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Exploring Existing Auction Formats For Obviously Strategy-Proofness

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Seoul National University

Dahae Jung

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Jinwoo Kim

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Graduate School of Economics

Seoul National University

Economics Major

Dahae Jung

Confirming the master's thesis written by

Dahae Jung

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Chair	____________ (인)
Vice Chair	____________ (인)
Examiner	____________ (인)

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Dahae Jung
Department of Economics
The Graduate School
Seoul National University

This research paper aims to investigate various existing auction formats, with a particular focus on evaluating their adherence to the notion of obviously strategy-proofness. The study evaluates the following auction formats in a multiple-object and private-values setting: the simultaneous ascending auction, Ausubel's ascending price clinching auction, and Mishra and Parkes's descending price clinching auction models. The analysis of this research is conducted in three parts: first, we determine whether the aforementioned auction formats are strategy-proof, and second, if it is strategy-proof, we move on to analyzing its obviously strategy-proofness. Lastly, if the auction is not obviously strategy-proof, we explore some of the restricted environments in which bidders have either unit demands or additive valuations to see if they can be obviously strategy-proof in the restricted environments. The results suggest that none of the auction formats mentioned above are obviously strategy-proof. The simultaneous ascending auction is not even strategy-proof. Ausubel's ascending price clinching auction and Mishra and Parkes's descending price clinching auction are strategy-proof. However, they fail to be obviously strategy-proof even in restricted environments.

Keywords: obviously strategy-proof, multiple-object, private values, clinching rule, simultaneous ascending auction, ascending price auction, descending price auction

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Introduction

Ausubel (2004) presents an effective solution to the challenge of designing an efficient ascending price auction for multiple objects. The author introduces the "clinching rule" in an ascending-bid auction, which encourages sincere bidding for all bidders. The sincere bidding by all bidders is an ex post perfect equilibrium[1] (Ausubel, 2004). Furthermore, when bidders exhibit diminishing marginal values, sincere bidding is a weakly dominant strategy for every bidder after every history under a certain information structure[2](Ausubel, 2004). However, it is important to note that even if Ausubel (2004) establishes the sincere bidding strategy as an ex post perfect equilibrium and a weakly dominant strategy, this does not automatically imply that it is an obviously strategy-proof mechanism. To clarify, a mechanism is obviously strategy-proof if there exists an extensive form game that is implemented through obviously dominant strategies by all participants. Since an obviously dominant strategy is a stronger concept than a dominant strategy, if there is no dominant strategy in an auction, the auction cannot be obviously strategy-proof. Additionally, if an equilibrium comprises the dominant strategies of all participants, it is also an ex post perfect equilibrium. However, the converse may not hold true. Consequently, when a sincere bidding strategy constitutes an ex post perfect equilibrium in an auction, it is necessary to first analyze whether it is also a dominant strategy for the bidders. If it is a dominant strategy, further analysis can be implemented to determine if it is an obviously dominant strategy. Only if all these criteria are satisfied can we conclude that the auction is obviously strategy-proof. In this paper, we apply this approach to three auction formats: simultaneous ascending auction, Ausubel's ascending price clinching auction, and Mishra and Parkes's descending price clinching auction. Our specific methodology involves initially examining whether sincere bidding is a dominant strategy. If it is, we then analyze whether it qualifies as an obviously dominant strategy. If it does, we conclude the auction is obviously strategy-proof. If it doesn't, we move on to exploring some restrictive environments where all bidders have either unit demands or additive valuations. This is to see if there exist some cases in which sincere bidding can become an obviously dominant strategy.

The identification of obviously strategy-proof mechanisms holds significant importance within auction theory, particularly when considering real-world scenarios where agents may have cognitive limitations. While strategy-proof mechanisms

[1]We cite Ausubel (2004) to define ex post perfect equilibrium as follows: the strategy n-tuple $\{\sigma_i\}_{i=1}^n$ is said to comprise an ex post perfect equilibrium if for every time t, following any history h_i^t, and for every realization $\{U_i\}_{i=1}^n$ of private information, the n-tuple of continuation strategies $\{\sigma_i(\cdot,\cdot|t,h_i^t,U_i)\}_{i=1}^n$ constitutes a Nash equilibrium of the game in which the realization of $\{U_i\}_{i=1}^n$ is common knowledge.

[2]It is *no bid information*, which is described in section 4.

are conventionally regarded as sufficient for achieving desirable outcomes, the presence of cognitive limitations among agents necessitates the exploration of obviously strategy-proof mechanisms. In fact, Shengwu Li's research in 2017 demonstrated that obviously strategy-proof mechanisms outperform their strategy-proof counterparts in attaining efficient outcomes within real-world contexts involving agents with cognitive limitations. This finding highlights the crucial role of identifying obviously strategy-proof mechanisms in bridging the gap between theoretical models and practical applications, as they offer a means to ensure the realization of efficient outcomes. Consequently, this paper aims to contribute to the existing body of knowledge by investigating which auction formats have the potential to deliver efficient outcomes in practical settings.

To begin with, the simultaneous ascending auction is a generalization of a static English auction for selling multiple goods (Peter Cramton, 2004). The simultaneous ascending auction is a dynamic auction in which multiple individual goods are sold simultaneously, and the auction progresses through multiple discrete rounds. The pricing rule of the simultaneous ascending auction is pay-as-bid. A sincere bidding strategy in simultaneous ascending auctions can be described as follows: when a bidder becomes the standing high bidder[3], they should stop bidding. On the other hand, when a bidder is not the standing high bidder, they should submit a bid slightly higher than the current standing high bid. The bidder will continue submitting new bids as long as they are not currently the standing high bidder, unless the price reaches their own valuation for the item. In general, simultaneous ascending auctions face an issue: the sincere bidding strategy is not an equilibrium strategy for bidders. Specifically, the auction is often subject to the problem of demand reduction or bid shading by bidders (Peter Cramton, 2004). That is, there are cases in which sincere bidding by bidders is suppressed. One typical example is as follows.

Suppose there are two licenses available in the auction, License A and License B, and two bidders participating, Bidder 1 and Bidder 2, with their valuations as shown in Table 1. Let $v(\cdot)$ represent the bidder's value function. Bidder 1 values each license at \$0 individually and \$90 for both licenses together, indicating that the two licenses are complements for Bidder 1. Bidder 2 values each license at \$50 individually and also values the combination of both licenses at \$50, indicating that the licenses are perfect substitutes for Bidder 2. Assuming that each bidder follows the sincere bidding strategy, which means they bid up to their own valuations whenever they are not the standing high bidder after each round, the auction will conclude with each bidder obtaining one license at a uniform price of \$45. However, it's important to note that Bidder 1 ends up with a payoff of -\$45, indicating that Bidder 1 has an incentive to deviate from the sincere bidding strategy. This situ-

[3]The standing high bidder is the one who submitted the highest price for the item.

ation is known as the exposure problem faced by Bidder 1. The exposure problem arises when some bidders have non-substitute preferences (Wellman et al., 2007). As a result, the simultaneous ascending auction fails to produce an efficient outcome in this case since sincere bidding is not an optimal strategy for Bidder 1.

Table 1: Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	0	0	90
Bidder 2	50	50	50

The previous example highlights the exposure problem that arises when a bidder has complementary preferences. Then, this prompts an important question: would bidders truthfully reveal their valuations if all bidders had substitute preferences instead? Regrettably, even when we confine the bidders' preferences to substitutes, employing a sincere bidding strategy does not lead to an equilibrium. Wellman et al. (2007) demonstrates that bidders have an incentive to reduce their demands even when the goods are substitutes for all bidders. Specifically, the paper establishes that when all bidders have weakly decreasing marginal utilities, the sincere bidding strategy is dominated by the demand reduction strategy. Consequently, it becomes evident that the simultaneous ascending auction fails to be a strategy-proof mechanism within a general preference setting. Thus, analyzing its status as an obviously strategy-proof mechanism in such a context is unnecessary. Instead, our focus shifts to examining some restricted environments where all bidders have either unit demands or additive valuations. We explore these environments because there are papers that demonstrate the sincere bidding strategy is a reasonable strategy in these restricted settings. First, Peters and Severinov (2006) showed that the sincere bidding strategy is a perfect Bayes-Nash equilibrium when all bidders have unit demands in a multi-unit auction environment. Second, Wellman et al. (2007) showed that the sincere bidding strategy is an equilibrium strategy when all bidders have additive valuations. Therefore, within these settings, we proceed to analyze whether sincere bidding emerges as a dominant strategy. If it does, we can move on to investigating whether it qualifies as an obviously dominant strategy. However, our findings suggest sincere bidding is not a dominant strategy even in these restrictive environments.

There is one more auction format that we explore in this paper. Motivated by Ausubel (2004), Mishra and Parkes (2007) applies the notion of "clinching" to a descending price auction and constructs the descending price clinching auction. Then, Mishra and Parkes (2007) shows that in the homogeneous items environment with non-increasing marginal values setting, the descending price clinching auction supports the sincere bidding strategy in an ex post Nash equilibrium. This implies that the auction is worth analyzing its obviously strategy-proofness. Hence we first investigate if sincere bidding is a dominant strategy within a general preference setting. If it is not, we then limit the environments to those where all bidders have either unit demands or additive valuations. Again, the restriction is imposed to see if the auction is obviously strategy-proof at least in the limited environments. Our findings suggest sincere bidding is a dominant strategy within a general preference setting. However, sincere bidding still fails to be an obviously dominant strategy even in restrictive environments.

Lastly, the paper is organized as follows. We first introduce the definition of an obviously strategy-proof mechanism. Then, in section 2, we introduce the common settings of all three auctions. In section 3, we investigate the simultaneous ascending auction. We introduce its specific auction rule first and show that sincere bidding is not a dominant strategy even in restrictive environments via counterexamples. In section 4, we explore Ausubel's ascending price clinching auction. We first introduce its auction rule. Then, we analyze its obviously strategy-proofness in various environments. By constructing various counterexamples, we demonstrate sincere bidding is not an obviously dominant strategy. Section 5 treats Mishra and Parkes's descending price clinching auction in the same manner. Lastly, section 6 concludes.

1.1 Background Information

1.1.1 Obviously Strategy-Proof Mechanism

A mechanism is obviously strategy-proof if there exists an extensive form game with obviously dominant strategies. Following definitions and notations are taken from Shengwu Li (2017) to define the obviously strategy-proof mechanism.

Definition 1 (Earliest Points of Departure).[4] The earliest points of departure are the information sets $I_i \in \alpha(S_i, S_i')$ such that S_i and S_i' diverge for the first time. We say two strategies S_i and S_i' diverge at $h \in I_i$ if $S_i(h) \neq S_i'(h)$ and h can be reached in two profiles (S_i, S_{-i}) and (S_i', S_{-i}), for some S_{-i}.

Definition 2 (Obviously Dominant). Given G and θ_i, S_i^* is *obviously dominant* if $\forall S_i'$, $\forall I_i \in \alpha(S_i^*, S_i')$:

$$sup_{h\in I_i, S_{-i}, d_c} U_i^G(h, S_i', S_{-i}, d_c, \theta_i) \leq inf_{h\in I_i, S_{-i}, d_c} U_i^G(h, S_i^*, S_{-i}, d_c, \theta_i).$$

[4] A slight adaptation may have been made from Shengwu Li (2017) to make the definition simple but it still builds upon Shengwu Li (2017).

Table 2: Notations

Name	Notation
histories	H (h: representative element)
information set for agent i	$\mathcal{I}_i(I_i$: representative element)
type for agent i	θ_i
strategy for agent i	S_i
realization of chance moves	d_c
game form	G
earliest points of departure	$\alpha(S_i, S_i')$

2 The Model

There are three auction formats we handle in the paper. Specific auction rules differ but there are some common settings among the three auction formats. We illustrate the specific auction rules for each auction format in separate sections and here, we only define some common settings among the three auction formats.

A seller wishes to allocate m homogeneous licenses represented by a set $M = \{1, \cdots, m\}$. Additionally, there are n bidders in a set $N = \{1, \cdots, n\}$. These bidders have non-increasing marginal values. Also, the utility functions are quasi-linear. Hence, the utility function for bidder $i \in N$ on a bundle $S \subseteq M$ is determined by the expression $U_i(S, p) = v_i(S) - p$, where $v_i(S)$ is bidder i's value on a bundle S and p represents the payment for the bundle S. Lastly, bidders have private values. That is, each bidder knows only the value of the licenses to oneself and knowing other bidders' valuation doesn't affect one's own valuation.

3 Simultaneous Ascending Auction

3.1 The Auction Rule

The auction rule of the simultaneous ascending auction is illustrated in Cramton (1998) and Wellman et al. (2007) as follows.

In this auction format, multiple individual goods are sold simultaneously, and the auction progresses through multiple discrete rounds. At the beginning of each new round, the standing high bid, along with the current winner in each auction, is announced. In each round, eligible bidders have the opportunity to submit a serious bid for any good. An eligible bidder is one who remains active from round to round by either submitting serious bids or by being the high bidder on sufficient goods. A serious bid for a good means that it exceeds the standing high bid for that particular good by at least a specified minimum increment. At the end of each round, the highest bid received for each good becomes the new standing high bid for that good. If no bids are received, the previous standing high bid remains unchanged.

The auction follows an all-or-nothing stopping rule, which means that the auction ends when no serious bids are received for any good. The pricing rule implemented is pay-as-bid, where the standing high bidders win the goods and pay the amount they bid.

3.2 Theoretical Analysis of Strategy-Proofness

3.2.1 The Unit Demand Environment

First, we handle an extreme case of substitutability; all bidders have unit demand valuations. Surely, if all bidders play sincere bidding strategies, the outcome is efficient. However, the sincere bidding strategy is not a dominant strategy. Consider the following example.

Suppose there are two licenses, which are License A and B. Also, there are two bidders, who are Bidder 1 and 2 in the auction. Each bidder has unit demand so licenses are perfect substitutes for them. Table 3 shows the valuations of each bidder on licenses. Now, suppose Bidder 2 plays the following strategy: Bidder 2 submits (\$1, License A) and (\$2, License B) in the first period. If Bidder 1 submits her bid for License A in the following period, Bidder 2 engages in the competition with Bidder 1 thereafter by submitting a new bid up to his valuation for License A. However, if Bidder 1 submits her bid only for License B in the following period, Bidder 2 stops submitting a new bid. If Bidder 2 plays this strategy, it is not optimal for Bidder 1 to play a sincere bidding strategy. If Bidder 1 plays the sincere bidding strategy, she should submit (\$2, License A) in the following period. However, the payoff of the resulting outcome will be strictly smaller than submitting a bid of (\$3, License B). Thus, the sincere bidding strategy is not a dominant strategy in simultaneous ascending auctions under the unit demand environment.

Table 3: Unit Demand Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	10	10	10
Bidder 2	15	15	15

3.2.2 The Additive Valuation Environment

Next, we consider a case in which all bidders have additive valuations. Still, the sincere bidding strategy is not a dominant strategy.

Consider a scenario where there are two licenses in the auction and two bidders with additive valuations. Table 4 displays their respective valuations. Suppose Bidder 2 adopts a strategy of randomly selecting a license to bid on and continues bidding unless the expected total payment surpasses his combined valuation for both licenses. When Bidder 2 employs this strategy, the sincere bidding strategy becomes suboptimal for Bidder 1. In other words, Bidder 1 can benefit from deviating from the sincere bidding strategy. To illustrate this, suppose Bidder 1 is currently the highest bidder for License A at a price of \$3, while Bidder 2 holds the highest bid for License B at \$36 during a specific stage of the auction. If Bidder 1 believes that Bidder 2's combined valuation for both licenses is close to \$36, then choosing to bid on License A, even though she is already the highest bidder, could be a clever decision. By submitting a bid of \$4 for License A, Bidder 1 can potentially end the auction and secure License A for herself. Additionally, she can eliminate the possibility of Bidder 2 bidding on License A in the next period and driving the price above \$4 if Bidder 1 were to bid on License B in the current period. Consequently, Bidder 1 successfully obtains License A at a price of \$4, while also avoiding the risk of Bidder 2 increasing the price beyond \$4. Therefore, even in a scenario where all bidders have additive valuations, it is evident that the sincere bidding strategy is not a dominant strategy.

In conclusion, the simultaneous ascending auctions are not an obviously strategy-proof mechanism even in some restrictive environments.

Table 4: Additive Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	15	15	30
Bidder 2	20	20	40

4 Ausubel's Ascending Price Clinching Auction

4.1 The Auction Rule

The auction rule of Ausubel's ascending price clinching auction is illustrated in Ausubel (2004) as follows.

The informational structure in this auction is characterized as "no bid information." Under no bid information, bidders are only aware of whether the auction is still open or not. Bidder i receives information denoted as $h_i^t = 1$ if $\sum_{j=1}^{n} x_j^{t-1} > m$, indicating that the sum of bidding quantities from all bidders in the previous period exceeds the total quantity of goods. Otherwise, bidder i receives $h_i^t = 0$. Regarding the observable histories, let H_i^t represent the set of all possible histories observable to bidder i at period t. Under "no bid information" setting, the set H_i^t includes bidder i's own bidding histories as well as information about whether the auction is still open or closed. That is, the inclusion of bidding histories and the status of the open or closed auction constitutes the observable information available to bidder i

in H_i^t.

Now, we illustrate how the auction proceeds. At each period t, the price is announced as $p^t = t$. Then, each bidder $i \in N$ responds to the price by bidding a quantity $x_i^t \in M_i$, where $M_i = \{0, 1, 2, \ldots, \lambda_i\}$ and $0 < \lambda_i \leq m$. The quantity x_i^t is subject to two constraints. The first constraint is the monotone activity rule, which requires that $x_i^t \leq x_i^{t-1}$ for all $t = 1, \ldots, T$ and all $i = 1, \ldots, n$. The second constraint is $x_i^t \geq C_i^{t-1}$ for all $t = 1, \ldots, T$ and all $i = 1, \ldots, n$, meaning that the bidding quantity should not be smaller than one's prior cumulative clinches, denoted as C_i^{t-1}. The cumulative clinch C_i^t at period t is calculated as $C_i^t = \max\{0, m - \sum_{j \neq i} x_j^t\}$ for all $t = 1, \ldots, T$ and all $i = 1, \ldots, n$, and $C_i^L = x_i^*$, where m represents the total quantity of goods, x_j^t denotes the bidding quantity of bidder j at period t, L represents the last auction round and x_i^* is the final quantity assigned to bidder i. After each bidder submits its bid at period t, the quantity of goods assigned to bidder i is determined by the current clinches, denoted as c_i^t, where $c_i^t = C_i^t - C_i^{t-1}$ for all $t = 1, \ldots, T$, and $c_i^0 = C_i^0$ for all $i = 1, \ldots, n$. If the sum of bidding quantities exceeds the total quantity of goods, i.e., $\sum_{i=1}^n x_i^t > m$, the next period t is incremented, and the procedure repeats. When the sum of bidding quantities equals the total quantity of goods or when it reaches the designated ending period L, the auction terminates. The payment of each bidder is determined by the periods in which the goods were clinched by them. For example, if bidder i clinches two units of goods at period t and $t+1$, the payment would be $t + (t+1) = 2t + 1$.

Lastly, the "sincere bidding strategy" of bidder i is defined as bidding x_i^t at every period t and after every history $_i^t$, where

$$x_i^t = min\{x_i^{t-1}, max\{Q_i^t, C_i^{t-1}\}\}, \tag{1}$$

for all $t = 1, \ldots, T$ and $x_i^0 = Q_i^0$. This means at each time t, bidder i submits the quantity x_i^t, which is the minimum of two values: the previous bidding quantity x_i^{t-1} and the maximum between the sincere demand $Q_i(p^t)$ and the prior cumulative clinch C_i^{t-1}. The sincere demand $Q_i(p)$ of bidder i at price p is defined as $Q_i(p) \equiv \inf\{\arg\max_{x_i \in M_i}\{v_i(x_i) - px_i\}\}$, where $v_i(x_i)$ represents the value of bidder i for a bidding quantity x_i.[5]

4.2 Theoretical Analysis of Obviously Strategy-Proofness

4.2.1 Efficiency of Sincere Bidding Strategy

Suppose there are five licenses for the auction. Bidders have a taste for more than one license. There are five bidders with values in the relevant range and suppose

[5]All the notations, definitions, and strategies are taken from Ausubel (2004).

bidders can get at most three licenses. Bidders' valuations for licenses are given in Table 5.

Table 6 shows the quantities each bidder bids at each price. When bidders play sincere bidding strategies defined in (1), it is Bidder 1 who gets clinched for a license for the first time. When the price reaches \$65, it is $\Sigma_{j \neq 1} x_j^{65} = 4 < 5$. So Bidder 1 clinches one license (5 - 4 = 1) at period 65. When the price reaches \$70, it is $\Sigma_{j \neq 1} x_j^{70} = 3$ and $\Sigma_{j \neq 3} x_j^{70} = 4$. So Bidder 1 clinches one more license while Bidder 3 also clinches one license at period 70. The auction proceeds in this manner until the price reaches \$80. The excess demand becomes zero at period 80.

The outcome of the auction is in Table 7. Bidder 1 clinched three licenses at prices $\$65, \$70$, and \$80 each. Bidder 3 clinched two licenses at prices \$70 and \$80 each. So the payoff of Bidder 1 is $\$123 + \$113 + \$86 - \$65 - \$70 - \$80 = \$107$ while it is $\$130 + \$128 - \$70 - \$80 = \$108$ for Bidder 3.

The outcome is efficient in that licenses are assigned to bidders who value them the most. Also, Ausubel (2004) shows sincere bidding is an ex post perfect equilibrium and a weakly dominant strategy for every bidder after every history under no bid information. However, sincere bidding is not an obviously dominant strategy within a general preference setting.

Table 5: Valuations of Bidders

Marginal Value	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5
1st unit	123	70	130	80	55
2nd unit	113	10	128	65	34
3rd unit	86	8	49	13	21

Table 6: Sincere Bidding by Bidders

Period (t)	Price (\$)	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5	History (h_i^t)
1	1	3	3	3	3	3	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
55	55	3	1	2	2	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
65	65	3	1	2	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
70	70	3	0	2	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80	80	3	0	2	0	0	1

Table 7: Outcome of Sincere Bidding by Bidders

	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5
Units won	3	0	2	0	0
Payment ($)	215	0	150	0	0
Payoff ($)	107	0	108	0	0

4.2.2 Absence of Obviously Dominant Strategy

We show $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1}) \geq sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$ does not hold for all S_1' and all $I_1 \in \alpha(S_1^*, S_1')$. Note that S_1^* is the sincere bidding strategy of Bidder 1 and valuations of bidders are identical to those in Table 5.

1. $inf_{h \in I_1, S_{-1}} U(h, S_1^*, S_{-1})$

While Bidder 1 plays the sincere bidding strategy, all other bidders can play any strategy. Suppose Bidder 3 and 4 play insincere bidding strategies. Bidder 3 and 4's insincere bidding strategy is bidding one unit more than its sincere bid up to a certain time period[6]. The bidders' bids at each period are in Table 8. At period 80, it is $\Sigma_{j \neq i} x_j^{80} = 4$ for $i = 1, 3$. Hence, Bidder 1 and 3 clinch a unit of licenses each at a price \$80. At price \$86, it is $\Sigma_{j \neq 1} x_j^{86} = 3$. Thus, Bidder 1 clinches one more license at a price \$86. Also, it is $\Sigma_{j \neq 3} x_j^{86} = 2$. So Bidder 3 clinches two more licenses at a price &86. In conclusion, the payoff of Bidder 1 is \$123 + \$113 - \$80 - \$86 = \$70. Note that this doesn't necessarily have to be an infimum of Bidder 1's payoff. We just need to find a case in which Bidder 1 gets a strictly higher payoff than this. The outcome of the auction is in Table 9.

2. $sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$

Now, Bidder 1 plays an insincere bidding strategy and all the other bidders can play any strategy. Suppose Bidder 1 chooses to bid sincerely only up to a certain period and submits insincere bids afterward. In this case, assume Bidder 1 submits its sincere bids only up to period 65 and reduces the quantity by a unit afterward. Then, the earliest point of departure is when the price reaches \$65. Assume all other bidders but Bidder 1 bid sincerely. Then, the bids of each bidder are submitted as in Table 10. When period reaches 65, it is $\Sigma_{j \neq i} x_j^{65} = 4$ for $i = 1, 3$. So Bidder 1 and 3 clinch 1 unit of license each at \$65. At period 70, it is $\Sigma_{j \neq i} x_j^{70} = 3$ for $i = 1, 3$ and $\Sigma_{j \neq 4} x_j^{70} = 4$. So Bidder 1 and 3 clinch one more unit while Bidder 4 clinches its first unit at \$70. Then, the auction ends. The payoff of Bidder 1 is $\$123 + \$113 - \$65 - \$70 = \$101$. The outcome is in Table 11.

[6] Suppose this period is arbitrarily and exogenously determined by bidders.

We showed it is $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1}) \leq 70 < 101 \leq sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$. This relation holds for any number of bidders and licenses in the auction. Hence, even if the sincere bidding strategy is a dominant strategy under no bid information, it is not an obviously dominant strategy. In conclusion, Ausubel's ascending-bid auction is not obviously strategy-proof within a general preference setting.

Table 8: Bidders' Bid at Each Period

Period (t)	Price ($)	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5	History (h_i^t)
1	1	3	3	3	3	3	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
65	65	3	1	3	2	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
70	70	3	0	3	2	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80	80	3	0	3	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
86	86	2	0	3	0	0	1

Table 9: Outcome of Sincere Bidding by Bidder 1

	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5
Units won	2	0	3	0	0
Payment ($)	166	0	252	0	0
Payoff ($)	70	0	55	0	0

Table 10: Bidders' Bid at Each Period

Period (t)	Price ($)	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5	History (h_i^t)
1	1	3	3	3	3	3	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
55	55	3	1	2	2	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
65	65	2	1	2	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
70	70	2	0	2	1	0	1

Table 11: Outcome of Insincere Bidding by Bidder 1

	Bidder 1	Bidder 2	Bidder 3	Bidder 4	Bidder 5
Units won	2	0	2	1	0
Payment ($)	135	0	135	70	0
Payoff ($)	101	0	123	5	0

4.2.3 The Unit Demand Environment

Here, we explore the unit demand environment to see if we can make Ausubel's ascending-bid auction obviously strategy-proof. Assume we have two homogeneous licenses in the market and three bidders with unit demands. The bidders' valuations are in Table 12.

1. $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1})$

We first explore a case where the earliest point of departure occurs in period 1. Suppose Bidder 2 plays an insincere bidding strategy, in which he submits two quantities until period 14. When the price reaches \$15, the auction ends since there exists no excess demand. In period 1, it is $x_1^1 = 1$. So Bidder 2 clinches a unit of licenses $(2 - 1 = 1)$ at price \$1. In period 15, it is $x_2^{15} = 1$. So Bidder 1 clinches a license at price \$15. As a result, the outcome is that each bidder clinches one unit of licenses with Bidder 1 paying \$15 and Bidder 2 paying \$1. Then, the payoff of Bidder 1 is \$5. Again, this doesn't necessarily have to be an infimum of Bidder 1's payoff as long as we find a case in which Bidder 1 gets a strictly higher payoff than this case. The bids of bidders are in Table 13 and the auction's outcome is in Table 14.

2. $sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$

Suppose Bidder 1 bids two quantities in the first period and reduces to one in the subsequent period. If Bidder 2 bids sincerely, the auction ends in period 2. The bidders' bids are in Table 15. Then, we have $x_2^1 = 1$ and $x_1^2 = 1$. So Bidder 1 clinches a unit of licenses in period 1 while Bidder 2 clinches a unit of licenses in period 2. Hence, the payment of Bidder 1 is \$1, resulting in a payoff of \$19. Refer to Table 16 for the specific outcome.

In conclusion, it is $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1}) \leq 5 < 19 \leq sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$. The ascending price clinching auction still fails to be obviously strategy-proof under the unit demand environment.

Table 12: Unit Demand Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	20	20	20
Bidder 2	15	15	15

Table 13: Bidders' Bid at Each Period

Period (t)	Price ($)	Bidder 1	Bidder 2	History (h_i^t)
1	1	1	2	1
⋮	⋮	⋮	⋮	⋮
14	14	1	2	1
15	15	1	1	1

Table 14: Outcome of Sincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	1	1
Payment ($)	15	1
Payoff ($)	5	14

Table 15: Bidders' Bid at Each Period

Period (t)	Price ($)	Bidder 1	Bidder 2	History (h_i^t)
1	1	2	1	1
2	2	1	1	1

Table 16: Outcome of Insincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	1	1
Payment ($)	1	2
Payoff ($)	19	13

4.2.4 The Additive Valuation Environment

When bidders exhibit additive valuations for the goods, the sincere bidding strategy is not a dominant strategy at all. Suppose there are two licenses, License A and B, and two bidders, Bidder 1 and 2, with their valuations in Table 17.

1. $U_1(h, S_1^*, S_{-1})$

Suppose Bidder 2's strategy is as follows: Bidder 2 reports its quantity as one after it clinches at least one license, regardless of its true demand. Then, the sincere bidding strategy is not optimal for Bidder 1. When Bidder 1 plays the sincere bidding strategy, the outcome of the auction is that Bidder 1 clinches two licenses in period 15 since it is $x_2^{15} = 0$ at period 15. Note that Bidder 2 opts out of the auction when the price reaches $15 since its valuation for a license is $15. Then, the payment of Bidder 1 is $15 + $15 = $30. So the payoff of Bidder 1 is $40 - $30 = $10. Refer to Table 18 for the bidders' bid at each period and Table 19 for the outcome of the auction.

2. $U_1(h, S_1', S_{-1})$

Given Bidder 2's strategy, Bidder 1 can increase its payoff by bidding insincerely. Suppose Bidder 1 reduces its quantity by one at an early period such as at period 5. Then, Bidder 2 clinches a license at period 5 and reduces its quantity to one afterward. Then, the outcome is that Bidder 1 clinches a unit of license at a price of \$6 while Bidder 2 clinches a unit of licenses at a price of \$5. Then, the payoff of Bidder 1 is \$20 - \$6 = \$14, yielding a higher payoff than when she bids sincerely. Refer to Table 20 for the bidders' bid at each period and Table 21 for a specific outcome of the auction.

As a result, it is $U_1(h, S_1^*, S_{-1}) < U_1(h, S_1', S_{-1})$. The sincere bidding is not a dominant strategy for Bidder 1. In conclusion, the ascending price clinching auction is not strategy-proof when bidders have additive valuations.

Table 17: Additive Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	20	20	40
Bidder 2	15	15	30

Table 18: Bidders' Bid at Each Period

Period (t)	Price (\$)	Bidder 1	Bidder 2	History (h_i^t)
1	1	2	2	1
⋮	⋮	⋮	⋮	⋮
14	14	2	2	1
15	15	2	0	1

Table 19: Outcome of Sincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	2	0
Payment (\$)	30	0
Payoff (\$)	10	0

Table 20: Bidders' Bid at Each Period

Period (t)	Price (\$)	Bidder 1	Bidder 2	History (h_i^t)
1	1	2	2	1
⋮	⋮	⋮	⋮	⋮
5	5	1	2	1
6	6	1	1	1

Table 21: Outcome of Insincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	1	1
Payment (\$)	6	5
Payoff (\$)	14	10

5 Mishra and Parkes's Descending Price Clinching Auction

5.1 The Auction Rule

The auction rule of Mishra and Parkes's descending price clinching auction is illustrated in Mishra and Parkes (2004) as follows.

The auction operates with a non-linear and non-anonymous pricing mechanism. The information structure in this auction is also characterized as "no bid information". As defined earlier, there are n bidders in a set $N = \{1, \dots, n\}$ and m homogeneous licenses in a set $M = \{1, \dots, m\}$. Let $N_{-i} = N \setminus \{i\}$ be the set of bidders without bidder i. Let $B = \{N, N_{-1}, \dots, N_{-n}\}$.

Then, the descending price clinching auction proceeds as follows[7]:

1. Initialize prices as $p_i^0(j) = q^0 j$ for all $i \in N$ and for all licenses $j \leq m$, where q^0 is a large integer. Set $t := 0$.

2. In iteration t of the auction with price vector p^t:

   (a) Collect the demand sets $D_i(p^t)$ of all $i \in N$ at p^t.

   - Let $v_i(j)$ denote the value of bidder $i \in N$ for $j \in M$ units of the licenses.
   - The demand set $D_i(p^t)$ is defined as

   $$D_i(p^t) = \{j \in \{0, 1, \dots, m\} : v_i(j) - p_i^t(j) \geq max_{0 \leq j' \leq m}\{v_i(j') - p_i^t(j')\}\}$$

   (b) Based on the demand sets of bidders at p^t, calculate the under-demand $\alpha(L, p^t)$ for every $L \in B$.

   - The under-demand is defined as $\alpha(L, p^t) := max(0, m - \Sigma_{i \in L} \overline{D}_i(p^t))$ for $L \in B$.
   - $\overline{D}_i(p^t)$ is the maximal demand of bidder i at price vector p^t, defined as the maximum number of units demanded.

   (c) If $\alpha(L, p^t) = 0$ for every $L \in B$ or $q^t = 0$ then go to step 4. Else, $q^{t+1} := q^t - 1$ and for every $i \in N$, set

   $$p_i^{t+1}(j) := \begin{cases} p_i^t(j) & for \forall j \leq \overline{D}_i(p^t) \\ p_i^t(\overline{D}_i(p^t)) + q^{t+1}(j - \overline{D}_i(p^t)) & otherwise. \end{cases}$$

   Set $t := t + 1$ and repeat from step i.

[7] We cite Mishra and Parkes (2004) to illustrate the auction procedure.

- q^t is the ask price in iteration t on each marginal unit over and above the current number of licenses demanded by bidders.

3. The auction terminates in current iteration T with price vector p^T. Let y_i denote the quantity allocated to bidder $i \in N$. Then, the final allocation $y^T \in Y(D(p^T))$.
   - $Y(D(p^T))$ is the set of provisional allocations at price vector p^T.
   - A provisional allocation is an admissible allocation that maximizes the revenue $\Sigma_{i\in N} p_i^T(y_i^T)$ among all admissible allocations.
   - An admissible allocation y at a price vector p is an allocation such that $y \in D_i(p) \cup \{0\}$.
4. For each bidder $i \in N$ with $y_i^T > 0$, his payment is $p_i^T(y_i^T) - [\pi^s(p^T) - \pi^s(p_{-i}^T)]$, where $\pi^s(p^T) = \Sigma_{i\in N} p_i^T(y_i^T)$.[8]

5.2 Theoretical Analysis of Obviously Strategy-Proofness

5.2.1 Absence of Obviously Dominant Strategy

We show that sincere bidding is a dominant strategy through the proof of Proposition 1 in the Appendix. Here, we analyze whether sincere bidding is an obviously dominant strategy or not. The bidders' valuations are in Table 22. Again, the infimum and supremum we find here may not necessarily be the true infimum and supremum. However, it won't change our conclusion as long as we show a case in which Bidder 1 bidding sincerely results in a worse outcome than a case where Bidder 1 bids insincerely.

1. $inf_{h\in I_1, S_{-1}} U_1(h, S_1^*, S_{-1})$

Suppose Bidder 2 reports his demand for licenses in periods 85 and 86. Bidder 2 may choose to submit his demand above his true valuations since this doesn't necessarily lead to negative payoffs after he wins any license. It is because of the VCG payment rule. After the auction ends, Bidder 2 gets both licenses. So it is $\pi(B) = 171, \pi(B_{-1}) = 171, \pi(B_{-2}) = 155$. Thus, the payment for each bidder is $p_1 = 0 - (171 - 171) = 0$ and $p_2 = 171 - (171 - 155) = 155$. So Bidder 1 ends up with a payoff of 0. Table 23 illustrates how the auction proceeds and Table 24 is the final outcome of the auction. Note that the parentheses around prices for a bidder in Table 23 indicate the quantity of units in his demand set in each iteration.

[8] This is the VCG payment rule.

2. $sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$

Here, we find a case where the payoff of Bidder 1 is strictly higher than 0. Suppose Bidder 1 submits her demand for each unit at prices of \$85 and \$80, whereas Bidder 2 submits his demand for two units at a price of \$60. The auction will terminate with Bidder 1 getting both licenses. We get $\pi(B) = 165, \pi(B_{-1}) = 120, \pi(B_{-2}) = 165$. So the payment of each bidder is $p_1 = 165 - (165 - 120) = 120$ and $p_2 = 0 - (165 - 165) = 0$. So the payoff of Bidder 1 is $80 + 75 - 120 = 35$. The auction proceeds as in Table 25 and the final outcome of the auction is in Table 26.

Thus, it is $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1}) \leq 0 < 35 \leq sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$. Mishra and Parkes' descending price clinching auction fails to be obviously strategy-proof in a general preference setting.

Table 22: Valuations of Bidders

Marginal Value	Bidder 1	Bidder 2
1st unit	80	78
2nd unit	75	60

Table 23: Outcome After Each Iteration

		Bidder 1		Bidder 2		$\alpha(\cdot)$		
Iteration	q^t	1	2	1	2	N	N_{-1}	N_{-2}
1	91	91	182	91	182	2	2	2
2	90	90	180	90	180	2	2	2
3	89	89	178	89	178	2	2	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
7	85	85	170	(85)	170	1	1	2
8	86	86	172	(85)	(171)	0	0	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
12	80	(80)	160	(85)	(171)	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
17	75	(80)	(155)	(85)	(171)	0	0	0

Table 24: Final Outcome of Sincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	0	2
Payment (\$)	0	155
Payoff (\$)	0	−17

Table 25: Outcome After Each Iteration

Iteration	q^t	Bidder 1		Bidder 2		$\alpha(\cdot)$		
		1	2	1	2	N	N_{-1}	N_{-2}
1	91	91	182	91	182	2	2	2
2	90	90	180	90	180	2	2	2
3	89	89	178	89	178	2	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
7	85	(85)	170	85	170	1	2	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
12	80	(85)	(165)	80	160	0	2	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
32	60	(85)	(165)	(60)	(120)	0	0	0

Table 26: Final Outcome of Insincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	2	0
Payment ($)	120	0
Payoff ($)	35	0

5.2.2 The Unit Demand Environment

Now, we explore a restrictive environment where all bidders exhibit unit demands. The bidders' valuations are in Table 27.

1. $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1})$

Consider a case where Bidder 2 submits his demand for a license at prices of \$15 and \$11 each. Then, the auction proceeds as in Table 28. The auction will end in iteration 17 since it is $q^{17} = 0$. Then, the final allocation is that Bidder 2 gets two licenses. As a result, we have $\pi(B) = 26, \pi(B_{-1}) = 26, \pi(B_{-2}) = 10$. So the payment of each bidder is $p_1 = 0 - (26 - 26) = 0$ and $p_2 = 26 - (26 - 10) = 10$. So Bidder 1 will end up with a payoff of 0. Table 29 illustrates the final outcome of this auction.

2. $sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$

Again, it is not necessary to find the true supremum of Bidder 1's utility. We just need to find a case where Bidder 1's payoff is higher than 0. Suppose Bidder 1 submits her demand at a price of \$16 while Bidder 2 bids sincerely. Then, the auction proceeds as in Table 30. The auction ends in iteration 17 since $q^{17} = 0$. The outcome of the auction is both bidders getting a license each. As a result, it is $\pi(B) = 31, \pi(B_{-1}) = 15, \pi(B_{-2}) = 16$. So the payment of each bidder is $p_1 = 16 - (31 - 15) = 0$ and $p_2 = 15 - (31 - 16) = 0$. So the payoff of Bidder 1 is 10. Refer to Table 31 for the final outcome.

In conclusion, it is $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1}) \leq 0 < 10 \leq sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$. So the auction still fails to be obviously strategy-proof in this environment.

Table 27: Unit Demand Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	10	10	10
Bidder 2	15	15	15

Table 28: Outcome After Each Iteration

		Bidder 1		Bidder 2		$\alpha(\cdot)$		
Iteration	q^t	1	2	1	2	N	N_{-1}	N_{-2}
1	16	16	32	16	32	2	2	2
2	15	15	30	(15)	30	1	1	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	11	11	22	(15)	(26)	0	0	2
7	10	(10)	20	(15)	(26)	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
17	0	(10)	0	(15)	(26)	0	0	1

Table 29: Final Outcome of Sincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	0	2
Payment ($)	0	10
Payoff ($)	0	5

Table 30: Outcome After Each Iteration

		Bidder 1		Bidder 2		$\alpha(\cdot)$		
Iteration	q^t	1	2	1	2	N	N_{-1}	N_{-2}
1	16	(16)	32	16	32	1	2	1
2	15	(16)	31	(15)	30	0	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
17	0	(16)	0	(15)	0	0	1	1

Table 31: Final Outcome of Insincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	1	1
Payment ($)	0	0
Payoff ($)	10	15

5.2.3 The Additive Valuation Environment

Now, let's consider a case where all bidders have additive valuations. Suppose there are two licenses in the auction and two bidders with their valuations in Table 32.

1. $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1})$

Suppose Bidder 2 submits his demand for licenses at prices of \$15 and \$14. Then, the auction ends in iteration 7 and the outcome is that Bidder 2 gets both licenses. Refer to Table 33 for the outcome after each iteration. We get $\pi(B) = 29, \pi(B_{-1}) = 29, \pi(B_{-2}) = 20$. So the payment of each bidder is $p_1 = 0-(29-29) = 0$ and $p_2 = 29 - (29 - 20) = 20$. Then, the payoff of Bidder 1 is 0. Refer to Table 34 for the final outcome of the auction.

2. $sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$

Now, Bidder 1 and 2 both bid insincerely. Suppose Bidder 1 submits its demands at prices \$15 and \$8 while Bidder 2 submits its demands at prices \$15 and \$9. The outcome after each iteration is in Table 35. Then, the auction ends in iteration 9. Then, the auctioneer will allocate a license to each bidder. It is $\pi(B) = 30, \pi(B_{-1}) = 24, \pi(B_{-2}) = 23$. So the payment of each bidder is $p_1 = 15 - (30 - 24) = 9$ and $p_2 = 15 - (30 - 23) = 8$. As a result, the payoff of Bidder 1 is 1, which is higher than 0. Refer to Table 36 for the final outcome of the auction.

As a result, it is $inf_{h \in I_1, S_{-1}} U_1(h, S_1^*, S_{-1}) \leq 0 < 1 \leq sup_{h \in I_1, S_{-1}} U_1(h, S_1', S_{-1})$. Again, even if we restrict the environment to which all bidders have additive valuations, the auction still fails to be obviously strategy-proof.

Table 32: Additive Valuations on License A and B

	v({A})	v({B})	v({A,B})
Bidder 1	10	10	20
Bidder 2	15	15	30

Table 33: Outcome After Each Iteration

		Bidder 1		Bidder 2			$\alpha(\cdot)$	
Iteration	q^t	1	2	1	2	N	N_{-1}	N_{-2}
1	16	16	32	16	32	2	2	2
2	15	15	30	(15)	30	1	1	2
3	14	14	28	(15)	(29)	0	0	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
7	10	(10)	(20)	(15)	(29)	0	0	0

Table 34: Final Outcome of Sincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	0	2
Payment (\$)	0	20
Payoff (\$)	0	10

Table 35: Outcome After Each Iteration

Iteration	q^t	Bidder 1		Bidder 2		$\alpha(\cdot)$		
		1	2	1	2	N	N_{-1}	N_{-2}
1	16	16	32	16	32	2	2	2
2	15	(15)	30	(15)	30	1	1	1
3	14	(15)	29	(15)	29	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	9	(15)	24	(15)	(24)	0	0	1
9	8	(15)	(23)	(15)	(24)	0	0	0

Table 36: Final Outcome of Insincere Bidding by Bidder 1

	Bidder 1	Bidder 2
Units won	1	1
Payment ($)	9	8
Payoff ($)	1	7

6 Conclusion

This paper examined the concept of obviously strategy-proof mechanism in the context of auction theory. The identification of such mechanisms is crucial, especially when considering real-world scenarios where agents may have cognitive limitations. Shengwu Li (2017) demonstrates that obviously strategy-proof mechanisms outperform their strategy-proof counterparts in real-world contexts involving agents with cognitive limitations, highlighting their importance in bridging the gap between theoretical models and practical applications.

In this paper, we applied an analytical approach to investigate the potential of three auction formats to deliver efficient outcomes in practical settings: the simultaneous ascending auction, Ausubel's ascending price clinching auction, and Mishra and Parkes's descending price clinching auction. We first examined whether the sincere bidding strategy is dominant in each auction format. Subsequently, we analyzed whether it qualifies as an obviously dominant strategy, which would indicate that the auction is obviously strategy-proof. Unfortunately, our findings indicate that none of the auction formats mentioned above can be considered obviously strategy-proof in general preference settings. The simultaneous ascending auction lacked a dominant strategy equilibrium altogether, rendering sincere bidding ineffective. In the case of Ausubel's ascending price clinching auction and Mishra and Parkes's descending price clinching auction, the sincere bidding strategy emerged as a dominant strategy. However, we were able to demonstrate through counterexamples that they do not meet the criteria for obviously strategy-proofness. These results remained consistent even when considering restrictive environments where all bidders have either unit demands or additive valuations.

The implications of these findings are significant. The absence of obviously strategy-proof mechanisms in these auction formats highlights the challenges in ensuring fair and efficient outcomes, particularly in real-world scenarios where cognitive limitations among agents exist. Although strategy-proof mechanisms have traditionally been considered sufficient, the presence of cognitive limitations necessitates the exploration of obviously strategy-proof mechanisms. While our analysis did not identify obviously strategy-proof mechanisms among the examined auction formats, this research contributes to the existing body of knowledge by shedding light on their limitations and providing insights into their potential shortcomings in practical settings. This paper will help inform future auction design decisions and emphasize the necessity to explore alternative auction formats or mechanisms that may offer greater potential for delivering obviously strategy-proofness and desirable outcomes.

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8 Appendix

8.1 Dominance of Sincere Bidding Strategy in Mishra and Parkes's Descending Price Clinching Auction

Proposition 1. *All bidders have a dominant strategy to announce their true valuations in Mishra and Parkes's descending price clinching auction.*

Setting

- Assumption 1: Bidders exhibit non-increasing marginal utilities.
- Assumption 2: There are two bidders and two homogeneous licenses in the auction.
  - This assumption is included only to make the analysis simple. Proposition 1 holds for any number of bidders and licenses in the auction.
- Assumption 3: Bidder 1 has unit demand.
  - This assumption is included only to make the analysis simple. Proposition 1 still holds even if Bidder 1 has multi-unit demand.
- Let v denote Bidder 1's true marginal valuation for a license she wins, if any.
- Let s denote the marginal price at which Bidder 1 reports her demand for a license.
- Let b_1^*, b_2^* each denote the marginal prices at which the other bidders report their demand for license(s).

Proof

We need to show that reporting one's demand at a marginal price v weakly dominates reporting at a marginal price s for any $s \neq v$. If so, reporting one's demand only at a marginal price v is a dominant strategy for Bidder 1.

1. **Case 1**: $s < v$
   (a) If $s > b_1^* > b_2^*$, then reporting a demand for a license at a price of either v or s will both result in Bidder 1 winning a license and payment is b_2^* in both cases.
   (b) If $v > b_1^* > s > b_2^*$, then reporting a demand for a license at a price of either v or s will both result in Bidder 1 winning a license and payment is b_2^* in both cases.

(c) If $v > b_1^* > b_2^* > s$, Bidder 1's payoff is 0. So Bidder 1 would be better off reporting her demand for a license at a price v, in which case her payoff is $v - b_2^* > 0$.

(d) If $b_1^* > v > s > b_2^*$, then reporting a demand for a license at a price of either v or s will both result in Bidder 1 winning a license and payment is b_2^* in both cases.

(e) If $b_1^* > v > b_2^* > s$, Bidder 1's payoff is 0. So Bidder 1 would be better off reporting her demand for a license at a price v, in which case her payoff is $v - b_2^* > 0$.

(f) If $b_1^* > b_2^* > v$, then reporting a demand for a license at a price of either v or s both result in a payoff of 0.

In conclusion, reporting a demand at a price of v is never worse and sometimes better than reporting a demand at a price of s. Hence, bidding v weakly dominates bidding s.

2. **Case 2**: $s > v$

(a) If $v > b_1^* > b_2^*$, then reporting a demand for a license at a price of either v or s will both result in Bidder 1 winning a license and payment is b_2^* in both cases.

(b) If $s > b_1^* > v > b_2^*$, then reporting a demand for a license at a price of either v or s will both result in Bidder 1 winning a license and payment is b_2^* in both cases.

(c) If $s > b_1^* > b_2^* > v$, Bidder 1's payoff is $v - b_2^* < 0$. So Bidder 1 would be better off reporting her demand for a license at a price v, in which case her payoff is 0.

(d) If $b_1^* > s > v > b_2^*$, then reporting a demand for a license at a price of either v or s will both result in Bidder 1 winning a license and payment is b_2^* in both cases.

(e) If $b_1^* > s > b_2^* > v$, Bidder 1's payoff is $v - b_2^* < 0$. So Bidder 1 would be better off reporting her demand for a license at a price v, in which case her payoff is 0.

(f) If $b_1^* > b_2^* > s$, then reporting a demand for a license at a price of either v or s both result in a payoff of 0.

In conclusion, reporting a demand at a price of v is still never worse and sometimes better than reporting a demand at a price of s. Hence, bidding v weakly dominates bidding s.

Since this is true for all $v \neq s$, reporting a demand for a license at a price v is a weakly dominant strategy. Note that this conclusion is robust to the number of licenses and bidders. Also, the conclusion still holds even if Bidder 1 has multi-unit demand.

$\blacksquare$

국문초록

이 논문은 기존에 존재하는 다양한 경매 방법들 중에서 명백 전략 증명 메커니즘이 존재하는지 분석하는것을 목적으로 한다. 해당 논문은 다품목, 개인가치 경매 중에서 동시오름경매, Ausubel의 오름가격 클린칭 경매, Mishra and Parkes의 내림가격 클린칭 경매를 분석한다. 분석은 크게 세 가지로 구성된다. 첫째, 앞서 언급된 경매들이 전략 증명 메커니즘인지 분석한다. 둘째, 전략 증명 메커니즘을 만족하는 경우 명백 전략 증명 메커니즘이 될 수 있는지 분석한다. 셋째, 명백 전략 증명 메커니즘을 만족하지 않는 경우 입찰자들의 수요에 제약 조건을 부과한 후 추가적인 분석을 진행한다. 이와 관련하여 두 가지 경우를 추가적으로 분석한다. 첫 번째는 모든 입찰자들이 물품을 한 단위만 수요하는 경우이다. 두 번째는 모든 입찰자들이 물품을 한 단위 갖는 것과 다수의 단위를 갖는 것이 무차별한 경우이다. 각 경우에서 앞서 언급한 경매들이 명백 전략 증명 메커니즘이 될 수 있는지 분석한다. 분석 결과 위 경매들은 모두 명백 전략 증명 메커니즘이 되지 않는 것으로 나타났다. 동시오름경매의 경우 전략 증명 메커니즘도 되지 않는 것으로 나타났다. Ausubel의 오름가격 클린칭 경매와 Mishra and Parkes의 내림가격 클린칭 경매는 전략 증명 메커니즘은 될 수 있지만 입찰자들의 수요에 제약조건을 부과해도 명백 전략 증명 메커니즘은 되지 않는 것으로 나타났다.

주요어: 명백 전략 증거, 다품목, 개인가치, 클린칭 규칙, 동시오름경매, 오름경매, 내림경매

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