공학석사학위논문

An Interval-Based Two-Stage Stochastic Optimization Model for the Unit Commitment Problem Under Demand Uncertainty

수요의 불확실성 하에서의 발전구간 기반 2단계 추계적 발전계획 모형

2023 년 2 월

서울대학교 대학원

산업공학과

정 호 진

An Interval-Based Two-Stage Stochastic Optimization Model for the Unit Commitment Problem Under Demand Uncertainty

수요의 불확실성 하에서의 발전구간 기반 2단계 추계적 발전계획 모형

지도교수 이 경 식

이 논문을 공학석사 학위논문으로 제출함

2022 년 12 월

서울대학교 대학원

산업공학과

정 호 진

정호진의 공학석사 학위논문을 인준함

2022 년 12 월

위 원 장	홍 성 필	(인)
부위원장	이 경 식	(인)
위 원	이 덕 주	(인)

Abstract

An Interval-Based Two-Stage Stochastic Optimization Model for the Unit Commitment Problem Under Demand Uncertainty

HOJIN JUNG
Department of Industrial Engineering
The Graduate School
Seoul National University

The unit commitment problem aims to find a minimum-cost on/off status and amount of generation for each generator while satisfying the electricity demand and operational requirements. To efficiently deal with demand uncertainty, the two-stage stochastic optimization models have been widely used in the literature, where the on/off status is decided in the first stage and the amount of generation is in the second stage. However, they often suffer from excessive computational burden as the number of demand scenarios increases. In this thesis, we propose an interval-based two-stage stochastic optimization model to mitigate the drawback under the period-wise independent demand assumption. In the model, an interval of a generator, which is a range of the amount of generation, is determined along with its on/off status for each period. It enables the second-stage problem to be decomposed in a period-wise manner, which reduces the need for a large number of scenarios.

We also propose a compact Benders reformulation by exploiting the property of the subproblem. Lastly, we show that the bounds on the expected costs can be obtained for the proposed model. Computational experiments were conducted to show the effectiveness and efficiency of the proposed model.

Keywords: Unit Commitment, Demand Uncertainty, Two-stage Stochastic Optimization Model, Interval, Benders Reformulation, Bounding Method

Student Number: 2021-23873

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Background

Unit Commitment (UC) is one of the most important optimization problems in power system operation. It is solved not only by the independent system operators (ISO) to establish an efficient and reliable plan but also by the generation companies (GENCO) to make the optimal bidding strategies. In South Korea, Korea Power Exchange (KPX), taking a role as a wholesaler, receives the capacity of GENCOs in the day-ahead market and determines the system marginal price (SMP) by solving a price-setting unit commitment problem [1]. When the system marginal price is determined, KPX announces to GENCOs how much they should generate. On the trading day, the real-time dispatch to balance the supply and demand is performed in KPX. In other words, even though the price-setting unit commitment problem determines the specific amount of generation for entering generators, it is controlled by KPX which monitors the customer demand and plans a feasible transmission plan in real-time. After the dispatch, uplifting payment is incurred for the excess generation. This is a unique market structure called the cost-based pool market. Figure 1.1 illustrates the electricity market trading process in South Korea.

To stabilize the price and supply of electricity in the market, ISO should forecast

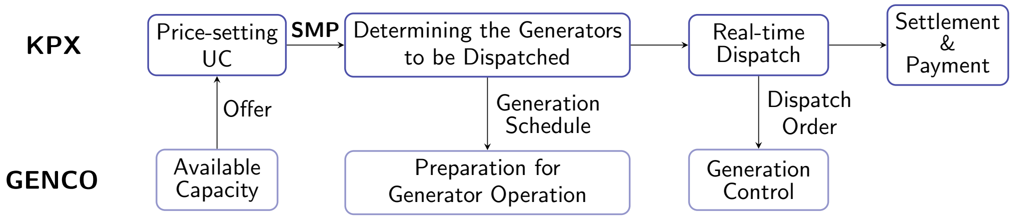


Figure 1.1: Cost-based Pool Market in South Korea

the demand of the customers and plan how to satisfy it. But the increasing penetration of renewable energy is becoming a challenging issue to achieve the goal. Since the amount of renewable generation significantly depends on uncontrollable factors such as solar irradiance and weather conditions, it magnifies the difficulty in forecasting the exact amount of generation the power plants need to produce, which is called the *net demand*, in the power system. For the case in South Korea, it may result in a high level of discrepancy between the planned amount of generation and the actual dispatch level, and eventually an excessive burden of uplift payment by KPX. Despite the efforts on increasing the accuracy of renewable generation forecasting [2], the forecasting models usually do not assure a consistent level of accuracy, and selecting or customizing the appropriate one might be a complicated and tedious task. Therefore, under the fundamental limitation in net demand forecasting, the unit commitment models to deal with demand uncertainty have attracted increasing attention.

1.2 Problem Description

In the unit commitment problem (UCP), a generation schedule for each generator, which indicates the on/off status and its amount of generation is decided for each time period during the planning horizon as shown in Figure 1.2. Typically, the planning horizon is 24 hours divided into periods of 1 hour. First, we illustrate the generator's operational requirements. They consist of the minimum up/down time limit, minimum/maximum power limit, and ramping limit. The minimum and maximum power limits bound the amount of generation when the generator is turned on. For example, suppose that the minimum and maximum power limits for a generator are 5 MW and 30 MW, respectively (see Figure 1.2). Then the amount of generation is no less than 5 MW and no more than 30 MW every time the generator is turned on.

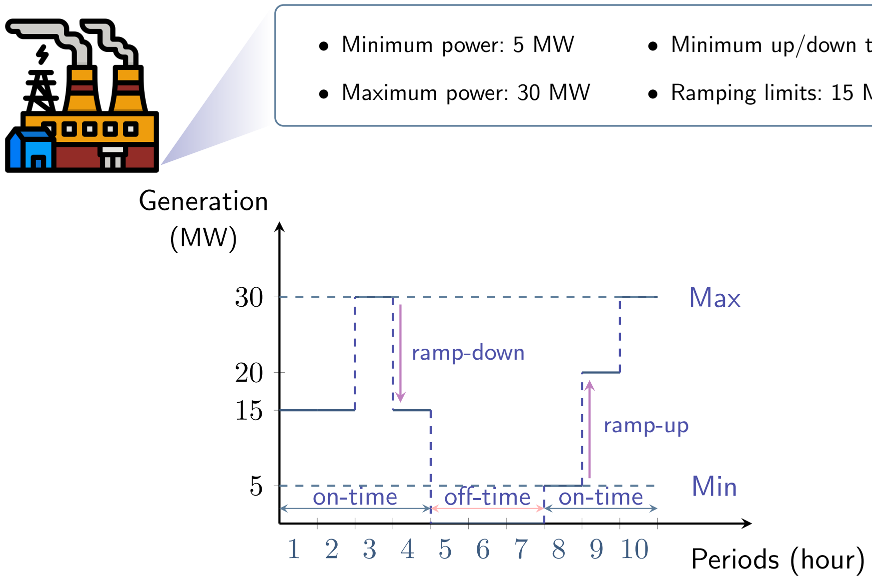


Figure 1.2: Operational Requirements in UC

Minimum up/down time refers to the number of minimum periods for which the generator must keep the on/off status once it is turned on after the off-status (start-up), or turned off after the on-status (shut-down). For example, suppose that the minimum up time and minimum down time of a generator is both 3 periods. If it starts up at period 1 and shuts down at period 5, then it must be turned on from period 1 to 3 and turned off from period 5 to 7.

Ramping limit is the limit on the difference in the amount of generation between two adjacent periods. If the ramping-up limit is 15 MW and its amount of generation in period 8 is 5 MW, its amount of generation in period 9 must not be larger than 20 MW.

A generation schedule has information on the on/off status and the amount of generation for each period. When the generator is turned on, a certain amount of fixed cost is incurred. In addition, there is an extra cost incurred for a generator that starts up called start-up cost. In terms of the amount of generation, it is well-known that the generation function of thermal generators is quadratic in general [3], but in this thesis, it is assumed that generation cost is linear with the amount of generation for ease of analysis. Because of the operational constraints described before, the total amount of generation may not satisfy the electricity demand for each period. We introduced the conventional supposition in the unit commitment models that the amount of unsatisfied demand is lost and a substantial penalty cost called load-shedding cost is incurred in order to ensure the feasibility of the model. In addition, we assume that all the generators are turned off for long enough periods before the beginning of the planning horizon so that they are available in the first period, and the ramping constraints in the first period are not considered.

In practice, the power system operators have to solve the UCP before the realization of the real-time demand. They usually forecast the electricity demand in the near future to get the nominal demand, but the inherent error in the demand forecasting model may cause an unexpected mismatch between the demand and supply. Hence, the proper consideration of the demand uncertainty in UCP is essential for both reliable and cost-efficient electricity supply in the power system. Furthermore, the increasing penetration of renewable energy in the power system makes it difficult to accurately capture the net demand. In other words, renewable generation is one of the most noteworthy uncertain elements in UC, and the UC models that consider the demand uncertainty can incorporate it.

In this thesis, we consider the UCP with stochastic demand, which follows the period-wise independent probability distribution. In general, the available information on the demand distribution that the demand forecasting model provides is the nominal demand and the forecasting error for each period, not the joint distribution of all the demands [4]. Therefore, the period-wise independence setting in the stochastic unit commitment (SUC) problem is not a too restrictive assumption. Under these constraints, the objective is to find an optimal generation schedule over the planning horizon that minimizes the expectation of the total operational cost, which consists of the start-up cost, fixed cost, generation cost, and load-shedding cost.

1.3 Literature Review

1.3.1 Unit Commitment Problem

UCP described in Chapter 1.2 is an integer optimization problem. Even for the single-period case, it is NP-hard [5], and recently it was proven to be strongly NP-hard in general [6]. In the 1980s-90s, dynamic programming and branch-and-bound technique were used as an exact method, whereas priority listing, tabu search, and simulated annealing were devised as a heuristic algorithm that gives a good-quality solution in a relatively short time [7]. Later, combined with the rapid enhancement of computing power and the development of the MIP solver, the polyhedral studies on the unit commitment problem to enhance the efficiency of LP relaxation-based branch-and-bound have been increased in the literature. They try to approximate or characterize the convex hull of the feasible region of the unit commitment problem by adding some facet-defining inequalities, or valid inequalities [8, 9, 10].

1.3.2 Unit Commitment Models Under Uncertainty

To deal with the demand uncertainty in unit commitment models, stochastic optimization models and robust optimization models have been commonly used in the literature. Stochastic optimization models assume that the uncertain parameter follows a given probability distribution. Among them, the chance-constrained model is often used which guarantees the solution to satisfy the constraints with a pre-specified probability [11]. Robust optimization models focus on the decision with minimum worst-case cost under the assumption that the uncertain parameter is contained in the so-called uncertainty set [12].

On the other hand, variants of these traditional optimization models under demand uncertainty have been proposed to overcome the disadvantage of the stochastic or robust optimization models or to reflect the context of the power system operation more suitably. The interval unit commitment (IUC) model, where the nominal demand is mainly used for planning as in the deterministic case, and the minimum and maximum generation level among the given scenarios in each period are used to consider the ramping limits in the worst-case [13]. It both avoids the computational burden of the SUC model and the inner optimization problem in the formulation of the RUC model. However, it only takes the ramp-feasibility for the artificial bound scenarios into consideration. Focusing on the actual operation in the given scenarios, the improved version of the IUC model where the worst-case ramping for each pair of two consecutive periods is considered is proposed in [14]. Since the basic SUC or RUC models do not fully describe the dynamics in the real power system operation, the UC models that consider the real-time adaptation of the generation under inter-temporal constraints have also been presented. In [15], the optimal amount of generation follows the affine decision rule based on the affine function of the realized value of demands for each node, and a similar approach in the distributionally robust optimization framework is also proposed in [16]. In order to guarantee that the planned generation schedule is always feasible in the real-world situation where the demand is sequentially realized, a modified range of generation is introduced in the first stage of the RUC model [17]. It is achieved by having the inner problem in the RUC model free of inter-temporal constraints.

1.3.3 Solution Approaches for the Stochastic Unit Commitment Model

Decomposition Methods

The SUC model, which we cover in this thesis, has an inherent difficulty that the mixed-integer stochastic programming models share. The expectation in the objective function makes it impossible to apply the algorithms developed for the integer programming models. To handle this problem, the traditional approach is to use sample average approximation (SAA). In SAA, the expectation is approximated by the average of the objective value under the samples, or so-called *scenarios*, from the distribution of the uncertain parameters to ease the problem. However, to guarantee convergence, a large enough number of scenarios should be considered [18], which increases the computational burden in solving the model. Hence, several kinds of decomposition techniques have been studied to reduce the computational burden. First, the Lagrangian relaxation approach has been used to decompose the problem for each scenario, or each generator. In the former case, the constraints that enforce for the on/off decision to be equal for each scenario are relaxed so that the original problem is decomposed for each scenario [19, 20]. In the latter case, the demand constraints are relaxed and the model is decomposed into a single-generator unit commitment problem [21, 22]. Another well-known method for the SUC model is the Benders decomposition. In the Benders decomposition method, the Benders cut is added only when it is turned out to be necessary by iteratively solving the subproblem. It exploits the fact that the subproblem is relatively easy when the decision in the previous stage is fixed. The application of the Benders decomposition to solve the SUC model showed its efficiency in various problem contexts [23, 24].

Bounds on the Expectations

Although SAA is widely used to deal with stochastic programming models, there have been some indirect approaches to handle the expectation, based on its bound that is relatively easy to be computed. Jensen inequality and Edmundson-Madansky inequality are the representative inequalities that give lower and upper bound on the expectation of a convex function of a random variable [25, 26]. These inequalities were strengthened by refining the given range of the random variable [27]. The bounds can serve as stopping criteria in the L-shaped method [28], and the inner linearization algorithm can also be developed based on the specific family of upper bound [29]. This bounding technique has been applied to deal with several kinds of stochastic programming models such as network interdiction problems [30] and appointment scheduling problems [31]. Recently, novel bounds and their usage in the algorithms have been devised in the context of the multi-stage stochastic programming models with a finite number of scenarios [32, 33, 34, 35].

1.4 Motivation and Contributions

Despite the efforts in the literature, the SUC models still suffer from handling a large number of scenarios. In the worse case where we do not know the demand distribution in advance but only the nominal value, we may consider three possible observations for each period: the low level; the medium level, which equals the nominal demand; and the high level. Then the number of whole possible demand scenarios with T periods is 3^T, which is highly unmanageable to consider in real operation. In addition, the conventional SUC models need extra consideration of ramping limits in the phase of real-time dispatch. The alternative models for the SUC model were proposed by [13] and [14] to alleviate this issue; however, they only consider the ramping constraints based on the scenarios, not the cost-efficient recourse actions for each scenario.

Therefore, we propose a novel optimization model, which we call the interval-based two-stage stochastic unit commitment (ITSUC) model based on the concept of an *interval*, which stands for the lower and upper generation limits of each generator in a period. By deciding the intervals of a generator along with its on/off status in the first stage, the amount of generation can be decided independently for each period in the second stage. This *period-wise decomposition property* of the ITSUC model makes it possible that demand samples can be considered independently for each period rather than considering a demand scenario over the whole planning horizon (the combination of samples for all periods), which significantly reduces the size of the optimization model compared to the conventional two-stage stochastic unit commitment model. The idea of an interval is similar to the concept of a *box* for the two-stage robust unit commitment model proposed in [17], and we utilize the concept to enhance the modeling capacity and to reduce the computational burden in the stochastic setting

By utilizing the period-wise decomposition property of the ITSUC model, we show that the tight upper and lower bounds for the ITSUC model under a given probability distribution of uncertain demand can be efficiently obtained, based on the results in [27]. The result gives one possible approach to deal with the ITSUC model not relying on approximating the objective function using a finite number of samples. To the best of our knowledge, the application of the bounds on the expected cost in UCP has rarely been investigated in the existing literature.

Since the ramping constraints do not need to be directly considered, the optimization problem corresponding to the second stage becomes easier to be solved than the conventional two-stage stochastic unit commitment model. In addition, we show that its dual problem has a polynomial number of possible optimal solutions. It leads to a compact Benders reformulation, which may further enhance the solvability of the proposed ITSUC model.

Through computational experiments, we investigated the effectiveness of the proposed ITSUC model and the efficiency of the proposed solution approaches. The proposed model was compared with the conventional TSUC model. The efficiency of the proposed Benders reformulation and the bounding method was also investigated. In addition, we tested several choices of predetermined interval candidates to give an insight for selecting them in practice.

1.5 Organization of the Thesis

The remainder of this thesis is organized as follows. In Chapter 2, we briefly describe the unit commitment problem and provide the formulation for the conventional TSUC model. In Chapter 3, we propose the ITSUC model and analyze the interval design method. Solution approaches for the proposed model are presented in Chapter 4. The computational results for the model and the solution approaches are discussed in Chapter 5. In Chapter 6, concluding remarks with some future research directions are given.

Chapter 2

Two-stage Stochastic Unit Commitment Model

In this chapter, we briefly discuss the conventional TSUC model before we go into the key idea of the proposed model. Its modeling framework and assumption are described in Chapter 2.1, and its mathematical formulation is given in Chapter 2.2.

2.1 Modeling Framework

A representative unit commitment model in the literature dealing with demand uncertainty is the two-stage stochastic unit commitment (TSUC) model. In the model, the expected operation cost under a given probability distribution of uncertain demand is minimized while assuming the two-stage decision framework which is illustrated in Figure 2.1. The on/off decisions over the whole time horizon are made before the realization of the demand, and the generation amount of each generator is decided corresponding to the demand realization. This assumption is rationalized by

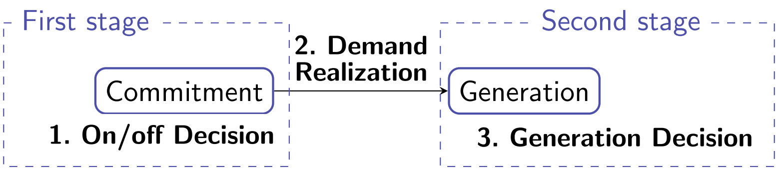


Figure 2.1: Two-stage Decision Framework for TSUC

the fact that the on/off status is harder to be changed each time the demand is realized. Because of the precedence relationship between decisions, the stage where the on/off decisions are made is called the *first stage*, and the stage where the amount of generation is determined is called the *second stage*.

2.2 Model Formulation

Under the modeling framework in 2.1, there are two modeling choices corresponding to the type of demand distribution. If the demand follows a continuous probability distribution, the corresponding TSUC model is called the *distribution-based model*. On the other hand, if the demand follows a discrete probability distribution, i.e. the number of the possible demand realizations is finite, then the corresponding TSUC model is called the *scenario-based model* since each of them is called the demand *scenario*. In this case, the expected cost becomes a weighted average of the cost corresponding to the probability of each scenario.

The notation for the formulation of the TSUC model is presented in Table 2.1. The vectors are represented by the associated letters with **bold** fonts for the remaining parts.

Table 2.1: Notation for TSUC Model

Sets and Indices	$\mathcal{H}$	Set of periods, $t \in \mathcal{H} = \{1, \dots, T\}$
	$\mathcal{G}$	Set of generators, $g \in \mathcal{G} = \{1, \dots, G\}$
	$\mathcal{S}$	Set of scenarios, $s \in \mathcal{S} = \{1, \dots, S\}$
Parameters	SUC_g	Start-up cost of generator g
	FC_g	Fixed cost of generator g
	VC_g	Variable cost coefficient of generator g
	$VoLL$	Value of lost load
	RUP_g	Ramp-up limit of generator g
	RDN_g	Ramp-down limit of generator g
	P_g^{min}	Minimum power limit for generator g
	P_g^{max}	Maximum power limit for generator g
	MUT_g	Minimum up time of generator g
	MDT_g	Minimum down time of generator g
	d_t (d_t^s)	Demand in period t (under scenario s)
Decision Variables	x_{gt}	1 if generator g is turned on in period t, 0 otherwise
	u_{gt}	1 if generator g starts up in time period t, 0 otherwise
	v_{gt}	1 if generator g shuts down in time period t, 0 otherwise
	p_{gt} (p_{gt}^s)	Power generation of generator g in time period t (under scenario s)
	ls_t (ls_t^s)	Load shedding in time period t (under scenario s)

Based on this notation, we can formulate the TSUC models. For the sake of clarity, we first provide the formulation of the distribution-based model as follows:

$(TSUC^D)$

$$\min \quad \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{H}} (SUC_g u_{gt} + FC_g x_{gt}) + \mathbb{E}_{\mathbf{d}}[C(\mathbf{x}, \mathbf{d})] \tag{2.1}$$

$$\text{s.t.} \quad u_{gt} - v_{gt} = x_{gt} - x_{g(t-1)}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}, \tag{2.2}$$

$$u_{g1} = x_{g1}, \qquad \forall g \in \mathcal{G}, \tag{2.3}$$

$$\sum_{t'=(t-MUT_g+1)^+}^{t} u_{gt'} \le x_{gt}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{2.4}$$

$$\sum_{t'=(t-MDT_g+1)^+}^{t} v_{gt'} \le 1 - x_{gt}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{2.5}$$

$$u_{gt}, v_{gt}, x_{gt} \in \{0, 1\}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{2.6}$$

, where

$$C(\mathbf{x},\mathbf{d}) := \min \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}} VC_g \cdot p_{gt} + \sum_{t\in\mathcal{H}} VoLL \cdot ls_t \tag{2.7}$$

$$\sum_{g\in\mathcal{G}} p_{gt} + ls_t \geq d_t, \qquad \forall t \in \mathcal{H}, \tag{2.8}$$

$$P_g^{min} x_{gt} \leq p_{gt} \leq P_g^{max} x_{gt}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{2.9}$$

$$p_{gt} - p_{g(t-1)} \leq RUP_g, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\} \tag{2.10}$$

$$p_{g(t-1)} - p_{gt} \leq RDN_g, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\} \tag{2.11}$$

$$p_{gt} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{2.12}$$

$$ls_t \geq 0, \qquad \forall t \in \mathcal{H} \tag{2.13}$$

The formulation ($TSUC^D$) comprises two parts. The outer part (2.1)-(2.6) corresponds to finding an optimal first-stage decision while the inner part (2.7)-(2.13) corresponds to finding an optimal second-stage decision. The inner minimization problem (2.7)-(2.13) is called the *second-stage problem*, and its optimal objective value is called *second-stage cost*, which is denoted by $C(\mathbf{x},\mathbf{d})$ in the above formulation to represent its dependency on the first-stage decision $\mathbf{x}$ and demand realization $\mathbf{d}$. Now we describe the model in more detail. The objective function (2.1) consists of start-up cost, fixed cost, and the expected second-stage cost. Constraints (2.2)-(2.3) describe the logical relationship between start-up and shut-down decisions and constraints (2.4)-(2.5) represent the minimum up/down time constraints, respectively. (2.7) is the objective function for the inner part, which consists of variable generation cost and load shedding cost. Constraints (2.8) are the demand constraints that enforce that either the sum of the amount of generation exceeds the demand in the period, or the unsatisfied demand is regarded as load shedding amount. Constraints

(2.9) restrict the amount of the generation by generator-specific power limits. Constraints (2.10)-(2.11) represent the ramping constraints between period $t-1$ and period t. Constraints (2.6) are the binary conditions for variables that indicates start-up and shut-down, whereas constraints (2.12)-(2.13) represent the nonnegativity conditions for the amount of generation and load shedding.

Similarly, the scenario-based TSUC model can be derived as follows:

$$
\begin{aligned}
&(TSUC^S) \\
&\min \quad \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{H}} (SUC_g u_{gt} + FC_g x_{gt}) + \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} C(\mathbf{x}, \mathbf{d}^s) && (2.14) \\
&\text{s.t.} \quad (2.2) - (2.6) \\
&\qquad \sum_{g \in \mathcal{G}} p^s_{gt} + ls^s_t \geq d^s_t, && \forall t \in \mathcal{H}, s \in \mathcal{S}, \quad (2.15) \\
&\qquad P^{min}_g x_{gt} \leq p^s_{gt} \leq P^{max}_g x_{gt}, && \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S}, \quad (2.16) \\
&\qquad p^s_{gt} - p^s_{g(t-1)} \leq RUP_g, && \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}, s \in \mathcal{S}, \quad (2.17) \\
&\qquad p^s_{g(t-1)} - p^s_{gt} \leq RDN_g, && \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}, s \in \mathcal{S}, \quad (2.18) \\
&\qquad p_{gt} \geq 0, && \forall g \in \mathcal{G}, t \in \mathcal{H}, \quad (2.19) \\
&\qquad ls_t \geq 0, && \forall t \in \mathcal{H} \quad (2.20)
\end{aligned}
$$

The objective function (2.14) represents the average cost over the given demand scenarios. Constraints (2.15)-(2.20) correspond to the constraint (2.8)-(2.13) in the $(TSUC^D)$, but they are imposed for each scenario.

Chapter 3

Interval-based Two-stage Stochastic Unit Commitment Model

In this chapter, we propose the ITSUC model and discuss its properties. The key idea of the model and the detailed description of its modeling framework are given in Chapter 3.1. The mathematical formulation of the model and a comparative analysis of the models are provided in Chapter 3.2.

3.1 Modeling Framework

In the ITSUC model, the range of the generation, called *interval*, is decided in the first stage in addition to the on/off decision of each generator. In the second stage, the amount of generation for the generators to be turned on must lie in the interval chosen in the first stage. This is different from the TSUC model described in Chapter 2.1 where the on/off decision in the first stage is the only restriction that is imposed in the second stage. Based on the number of possible intervals that can be selected in each period, we propose two interval design methods for the first-stage interval decision, *finite interval design* and *infinite interval design*. Each interval design method will be further discussed in Chapter 3.1.1 and Chapter 3.1.2.

The main purpose of the interval is to consider the ramping constraints in the first

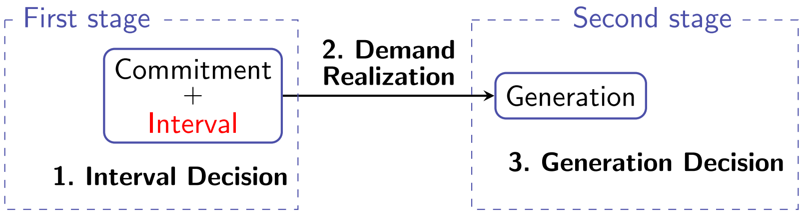


Figure 3.1: Two-stage Decision Framework for ITSUC

stage. If the intervals are carefully selected, the change of generation in the second stage cannot exceed the ramping limit. For example, if the amount of generation for the generator with 30 MW ramping limits is controlled by the interval [20, 40] at the period $t-1$ and [30, 50] at the period t, the maximum amount of ramp-up is $50-20=30$ MW and maximum amount of ramp-down is $40-30=10$ MW. Since these are not greater than the ramping limit of the generator, the ramping constraint in the second period for this generator is obviously satisfied. In other words, the ramping constraints in the second-stage problem can be taken out of consideration once the choice of intervals already implies the ramping limit. When any amount of generation in an interval I' in the next period does not violate the ramping constraint for the current interval I, we call I' is *reachable* from I. For example, in the case described above, [30,50] is reachable from [20,40] for the generator g. The more concise definitions related with the concept of the interval will be described in the following subchapter.

The reachability of the intervals determined in the first stage gives a distinctive property to the ITSUC model. Figure 3.3 and Figure 3.4 show the difference in the structure of the second-stage problem between TSUC and ITSUC. In Figure 3.3, the amount of generation is not only restricted between the minimum and maximum power limit but also the minimum and maximum generation level determined by the

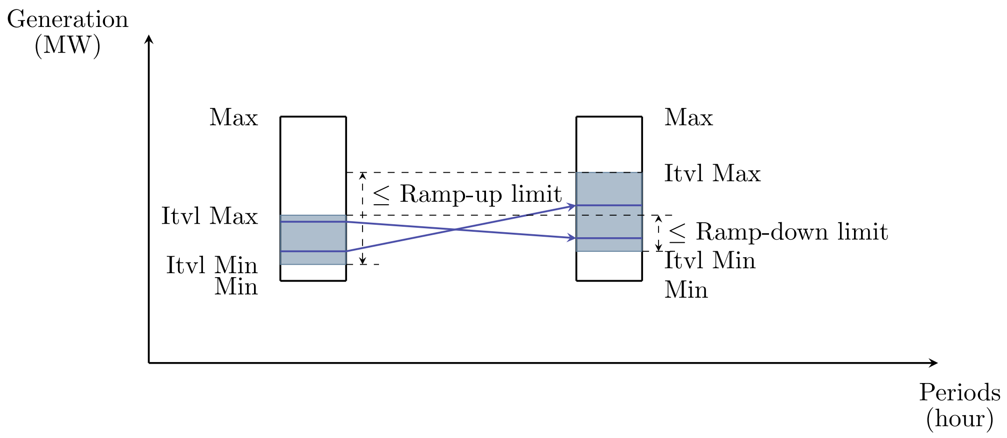


Figure 3.2: Example of Reachable Intervals

ramping limits. Therefore, the decision on the amount of generation in the TSUC model depends on that in the adjacent periods. However, in the case of the ITSUC model, the consideration of this inter-temporal dependency is no longer needed. As illustrated in Figure 3.4, the range of generation is further restricted due to the reachability constraints in the first stage. Since any adjacent selected intervals are reachable, the inter-temporal relationship of the amount of generation caused by the ramping constraints breaks down. Hence, the amount of generation can be decided separately in each period, which we call the *period-wise decomposition property*.

3.1.1 Finite Interval Design

In the finite interval design, the candidates for the interval decision should be predetermined for each generator. In practice, a decision maker who tries to apply the ITSUC model with finite interval design should decide how many and which intervals to introduce in the model. Setting the set of intervals properly is an important task for the model since it affects the performance and computation time of the model.

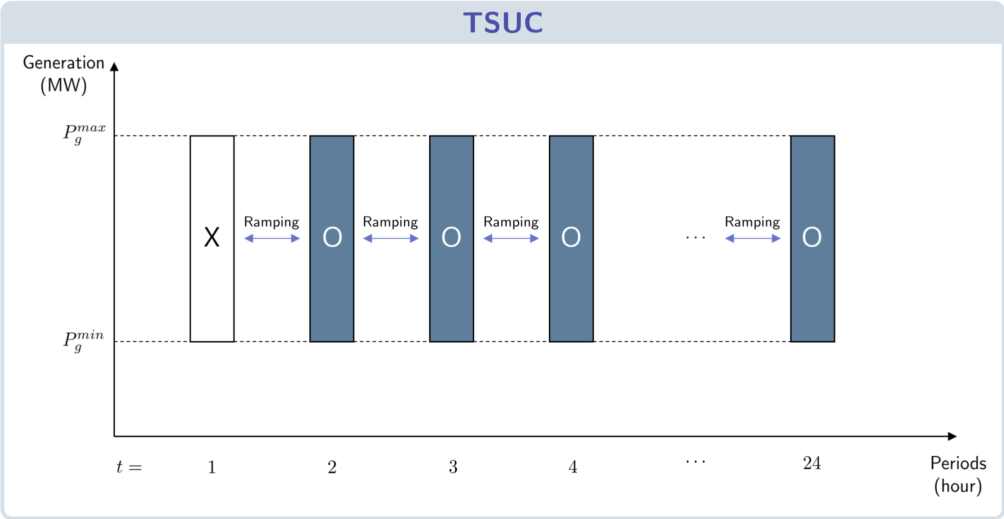


Figure 3.3: Inter-temporal Dependence of TSUC

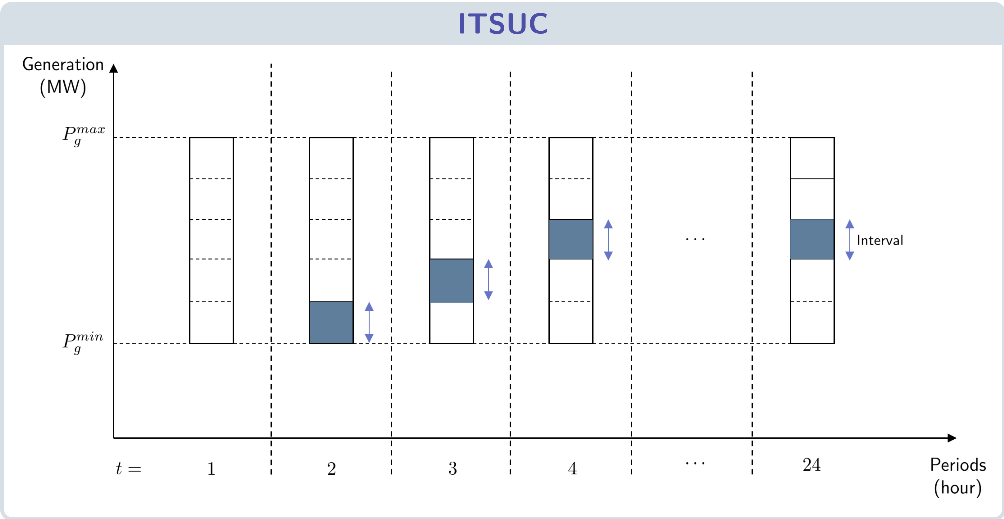


Figure 3.4: Period-wise Independence of ITSUC

Furthermore, the set of intervals chosen carelessly may be impossible to be used if one of them does not have any reachable interval nor can it shut down. In this subchapter, we will provide the mathematical definition of the concept of reachability and derive the condition for the set of intervals to be *proper*. In addition, we will suggest a simple and intuitive method to construct a proper set of intervals.

To establish the condition for the given set of intervals of a generator being able to yield a generation schedule that satisfies the ramping constraints, we here present the formal definition of *reachability*.

Definition 3.1. *(Reachable interval) Let* $I_g = [\underline{I}_g, \bar{I}_g]$ *and* $I'_g = [\underline{I}'_g, \bar{I}'_g]$ *be two intervals of a generator* $g \in \mathcal{G}$*. Then, we call that* I'_g *is* ***reachable*** *from* I_g *if* $\bar{I}'_g - \underline{I}_g \leq RUP_g$ *and* $\bar{I}_g - \underline{I}'_g \leq RDN_g$*.*

According to the Definition 3.1, if I'_g is reachable from I_g for $g \in \mathcal{G}$, it implies that the maximum difference in the amount of generation between two intervals cannot exceed the ramping limits, RDN_g or RUP_g.

Proposition 3.2. *(Reachability implies ramping limits) Suppose that interval* I'_g *is reachable from interval* I *for generator* $g \in \mathcal{G}$*. For any* $p \in I_g$ *and* $p' \in I'_g$*,* $p' - p \leq RUP_g$ *and* $p - p' \leq RDN_g$*.*

Proof. $p' - p \leq \bar{I}'_g - \underline{I}_g \leq RUP_g$ and $p - p' \leq \bar{I}_g - \underline{I}'_g \leq RDN_g$. □

By Proposition 3.2, given a set of intervals, there might exist an interval that does not have any reachable interval. In this case, that interval may be excluded from the set of intervals. Also, to consider all the possible amounts of generation between the minimum and maximum power limit, the intervals should cover the range of generation. If a set of intervals satisfies these two conditions, we call it a proper set of intervals.

Definition 3.3. *(Proper set of intervals) A set of intervals $\mathcal{I}_g$ is* ***proper*** *if*

1. $\bigcup_{I_g \in \mathcal{I}_g} I_g = [P_g^{min}, P_g^{max}]$
2. *For every $I_g \in \mathcal{I}_g$, there exists an interval $I_g^* \in \mathcal{I}_g$ reachable from I_g.*

The definition above is not a necessary condition to be used as a set of intervals in the ITSUC model. Instead, it indicates that the set of intervals satisfying the conditions above is a reasonable choice in that they do not excessively limit the range of generation. For example, if there exists no reachable interval for any interval in $\mathcal{I}_g$, then the only feasible generation schedule for generator g is to turn off until the end of the horizon. This may be too conservative and restrictive a modeling choice for the power system operator and may even cause a large amount of load-shedding cost which might have been avoided.

Next, we present a generic method for constructing a proper set of intervals, which we call *overlapping uniform interval design method.* The method has two control parameters, δ, and ℓ, and allows overlapping of intervals. The former indicates a difference between the minimum power limit of two consecutive intervals, and the latter indicates a size of an interval, which is the difference between the minimum and maximum power limits of each interval.

Definition 3.4. *Let δ, ℓ be positive real number such that $\delta \leq \ell \leq \min\{RUP_g,\ RDN_g\}$. From P_g^{min}, constant step size δ and size of intervals ℓ corresponds to the following proper set of intervals,*

$$\mathcal{I}_g = \left\{ k = 1, \ldots, \left\lceil \frac{P_g^{max} - P_g^{min} - \ell}{\delta} \right\rceil + 1 : [P_g^{min} + (k-1)\cdot\delta,\ \min\{P_g^{max}, P_g^{min} + (k-1)\cdot\delta + \ell\}] \right\}.$$

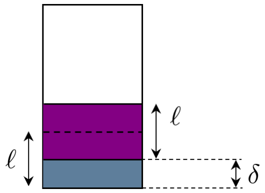


Figure 3.5: Example of Overlapping Intervals

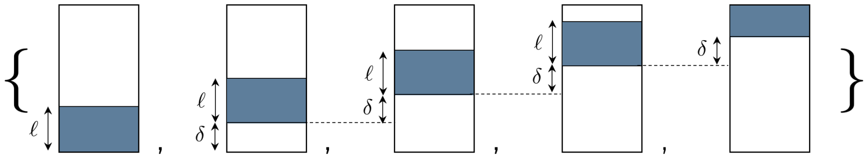


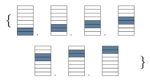
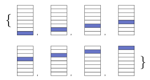
Figure 3.6: Illustration of Overlapping Uniform Interval Design

*Then, $\mathcal{I}_g$ is called **overlapping uniform interval design**.*

Figure 3.5 and Figure 3.6 illustrate how interval candidates are determined under the overlapping uniform interval design. In Figure 3.5, there are two overlapping intervals with the same length ℓ, the gap of which is δ. The set of interval candidates under the overlapping uniform design with parameters δ, ℓ is given in Figure 3.6. A specific example of the design is given in Table 3.1. Suppose that the ramping limits are half of the difference between the minimum and maximum power limits, i.e. $RUP_g = RDN_g = 0.5 \cdot (P_g^{max} - P_g^{min})$. If $\delta = \beta_s \cdot RUP_g$ and $\ell = \beta_l \cdot RUP_g$, the set of intervals can be designed differently according to the choice of β_s and β_l as shown in Table 3.1.

The parameters δ and ℓ play an important role in tuning the level of restriction of power generation. Their impact on the computation time and the planning cost is shown in Chapter 5.3.5 by controlling these parameters.

Table 3.1: Examples of Overlapping Uniform Interval Design

γ	β_s	β_l	Design
0.5	0.5	0.5	
0.5	0.25	0.5	
0.5	0.25	0.25	

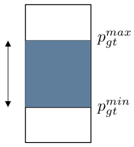


Figure 3.7: Example of Interval in Infinite Interval Design

3.1.2 Infinite Interval Design

In the finite interval design, we assumed that the number of intervals that can be selected in the first stage is finite. However, there might be a better choice of interval which is not included in the candidates. If we do not limit them to a finite set of intervals, a more efficient on/off decision may become possible. In an extreme case, every interval within the unit-specific power limit may be available, as illustrated in Figure 3.7. We call this infinite interval design since there are infinite numbers of intervals that can be possibly selected, as illustrated in Figure 3.8. It is in line with the RUC model proposed in [17] where the lower and upper limits of the interval can be decided in the first stage.

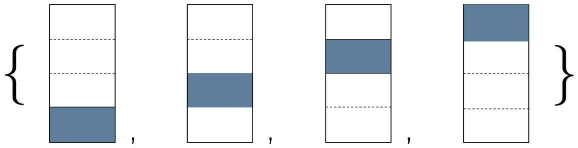


Figure 3.8: Illustration of Infinite Interval Design

Table 3.2: Notation for ITSUC Model

Sets and Indices	$\mathcal{I}_g$	Set of intervals for generator $g \in \mathcal{G}$
	$\mathcal{K}_g$	Set of indices for set of intervals $\mathcal{I}_g$
	$\mathcal{RI}_g(k)$	Set of reachable intervals for generator $g \in \mathcal{G}$ from kth interval
Parameters	P_{gk}^{min}	Minimum power limit of interval I_{gk} for generator g
	P_{gk}^{max}	Maximum power limit of interval I_{gk} for generator g
Decision Variables	b_{gtk}	1 if generator g is in interval I_k in period t, 0 otherwise.
	p_{gt}^{min}	the minimum generation of generator g in period t
	p_{gt}^{max}	the maximum generation of generator g in period t

3.2 Model Formulation

3.2.1 Finite Interval Design

The ITSUC model with finite interval design will be represented as *finite-interval-based two-stage stochastic unit commitment* (FITSUC) model in the rest of the thesis. Before we provide the formulation of the FITSUC model, we additionally introduce some notations regarding intervals as summarized in Table 3.2. For each generator $g \in \mathcal{G}$, the kth interval (for $k \in \mathcal{K}_g$) I_{gk} has two parameters, P_{gk}^{min} and P_{gk}^{max}, each of which indicates the minimum and maximum generation amount that an interval k is selected. Then, $\mathcal{I}_g := \{I_{gk} : k \in \mathcal{K}_g\}$ denotes the set of intervals and $\mathcal{RI}_g(k)$ denotes the set of reachable intervals from $k \in \mathcal{K}_g$. Note that the singleton $\{0\}$ $(= [0,0])$ indicates the off status of the generator and $I_{g0} = \{0\}$, i.e. $P_{g0}^{min} = P_{g0}^{max} = 0, \forall g \in \mathcal{G}$. Then, the formulation of the distribution-based FITSUC model can be derived as follows:

$$(FITSUC^D)$$

$$\min \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}}(SUC_g u_{gt} + FC_g x_{gt}) + \sum_{t\in\mathcal{H}} \mathbb{E}_{d_t}[C_t^F(\mathbf{b}, d_t)] \tag{3.1}$$

$$\text{s.t.} \quad u_{gt} - v_{gt} = b_{g(t-1)0} - b_{gt0}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}, \tag{3.2}$$

$$u_{g1} = 1 - b_{g10}, \qquad \forall g \in \mathcal{G}, \tag{3.3}$$

$$\sum_{t'=(t-MUT_g+1)^+}^{t} u_{gt'} \leq 1 - b_{gt0}, \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{3.4}$$

$$\sum_{t'=(t-MDT_g+1)^+}^{t} v_{gt'} \leq b_{gt0}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{3.5}$$

$$\sum_{k\in\mathcal{K}_g} b_{gtk} = 1, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{3.6}$$

$$\sum_{k'\in\mathcal{RI}_g(k)} b_{gtk'} \geq b_{g(t-1)k}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}, k \in \mathcal{K}_g, \tag{3.7}$$

$$u_{gt}, v_{gt}, x_{gt} \in \{0, 1\}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{3.8}$$

$$b_{gtk} \in \{0, 1\}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, k \in \mathcal{K}_g, \tag{3.9}$$

, where

$$C_t^F(\mathbf{b}, d_t) := \min \quad \sum_{g\in\mathcal{G}} VC_g \cdot p_{gt} + VoLL \cdot ls_t \tag{3.10}$$

$$\text{s.t.} \quad \sum_{g\in\mathcal{G}} p_{gt} + ls_t \geq d_t, \tag{3.11}$$

$$\sum_{k\in\mathcal{K}_g} P_{gk}^{min} b_{gtk} \leq p_{gt} \leq \sum_{k\in\mathcal{K}_g} P_{gk}^{max} b_{gtk}, \forall g \in \mathcal{G}, \tag{3.12}$$

$$p_{gt} \geq 0, \qquad \forall g \in \mathcal{G}, \tag{3.13}$$

$$ls_t \geq 0 \tag{3.14}$$

The objective function (3.1) consists of the start-up cost, fixed cost, and the expected second-stage cost, which is decomposed for each period into $C_t^F(\mathbf{b}, \mathbf{d})$. It could be done due to the period-wise decomposition property of the ITSUC model illustrated in Chapter 3.1. Constraints (3.2)-(3.5) correspond to the constraints (2.2)-(2.5) in $(TSUC^D)$ where the variable x_{gt} is eliminated by the equation $x_{gt} = 1 - b_{gt0}, \quad \forall g \in \mathcal{G}, t \in \mathcal{H}$. Constraints (3.6) indicate that exactly one of the intervals (including $\{0\}$) should be selected for each generator and period. Constraints (3.7) represent that only the reachable intervals from the interval chosen before are available. Constraints (3.8) are the binary conditions for the variables that indicate start-up and shut-down and constraints (3.9) are those for the variables that indicate the interval decision.

In the second-stage problem, the objective function (3.10) corresponds to (2.7), and constraints (3.11)-(3.14) correspond to (2.7)-(2.9), (2.12)-(2.13), respectively, except that the period is fixed by t and the amount of generation is restricted in the interval by constraints (3.12). The ramping constraints (2.10)-(2.11) are not included here explicitly since they are indirectly considered by the reachability constraints (3.7) in the first stage.

Similarly, the scenario-based FITSUC model can be derived as follows:

$$(FITSUC^S)$$

$$\min \quad \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{H}} (SUC_g u_{gt} + FC_g x_{gt}) \tag{3.15}$$

$$+ \sum_{t \in \mathcal{H}} \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \big(\sum_{g \in \mathcal{G}} VC_g \cdot p_{gt}^s + VoLL \cdot ls_t^s \big) \tag{3.16}$$

$$\text{s.t.} \quad (3.2) - (3.9)$$

$$\sum_{g\in\mathcal{G}} p_{gt}^s + ls_t^s \geq d_t^s, \qquad \forall t\in\mathcal{H}, s\in\mathcal{S}, \tag{3.17}$$

$$\sum_{k\in\mathcal{K}_g} P_{gk}^{min} b_{gtk} \leq p_{gt}^s \leq \sum_{k\in\mathcal{K}_g} P_{gk}^{max} b_{gtk}, \qquad \forall g\in\mathcal{G}, t\in\mathcal{H}, s\in\mathcal{S}, \tag{3.18}$$

$$p_{gt}^s \geq 0, \qquad \forall g\in\mathcal{G}, t\in\mathcal{H}, s\in\mathcal{S} \tag{3.19}$$

$$ls_t^s \geq 0, \qquad \forall t\in\mathcal{H}, s\in\mathcal{S} \tag{3.20}$$

The objective function (3.16) represents the average cost over the given demand scenarios. Constraints (3.17)-(3.20) corresponds to the constraints (3.11)-(3.14) in the ($FITSUC^D$), but they are imposed for each scenario.

3.2.2 Infinite Interval Design

The ITSUC model with infinite interval design will be represented as *infinite-interval-based two-stage stochastic unit commitment* (IITSUC) model in the rest of the thesis. As an additional notation, we introduce new decision variables p_{gt}^{min} and p_{gt}^{max} which indicates the minimum and maximum power limits of the interval for generator $g\in\mathcal{G}$ in period $t\in\mathcal{H}$. Then, the formulation of the distribution-based IITSUC model can be presented as follows:

$$(IITSUC^D)$$

$$\min \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}} (SUC_g u_{gt} + FC_g x_{gt}) + \sum_{t\in\mathcal{H}} \mathbb{E}_{d_t}[C_t^I(\mathbf{b}, d_t)] \tag{3.21}$$

$$\text{s.t.} \quad (2.2)-(2.6), (3.8)$$

$$p_{gt}^{max} - p_{g(t-1)}^{min} \leq RUP_g, \qquad \forall g\in\mathcal{G}, t\in\mathcal{H}\setminus\{1\}, \tag{3.22}$$

$$p_{g(t-1)}^{max} - p_{gt}^{min} \leq RDN_g, \qquad \forall g\in\mathcal{G}, t\in\mathcal{H}\setminus\{1\}, \tag{3.23}$$

$$P_g^{min} x_{gt} \leq p_{gt}^{min} \leq p_{gt}^{max} \leq P_g^{max} x_{gt}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{3.24}$$

$$p_{gt}^{min}, p_{gt}^{max} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, \tag{3.25}$$

, where

$$\begin{aligned} C_t^I(\mathbf{b}, \mathbf{d}) := \min \quad & (3.10) \\ \text{s.t.} \quad & (3.11), (3.13) - (3.14) \\ & p_{gt}^{min} \leq p_{gt} \leq p_{gt}^{max}, \quad \forall g \in \mathcal{G}, t \in \mathcal{H} \end{aligned} \tag{3.26}$$

Most of the elements in the above formulation are same as those in $(TSUC^D)$ or $(FITSUC^D)$. To avoid redundancies, we focus on the remaining part. Constraints (3.22)-(3.23) indicate that the gap between the minimum power limit and maximum power limit imposed by the interval should not exceed the ramping limit. (3.24) restricts the limits of the interval within the unit-specific power limit and set the amount of generation to zero if the generator is turned off. Constraints (3.25) represents the nonnegativity conditions for the variables that indicate limits of the interval. In the period-wise second-stage problem of the IITSUC model, whose cost function is denoted by $C_t^I(\mathbf{b}, \mathbf{d})$, the only difference from that of the FITSUC model is the constraint (3.26) that enforces the amount of generation to be within the interval determined in the first stage.

Similarly, the scenario-based IITSUC model can be derived as follows:

$$\begin{aligned} & (IITSUC^S) \\ & \min \;\; (3.16) \\ & \text{s.t.} \;\; (2.2) - (2.6), (3.17), (3.19) - (3.20), (3.22) - (3.25) \end{aligned}$$

$$p_{gt}^{min} \leq p_{gt}^{s} \leq p_{gt}^{max}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S}, \tag{3.27}$$

Constraints (3.27) represents that the amount of generation should be within the interval. Similarly to the $(TSUC^S)$ and $(FITSUC^S)$, all the constraints in the second-stage problem are imposed for each scenario.

3.3 Comparison of the Objective Values

Compared with the conventional TSUC model formulated in Chapter 2.2, the infinite-interval-based model can be considered as a restriction of it, in that it has a more restrictive range of the amount of generation than the TSUC model. Hence, the optimal cost of $(IITSUC^S)$ is less than or equal to that of $(TSUC^S)$ since every feasible solution for $(IITSUC^S)$ is a feasible solution for $(TSUC^S)$. Similarly, the optimal cost of $(FITSUC^S)$ is not less than that of $(IITSUC^S)$. Proposition 3.5 summarizes the comparison result of the TSUC and ITSUC models.

Proposition 3.5. *(Comparison of the optimal cost between $FITSUC^S$, $IITSUC^S$, and $TSUC^S$)*
Let $z_{\mathcal{C}}$ be the optimal objective value for an instance $\mathcal{C}$, and $\mathcal{FI}(\mathcal{U})$ and $\mathcal{II}(\mathcal{U})$ denote the $FITSUC^S$ instance and $IITSUC^S$ instance corresponding to a $TSUC^S$ instance $\mathcal{U}$, respectively. Then, the following statement holds.

$$z_{\mathcal{FI}(\mathcal{U})} \geq z_{\mathcal{II}(\mathcal{U})} \geq z_{\mathcal{U}}$$

Proof.

(i) $z_{\mathcal{FI}(\mathcal{U})} \geq z_{\mathcal{II}(\mathcal{U})}$

Let $Q^{\mathcal{FI}(\mathcal{U})}$ and $Q^{\mathcal{II}(\mathcal{U})}$ denote the set of feasible solutions of $\mathcal{FI}(\mathcal{U})$ and $\mathcal{II}(\mathcal{U})$, respectively. It suffices to show that for any $(\mathbf{x}, \mathbf{b}, \mathbf{u}, \mathbf{v}, \mathbf{p}, \mathbf{ls}) \in Q^{\mathcal{FI}(\mathcal{U})}$, there exists $(\mathbf{x}, \mathbf{u}, \mathbf{v}, \mathbf{p}^{\mathbf{min}}, \mathbf{p}^{\mathbf{max}}, \mathbf{p}, \mathbf{ls}) \in Q^{\mathcal{II}(\mathcal{U})}$ with the same objective value. Consider $(\mathbf{b}, \mathbf{u}, \mathbf{v}, \mathbf{p}, \mathbf{ls}) \in Q^{\mathcal{FI}(\mathcal{U})}$. We construct $(\tilde{\mathbf{x}}, \tilde{\mathbf{u}}, \tilde{\mathbf{v}}, \tilde{\mathbf{p}}, \tilde{\mathbf{ls}}) \in Q^{\mathcal{II}(\mathcal{U})}$ as follows:

$$\begin{aligned}
\tilde{x}_{gt} &= 1 - b_{gt0}, && \forall g \in \mathcal{G}, t \in \mathcal{H},\\
\tilde{u}_{gt} &= u_{gt}, && \forall g \in \mathcal{G}, t \in \mathcal{H},\\
\tilde{v}_{gt} &= v_{gt}, && \forall g \in \mathcal{G}, t \in \mathcal{H},\\
\tilde{p}_{gt}^{min} &= \sum_{k \in \mathcal{K}_g} P_{gk}^{min} b_{gtk}, && \forall g \in \mathcal{G}, t \in \mathcal{H},\\
\tilde{p}_{gt}^{max} &= \sum_{k \in \mathcal{K}_g} P_{gk}^{max} b_{gtk}, && \forall g \in \mathcal{G}, t \in \mathcal{H},\\
\tilde{p}_{gt}^{s} &= p_{gt}^{s}, && \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S},\\
\tilde{ls}_{t}^{s} &= ls_{t}^{s}, && \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S}
\end{aligned}$$

Constraints (3.6) and (3.9) indicate that there exists a unique $k \in \mathcal{K}_g$ such that $b_{gtk} = 1$ for all $g \in \mathcal{G}$ and $t \in \mathcal{H}$. Let $k(g,t)$ denote such k. Then, $\tilde{p}_{gt}^{min} = P_{gk(g,t)}^{min}$ and $\tilde{p}_{gt}^{max} = P_{gk(g,t)}^{max}$ for all $g \in \mathcal{G}$ and $t \in \mathcal{H}$. In addition, the constraints (3.7) indicate that $k(g,t) \in \mathcal{RI}_g(k(g,t-1)), \forall t \in \mathcal{H} \setminus \{1\}$. Therefore,

$$\begin{aligned}
\tilde{p}_{gt}^{max} - \tilde{p}_{g(t-1)}^{min} &= P_{gk(g,t)}^{max} - P_{gk(g,t-1)}^{min} \leq RUP_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}\\
\tilde{p}_{g(t-1)}^{max} - \tilde{p}_{gt}^{min} &= P_{gk(g,t-1)}^{max} - P_{gk(g,t)}^{min} \leq RDN_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}
\end{aligned}$$

and the constraints (3.22), (3.23) in $\mathcal{II}(\mathcal{U})$ are satisfied. Besides,

$$\tilde{p}^s_{gt} \geq \sum_{k \in \mathcal{K}_g} P^{min}_{gk} b_{gtk} = \tilde{p}^{min}_{gt}$$

$$\tilde{p}^s_{gt} \leq \sum_{k \in \mathcal{K}_g} P^{max}_{gk} b_{gtk} = \tilde{p}^{max}_{gt}$$

Hence the constraints (3.27) in $\mathcal{II}(\mathcal{U})$ are satisfied. Similarly, the other constraints in $\mathcal{II}(\mathcal{U})$ are satisfied by construction, and so $(\tilde{\mathbf{x}}, \tilde{\mathbf{u}}, \tilde{\mathbf{v}}, \tilde{\mathbf{p}}, \tilde{\mathbf{ls}}) \in Q^{\mathcal{II}(\mathcal{U})}$. Since the objective of $\mathcal{FI}(\mathcal{U})$ and $\mathcal{II}(\mathcal{U})$ are identical, we have $z_{\mathcal{FI}(\mathcal{U})} \geq z_{\mathcal{II}(\mathcal{U})}$.

(ii) $z_{\mathcal{II}(\mathcal{U})} \geq z_{\mathcal{U}}$

Let $Q^{\mathcal{U}}$ denote the set of feasible solutions of $\mathcal{II}(\mathcal{U})$. It suffices to show that for any $(\mathbf{x}, \mathbf{u}, \mathbf{v}, \mathbf{p}, \mathbf{p^{min}}, \mathbf{p^{max}}, \mathbf{ls}) \in Q^{\mathcal{II}(\mathcal{U})}$, $(\mathbf{x}, \mathbf{u}, \mathbf{v}, \mathbf{p}, \mathbf{ls}) \in Q^{\mathcal{U}}$ and both have the same objective value. The constraints (3.24) and (3.27) imply that

$$P^{min}_g x_{gt} \leq p^{min}_{gt} \leq p^s_{gt} \leq p^{max}_{gt} \leq P^{max}_g x_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S}$$

and the constraints (2.16) are satisfied. Besides,

$$p^s_{gt} - p^s_{g(t-1)} \leq p^{max}_{gt} - p^{min}_{g(t-1)} \leq RUP_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}$$

$$p^s_{g(t-1)} - p^s_{gt} \leq p^{max}_{g(t-1)} - p^{min}_{gt} \leq RDN_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{H} \setminus \{1\}$$

Hence, the ramping constraints (2.17)-(2.18) in $\mathcal{U}$ are satisfied. Similarly, the other constraints are satisfied by construction, and so $(\mathbf{x}, \mathbf{u}, \mathbf{v}, \mathbf{p}, \mathbf{ls}) \in Q^{\mathcal{U}}$. Since the objective of $\mathcal{II}(\mathcal{U})$ and $\mathcal{U}$ are identical, we have $z_{\mathcal{II}(\mathcal{U})} \geq z_{\mathcal{U}}$. □

Chapter 4

Solution Approaches

As mentioned in Chapter 1.4, there is still a limitation on efficiently solving the TSUC model. In this chapter, we give two solution approaches for the ITSUC models, one for the distribution-based model, and the other for the scenario-based model. The former is the bounding method described in Chapter 4.1, and the latter is the Benders reformulation presented in Chapter 4.2.

4.1 Bounding Method

As shown in the formulation of the distribution-based ITSUC models in Chapter 3.2, the expectation of the second-stage cost function can be decomposed into several period-wise cost terms. If we know the lower and upper bounds of the demand, i.e. $d_t \in [a_t, b_t]$, $\forall t \in \mathcal{H}$, then we can apply the tightened lower and upper bound derived from the Jensen inequality and the Edmundson-Madansky inequality [27]. The Jensen inequality is a well-known inequality that holds for any convex function, which demonstrates that the function value at the mean of a random variable is less than or equal to the expectation of the function. Since the second-stage cost function in the ITSUC model is a convex function by LP duality, applying Jensen inequality gives an optimization problem whose optimal objective value is a valid lower bound of the original problem. Similarly, the Edmundson-Madansky inequality, an inequality that holds for the convex function of the bounded random variable, is applicable to the ITSUC model in order to obtain a valid upper bound [26]. However, even though the gap between the lower and upper bound is a useful indicator of the solution quality, it may be too large to be used as an alternative to the traditional SAA method. To overcome this limitation, we used the enhanced bound proposed in [27]. They devised a method to obtain tighter bounds by sequentially applying these classic Jensen and Edmundson-Madansky bounds.

We suppose that there exists a lower and upper bound. To introduce its application to the ITSUC model, we suppose that there exists a lower and upper bound of demand in each period t, i.e. $a_t \leq d_t \leq b_t,\ \forall t \in \mathcal{H}$ since Edmundson-Madansky inequality makes sense only if the random variable is bounded. Let $f_t(\cdot)$, $F_t(\cdot)$ represents the density function and distribution function of d_t, respectively, and $C_t(\mathbf{b}, d_t)$ denote either $C_t^F(\mathbf{b}, d_t)$ or $C_t^I(\mathbf{b}, d_t)$. Then, the following proposition holds.

Proposition 4.1. *(Bounds on the expectation of second-stage cost) Let* $a_t = d_{t0} < d_{t1} < \cdots < d_{tm} = b_t$ *be arbitrary points in* $[a_t, b_t]$.

$$\sum_{t=1}^{T}\sum_{i=1}^{m} \alpha_{ti} C_t(\mathbf{b}, m_{ti}) \leq \sum_{t=1}^{T} \mathbb{E}_{\mathbf{d}}[C_t(\mathbf{b}, d_t)] \leq \sum_{t=1}^{T}\sum_{i=0}^{m} \beta_{ti} C_t(\mathbf{b}, d_{ti})$$

, where

$$\alpha_{ti} := F_t(d_{ti}) - F_t(d_{t(i-1)}),\ \forall i \in \{1, \ldots, m\},\ \alpha_{t0} = \alpha_{t(m+1)} = 0,$$

$$m_{ti} := \mathbb{E}\big[d_t | d_t \in [d_{t(i-1)}, d_{ti}]\big],\ \forall i \in \{1, \ldots, m\},$$

$$\beta_{ti} := \alpha_{ti} \cdot \Big(\frac{m_{ti} - d_{t(i-1)}}{d_{ti} - d_{t(i-1)}}\Big) + \alpha_{t(i+1)} \cdot \Big(\frac{d_{t(i+1)} - m_{t(i+1)}}{d_{t(i+1)} - d_{ti}}\Big),\ \forall i \in \{0, \ldots, m\}$$

Proof. Since $C_t(\mathbf{b}, d_t)$ in a convex function in d_t [36], the statement immediately follows from Theorem 2 and 3 in [27]. $\square$

In Proposition 4.1, m_{ti}, the conditional mean in the interval $[d_{t(i-1)}, d_{ti}]$, for a lower bound may not be directly computed because of the difficulty of calculating the integral. However, if the demand follows the multivariate truncated normal distribution, denoted as $\mathbf{d} \sim \mathcal{N}(\mu, \Sigma, \mathbf{a}, \mathbf{b})$, m_{ti} can be computed as follows:

$$
\begin{aligned}
m_{ti} := \mathbb{E}\big[d_t | d_t \in [d_{t(i-1)}, d_{ti}]\big] &= \frac{\int_{d_{t(i-1)}}^{d_{ti}} x f_t(x) dx}{\int_{d_{t(i-1)}}^{d_{ti}} f_t(x) dx} = \frac{\int_{d_{t(i-1)}}^{d_{ti}} x \cdot \frac{f_t(x)}{\Phi(b)-\Phi(a)} dx}{\frac{\Phi_t(d_{ti})-\Phi_t(d_{t(i-1)})}{\Phi(b)-\Phi(a)}} \\
&= \frac{\mu_t(\Phi_t(d_{ti}) - \Phi_t(d_{t(i-1)})) + \int_{d_{t(i-1)}}^{d_{ti}} (x-\mu_t)\phi_t(x) dx}{\Phi_t(d_{ti}) - \Phi_t(d_{t(i-1)})} \\
&= \mu_t - \sigma_t^2 \cdot \frac{\phi_t(d_{ti}) - \phi_t(d_{t(i-1)})}{\Phi_t(d_{ti}) - \Phi_t(d_{t(i-1)})} \qquad , \forall i \in \{1, \ldots, m\}
\end{aligned}
$$

, where $\phi(\cdot)$, $\Phi(\cdot)$ denote the density function and distribution function of the normal distribution $\mathcal{N}(\mu_t, \sigma_t^2)$, respectively. Replacing the expectation part with the upper bound represented in Proposition 4.1, the surrogate models that provide lower and upper bounds for the FITSUC model are as follows:

$$
\begin{aligned}
&(FITSUC_{LB}^{D}) \\
&\min \quad \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{H}} (SUC_g u_{gt} + FC_g(1 - b_{gt0})) + \sum_{t \in \mathcal{H}} \sum_{i=1}^{m} \alpha_{ti} C_t^F(\mathbf{b}, m_{ti}) \\
&\text{s.t.} \quad (3.2) - (3.9)
\end{aligned}
$$

$(FITSUC^D_{UB})$

$$\min \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}}(SUC_g u_{gt} + FC_g(1-b_{gt0})) + \sum_{t\in\mathcal{H}}\sum_{i=0}^{m}\beta_{ti}C_t^F(\mathbf{b}, d_{ti})$$

$$\text{s.t.} \quad (3.2)-(3.9)$$

According to Proposition 4.1, The optimal objective value of $(FITSUC^D_{LB})$ is a valid lower bound for the ITSUC model, while $(FITSUC^D_{UB})$ gives an upper bound. If we solve both $(FITSUC^D_{LB})$ and $(FITSUC^D_{UB})$, the solution of $FITSUC^D_{UB}$ can be regarded as the optimal solution of $FITSUC^D$ at the certain optimality tolerance level, which is the gap between the objective values of those two. The same arguments hold for the IITSUC model and the formulation for the corresponding surrogate models are as follows:

$(IITSUC^D_{LB})$

$$\min \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}}(SUC_g u_{gt} + FC_g(1-b_{gt0})) + \sum_{t\in\mathcal{H}}\sum_{i=1}^{m}\alpha_{ti}C_t^I(\mathbf{b}, m_{ti})$$

$$\text{s.t.} \quad (2.2)-(2.6),(3.8),(3.22)-(3.25)$$

$(IITSUC^D_{UB})$

$$\min \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}}(SUC_g u_{gt} + FC_g(1-b_{gt0})) + \sum_{t\in\mathcal{H}}\sum_{i=0}^{m}\beta_{ti}C_t^I(\mathbf{b}, d_{ti})$$

$$\text{s.t.} \quad (2.2)-(2.6),(3.8),(3.22)-(3.25)$$

For the conventional TSUC model, the argument similar to Proposition 4.1 holds when the demand is period-wise independent, but the bound contains m^T cost function terms which are much more than $m \times T$ terms in the ITSUC model. For this reason, to the best of our knowledge, the application of this method to the TSUC model has not been reported.

In the objective of the surrogate models, α_{ti} and β_{ti} can be interpreted as the weights of each cost function value used for the enhanced Jensen bound and Edmundson-Madansky bound, respectively. Even more, the sum of those weights equals to 1, i.e. $\sum_{i=1}^{m} \alpha_{ti} = \sum_{i=0}^{m} \beta_{ti} = 1, \forall t \in \mathcal{H}$. In this point of view, these two bounds can be viewed as the results of the SAA method if the arbitrary demand points $d_{ti}, i \in \{1, \ldots, m\}$ or the conditional mean m_{ti} were sampled from the demand distribution. In addition, the gap of the bounds can be closed as much as it is desired by increasing m, the number of sub-intervals, in the surrogate models. Therefore, the proposed bounding method can be viewed as a discretization-based approximation method of the second-stage expected cost. The computational comparison results of the SAA method and the bounding method are given in Chapter 5.

4.2 Benders Reformulation

The formulation of the ITSUC model presented in Chapter 3 has $O(GTS)$ variables and $O(TS)$ constraints when the maximum number of demand samples among all the periods is denoted by S. Since the number of variables and constraints are both linear with the number of scenarios, the problem may become much harder to be solved even with a slight increase in demand scenarios. To deal with this scalability issue, Benders decomposition is widely used [37]. Benders decomposition is a general solution approach to linear programming or integer programming which can exploit the stage-wise decision framework in the stochastic programming model. In this subchapter, we will obtain the so-called Benders reformulation of the ITSUC model by characterizing a set of dual feasible solutions with polynomial cardinality containing the optimal dual solution to the second-stage problem.

Omitting the superscript s, the second-stage problem for period t in the IITSUC model can be written as follows:

$$
\begin{aligned}
\min \quad & \sum_{g \in \mathcal{G}} VC_g \cdot p_{gt} + VoLL \cdot ls_t & & \text{dual var.} \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_{gt} + ls_t \geq d_t, & & (\lambda_t) \\
& p_{gt} \geq p_{gt}^{min}, & \forall g \in \mathcal{G}, \quad & (\alpha_{gt}) \\
& p_{gt} \leq p_{gt}^{max}, & \forall g \in \mathcal{G}, \quad & (\beta_{gt}) \\
& ls_t \geq 0
\end{aligned}
\tag{4.1}
$$

According to the LP duality, (4.1) is equivalent to its following dual problem.

$$
\begin{aligned}
\max \quad & \sum_{g \in \mathcal{G}} (p_{gt}^{min} \alpha_{gt} + p_{gt}^{max} \beta_{gt}) + d_t \lambda_t \\
\text{s.t.} \quad & \lambda_t + \alpha_{gt} + \beta_{gt} = VC_g, & \forall g \in \mathcal{G}, \\
& \lambda_t \leq VoLL, \\
& \alpha_{gt} \geq 0, \beta_{gt} \leq 0, & \forall g \in \mathcal{G}, \\
& \lambda_t \geq 0
\end{aligned}
\tag{4.2}
$$

Since the ramping constraints are no longer needed, (4.1) is equivalent to LP relaxation of 0-1 knapsack problem. It can be easily solved by sorting the items by their cost coefficient and taking the largest value possible for each item in ascending order until the cumulative sum of item weights reached the capacity [38]. Hence, the optimal solution for (4.1) can be easily obtained by sorting the generators by their variable cost coefficient VC_g. Using this property, we can find out the whole possible optimal dual solution of (4.2) as presented in Proposition 4.2.

Proposition 4.2. *(Optimal dual solution of second-stage problem) Suppose that $VoLL > VC_g \geq 0, \forall g \in \mathcal{G}$. Then, set $\mathcal{D} := \mathcal{D}_1 \cup \mathcal{D}_2 \cup \mathcal{D}_3$ contains an optimal solution of dual problem (4.2) for every realization of d_t.*

$$\mathcal{D}_1 := \{((\mathbf{VC} - VC_g \cdot \mathbb{1})^+, (\mathbf{VC} - VC_g \cdot \mathbb{1})^-, VC_g) : g \in \mathcal{G}\}$$

$$\mathcal{D}_2 := \{(\mathbf{VC}, \mathbf{0}, 0)\}$$

$$\mathcal{D}_3 := \{(\mathbf{0}, \mathbf{VC} - VoLL \cdot \mathbb{1}, VoLL \cdot \mathbb{1})\}$$

Proof. Let $i(g)$ and $g(i)$ denote the rank of generator g and ith generator, respectively, when sorting all $g \in \mathcal{G}$ by VC_g in ascending order. We will show that the optimal amount of generation and load shedding for (4.1) can be found if the residual demand $d_t - \sum_{g\in\mathcal{G}} p_{gt}^{min}$ is satisfied by running the generators with small variable cost coefficient as much as possible according to the sorted order. Let $s_{j,t} := \sum\limits_{g\in\mathcal{G}} p_{gt}^{min} + \sum\limits_{i=1}^{j}(p_{g(i)t}^{max} - p_{g(i)t}^{min}),\ \forall j \in \{1,\ldots,|\mathcal{G}|\}$ and $s_{0,t} := \sum\limits_{g\in\mathcal{G}} p_{gt}^{min}$. Since $s_{j_1,t} \geq s_{j_2,t} \geq 0,\ \forall j_1 > j_2 \geq 0$, exactly one of the following statements is true.

(i) $\exists g* \in \mathcal{G}$ such that $s_{i(g*)-1,t} \leq d_t < s_{i(g*),t}$ (ii) $d_t \geq \sum_{g \in \mathcal{G}} p_{gt}^{max}$ (iii) $d_t \leq \sum_{g \in \mathcal{G}} p_{gt}^{min}$.

Case 1. $\exists g* \in \mathcal{G}$ such that $s_{i(g*)-1,t} \leq d_t < s_{i(g*),t}$.

Let

$$p^*_{g(i)t} = \begin{cases} p_{gt}^{max} & \big(i > i(g^*)\big) \\ d_t - s_{i(g^*)-1,t} & \big(i = i(g^*)\big) \\ p_{gt}^{min} & \big(i < i(g^*)\big) \end{cases} \qquad \begin{aligned} \alpha^*_{gt} &= (VC_g - VC_{g*})^+ \\ \beta^*_{gt} &= (VC_g - VC_{g*})^- \\ \lambda^*_t &= VC^*_g \end{aligned}$$

$$ls^*_t = 0$$

for every $g \in \mathcal{G}$. Then,

$$\begin{aligned} &\sum_{g \in \mathcal{G}} VC_g p^*_{gt} + VoLL \cdot ls^*_t \\ &= \sum_{i=1}^{i(g*)-1} VC_{g(i)} \cdot p_{g(i)t}^{max} + VC_{g*} \cdot (d_t - \sum_{i=1}^{i(g*)-1} p_{g(i)t}^{max} - \sum_{i=i(g*)+1}^{|\mathcal{G}|} p_{g(i)t}^{min}) \\ &+ \sum_{i=i(g*)+1}^{|\mathcal{G}|} VC_{g(i)} \cdot p_{g(i)t}^{min} \\ &= \sum_{g \in \mathcal{G}} p_{gt}^{min}(VC_g - VC_{g*})^+ + p_{gt}^{max}(VC_g - VC_{g*})^- + d_t VC_{g*} \\ &= \sum_{g \in \mathcal{G}} p_{gt}^{min} \alpha^*_{gt} + p_{gt}^{max} \beta^*_{gt} + d_t \lambda^*_t \end{aligned} \tag{4.3}$$

Case 2. $d_t \geq \sum_{g \in \mathcal{G}} p_{gt}^{max}$.

Let

$$p^*_{g(i)t} = p_{gt}^{max} \qquad \alpha^*_{gt} = 0$$

$$ls^*_t = d_t - \sum_{g \in \mathcal{G}} p_{gt}^{max} \qquad \beta^*_{gt} = VC_g - VoLL$$

$$\lambda^*_t = VoLL$$

for every $g \in \mathcal{G}$. Then,

$$\begin{aligned}
&\sum_{g \in \mathcal{G}} VC_g p_{gt}^{max} + VoLL \cdot (d_t - \sum_{g \in \mathcal{G}} p_{gt}^{max}) \\
&= \sum_{g \in \mathcal{G}} (VC_g - VoLL) p_{gt}^{max} + d_t \cdot VoLL \\
&= \sum_{g \in \mathcal{G}} p_{gt}^{min} \alpha^*_{gt} + p_{gt}^{max} \beta^*_{gt} + d_t \lambda^*_t
\end{aligned} \tag{4.4}$$

Case 3. $d_t \leq \sum_{g \in \mathcal{G}} p_{gt}^{min}$.

Let

$$p^*_{g(i)t} = p_{gt}^{min} \qquad \alpha^*_{gt} = VC_g$$

$$ls^*_t = 0 \qquad \beta^*_{gt} = 0$$

$$\lambda^*_t = 0$$

for every $g \in \mathcal{G}$. Then,

$$\sum_{g \in \mathcal{G}} VC_g p_{gt}^{min} = \sum_{g \in \mathcal{G}} p_{gt}^{min} \alpha^*_{gt} + p_{gt}^{max} \beta^*_{gt} + d_t \lambda^*_t, \quad \forall g \in \mathcal{G}, t \in \mathcal{H} \tag{4.5}$$

For all cases, ($\mathbf{p}^*$, $\mathbf{ls}^*$) is a primal feasible solution for (4.1), (α^*, β^*, λ^*) is a dual feasible solution for (4.2). Since the primal and dual objective values for these solutions coincide, the former is an optimal solution for (4.1), and the latter is for (4.2) by the duality of linear programming [36]. □

Let η_{ts} denote the optimal cost of (4.2). Then according to Proposition 4.2,

$$\eta_{ts} = \min_{(\alpha,\beta,\lambda)\in\mathcal{Y}} \sum_{g\in\mathcal{G}} (p_{gt}^{min}\alpha_{gt} + p_{gt}^{max}\beta_{gt}) + d_t\lambda_t = \min_{(\alpha,\beta,\lambda)\in\mathcal{D}} \sum_{g\in\mathcal{G}} (p_{gt}^{min}\alpha_{gt} + p_{gt}^{max}\beta_{gt}) + d_t\lambda_t \tag{4.6}$$

, where $\mathcal{Y}$ denotes the feasible region of (4.2).

Hence, the Benders reformulation of the scenario-based IITSUC model can be simplified as follows:

$$(IITSUC_{BR}^{S})$$

$$\min \quad \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{H}} (SUC_g u_{gt} + FC_g x_{gt}) + \sum_{s\in\mathcal{S}} p^s \sum_{t\in\mathcal{H}} \eta_{ts}$$

$$\text{s.t.} \quad (2.2) - (2.6)$$

$$\eta_{ts} \geq \sum_{g'\in\mathcal{G}} \left(p_{g't}^{min} (VC_{g'} - VC_g)^+ + p_{g't}^{max} (VC_{g'} - VC_g)^- \right) + d_t^s \cdot VC_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S}, \tag{4.7}$$

$$\eta_{ts} \geq \sum_{g'\in\mathcal{G}} p_{g't}^{min} \cdot VC_{g'}, \quad \forall t \in \mathcal{H}, s \in \mathcal{S}, \tag{4.8}$$

$$\eta_{ts} \geq \sum_{g'\in\mathcal{G}} \left(p_{g't}^{max} \cdot (VC_{g'} - VoLL) \right) + d_t^s \cdot VoLL, \quad \forall t \in \mathcal{H}, s \in \mathcal{S} \tag{4.9}$$

In a similar way, the Benders reformulation for the FITSUC model can be derived by putting $p_{gt}^{min} := \sum_{k\in\mathcal{K}_g\setminus\{0\}} P_{gk}^{min} b_{gtk}$ and $p_{gt}^{max} := \sum_{k\in\mathcal{K}_g\setminus\{0\}} P_{gk}^{max} b_{gtk}$.

$(FITSUC_{BR}^{S})$

$$\min \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{H}}(SUC_g u_{gt} + FC_g x_{gt}) + \sum_{s\in\mathcal{S}} p^s \sum_{t\in\mathcal{H}} \eta_{ts}$$

s.t. (2.2) − (2.6)

$$\eta_{ts} \geq \sum_{g'\in\mathcal{G}, k\in\mathcal{K}_{g'}\setminus\{0\}} \Big(P_{g'k}^{min}(VC_{g'} - VC_g)^+ + P_{g'k}^{max}(VC_{g'} - VC_g)^-\Big) b_{g'tk} + d_t^s \cdot VC_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{H}, s \in \mathcal{S}, \tag{4.10}$$

$$\eta_{ts} \geq \sum_{g'\in\mathcal{G}, k\in\mathcal{K}_{g'}\setminus\{0\}} P_{g'k}^{min} VC_{g'} \cdot b_{g'tk}, \quad \forall t \in \mathcal{H}, s \in \mathcal{S}, \tag{4.11}$$

$$\eta_{ts} \geq \sum_{g'\in\mathcal{G}, k\in\mathcal{K}_{g'}\setminus\{0\}} \big(P_{g'k}^{max} \cdot (VC_{g'} - VoLL)\big) + d_t^s \cdot VoLL, \quad \forall t \in \mathcal{H}, s \in \mathcal{S} \tag{4.12}$$

The constraints (4.7)-(4.9), or the constraints (4.10)-(4.12) each correspond to the Benders cut obtained from $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{D}_3$. The size of the set $\mathcal{D}$ in Proposition 4.2 is $|\mathcal{G}| + 2$, which is polynomial to the input size of the problem. Hence, $(FITSUC_{BR}^{S})$ and $(IITSUC_{BR}^{S})$ are the *compact* Benders formulation of the IT-SUC model. In general, it can be hardly obtained since the traditional form of the Benders reformulation itself has constraints at least as much as the number of extreme points in the dual feasible set. However, in this case, since we know that there exists a dual optimal solution in the set $\mathcal{D}$ according to the Proposition 4.2, the other extreme points do not need to be considered and the formulation can be much more simplified. The computational comparison results of the original formulation and the Benders reformulation are presented in Chapter 5.

Chapter 5

Computational Experiments

We have proposed the ITSUC model and analyzed its useful properties such as period-wise decomposition, compact Benders reformulation, and the applicability of the bounding method. The computational experiment results for the model and their efficiency is discussed in this chapter. The instance and control parameters we used for the experiments are described in Chapter 5.1. In Chapter 5.2, the evaluation method for the models is presented, and in Chapter 5.3, the computational results for the ITSUC model and solution approaches are illustrated.

5.1 Experiment Setting

We conducted all the experiments based on the 10-generator daily UC instance with 24 time periods from [39]. In terms of demand, we used the values multiplied by 75% from those in the instance, to avoid the load-shedding cost dominating the overall cost. The value of lost load ($VoLL$) was fixed by \$1,000. The ramping limits of the instance were controlled by the parameter γ for the analysis of its impact on the performance of the model. For each generator g, its ramping limits are determined by its length of generation range ($P_g^{max} - P_g^{min}$) multiplied by $\gamma \in [0, 1]$, i.e. $RUP_g = RDN_g = \gamma \cdot (P_g^{max} - P_g^{min})$. For the FITSUC model, the overlapping uniform interval design introduced in Chapter 3.1.1 was used. Since the validity of interval design and the number of reachable intervals are closely related with the ramping limits, we set the step size δ and the size of intervals ℓ to be proportional to RUP_g with coefficient $\beta_d \in [0,1]$ and $\beta_l \in [0,1]$, i.e. $\delta = \beta_d \cdot RUP_g$ and $\ell = \beta_l \cdot RUP_g$. β_d and β_l denote the control parameters for the overlapping uniform interval design.

Since we used the deterministic instance, the nominal demand was regarded as the mean of the demand, and the standard deviation of the demand was controlled by the coefficient of variation, CV for short. If the nominal demand for period t is denoted by $\bar{d}_t$, the demand for each period was sampled independently from the truncated normal distribution $d_t \overset{\text{i.i.d}}{\sim} N(\bar{d}_t, \sigma_t^2, (\bar{d}_t - 4\sigma_t)^+, \bar{d}_t + 4\sigma_t)$, where $\sigma_t = CV \cdot \bar{d}_t,\ \forall t \in \mathcal{H}$ based on the inverse transform sampling. Each combination of the demand samples over the time horizon became a demand scenario in $\mathcal{S}$. We conducted the experiment for $|\mathcal{S}| \in \{10, 50, 100, 500, 1{,}000\}$ and $|\mathcal{S}| = 500$ was set as the default value.

The ramping limits were controlled by $\gamma \in \{0.25, 0.5, 0.75\}$ and $\gamma = 0.5$ was set as the default value. Three types of overlapping uniform interval designs with $(\beta_d, \beta_l) \in \{(0.25, 0.25), (0.25, 0, 5), (0.5, 0.5)\}$ were used in the experiments. The set of interval candidates for $\gamma = 0.5$ is illustrated in Figure 3.6. $(\beta_d, \beta_l) = (0.5, 0.5)$ was set as the default values among them. All the models and methods were implemented with Xpress 8.14 and tested with Intel(R) Core(TM) i7-8700 CPU @ 3.20GHz. The time limits were set by 1,200 seconds. For the results where the time limit was reached, we regarded the computation time as equal to the time limit.

5.2 Evaluation Method

We evaluated the TSUC and ITSUC models in the context of real power system operation. Since the demand scenarios considered in the models may not be realized in the real world, it is not the amount of generation for those scenarios but the on/off status or interval that should be evaluated. Here we briefly illustrate the evaluation process for the case of the ITSUC model. First, we solve the model and fix the interval solution. Next, for each period starting t from 1 to T, the demand is realized and the optimal amount of generation for each generator is determined inside the interval. It can be easily found by the method demonstrated in Chapter 4.2. A similar evaluation procedure was performed for the TSUC model, where only the first-stage solution was changed from interval to on/off status. For each model, this procedure was repeated for $10,000$ test scenarios, which were sampled separately from the scenarios sampled for solving the models.

5.3 Experiment Results

5.3.1 Performance Comparison of the Models

We conducted a comparative analysis between the proposed ITSUC model and the conventional TSUC model. Figures 5.1 and 5.3 show the comparison results according to the number of scenarios when $CV = 0.1$. We omitted the result for the case of 1,000 scenarios because the ITSUC model could not be solved within the time limit. Figures 5.1 and 5.2 show the results in terms of the operation costs. $AvgCost$ and $StdCost$ denote the average and standard deviation, respectively, of the total operation cost incurred in the repeated simulation of the real power system operation

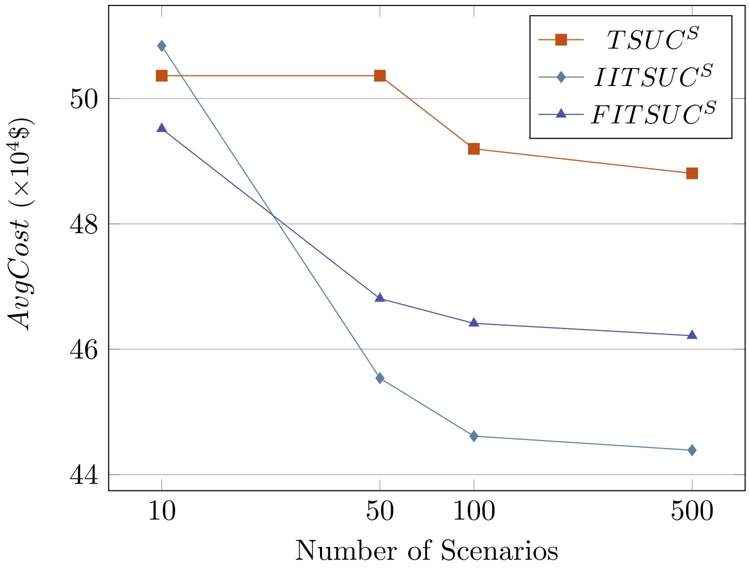


Figure 5.1: Operation Cost Comparison Between TSUC and ITSUC

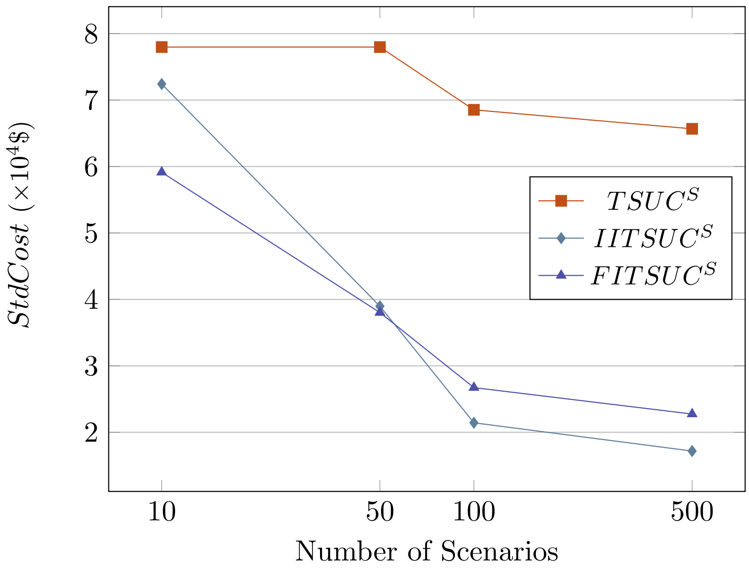


Figure 5.2: Standard Deviation Comparison Between TSUC and ITSUC

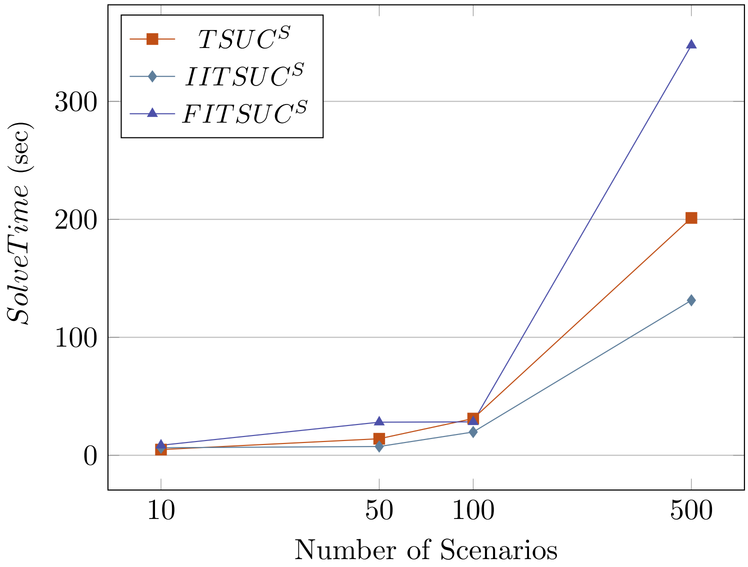


Figure 5.3: Computation Time Comparison Between TSUC and ITSUC

described in Chapter 5.2. Except for the case of 10 scenarios, the ITSUC models showed lower average operation costs than the TSUC model. In addition, the ITSUC model showed a less standard deviation of the operation costs than the TSUC model. It suggests that the proposed model might be a better choice than the conventional TSUC model for the purpose of efficient and reliable operation. Compared with the infinite interval design, the finite interval design performed better with 10 scenarios. It may be because demand uncertainty in the test phase cannot be considered properly with a small number of scenarios, which is more or less alleviated by the robust solution from the FITSUC model. $SolveTime$ in Figure 5.3 stands for the time for the MIP solver to solve the model. As the number of scenarios increases, the computation time to solve the model showed a tendency to increase. The FITSUC model showed a rapid increase in the computation because it introduces extra binary

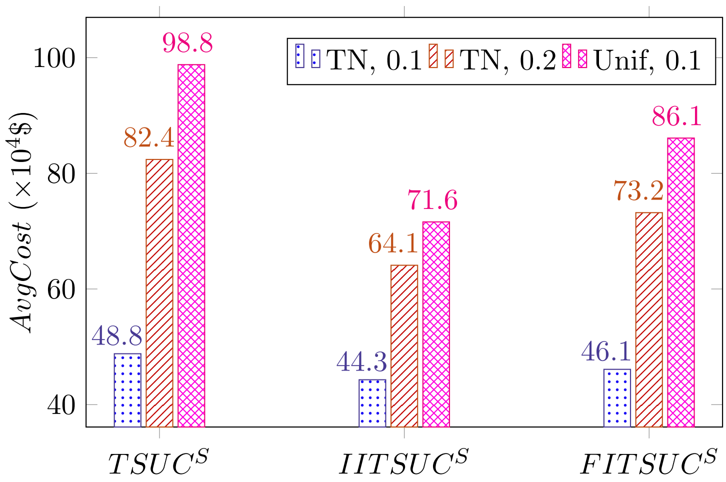


Figure 5.4: Operation Cost Under the Various Demand Distributions

variables for the interval decision. The IITSUC model showed computational benefit over the TSUC model when the number of scenarios become large. This tendency might be because the IITSUC model has a smaller feasible region than the TSUC model so the heuristic algorithm in the MIP solver worked more efficiently for the former than the latter. To conclude, the infinite interval design showed the most gentle increase in the computation time among the three models.

5.3.2 Robustness Test on the Different Demand Distributions

When operating the power system based on the solution from the UC model, the demand may not follow the probability distribution in the planning phase. In this case, the demand might not be completely satisfied in spite of the full utilization of the generators planned to be operated; hence, the total operation cost and the load-shedding cost may be significantly increased. To examine how well the schedule from each model can react to the unexpected demand realization, we controlled the

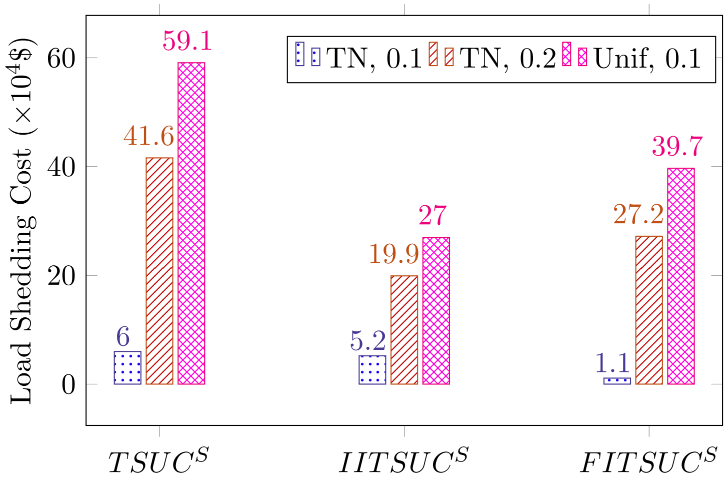


Figure 5.5: Load-shedding Cost Under the Various Demand Distributions

standard deviation by CV, and the type of demand distribution. Specifically, we used the demand scenarios sampling from the truncated normal distribution with $CV = 0.1$ (TN, 0.1) when solving the model, and tested on the different demand distributions such as the truncated normal distribution with $CV = 0.2$ (TN, 0.2), or the uniform distribution with $CV = 0.1$ (Unif, 0.1). Figures 5.4 and 5.5 show the average total operation cost and load-shedding cost, respectively, under the three types of distribution. One is the distribution that was expected when solving the model, while the other two are not. Both the operation cost and the load-shedding cost naturally turned out to be larger in the latter case than in the former case. In terms of the cost under the unexpected demand distribution, the ITSUC model could perform better than the TSUC model. In addition, the IITSUC model showed less operation and load-shedding cost than the FITSUC model. The performance comparison based on the rate of increase of the cost yields the same hierarchy of the models.

5.3.3 Efficiency of the Bounding Method

In Chapter 4.1, we introduced the bounding method for the ITSUC model and observed that the gap between the optimal objective value of the surrogate models can be closed as the number of sub-intervals increases. Here we report the efficiency of the bounding method compared with the SAA method. For a fair comparison, we evaluated the expected cost, denoted by $ObjVal^D$, corresponding to the interval solution obtained from the SAA method. It could be easily computed by applying the bounding method for the fixed interval solution since the resulting surrogate models are both linear programs. To construct the sub-intervals, we equally subdivided the range of generation. We controlled the number of sub-intervals m just as we did for the number of scenarios $|\mathcal{S}|$. Figures 5.6 and 5.7 show the cost and time comparison results between the SAA method and the bounding method. In Figure 5.6, the bounding method could obtain an optimal interval solution for $FITSUC^D$ within 0.1% optimality gap with only 50 sub-intervals. In contrast, the SAA method could reach that gap only with at least 500 scenarios. It suggests that the systematic method of tightening the expectation can be more effective than the random sampling from the demand distribution. In terms of the time to solve the models, both the scenario-based model from the SAA method and the surrogate model from the bounding method showed a similar trend in computation time as the scenarios, or the demand samples, increased. The reason for this trend is that the $|\mathcal{S}|$ and m have nearly the same effect on the size of the optimization problem.

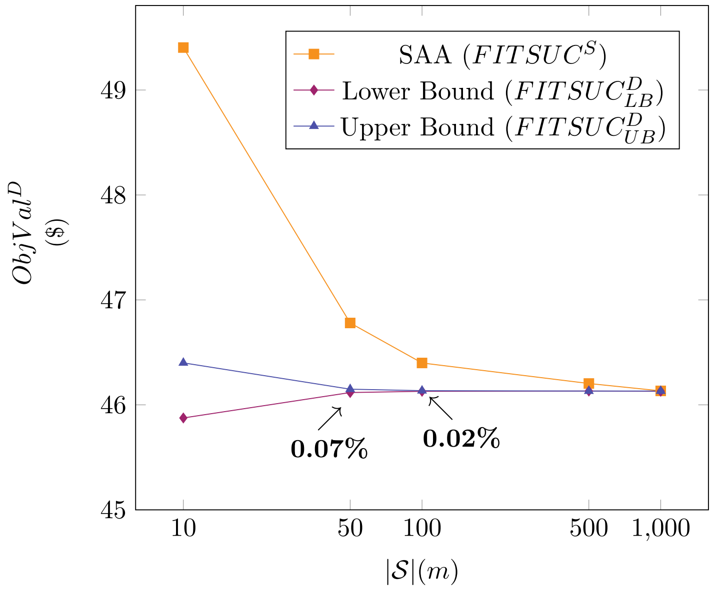


Figure 5.6: Expected Cost Comparison Between SAA and Bounding Method

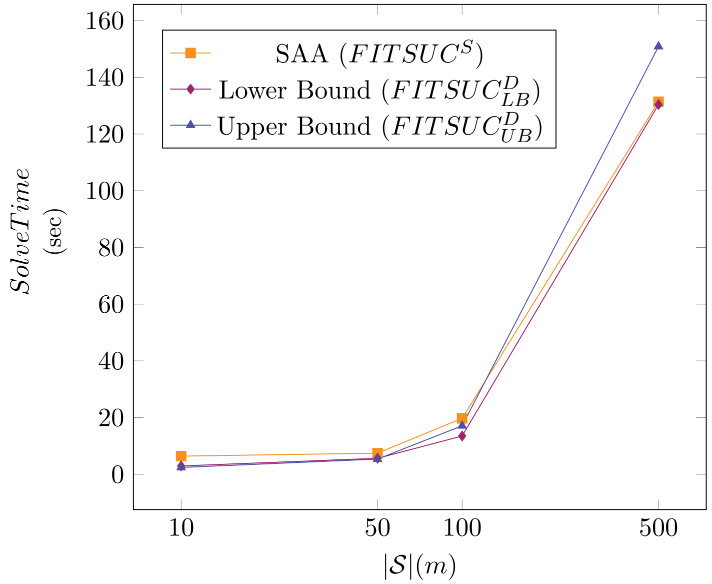


Figure 5.7: Computation Time Comparison Between SAA and Bounding Method

Table 5.1: Computation Time (sec) of Extensive Formulation

	$FITSUC^S$			$IITSUC^S$
	(β_s, β_l)			
$\|\mathcal{S}\|$	(0.25, 0.25)	(0.25, 0.5)	(0.5, 0.5)	∞
10	48.06	80.39	61.03	3.31
50	23.73	57.34	45.69	6.38
100	16.82	27.85	25.54	12.61
500	61.97	127.03	171.92	98.51
1000	278.70	504.78*	679.48*	219.00

*: Time limit was reached for $CV = 0.1$

Table 5.2: Computation Time (sec) of Benders Reformulation

	$FITSUC_{BR}^S$			$IITSUC_{BR}^S$
	(β_s, β_l)			
$\|\mathcal{S}\|$	(0.25, 0.25)	(0.25, 0.5)	(0.5, 0.5)	∞
10	102.25	423.86	95.90	4.10
50	27.67	79.53	38.05	6.31
100	29.92	25.77	32.05	11.91
500	86.76	98.68	128.17	60.35
1000	254.43	398.58	535.67**	104.38

**: Out of memory for $CV = 0.1$

5.3.4 Efficiency of the Benders Reformulation

We presented a compact Benders reformulation in Chapter 4.2. To investigate its potential computational gain, we compared between the extensive formulation and the Benders reformulation. The comparison results of computation time between the extensive formulation and the Benders reformulation are shown in Tables 5.1 and 5.2. Each number represents the average computation time for $CV \in \{0.1, 0.2, 0.3\}$. Even though neither of the two formulations could completely dominate the others under the finite interval design, $(FITSUC_{BR}^{S})$ showed its efficiency with a relatively large number of scenarios. For the IITSUC model, the computation time of $(IITSUC_{BR}^{S})$ increased more gently than $(IITSUC^{S})$ as $|\mathcal{S}|$ increases. It can be concluded that the impact of the choice between the two formulations on the computation time is more significant in the IITSUC model than in the FITSUC model.

5.3.5 Comparative Analysis of Various Interval Design Methods

In Chapter 3.1.1, we mentioned that the choice of the appropriate interval design is important for the FITSUC model. Here we present the experiment results for the various interval designs to test their effects on the performance of the model. We controlled the parameters β_s and β_l to specify the step size δ and the length ℓ in the overlapping uniform interval design. In addition, several levels of γ, the ramping limits control parameter, were used to test the impact of the ramping limits on the performance of the interval design. Figure 5.8 shows the average operation cost and the computation time, respectively, based on the interval decision with different values of β_l and β_s for the case of $|\mathcal{S}| = 500$. Among the tested finite interval designs, $(\beta_s, \beta_l) = (0.25, 0.25)$ was outperformed by the other two, both in terms of the operation cost and the computation time. It can be explained by the extensive restriction on the generation due to the numerous intervals, and consequently a large number of additional binary variables. For the other two designs, $(\beta_s, \beta_l) = (0.25, 0.5)$ showed slightly better cost than $(\beta_s, \beta_l) = (0.5, 0.5)$. It may be because the former allows the overlapping of the intervals, whereas the latter does not. Even more, the computation time of $(\beta_s, \beta_l) = (0.25, 0.5)$ was less than that of the $(\beta_s, \beta_l) = (0.5, 0.5)$.

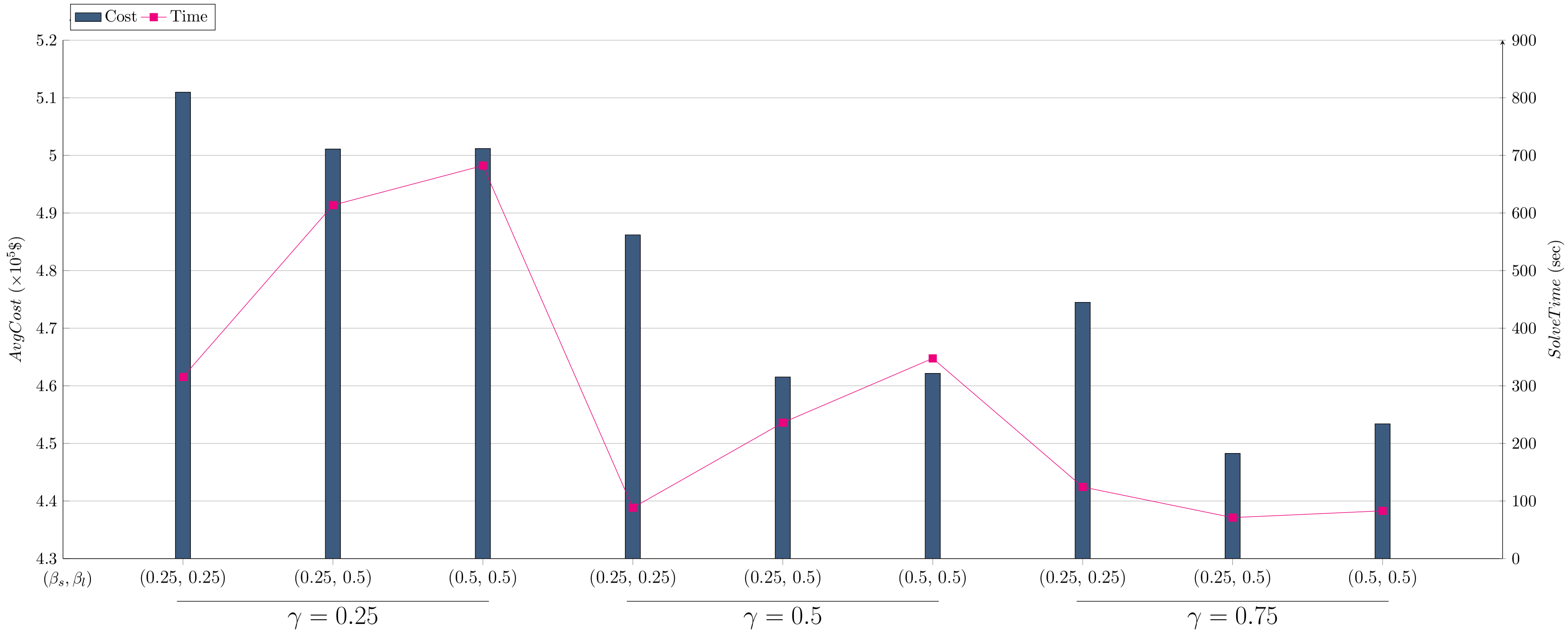


Figure 5.8: Comparison Between Various Interval Design Methods

This tendency was consistent regardless of the ramping limits. In conclusion, the interval design with $(\beta_s, \beta_l) = (0.25, 0.5)$ is discovered to be the best choice in the above experiment settings.

Chapter 6

Conclusion

In this thesis, we have proposed the ITSUC model to mitigate the drawbacks of the conventional TSUC model. In the model, an interval is chosen along with on/off decisions in the first stage. We suggested two interval selection schemes for the ITSUC model and their corresponding mathematical formulations. Since the interval decision already considers the ramping constraints in the ITSUC model, the second-stage problem can be decomposed for each period. We discovered that the bounds on the expected second-stage cost can be efficiently used to solve the ITSUC model. In addition, we used the period-wise decomposition property to derive a compact Benders reformulation in the ITSUC model. Through the computational experiments, the robustness of the solutions of the proposed model and the efficiency of the proposed solution approach are evaluated. The effect of interval design on computational performance is also analyzed. The applicability of our model can be further investigated in more practical settings. For example, the network structure of the power system can be additionally considered in the model when the transmission line has a certain level of flow limit. Also, our model focuses on the unit commitment problem with thermal generators, but other types of generators such as hydro generators or water pump generators, and their own operational constraints can be considered in the model. Lastly, the model will describe the real-world cost structure more precisely if the piece-wise linear or quadratic generation cost function is assumed in the model. In an algorithmic view, the efficient Benders decomposition algorithm in the branch-and-bound framework can be applied and compared with the proposed model. In addition, valid inequalities related with the interval decision can be developed to strengthen the linear programming relaxation bound. In terms of the application of our model, a hybrid model where the interval decision can be different for each group of scenarios could be devised.

Bibliography

[1] K.-S. Lee and S.-H. Song, "An explicit column generation algorithm for the profit based unit commitment problem in electric power industry," *IE interfaces*, vol. 20, no. 2, pp. 186–194, 2007.

[2] A. Ahmed and M. Khalid, "A review on the selected applications of forecasting models in renewable power systems," *Renewable and Sustainable Energy Reviews*, vol. 100, pp. 9–21, 2019.

[3] J. A. Momoh, R. Adapa, and M. El-Hawary, "A review of selected optimal power flow literature to 1993. i. nonlinear and quadratic programming approaches," *IEEE Transactions on Power Systems*, vol. 14, no. 1, pp. 96–104, 1999.

[4] I. Ghalehkhondabi, E. Ardjmand, G. R. Weckman, and W. A. Young, "An overview of energy demand forecasting methods published in 2005–2015," *Energy Systems*, vol. 8, no. 2, pp. 411–447, 2017.

[5] C.-L. Tseng, *On power system generation unit commitment problems.* University of California, Berkeley, 1996.

[6] P. Bendotti, P. Fouilhoux, and C. Rottner, "On the complexity of the unit commitment problem," *Annals of Operations Research*, vol. 274, no. 1, pp. 119–130, 2019.

[7] N. P. Padhy, "Unit commitment-a bibliographical survey," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 1196–1205, 2004.

[8] J. Ostrowski, M. F. Anjos, and A. Vannelli, "Tight mixed integer linear programming formulations for the unit commitment problem," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 39–46, 2011.

[9] J. Lee, J. Leung, and F. Margot, "Min-up/min-down polytopes," *Discrete Optimization*, vol. 1, no. 1, pp. 77–85, 2004.

[10] A. Frangioni and C. Gentile, "Perspective cuts for a class of convex 0–1 mixed integer programs," *Mathematical Programming*, vol. 106, no. 2, pp. 225–236, 2006.

[11] J. R. Birge and F. Louveaux, *Introduction to stochastic programming.* Springer Science & Business Media, 2011.

[12] C. Zhao, J. Wang, J.-P. Watson, and Y. Guan, "Multi-stage robust unit commitment considering wind and demand response uncertainties," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2708–2717, 2013.

[13] Y. Wang, Q. Xia, and C. Kang, "Unit commitment with volatile node injections by using interval optimization," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1705–1713, 2011.

[14] H. Pandžić, Y. Dvorkin, T. Qiu, Y. Wang, and D. S. Kirschen, "Toward cost-efficient and reliable unit commitment under uncertainty," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 970–982, 2015.

[15] Á. Lorca, X. A. Sun, E. Litvinov, and T. Zheng, "Multistage adaptive robust optimization for the unit commitment problem," *Operations Research*, vol. 64, no. 1, pp. 32–51, 2016.

[16] P. Xiong, P. Jirutitijaroen, and C. Singh, "A distributionally robust optimization model for unit commitment considering uncertain wind power generation," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 39–49, 2016.

[17] Y. Cho, T. Ishizaki, N. Ramdani, and J.-i. Imura, "Box-based temporal decomposition of multi-period economic dispatch for two-stage robust unit commitment," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 3109–3118, 2019.

[18] A. J. Kleywegt, A. Shapiro, and T. Homem-de Mello, "The sample average approximation method for stochastic discrete optimization," *SIAM Journal on Optimization*, vol. 12, no. 2, pp. 479–502, 2002.

[19] P. Carpentier, G. Gohen, J.-C. Culioli, and A. Renaud, "Stochastic optimization of unit commitment: a new decomposition framework," *IEEE Transactions on Power Systems*, vol. 11, no. 2, pp. 1067–1073, 1996.

[20] A. Papavasiliou and S. S. Oren, "Multiarea stochastic unit commitment for high wind penetration in a transmission constrained network," *Operations Research*, vol. 61, no. 3, pp. 578–592, 2013.

[21] S. Takriti, J. R. Birge, and E. Long, "A stochastic model for the unit commitment problem," *IEEE Transactions on Power Systems*, vol. 11, no. 3, pp. 1497–1508, 1996.

[22] M. P. Nowak and W. Römisch, “Stochastic lagrangian relaxation applied to power scheduling in a hydro-thermal system under uncertainty,” *Annals of Operations Research*, vol. 100, no. 1, pp. 251–272, 2000.

[23] Q. P. Zheng, J. Wang, P. M. Pardalos, and Y. Guan, “A decomposition approach to the two-stage stochastic unit commitment problem,” *Annals of Operations Research*, vol. 210, no. 1, pp. 387–410, 2013.

[24] Y. Huang, Q. P. Zheng, and J. Wang, “Two-stage stochastic unit commitment model including non-generation resources with conditional value-at-risk constraints,” *Electric Power Systems Research*, vol. 116, pp. 427–438, 2014.

[25] J. L. W. V. Jensen, “Sur les fonctions convexes et les inégalités entre les valeurs moyennes,” *Acta mathematica*, vol. 30, no. 1, pp. 175–193, 1906.

[26] A. Madansky, “Bounds on the expectation of a convex function of a multivariate random variable,” *The Annals of Mathematical Statistics*, vol. 30, no. 3, pp. 743–746, 1959.

[27] C. Huang, W. T. Ziemba, and A. Ben-Tal, “Bounds on the expectation of a convex function of a random variable: With applications to stochastic programming,” *Operations Research*, vol. 25, no. 2, pp. 315–325, 1977.

[28] J. R. Birge, “Using sequential approximations in the l-shaped and generalized programming algorithms for stochastic linear programs,” tech. rep., 1983.

[29] J. R. Birge and R. J.-B. Wets, “Sublinear upper bounds for stochastic programs with recourse,” *Mathematical Programming*, vol. 43, no. 1, pp. 131–149, 1989.

[30] K. J. Cormican, D. P. Morton, and R. K. Wood, "Stochastic network interdiction," *Operations Research*, vol. 46, no. 2, pp. 184–197, 1998.

[31] B. Denton and D. Gupta, "A sequential bounding approach for optimal appointment scheduling," *IIE transactions*, vol. 35, no. 11, pp. 1003–1016, 2003.

[32] B. Sandıkçı, N. Kong, and A. J. Schaefer, "A hierarchy of bounds for stochastic mixed-integer programs," *Mathematical Programming*, vol. 138, no. 1, pp. 253–272, 2013.

[33] F. Maggioni, E. Allevi, and M. Bertocchi, "Monotonic bounds in multistage mixed-integer stochastic programming," *Computational Management Science*, vol. 13, no. 3, pp. 423–457, 2016.

[34] B. Sandıkçı and O. Y. Özaltin, "A scalable bounding method for multistage stochastic programs," *SIAM Journal on Optimization*, vol. 27, no. 3, pp. 1772–1800, 2017.

[35] I. Bakir, N. Boland, B. Dandurand, and A. Erera, "Sampling scenario set partition dual bounds for multistage stochastic programs," *INFORMS Journal on Computing*, vol. 32, no. 1, pp. 145–163, 2020.

[36] D. Bertsimas and J. N. Tsitsiklis, *Introduction to linear optimization*, vol. 6. Athena Scientific Belmont, MA, 1997.

[37] Q. P. Zheng, J. Wang, and A. L. Liu, "Stochastic optimization for unit commitment—a review," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 1913–1924, 2014.

[38] H. Kellerer, U. Pferschy, and D. Pisinger, "Multidimensional knapsack problems," in *Knapsack problems*, pp. 235–283, Springer, 2004.

[39] S. A. Kazarlis, A. Bakirtzis, and V. Petridis, "A genetic algorithm solution to the unit commitment problem," *IEEE Transactions on Power Systems*, vol. 11, no. 1, pp. 83–92, 1996.

[40] Y. An and B. Zeng, "Exploring the modeling capacity of two-stage robust optimization: Variants of robust unit commitment model," *IEEE transactions on Power Systems*, vol. 30, no. 1, pp. 109–122, 2014.

[41] J. F. Benders, "Partitioning procedures for solving mixed-variables programming problems," *Numerische mathematik*, vol. 4, no. 1, pp. 238–252, 1962.

[42] N. Dupin, "Tighter mip formulations for the discretised unit commitment problem with min-stop ramping constraints," *EURO Journal on Computational Optimization*, vol. 5, no. 1-2, pp. 149–176, 2017.

[43] A. Shapiro, D. Dentcheva, and A. Ruszczynski, *Lectures on stochastic programming: modeling and theory.* SIAM, 2021.

[44] J. H. Stock, M. W. Watson, *et al.*, *Introduction to econometrics*, vol. 104. Addison Wesley Boston, 2003.

[45] N. Zhang, C. Kang, Q. Xia, and J. Liang, "Modeling conditional forecast error for wind power in generation scheduling," *IEEE Transactions on Power Systems*, vol. 29, no. 3, pp. 1316–1324, 2013.

[46] Z.-S. Zhang, Y.-Z. Sun, D. W. Gao, J. Lin, and L. Cheng, "A versatile probability distribution model for wind power forecast errors and its application

in economic dispatch," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 3114–3125, 2013.

국문초록

발전계획 문제는 전력 수요와 운영상의 제약을 만족하면서 전체 운영 비용을 최소로 하는 발전기별 운전상태를 찾는 것을 목표로 한다. 수요의 불확실성에 효율적으로 대처하기 위해, 1단계에서 운전상태를 결정하고 2단계에서 발전량을 결정하는 2단계 추계적 최적화 모형이 문헌에서 널리 사용되어 왔으나, 수요 시나리오의 수가 증가함에 따라 과도한 계산 부담을 겪는 경우가 많다. 본 논문에서는 이러한 단점을 보완하기 위해 수요가 시점별로 독립이라는 가정 하에서 발전구간 기반 2단계 추계적 최적화 모형을 제안한다. 해당 모형에서는 각 시점별로 발전기의 운전 여부뿐만 아니라 발전구간, 즉 발전량의 범위 또한 함께 결정된다. 이는 2단계 문제가 시점별로 분해될 수 있도록 하고, 많은 수의 시나리오가 필요하지 않게끔 해준다. 또한, 부문제의 성질을 이용하여 압축된 벤더스 모형을 제안한다. 마지막으로, 제안한 모형에 대해 기대비용의 한계치를 구할 수 있음을 보인다. 수치적 실험을 수행하여 제안한 모형의 효과성과 효율성을 입증하였다.

주요어: 발전계획, 수요의 불확실성, 2단계 추계적 최적화 모형, 발전구간, 벤더스 모형, 기대비용의 상·하한

학번: 2021-23873