1.03E-02	18.2	464.5	1.95E-01	2.388	6.814	28.69	1.577	5.29E-02	7.62E+00	2.67E+00	4.03E-01	1.41E-01
390	1.35	0.001001	1.14E-02	18.2	488.2	2.03E-01	2.224	3.261	21.42	1.177	5.62E-02	8.18E+00	5.58E+00	4.60E-01	3.14E-01

390	1.35	0.001001	1.14E-02	18.2	500.7	2.50E-01	2.002	3.313	20.65	1.135	4.56E-02	9.09E+00	5.49E+00	4.14E-01	2.50E-01
327.5	1.35	0.001108	7.26E-03	9.77	751.7	5.09E-01	1.476	3.887	19.42	1.988	1.43E-02	6.62E+00	2.51E+00	9.43E-02	3.58E-02
327.5	1.35	0.001108	7.26E-03	9.77	622.3	3.49E-01	1.783	4.54	13.04	1.335	2.08E-02	5.48E+00	2.15E+00	1.14E-01	4.48E-02
327.5	1.35	0.001108	7.26E-03	9.77	464.5	1.95E-01	2.388	6.814	27.35	2.799	3.73E-02	4.09E+00	1.43E+00	1.53E-01	5.35E-02
327.5	1.35	0.001001	8.04E-03	9.77	500.7	2.50E-01	2.002	3.313	14.77	1.512	3.21E-02	4.88E+00	2.95E+00	1.57E-01	9.48E-02
196	1.35	0.001108	2.60E-03	7.36	751.7	5.09E-01	1.476	3.887	11.16	1.517	5.10E-03	4.99E+00	1.89E+00	2.55E-02	9.67E-03
196	1.35	0.001108	2.60E-03	7.36	622.3	3.49E-01	1.783	4.54	4.18	0.568	7.45E-03	4.13E+00	1.62E+00	3.07E-02	1.21E-02
196	1.35	0.001108	2.60E-03	7.36	464.5	1.95E-01	2.388	6.814	26.07	3.542	1.34E-02	3.08E+00	1.08E+00	4.12E-02	1.44E-02

국문초록

관성입자의 침강속도에 난류가 미치는 영향에 대한 실험연구

서울대학교 대학원

건설환경공학부

백 승 준

기존의 입자추적모델은 주변 유체의 유속과 입자의 정지 수체에 서의 침강속도 그리고 분산과 확산 효과를 나타내기 위해 정규 분포를 따르는 임의 값의 선형 합으로 입자의 연직 방향 속도를 예측하다. 하지 만 많은 선행 연구들은 난류 흐름에서 입자의 최종 침강속도가 변한다는 것을 제시해왔다. 따라서 입자추적모델의 입자 수송(particle transport)에 대한 정확도 향상을 위해, 주변 유체에 의한 이송(advection), 정지 수체 에서의 입자 침강속도, 그리고 난류 흐름에서의 침강속도 변화 간의 상 호작용에 대해 조사할 필요가 있다. 이를 위해, 본 연구에서는 수치 모의 와 실험실 실험이 수행되었다. 먼저, 입자의 침강 방향과 평행한 이송이 작용할 때 침강속도에 미치는 영향을 평가하기 위해, 정상류에서 입자의 침강 거동에 대한 수치 모의가 수행되었다. 그 결과, 이송의 영향을 받은 침강 속도는 정지 수체에서의 침강속도와 주변 유체의 유속의 중첩을 통 해 계산된 것과 같았으며, 이는 유체 내의 관성입자의 거동에 대한 운동 방정식을 통해, 유체에 대한 입자의 상대속도가 입자 조건에 따라 일정 하기 때문임을 확인하였다. 다음으로, 침강 속도에 난류가 미치는 영향을 조사하기 위해 입자의 침강과 수직 방향으로 유체가 이동하는 개수로 흐 름에서의 실험과 침강과 평행한 방향으로 유체가 이동하는 연직순환수로 (Vertical Recirculation Tube; VeRT) 실험을 진행하였다. 두 가지 실험에서, 실험 입자의 속도는 PTV(Particle Tracking Velocimetry) 기법을 통해 측정되 었으며, 유체의 속도와 난류는 PIV(Particle Image Velocimetry) 기법을 통해 측정되었다. 특히, 본 연구에서는 여러 개의 입자를 함께 추적 가능한 PTV 알고리즘을 구축하여 사용하였다. 실험 결과는 입자의 침강속도가 일반적으로 정지 수체보다 난류 흐름에서 더 빠르다는 것을 보여주었고, 그 침강속도 변화에 어떤 인자가 종속적인지를 Stokes 수, Rouse 수 등 입자 및 난류 특성을 함께 나타내는 몇 가지 인자들을 대상으로 조사하 였다. 그 결과, Stokes 수와 Rouse 수를 곱하여 입자와 난류 특성의 길이 차원 비를 나타내는 *SvSt*가 해당 값이 증가함에 따라 침강 속도 변화율 이 감소하는 형태를 다른 인자들에 비해 명확하게 보여주었다. 따라서 *SvSt*가 난류 흐름에서 관성입자의 침강속도 변화에 난류가 미치는 영향 을 설명할 수 있는 가장 지배적인 인자로 사용될 수 있음을 확인하였다.

결론적으로, 본 연구에서 수행된 실험들과 선행 연구의 결과로부 터 난류 흐름에서 침강 속도는 대체로 증가함을 관측하였다. 따라서 기 존의 입자추적모델은 입자의 연직방향 유속을 과소산정할 수 있으며, 이 에 따라 입자의 수송 거리를 과대산정할 수 있다. 그러므로, 특정한 입자 및 흐름 조건에서 침강 속도의 변화를 예측할 수 있도록 추가적인 연구 를 수행하고, 이를 입자추적모델의 정확도 향상을 위해 입자 거동 해석 에 반영할 필요가 있다.

주요어: 관성입자, 침강속도, 난류, 입자추적모델

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132