Differential time and money pricing as a mechanism for in-kind redistribution

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We propose that differential pricing can be used to implement the distributional goal of "specific egalitarianism", or that allocation of a good be independent of income, but increasing in relative strength of preference or need. Governments could provide the good at multiple "outlets" offering different money and time prices. Individuals would self-select based on time opportunity cost. We show that differential pricing achieves specific egalitarianism more efficiently than tax-funded uniform public provision as the 1) relative importance of the good rises, 2) elasticity of substitution between goods falls, 3) variation in preferences increases and 4) proportion of the poor falls or income inequality rises.

Keywords: In-kind provision, Specific egalitarianism, Differential pricing
JEL Classifications: D30, D45, H31, I18

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For helpful comments we wish to thank Michael Hoy, and participants at the CPEG session of the Canadian Economics Association Meeting in Calgary, as well as seminar participants in the Economics Departments at the University of Guelph and New South Wales. Conference travel funding from the Business School of the University of Canterbury is gratefully acknowledged.

[**Seoul Journal of Economics** 2023, Vol. 36, No. 4]
DOI: 10.22904/sje.2023.36.4.001

I. Introduction

Economists have traditionally been wary of distributional concerns over private goods, or of publicly providing private goods for redistributive purposes. It has commonly been argued that distributional concerns grounded in utilitarian social welfare could be met at least cost by transferring *income* from rich to poor, and then allowing market prices to allocate resources to their most valued uses (the Second Welfare Theorem). Nevertheless, two factors have increased the attention economists have paid to in-kind provision over the past forty years.

First, good-specific distributional concerns, particularly over items “essential to life and citizenship,” have proven remarkably robust over time (Tobin 1970). For example, compulsory public health insurance in Canada, implemented federally in 1968, was reviewed in 2002 and justified in part on the basis that Canadians want the poor to have the same access to health care as the rich (Romanow Commission 2002, p. xvi). The enduring popularity of the Food Stamp program in the United States has ensured its survival through welfare reform, while conversely, pricing schemes for roads, national parks or residential water use often face political resistance based on distributional concerns (Small and Yan 2001; Park *et al.* 2010; Rogers *et al.* 2002). This has prompted some economists to suggest that good-specific distributional concerns be taken seriously as a public policy objective (Tobin 1970; Weitzman 1977; Rosen 2002a, p. 175).

Second, the incorporation of imperfect information into public economics has shown that egalitarian in-kind provision of some types of goods can actually increase utilitarian social welfare. Goods such as health insurance may not be available to all in private markets if providers cannot distinguish high and low risk individuals (Rothschild and Stiglitz 1976). Similarly, governments may be unable to distinguish high from low ability workers, and thus face restrictions on the degree of income redistribution possible through optimally designed tax systems. For individuals with high ability may mimic those with low ability in order to avoid taxes or qualify for cash transfers (Blackorby and Donaldson 1988; Boadway and Marchand 1995; Blomquist and Christiansen 1995). These authors have used unknown risk or ability to explore conditions where social welfare would be higher if certain private goods were publicly provided at a uniform level to all.

In this paper, we are agnostic as to the basis for good-specific egalitarianism, or the domain of goods to which it might apply. Instead, we ask how good-specific distributional concerns could be achieved at least cost in efficiency. The good in question could be some types of health care, access to popular public recreation sites, roading access, or provision of compulsory government services. We restrict our attention, however, to goods for which resale is infeasible. We propose a differential pricing mechanism, where the government could make the target good available at alternative "outlets" charging different time and money price combinations. By setting the money price of the good at each outlet appropriately, the government can ensure that individuals self-select outlets by their earnings ability.[1] With outlets stocked proportionately, individuals of each ability level would then purchase the same quantity of the target good on average, while those who valued the good more relative to other goods would purchase more of it than those who valued it less, regardless of income.

Of course, any use of time as a rationing device involves the waste of an otherwise valuable resource. There is then inescapably an efficiency cost to achieving specific egalitarianism using differential pricing. We show, however, that the cost of specific egalitarianism may be less using differential pricing than more conventional instruments, such as uniform public provision funded by proportional income tax, with or without additional optional private purchase. In particular, differential pricing is likely to be more efficient than tax-funded uniform provision as 1) the relative importance of the target good in people's utilities rises, 2) the elasticity of substitution between the target good and other goods falls, 3) heterogeneity of preference for the target good increases and 4) income inequality rises or the proportion of the poor falls.

The layout of the paper is as follows. Section 2 provides a brief review of the literature on the distributional and efficiency aspects of queuing as an allocation mechanism. Section 3 provides a formal model of the differential pricing mechanism. Section 4 compares social welfare under this mechanism with tax funded uniform provision. We conclude in Section 5.

[1] The need for the government to set prices at both outlets differentiates our paper from the recent literature on the welfare or stability properties of parallel public and private provision of goods (Cuff *et al.* 2010; Buckley *et al.* 2010; Lulfesmann and Myers 2011).

II. Equitably Inefficient Queues[2]

Allocating goods that are "essential to life and citizenship" using queues is often seen as fairer than using price because time is more evenly distributed than income (Nichols *et al.* 1971; Barzel 1974; O'Shaughnessy 2000; and Alexeev and Leitzel 2001). Yet allocating goods by time rather than price creates two major costs in efficiency. First, buyers who wait in line are surrendering a valuable resource, time, that unlike money does not get transferred to the seller. The opportunity cost of that lost time may be leisure, but also forgone production. Thus, widespread queuing for goods in an economy would ultimately make fewer of these goods available. Second, the time price of queuing penalizes those with a higher opportunity cost of time. When compared to pricing, queuing will thus transfer goods from some who value them more to others who value them less (Tobin 1970; Suen 1989; O'Shaugnessy 2000). This is why economists have generally recommended meeting distributional concerns at a general level with a tax and transfer system, and then allocating private goods by price, or congestible public services with user fees set at marginal social cost (Rosen 2002b).

On the other hand, tax and transfer systems carry their own distortions in work disincentives (Tobin 1970; Bucovetsky 1984) and imperfect targeting of the truly needy *vs.* the opportunistic (Alexeev and Leitzel 2001). Similarly, user fees for congestible public services may have regressive distributional effects (Nichols *et al.* 1971). In response, a number of studies have compared the efficiency of alternative re-distributional instruments, such as tax/transfers, in-kind transfers, queuing, or rationing with resale (Bucovetsky 1984; Sah 1987; Blackorby and Donaldson 1988; Polterovich 1993; O'Shaugnessy 2000; Alexeev and Leitzel 2001). In general, when re-sale is not practical, the inefficiency of queuing must be traded-off against the inefficiency of allocating uniform quantities of a good to heterogeneous people.

Our approach begins with the key insight by Nichols *et al.* (1971) that if people could *choose* whether to pay by money or by time, much of the re-distributional potential of queuing could be preserved, and its

[2] This section draws heavily on Clark and Kim (2007), which focuses exclusively on the re-distributional properties of differential pricing.

inefficiency reduced. Indeed, private firms with a degree of monopoly power commonly offer goods with a variety of price / wait combinations as a form of second-degree price discrimination in order to increase profits (Donaldson and Eaton 1981; Tirole 1988). Governments could do the same with a target good, but to pursue distributional ends. In practice, this is already being tried in applications such as "value pricing" of roads, where motorists can choose between free but congested roadways, or priced but less-congested roadways (Small and Yan 2001; Liu and McDonald 1999). Faced with a choice, low wage individuals would self-select to pay partly by (low) price and partly by time, while those with a high wage would self-select to pay only by money. If wage captures the opportunity cost of time, and differences in wages reflect differences in marginal product, then the time lost in queues would have low foregone cost in wages and production. The costly and error-prone apparatus of means testing individuals would be unnecessary. Nichols *et al.* provided no formal model of differential pricing, but O'Shaugnessy (2000) and Alexeev and Leitzel (2001) do when comparing social welfare under such a system with that under conventional tax and transfer systems. Both of the latter studies assume, however, that preferences are identical across the population. They also model economies in which only a single good is produced, and thus eligible for redistribution.

Though independently derived, we formalize the differential pricing mechanism proposed by Nichols et al. and show that it can make consumption of any particular good independent of income, but dependent on relative strength of preference or need. We then illustrate conditions under which it can do this with greater efficiency than public uniform provision, with or without allowance for private purchase.

III. A Model of Redistribution through Differential Pricing

Consider an economy of $i = 1,\ldots,N$ people, each of whom has a continuously differentiable and strictly quasi-concave utility function over leisure ℓ, a composite commodity y, and a "target" good g whose distribution is of concern to a policy maker, $U(\ell, y, \theta_i g)$.[3] While each

[3] Our preferences over 3 goods are more general than in Alexeev and Leitzel (2001), who assume equal weight Cobb-Douglas preferences between a single good and leisure, and O'Shaugnessy (2000), who assumes general concave utility

individual values g, some have a stronger preference (or need) for it relative to l and y than others. Thus, θ_i represents i's relative strength of preference for the target good ("strong" or "regular"), where $\theta_S > \theta_R$. Across the population, we denote by $0 \leq s \leq 1$ the proportion of individuals with a strong preference θ_S for g, and assume initially that it is identical among low and high wage individuals, or $s_L = s_H = s$. We discuss the relaxation of this assumption in the final section of the paper.

Individuals earn income from their choice of labor hours L, which they spend at competitive firms producing either? Y or G.[4] Workers are paid a wage equal to the value of their marginal product. We assume for simplicity that production technology is identical in the Y and G sectors, and that an individual's marginal product is the same at both. Income inequality arises in part because of differences in taste (θ_i), but mostly because of exogenous differences in ability. N_H of N workers have a high ability and marginal product, and so receive a high wage w_H. The remaining N_L workers ($= N - N_H$) have a low ability, marginal product, and wage w_L. Workers of high and low ability divide their labor hours between production sectors according to $L_{i,H} = L^G_{i,H} + L^Y_{i,H}$ and $L_{i,L} = L^G_{i,L} + L^Y_{i,L}$ respectively, where $i = R$ or S.

The prices individuals face are as follows. The price of leisure is a person's wage, w_j ($j = L$ or H), while the price of y is normalized to 1. Under the mechanism we propose, the *full* money and time price of the target good g at a given outlet is $P_{g,j} = w_j h + p$, where p is the money price per unit, and h is the hours of waiting time required per unit.[5] With these prices and income, each person faces a budget constraint $w_j L = pg + y$. Individuals also face a time constraint $\ell + L + hg = T$, as they have an (identical) time endowment T that can be spent working L, in leisure ℓ, or in line (hg). These constraints can be combined as $w_j \ell$

between one good and leisure.

[4] We adopt the convention of lowercase letters for demand, and uppercase letters for supply.

[5] We assume individuals must queue once per unit purchased, and that all outlet patrons wait an identical period of time per unit purchased. This is a common way of modeling queuing (Barzel 1974; Sah 1987; Suen 1989; Polterovich 1993; O'Shaughnessy 2000; Alexeev and Leitzel 2001). Alternatives are to model queuing time as dependent on show-up time (Holt and Sherman 1982), or as independent of quantity purchased (Weitzman 1991).

$+ y + P_{g,j}g \leq w_jT$. As shown in Appendix 1, individuals will choose the g outlet offering the lowest full price given their opportunity cost of time w_j. Conditional on this choice of outlet, an individual's problem is:

$$\underset{\ell,y,g}{Max}U\left(\ell, y, \theta_i g\right)$$

$$\text{s.t. } w_j\ell + y + P_{g,j}g \leq w_jT \text{ , where } i = R \text{ or } S \text{ and } j = L \text{ or } H \quad (1)$$

With interior solutions ($\ell^*_{i,j} > 0$, $y^*_{i,j} > 0$, and $g^*_{i,j} > 0$), the corresponding first order conditions are:

$$U_l(\ell^*_{i,j}, y^*_{i,j}, \theta_i g^*_{i,j}) = \lambda w_j$$

$$U_y(\ell^*_{i,j}, y^*_{i,j}, \theta_i g^*_{i,j}) = \lambda \text{ , and} \quad (2)$$

$$U_g(\ell^*_{i,j}, y^*_{i,j}, \theta_i g^*_{i,j})\theta_i = \lambda P_{g,j},$$

where λ is a Lagrange multiplier and $i = R$ *or* S, and $j = L$ *or* H.

We note from the individual's corresponding indirect utility function $V_{i,j} = V(w_j, P_{g,j}, T, \theta_i)$ that he/she is indifferent as to the composition of g's full price, $P_{g,j}$, between the time (h) and money (p) components.

Firms operate in perfectly competitive markets, producing either Y or G by employing workers with both ability levels and tastes using constant returns technology. With constant (exogenous) marginal products and the price of y set to one, wages adjust to equal marginal product, and so the price of both leisure and y are determined. With identical technology in the G sector as in Y, the competitive price of g without re-distribution would equal that of y, 1.

Under our differential pricing mechanism, however, the policy maker would purchase all G produced by firms at cost, and sell it at a higher money price $p_H > 1$ at one outlet, and at a lower money price p_L at a second outlet.[6] Note that the policy maker could even set the low price p_L negative, functioning as a unit subsidy funded from the tax at the

[6] We assume the policy maker can successfully prevent a black market in G from forming directly between producers and individual buyers. Alternatively, the government could allow private firms to sell G directly to individuals, offering a subsidy to those who sell at a prescribed below-market price, and taxing those who sell it at market price.

high price outlet. Separation of buyers by wage requires the incentive compatibility constraints that members of both wage groups find the full price at their own outlet lower than at the alternative, given their opportunity cost of time. We argue that by stocking each outlet with a supply of g in proportion to the distribution of ability types in the population, and setting money prices at each outlet so as to maximize social welfare subject to incentive compatibility constraints, a policy maker can satisfy "specific egalitarianism." As we define it, specific egalitarianism requires that 1) consumption of g is equalized across the average low and high wage person, or

$$g^*_L \equiv (1-s)g^*_{R,L} + sg^*_{S,L} = g^*_H \equiv (1-s)g^*_{R,H} + sg^*_{S,H} \tag{3}$$

and that 2) individuals with a strong preference or need for g will receive as much or more of it than individuals with a relatively weak preference, regardless of income, or

$$g^*_{S,j} \geq g^*_{R,j},\ g^*_{S,L} \geq g^*_{R,H},\ g^*_{S,H} \geq g^*_{R,L} \tag{4}$$

More formally,

Claim 1: *The consumption of g that results under a differential pricing mechanism that meets the incentive compatibility constraints* ($P_{g,H} \leq w_H h_L + p_L$ *and* $P_{g,L} \leq w_L h_H + p_H$) *will satisfy specific egalitarianism.*

The claim that differential pricing achieves (3) is trivially satisfied. With supply at each outlet pre-set in proportion to distribution of wage types, the separation of buyers by wage induced by the incentive compatibility constraints will induce $g^*_L = g^*_H$ and thus, consumption of g will be equalized across the average low and high wage person. To prove that differential pricing achieves (4) is only slightly more involved. For a given wage group at a given outlet, g is increasing in θ, or $\partial g^*_{i,j}/\partial\theta_i > 0$. Thus, within an outlet, $g^*_{R,j} < g^*_{S,j}$. Next, any pricing solution that satisfies (3) equalizes average demand across outlets, say at g^*. It follows from $\partial g^*_{i,j}/\partial\theta_i > 0$ that at the low wage outlet, for any distribution of s, $g^*_{R,L} < g^* < g^*_{S,L}$. Similarly, for any distribution of s at the high wage outlet, $g^*_{R,H} < g^* < g^*_{S,H}$. Thus, an individual with a high need for g will always purchase an above-average amount, regardless of income, while an individual with a low need for g will always purchase a below-average amount. (4) will be satisfied.

Note that various money price pairs at the two outlets could achieve specific egalitarianism ((3) and (4)) so long as they satisfied the incentive compatibility constraints. We turn therefore to how a policy maker should choose an optimal pair of money prices in order to maximise social welfare while meeting incentive compatibility constraints. The formal problem is:

$$\underset{p_H,h_H,p_L,h_L}{Max} \; SW = (1-s)N_L V_{R,L} + sN_L V_{S,L} + (1-s)N_H V_{R,H} + sN_H V_{S,H} \quad (5)$$

subject to

$$(p_H - 1)N_H = (1 - p_L)N_L \quad (6)$$

$$P_{g,H} \leq w_H h_L + p_L \quad (7a)$$

$$P_{g,L} \leq w_L h_H + p_H \quad (7b)$$

$$(1-s)N_L g^*{}_{R,L} + sN_L g^*{}_{S,L} + (1-s)N_H g^*{}_{R,H} + sN_H g^*{}_{S,H} =$$

$$(1-s)N_L w_L L^G{}_{R,L} + sN_L w_L L^G{}_{S,L} + (1-s)N_H w_H L^G{}_{R,H} + sN_H w_H L^G{}_{S,H} \quad (8)$$

Constraint (6) is a reduced form of the policy maker's balanced budget condition. Equations (7a) and (7b) are incentive compatibility constraints for low and high ability individuals to remain at their own outlets and (8) is the economy's resource constraint for the demand and supply of *g*.[7]

Claim 2: *The policy maker will set the money price at the low wage outlet just low enough to create the minimum queue needed to keep high wage individuals out of that outlet. He will set the money price at the high wage outlet so as to clear it without queuing.*

[7] A single resource constraint for *g* is sufficient because two of the three prices in the model have already been determined. Alternatively, the resource constraint for *y* will be identical.

To prove Claim 2, we characterize in steps how the optimal prices are chosen. First, the expression for p_H in the balanced budget condition (6) can be substituted into the equalized average consumption equation (3). The money price p_L that induces (3) can be expressed as an implicit function of h_L and h_H, as then from (6) can p_H. $P_{g,H}$ and $P_{g,L}$ in social welfare (5) can then be expressed using these implicit functions of h_L and h_H. Compacting the notation in (5) yields

$$\underset{h_H, h_L}{SW} = \sum_{i=R,S} \sum_{j=L,H} V(w_j, P_{g,j}(h_L, h_H, s, N_j), T, \theta_i) \tag{9}$$

Next, we show that social welfare in (9) falls in both high and low wage queuing times, so that the policy maker will maximize it by setting both h_H and h_L as low as possible, subject to the separation constraints (7a) and (7b). First, given some optimal queuing time at the high ability outlet, h_H^*, we claim (9) will fall in h_L. This is because as h_L rises, the full price at the low ability outlet must also rise. Why? If h_L rises, $P_{g,L}$ can remain constant or fall only if the money price p_L falls, which from budget balance (6) would require the policy maker to raise p_H. Yet from (3), a higher p_H given h_H^* would reduce demand for g among those with high ability. To satisfy (7a) and (7b) and equalized average consumption (3), this would have to be matched with a drop in demand by those with low ability, which cannot occur if $P_{g,L}$ has fallen or remained constant. Thus, a rise in h_L must raise $P_{g,L}$, lower the indirect utility of low ability individuals, and so lower social welfare. h_L^* will thus be set at the minimum level consistent with keeping the low wage outlet unattractive to those with a high wage, or from (7a), $h_L = (P_{g,H} - p_L)/w_H$. We can further show from the numerator that this time price will have to be positive. This follows because $P_{g,H}$ must exceed $P_{g,L}$ for (3) to be satisfied, since it can be shown that $\partial g_{i,j}^* / \partial w_j > 0$ as an income effect when prices are held fixed, and that $\partial g_{i,j} / \partial P_{g,j} < 0$. Yet $P_{g,H}$ cannot exceed $P_{g,L}$ if $h_L \leq 0$, since setting it minimally implies $P_{g,H} = w_H h_L + p_L \leq w_L h_L + p_L = P_{g,L}$.

While queuing is unavoidable at the low wage outlet to keep high wage individuals away, none is needed at the high wage outlet. Using the same line of reasoning, given an optimal h_L^*, social welfare will also be falling in h_H, for the analogous reason that when h_H rises, the full price $P_{g,H}$ must also rise, lowering the indirect utility of high wage individuals. But unlike for low wage individuals, (7b) will still be satisfied if $h_H = 0$, or $p_H \geq P_{g,L}$, because (7a) becomes $w_H h_L^* + p_L = P_{g,H} = p_H$, which exceeds $w_L h_L^* + p_L = P_{g,L}$. To summarize, $h_H^* = 0$, and h_L^* can be re-expressed as $(p_H - p_L) / w_H$.

Substituting these optimal queuing times into the resource constraint (8), the technical conditions are satisfied to ensure there exist unique money prices p_H and p_L that the policy maker can choose to ensure a feasible allocation.[8]

The reader may gain intuition concerning the planner's optimal pricing policy $\{p_H^*, h_h^*, p_L^*, h_L^*\}$ from Figure 1. The figure illustrates the time/money price pairs for g that would yield the same full price for high wage individuals, including the socially optimal pair at point *A*, where $h_H^* = 0$ and p_H^*. It also shows the time/money price pairs that would yield low wage individuals the same full price, including the optimal pair at point *B*, h_L^* and p_L^*.[9] *The optimal time/money prices at points A* and *B* are incentive compatible, in that no individual from either wage group would be better off by going to the outlet targeted to the other. Yet inefficiency is minimised by making the least use of queues possible.

Once the policy maker has determined the optimal prices, the total quantity of *G* that will be sold at each outlet follows easily, and thus the total quantity that must be purchased from the *G* production sector. As mentioned, *G* will need to be allocated to each outlet in proportion to the distribution of ability types in the population. Our assumption of identical, constant returns technology in the *G* and *Y* production sectors means that the precise allocation of individuals' labor hours across the two sectors is under-identified.[10] See Appendix 2 for details.

[8] However allocated between *G* and *Y* production, labor hours supplied can be represented as a function of full price $P_{g,j}$ via each individual's time constraint, $T - \ell - hg$. By (3), $P_{g,L}$ can further be written as an increasing function of $P_{g,H}$,, such that the entire resource constraint can be written as a function of $P_{g,H}$. Uniqueness of full price is then satisfied in that the entire expression is monotonically decreasing in $P_{g,H}$, with infinite excess demand at zero price, and finite excess supply at infinite price.

[9] As drawn, the "isoprice" lines for high and low ability individuals cross in the positive quadrant, so that the time and money prices for low ability individuals are both positive. In cases where $\rho < 0$ the intersection may occur at a positive time price, but *negative* money price (or subsidy). See Clark and Kim (2007) for further details.

[10] One plausible feasible allocation of labor across sectors would be for everyone to work in the production of *Y* in proportion to his or her expenditures on y out of total expenditures.

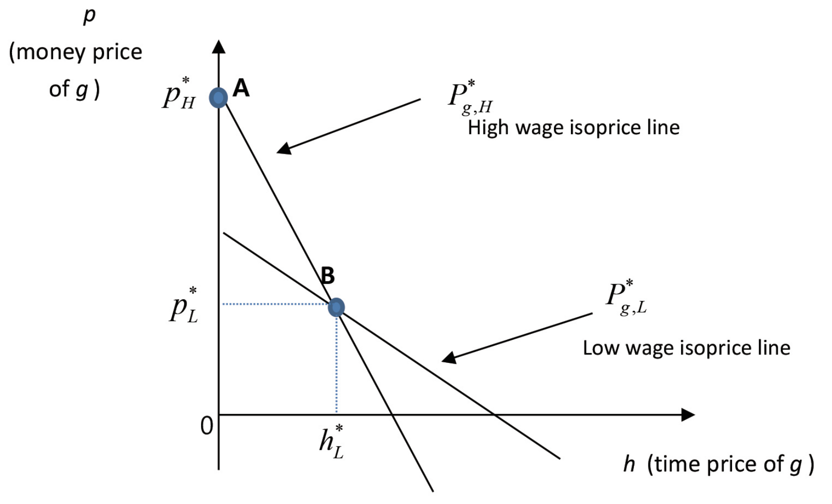


Figure 1
Illustration of efficient separating pricing for two income levels

Our characterization of the differential pricing mechanism for in-kind redistribution is complete. By commanding exclusive purchase rights over a target good and setting a high money price at one outlet, and a low money price with queue at another, a policy maker can ensure that consumption of the good is equalized across wage groups, while increasing in relative need. This is done at least cost to efficiency by setting the money price at the low wage outlet high enough that the resulting queue is the smallest necessary to keep high wage earners out of the outlet. The money price at the high wage outlet is set high enough to clear that outlet without queuing.

Claim 3: *A policy maker who did not satisfy incentive compatibility constraints (7a) and (7b) could achieve higher social welfare, but not achieve specific egalitarianism.*

A social planner setting prices to maximize (5) without incentive compatibility constraints (7a) (7b) and budget constraint (6) would choose a single price in a single outlet for both high and low wage groups that would eliminate queuing. This "first best" social welfare would be as much or higher than the "second best" under differential pricing, since it would be the solution to a less constrained problem. However, the unconstrained market-clearing money price would be positively associated with the wage level ($\partial p^*/\partial w > 0$), since consumption g is increasing in w. Thus the single money price would be intermediate to the two market clearing full prices of g under differential pricing. Graphically, the unconstrained single money price would lie between the two vertical intercepts of Figure 1, since it would clear the market for the average wage level. It follows that the consumption of g by high wage people would be higher without differential pricing, the consumption of g by low wage people would be lower, and equalized average consumption would not hold.

To evaluate the relative efficiency cost of achieving specific egalitarianism using differential pricing (our "second best"), we consider next two plausible alternative mechanisms. Each mechanism can (just) achieve specific egalitarianism as we have defined it in (3) and (4), without requiring the government to identify the ability status of a given individual. The first is uniform public provision of the target good, funded through a proportional income tax, with additional private purchase of g allowed. The second mechanism is uniform provision funded through proportional taxation, with additional purchase of g not allowed. We then identify the conditions under which our mechanism can achieve specific egalitarianism more efficiently than the more efficient of these alternatives.

IV. Uniform Provision with Proportional Income Tax

Suppose that instead of differential pricing, the policy maker were to provide every individual with a uniform quantity of g, funded by a proportional income tax t on labor income. This mechanism would additionally require the policy maker to know each individual's total labor income, but not their individual ability or wage. Along with public provision, individuals could be free to purchase additional units of g privately (as with school vouchers), or not (as with public health insurance in Canada). As we shall see, uniform provision funded by income tax replaces the inefficiency of queuing time and g price distortions with labor/leisure price distortions and under- or over-provision of the target good. It will turn out that specific egalitarianism is more costly to achieve in efficiency terms when private purchase *is* allowed, because the government must provide sufficient g to crowd out all private purchases. Our primary comparison will therefore be between differential pricing and no-purchase uniform provision.

A. Uniform Provision Characterised

To compare the efficiency cost of achieving specific egalitarianism using differential pricing vs. tax-funded uniform provision, we resort to a more specific utility function, constant elasticity of substitution (CES), or $U(\ell, y, \theta_i g) = \left(\ell^{\rho} + y^{\rho} + \theta_i g^{\rho}\right)^{\frac{1}{\rho}}$. Here $\rho(< 1)$ represents an individual's elasticity of substitution between the target good and other goods, and can range from almost perfect flexibility ($\rho \rightarrow 1$), to Cobb Douglas ($\rho = 0$), to Leontief ($\rho \rightarrow -\infty$). We consider first the case where the government provides $\tilde{g}$ units of the target good to everyone, but allows individuals to purchase additional units $\hat{g}$. An individual would face the problem:

$$\underset{\ell,y,g}{Max} U = \left(\ell^{\rho} + y^{\rho} + \theta_i(\tilde{g} + \hat{g})^{\rho}\right)^{1/\rho}$$

$$\text{s.t. } y + \hat{g} = (1-t)w_j L,$$

$$\ell + L = T \qquad \text{where } i = R \text{ or } S \text{ and } j = L \text{ or } H \tag{10}$$

With CES utility, an individual's demand function for additional $\hat{g}$ would be

$$\hat{g}^{*}_{i,j} = \left(\frac{(1-t)w_j T - \tilde{g}\theta_i^{\frac{1}{\rho-1}}(1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}})}{\theta_i^{\frac{1}{\rho-1}}(1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}}) + 1} \right) \quad i = R \text{ or } S \text{ and } j = L \text{ or } H \tag{11}$$

Note from (11) that there is a quantity of publicly provided $\tilde{g}$ that would just crowd out an individual's private purchase. It can easily be shown that this quantity is rising in both w and θ, or that $\partial\hat{g}/\partial w_j > 0$ and $\partial\hat{g}/\partial\theta_i > 0$. As we show in Appendix 3, a policy maker wishing to ensure specific egalitarianism would have to set $\tilde{g}$ at the "highest common denominator," or at the level desired by those with the highest wage and strength of preference for the target good. From the numerator of (11), the policy maker seeking to satisfy specific egalitarianism will thus face a constraint on maximising social welfare:

$$\tilde{g} \geq \left(\frac{(1-t)w_H \theta_S^{\frac{1}{1-\rho}} T}{(1 + ((1-t)w_H)^{\frac{\rho}{\rho-1}})} \right) \tag{12}$$

Achieving specific egalitarianism would come at a high cost in efficiency under this mechanism, because the government must impose on everyone the tax/provision trade-off that would be chosen by the keenest, wealthiest individual. This inefficiency could be reduced if the policy maker could restrict private purchase of g, and provide instead an "average" amount $\bar{g}$ to all at a lower tax rate. More formally, a policy maker could achieve higher social welfare if he did not have to face constraint (12). For purposes of efficiency comparison, therefore, we concentrate on the case of uniform provision without additional private purchase.

With consumption of g determined only by public provision $\bar{g}$, an individual with θ_i and w_j would face the problem:

$$\underset{\ell,y}{Max} U = \left(\ell^{\rho} + y^{\rho} + \theta_i \bar{g}^{\rho} \right)^{1/\rho}$$

s.t. $y = (1-t)w_j L$,

$$\ell + L = T \qquad \text{where } i = R \text{ or } S \text{ and or } H \tag{13}$$

The individual's resulting demand functions would be

$$\ell^*_{i,j} = \left(\frac{((1-t)w_j)^{\frac{\rho}{\rho-1}} T}{1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}}} \right),$$

$$y^*_{i,j} = \left(\frac{(1-t)w_j T}{1 + ((1-t)w_j)^{\frac{\rho}{\rho-1}}} \right) \tag{14}$$

Substituting these demands and $\bar{g}$ into utility would lead to indirect utility $V_{i,j} = (\ell^{*\ \rho}_{i,j} + y^{*\ \rho}_{i,j} + \theta_i \bar{g}_{i,j}{}^{\rho})^{1/\rho}$

Here, since everyone must consume an identical g, the policy maker will automatically (weakly) satisfy specific egalitarianism, whatever the level of provision. The policy maker will choose t and $\bar{g}$ to solve:

$$\underset{t,\bar{g}}{Max}\; SW = (1-s)N_L V_{R,L} + sN_L V_{S,L} + (1-s)N_H V_{R,H} + sN_H V_{S,H} \tag{15}$$

subject to

$$N_L t w_L L_L + N_H t w_H L_H = (N_L + N_H)\bar{g} \tag{16}$$

(16) represents the government budget balance. With private markets operating only for y and all prices but t given, a resource constraint is redundant, and the exact allocation of labor across G and Y production is under-identified as before. The technical conditions are satisfied to ensure that one or more ($\bar{g}^*$, t^*) pairs exist that solve this problem, and can be compared to find a global maximum.[11] Intuitively, the policy maker will choose the g/t tradeoff that would be chosen by the average person, weighted by the distribution of wage and preference strengths in the population. As mentioned, social welfare from tax-funded uniform provision will be at least as high without private purchase and with, so we shall concentrate on the more efficient option for comparison with differential pricing.

B. Comparing Policies

Ideally, we would like to make a global comparison of social welfare when specific egalitarianism is achieved using differential pricing vs. uniform provision without private purchase. That is, we would like to compare

$$\sum_{i=R,S}\sum_{j=L,H} N_{i,j}V_{i,j}(P^*_{g,j}) \text{ and } \sum_{i=R,S}\sum_{j=L,H} N_{i,j}V_{i,j}(\bar{g}^*, t^*) \tag{17}$$

Such a comparison is complicated by the fact that, even with CES utility, we cannot derive closed form solutions for policy variables. We can, however, derive closed form solutions and make comparisons when the elasticity of substitution between goods, ρ, is zero (Cobb-Douglas). We then rely on simulations to compare policies for other values of ρ.

As ρ approaches zero, an individual's CES preferences converge to

[11] The constraint (16) is closed, and the objective function (15) is bounded for all values contained in the constraint.

$$U(\ell, y, g) = \ell^{\frac{1}{2+\theta_i}} y^{\frac{1}{2+\theta_i}} g^{\frac{\theta_i}{2+\theta_i}} \qquad \text{for } i = R \text{ or } S \tag{18}$$

Under differential pricing, the individual's demand functions from problem (1) become:

$$\ell_{i,j}{}^* = \left(\frac{T}{2+\theta_i}\right), \; y_{i,j}{}^* = \left(\frac{w_j T}{2+\theta_i}\right), \text{ and } g_{i,j}{}^* = \left(\frac{\theta_i w_j T}{P_{g,j}(2+\theta_i)}\right) \tag{19}$$

There are closed form solutions for the $P_{g,j}$ in (19), which take the simple form $P_{g,H} = p_H = N/N_H$, and $P_{g,L} = w_L h_L + 0 = w_L(p_H - 0)/ w_H = w_L N/ w_H N_H$. Graphically, with Cobb- Douglas preferences, the two isoprice curves of $P_{g,j}$ in Figure 1 will cross at the horizontal axis with $p_L = 0$ since $\partial h^*/\partial w_j = 0$ when $\rho = 0$. That is, with a Cobb-Douglas degree of substitution between goods, the low wage outlet will charge a zero price and rely completely on queuing to deter high wage individuals.[12]

In contrast, under uniform provision without private purchase the policy maker's optimal choice of $\bar{g}$ and t become:

$$\bar{g}^* = \left(\frac{\bar{\theta}\bar{w}\, T}{2(1+\bar{\theta})}\right), \; t^* = \frac{\bar{\theta}}{1+\bar{\theta}} \tag{20}$$

where $\bar{w}$ and $\bar{\theta}$ are the weighted averages $[(N_L/N)w_L + (N_H/N)w_H]$ and $(1-s)\theta_R + s\theta_s$, respectively. The individual's after-tax demand functions from (13) become

$$\ell^*_{i,j} = \left(\frac{T}{2}\right), \; y^*_{i,j} = \left(\frac{w_j T}{2(1+\bar{\theta})}\right) \tag{21}$$

We can now identify the variables that determine which policy achieves higher social welfare.

a) The Importance of the Target Good

The first variable of interest is θ, the weight that individuals place on the target good relative to other goods. To simplify the comparison,

[12] Less (greater) willingness to substitute would require greater (lesser) queuing time and a negative (positive) money price at the low wage outlet.

we assume initially that preferences for g are homogeneous at θ, and later consider the effect of a mean preserving spread. As we show in Appendix 4, differential pricing will achieve specific egalitarianism more efficiently than uniform provision if and only if θ and wage disparity are related as follows:

$$\sum_{j=L,H} N_j V_j(P^*_{g,j}) \geq \sum_{j=L,H} N_j V_j(\bar{g}^*, t^*)$$

$$\Leftrightarrow \left(\frac{4(1+\theta)}{(2+\theta)^2}\right)^{\frac{1}{2+\theta}} \left(\frac{\frac{w_H}{w_L}\frac{N_H}{N}}{(\frac{w_H}{w_L}\frac{N_H}{N}+\frac{N_L}{N})} \frac{2(1+\theta)}{2+\theta} \right)^{\frac{\theta}{2+\theta}} \geq 1 \qquad (22)$$

As we prove in Appendix 4, for a given ratio of wages, the left hand side of the inequality in (22) is rising in θ. Thus, for a given wage disparity, differential pricing will generate higher social welfare than uniform provision if θ is sufficiently high. That is, there will exist a θ^* at which the policies yield equal welfare, and above which differential pricing will dominate. Intuitively, this is because the tax rate needed to support uniform provision rises in θ more rapidly than do changes in the relative price of g across outlets. As a result, as θ rises, uniform provision creates more substantial distortions to labor supply and consumption decisions than differential pricing when compared against a benchmark of no redistribution.

b) Disparities in Wage and the Proportion of Low Ability Individuals

The second two variables of interest are the degree of wage disparity and the proportion of low *vs.* high ability individuals in the population. Returning to (22), as proven in Appendix 4, differential pricing becomes more efficient than uniform provision as the ratio of high to low wage increases, or as the proportion of low ability individuals falls, all else constant. Intuitively, less queuing time and target good price distortions are required to keep high wage individuals out of the low wage outlet as relative wage differentials grow, or as fewer individuals need to be subsidized, thus raising the relative efficiency of differential pricing to uniform provision.

c) Disparities in Preference

The next variable of interest is the degree of variation in taste for g relative to other goods. As we show in Appendix 5, when $\rho = 0$ any mean preserving spread in θ_R and θ_S around a homogeneous θ improves the relative efficiency of differential pricing to uniform provision. We saw in Section a) that differential pricing will yield higher social welfare than uniform provision under preference homogeneity if θ is sufficiently high (θ^*). Preference heterogeneity will thus increase the efficiency advantage of differential pricing over uniform provision if it introduces a mean-preserving spread in preferences around θ^*. It could also tip the balance in favour of differential pricing for mean-preserving spreads in preferences around lower values of θ. Intuitively, preference heterogeneity favours differential pricing over uniform provision because the importance of allowing unequal consumption of g grows. Under uniform provision heterogeneous individuals must pay the same tax, receive the same g, and so choose identical labor/leisure and composite consumption.

d) The Elasticity of Substitution Between Goods

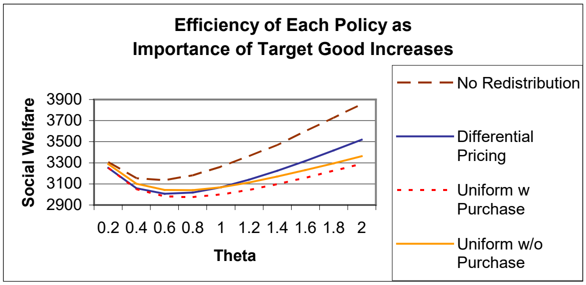
Our final variable of interest is individuals' elasticity of substitution between goods. We cannot make general welfare comparisons between policies for values of ρ other than zero, because we cannot derive closed form solutions for the optimal policy instruments. However, simulations under diverse parameters show that differential pricing yields higher social welfare than uniform provision as individuals grow less willing to substitute other goods for the target good. Our partial intuition for this is as follows. With a high elasticity of substitution, low wage individuals take far more leisure under tax-funded uniform provision than differential pricing, because income tax discourages work effort, and because they need not spend time queuing for g. With little difference in g or y consumption between mechanisms, the poor are thus better off under uniform provision. With a lower elasticity of substitution, however, the poor lose the "leisure premium" under uniform provision, and receive noticeably less g. This is because the policy maker must offer a negative price (subsidy) at the low income outlet as $\rho \to -\infty$ to compensate low income individuals for the long waiting time required to keep high income individuals out of the outlet. These subsidies encourage the poor to consume more g and leisure, while still purchasing y. The poor thus become better off under differential pricing.

High wage individuals face a reverse welfare ranking. With a high elasticity of substitution, they receive a "y premium" under differential pricing, with little difference in the other goods. This is because differential pricing raises the price of g, providing substitution incentives towards y, and because the income tax under uniform provision discourages labor that makes y affordable. The rich thus prefer differential pricing when ρ is high. With a low elasticity, however, the rich lose the "y premium" under differential pricing, and purchase more g. It appears that the large price differential for g set to ensure separation as ρ falls is not sufficient to turn the rich from purchasing g to purchasing y. Thus the rich become better off under uniform provision. Despite the asymmetry in welfare rankings for the rich and poor, overall social welfare rankings of the policies track those for the poor, because of a diminishing marginal utility of consumption. Thus differential pricing becomes socially preferable as the elasticity of substitution between goods falls.

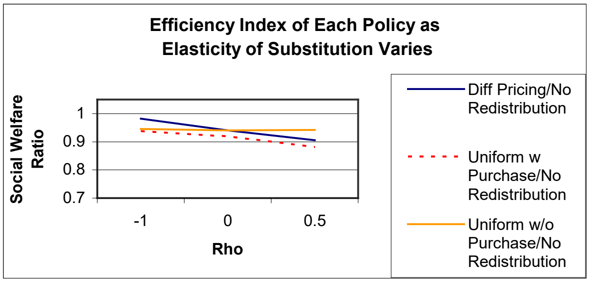
We illustrate all of these results in simulations in Figure 2. In window (a), we compare social welfare under a first best pricing scheme that does not meet specific egalitarianism, with that under our three mechanisms that do achieve it. Window (a) varies the value of θ, and shows that a high value of (homogeneous) θ favours differential pricing over uniform provision with or without private purchase. In the simulation we selected a wage ratio (5.4 to 1) that makes differential pricing and uniform provision without private purchase equivalent in welfare at $\theta^* = 1$. In window (b), we illustrate how changing people's elasticity of substitution between goods away from $\rho = 0$ affects the relative efficiency of the three policies, with first best social welfare as the denominator. The results are illustrated with homogenous preferences, a wage ratio of 5.4 to 1, and a value of θ that makes differential pricing and uniform provision welfare equivalent when $\rho = 0$. As window (b) illustrates, a low value of ρ favours differential pricing.

As window (c) illustrates, an increase in the wage disparity ratio from 5.4 to 15 increases the range in ρ over which differential pricing dominates uniform provision of either type, while a decrease in wage disparity from 5.4 to 3 reduces it.[13] Finally, window (d) illustrates that

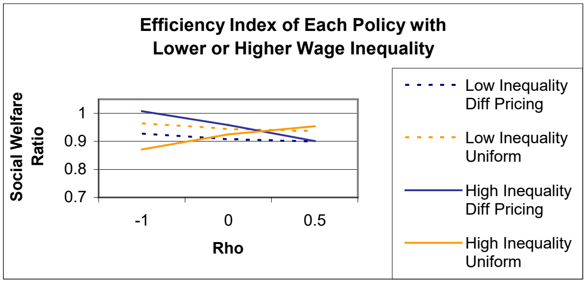
[13] While we do not illustrate it, we get analogous results if the wage disparity



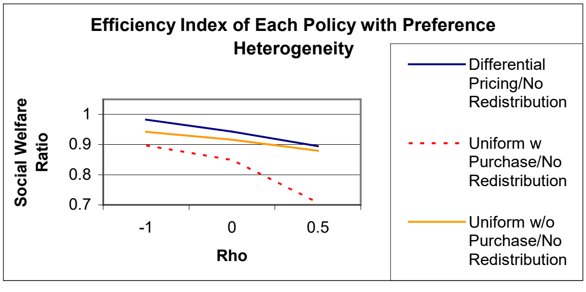
(a)



(b)



(c)



(d)

FIGURE 2

SIMULATIONS FOR $N_H = N_L = 100$, $s_L = s_H = .5$, $T = 24$, $w_L = 1$, $w_H = 5.4$, $\theta^* = 1$.

the introduction of mean-preserving heterogeneity in relative taste for g, from $\theta^{*} = 1$ to $\theta_R = .5$ and $\theta_S = 1.5$, also increases the range in ρ over which differential pricing dominates uniform provision.

V. Discussion and Conclusion

In this paper, we have considered how a policy maker could efficiently achieve “specific egalitarianism”, or make consumption of a good “essential to life or citizenship” independent of income, but increasing in relative strength of preference or need. We have assumed that the good in question cannot feasibly be on-sold, and that the policy maker’s information is limited to the distribution of wages and preference strengths in the population, rather than the earnings ability of any individual.[14] The policy maker would be exclusive purchaser of the good of interest from competitive producers, and would then make it available at outlets charging different money and time prices. A below-cost money price at one outlet would be accompanied by a positive time price, which would be set just high enough to make high wage individuals better off purchasing the good at the higher price outlet with no wait. These prices could be set to ensure that consumption of the target good was equalized across wage groups, while increasing in relative need, regardless of income.

Redistribution by differential pricing carries the efficiency cost of alterations to relative prices, and of time lost in queues that is not transferred to sellers. However, we show that this cost may be less than that under more conventional policies, such as uniform provision financed by proportional income taxation, with or without private purchase. For income tax also distorts relative prices, and uniform provision ignores differences in relative preferences for the target good. Allowing optional private purchase to accompany uniform provision actually makes specific egalitarianism *more* costly to achieve, because the policy maker must tax and spend enough to crowd out all private purchase of the target good.

remains at 5.4, and we reduce N_L / N_H from 1 to 1/3, or raise it to 3. A decrease in the proportion of the poor increases the range of ρ over which differential pricing achieves specific egalitarianism more efficiently than uniform provision.

[14] Our alternative uniform provision policies require knowledge of each person’s total income.

Even without private purchase, we find that uniform provision is likely to be a more costly way to achieve specific egalitarianism than differential pricing as 1) the relative importance of the target good rises, 2) the elasticity of substitution between the target good and other goods falls, 3) heterogeneity of preference for the target good rises, and 4) wage inequality increases or the proportion of the poor falls. Furthermore, uniform provision satisfies specific egalitarianism in letter but not in spirit, as consumption of the target good does not strictly increase in strength of need.

We note that real world examples of differential pricing, whether in health care, highway and ferry tolls, camping or park use permits, postal services, or immigration processing offer at most a few price/time combinations. This despite the fact that incomes (and abilities) follow a wide distribution. Nonetheless, with judicious use, even a few money/time price combinations will greatly diminish the disparity of income of individuals per outlet, and thus the inequality of consumption caused by such disparity.

Our proposal suffers from several limitations. First, as Nichols *et al.* (1971) observed, the existence of non-labor income uncorrelated with wage raises the possibility that, *e.g.*, wealthy retirees might choose outlets targeted to the poor. Second, the static nature of our model precludes its application to goods whose value to an individual would depreciate during the optimal queuing time, such as acute surgery. Perhaps most importantly, our mechanism was modelled with the restrictive assumption that the distribution of preferences for the target good is identical across income groups ($s_H = s_L = s$). Our mechanism still functions when the distribution of tastes varies with income, but two problems emerge as this variance grows. First, the ethical constraint that consumption be equalized across income groups (3) may lose its appeal, as it begins to penalize individuals in income groups who happen to have a higher proportion of strong preference for the good. Adjusting the equalization requirement to account for each income group's s could address this problem, but might jeopardise the second constraint (4) that consumption of g be non-decreasing in relative strength of preference, regardless of income. A second problem as s diverges across income groups is that the existence of feasible prices with a non-negative queuing time for low wage individuals can no longer be guaranteed. As the disparity in s across wage groups grows, our mechanism's tolerance for extreme disparities between strong and weak preference or relative size of wage groups is reduced.[15]

With these caveats in mind, we have provided a mechanism that can achieve specific egalitarianism without compulsory queues or income or ability tests, while respecting differences in people's relative preferences or needs.

Appendix 1: Proof that individuals are best off choosing the outlet with the lowest full price.

We claim that an individual will choose the target good outlet that offers the lowest full price given his wage. (He adjusts his time allocation between work and queuing accordingly).

Proof: Let the combined money and time budget set

$B = \{\ell, y, g : w_j\ell + y + P_{g,j}g \leq w_jT)$ for the price vector $(w_j,\ P_{g,j})$ and $B' = \{\ell, y, g : w_j\ell + y + P'_{g,j}g \leq w_jT)$ for $(w_j,\ P'_{g,j})$ and $P_{g,j} \leq P'_{g,j}$. Then, B' is contained in B. Hence, the maximum utility $V_{i,j}$ over B is at least as big as $V'_{i,j}$ over B'.

Appendix 2: Proof that the allocation of labor hours across production sectors is under-identified.

Define by $\alpha_{i,j}$ the proportion of individual i,j's total chosen labor hours devoted to production in the Y sector. For any arbitrary set of $\alpha_{i,j}$, total production of G and Y is given by

$$\sum_{i=R,S}\sum_{j=L,H} N_{i,j}w_j(1-\alpha_{i,j})L_{i,j} + \sum_{i=R,S}\sum_{j=L,H} N_{i,j}w_j\alpha_{i,j}L_{i,j} = \sum_{i=R,S}\sum_{j=L,H} N_{i,j}w_jL_{i,j} \tag{A.1}$$

$$= \sum_{i=R,S}\sum_{j=L,H} N_{i,j}[g_{i,j}p_j + y_{i,j}] = N_Hp_H[(1-s)g_{R,H} + sg_{S,H}] + N_Lp_L[(1-s)g_{R,L} + sg_{S,L}]$$
$$+N_H[(1-s)y_{R,H} + sy_{S,H}] + N_L[(1-s)y_{R,L} + sy_{S,L}] \tag{A.2}$$

The equality between (A.1) and (A.2) can be induced from a binding budget constraint $(w_jL_{i,j} = p_jg_{i,j} + y_{i,j})$. With equalized average consumption (3) achieved, this can be written as $[N_Hp_H + N_Lp_L\ ]g^*(\equiv (1-s)$

[15] For example, when $\rho = 0$, $w_L = 1$ and $w_H = 5.4$, all possible combinations of s_L and s_H are feasible when $N_L/N = N_H/N = .5$ and $\theta_S = 1.5$ and $\theta_R = .5$. If, however, $N_L/N = .9$ and $N_H/N = .1$, then if $s_H = 1$, s_L must exceed .083 for h_L to be non-negative. Or returning to the baseline, if $\theta_R = .1$ and $\theta_s = 1.9$, then if $s_H = 0$, s_L cannot exceed .476 for h_L to be non-negative.

$g^*_{R,j} + sg^*_{S,j}) + \sum_{i=R,S} \sum_{j=L,H} N_{i,j}y_{i,j}$.

Substituting in $p_H = \frac{N}{N_H} - \frac{N_L}{N_H} p_L$ from (6),

$$= [N_H + N_L]g^* + \sum_{i=R,S} \sum_{j=L,H} N_{i,j}y_{i,j} = \sum_{i=R,S} \sum_{j=L,H} N_{i,j}g_{i,j} + \sum_{i=R,S} \sum_{j=L,H} N_{i,j}y_{i,j} \quad \text{(A.3)}$$

Appendix 3: Proof that for the policy maker to achieve specific egalitarianism under uniform provision with private purchase, it is both necessary and sufficient to crowd out all private purchase of g.

Sufficiency:

Define as $\tilde{g}_{i,j}$ the level of uniform provision that would crowd out private purchase by an individual of preference type i and wage j. From the rhs of (12) it can be shown that $\partial\tilde{g}_{i,j}/\partial w_j > 0$ and $\partial\tilde{g}_{i,j}/\partial\theta_i > 0$. It follows that if $\tilde{g}$ is set high enough to crowd out the maximum $\tilde{g}_{i,j}$, namely $\tilde{g} \geq \tilde{g}_{S,H}$, every individual will consume only $\tilde{g}$, (or set $\hat{g}_{i,j} = 0$) and both (3) and (4) will be weakly but trivially satisfied.

Necessity:

We show that under all possible cases, $\tilde{g}$ must be set $\geq \tilde{g}_{S,H}$ or else (3) or (4) will be violated. From $\partial\tilde{g}_{i,j}/\partial w_j > 0$ and $\partial\tilde{g}_{i,j}/\partial\theta_i > 0$ it follows that $\tilde{g}_{R,L} < \tilde{g}_{R,H}$ and $\tilde{g}_{S,L} < \tilde{g}_{S,H}$. As well, from (12) it can be shown that $\hat{g}_{R,L} < g_{R,H}$ and $\hat{g}_{S,L} < g_{S,H}$ for any positive $\hat{g}_{i,j}$.

Case I: $\tilde{g}_{R,L} < \tilde{g}_{R,H} \leq \tilde{g}_{S,L} < \tilde{g}_{S,H}$.

If $\tilde{g} \leq \tilde{g}_{R,L}$ then the lhs of (3) will be $(1 - s)(\tilde{g} + \hat{g}_{R,L}) + s(\tilde{g} + \hat{g}_{S,L})$, and the rhs will be $(1 - s)(\tilde{g} + \hat{g}_{R,H}) + s(\tilde{g} + \hat{g}_{S,H})$. Since $\hat{g}_{R,H} > \hat{g}_{R,L}$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, average consumption across the two wage groups will not be equal.

If $\hat{g}_{R,L} < \hat{g} \leq \hat{g}_{R,H}$ then the lhs of (3) will be $(1 - s)\tilde{g} + s(\tilde{g} + \hat{g}_{S,L})$, and the rhs will be $(1 - s)(\tilde{g} + \hat{g}_{R,H}) + s(\tilde{g} + \hat{g}_{S,H})$. Again, since $\hat{g}_{R,H} \geq 0$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, equality will not hold.

If $\tilde{g}_{R,H} < \tilde{g} \leq \tilde{g}_{S,L}$ then the lhs of (3) will be $(1 - s)\tilde{g} + s(\tilde{g} + \hat{g}_{S,L})$ and the rhs will be $(1 - s)\tilde{g} + s(\tilde{g} + \hat{g}_{S,H})$. Equality will not hold.

If $\tilde{g}_{S,L} < \tilde{g} < \tilde{g}_{S,H}$ then (3) reduces to $\tilde{g} < (1 - s)\tilde{g} + s(\tilde{g} + \hat{g}_{S,H})$, or inequality.

If $\tilde{g}_{S,H} \leq \tilde{g}$, then (3) reduces to $\tilde{g} = \tilde{g}$. Only here is equality satisfied for any s and
(4) is (weakly) satisfied.

Case II: $\tilde{g}_{R,L} < \tilde{g}_{S,L} < \tilde{g}_{R,H} < \tilde{g}_{S,H}$.

If $\tilde{g} \leq \tilde{g}_{R,L}$ then the lhs of (3) will be $(1 - s)(\tilde{g} + \hat{g}_{R,L}) + s(\tilde{g} + \hat{g}_{S,L})$, and the rhs will be $(1 - s)(\tilde{g} + \hat{g}_{R,H}) + s(\tilde{g} + \hat{g}_{S,H})$. Since $\hat{g}_{R,H} > \hat{g}_{R,L}$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, average consumption across the two wage groups will not be equal.

If $\tilde{g} \leq \hat{g}_{S,L}$ then the lhs of (3) will be $(1 - s)\tilde{g} + s(\tilde{g} + \hat{g}_{S,L})$, and the rhs of (3) will be $(1 - s)(\tilde{g} + \hat{g}_{R,H}) + s(\tilde{g} + \hat{g}_{S,H})$. Since $\hat{g}_{R,H} > 0$ and $\hat{g}_{S,H} > \hat{g}_{S,L}$, average consumption across the two wage groups will not be equal.

If $\tilde{g}_{S,L} \leq \tilde{g} < \tilde{g}_{R,H}$, then the lhs of (3) reduces to $\tilde{g}$, and the rhs of (3) will be $(1 - s)(\tilde{g} + \hat{g}_{R,H}) + s(\tilde{g} + \hat{g}_{S,H})$. Since $\hat{g}_{R,H} > 0$ and $\hat{g}_{S,H} > 0$, average consumption across the two wage groups will not be equal.

If $\tilde{g}_{R,H} \leq \tilde{g} < \tilde{g}_{S,H}$, then using (3), $\tilde{g} < (1 - s)\tilde{g} + s(\tilde{g} + \hat{g}_{S,H})$, or consumption will not be equalized across wage groups.

If $\tilde{g}_{S,H} \leq \tilde{g}$. Only here is consumption equalized across wage groups and (4) is (weakly) satisfied.

Appendix 4: Derivation of the condition for a welfare comparison between differential pricing and uniform provision without private purchase when preferences are homogeneous.

Again defining $w_H = aw_L$, where $a \geq 1$, then under either policy,

$$\sum_{j=L,H} N_j V_j = N_L V_L + N_H V_H = N_L V_L + N_H a^{\frac{1}{2+\theta}} V_L = [N_L + N_H a^{\frac{1}{2+\theta}}] V_L \qquad \text{(A.4)}$$

Thus, $\sum_{j=L,H} N_j V_j(P^*_{g,j}) \geq \sum_{j=L,H} N_j V_j(\bar{g}^*, t^*)$ if and only if $V_L(P^*_{g,L}) \geq V_L(\bar{g}^*, t^*)$ (A.5)

or substituting in the functional forms, if and only if

$$\left(\frac{T}{2+\theta}\right)^{\frac{1}{2+\theta}} \left(\frac{w_L T}{2+\theta}\right)^{\frac{1}{2+\theta}} \left(\frac{\theta}{P^*_{g,L}} \frac{w_L T}{2+\theta}\right)^{\frac{\theta}{2+\theta}} \geq \left(\frac{T}{2}\right)^{\frac{1}{2+\theta}} \left(\frac{1}{1+\theta} \frac{w_L T}{2}\right)^{\frac{1}{2+\theta}} \left(\frac{\theta}{1+\theta} \frac{\bar{w} T}{2}\right)^{\frac{\theta}{2+\theta}} \qquad \text{(A.6)}$$

where $\bar{w} = [(N_L / N)w_L + (N_H / N)aw_L]$. Simplifying, this is equivalent to

$$\left(\frac{4(1+\theta)}{(2+\theta)^2}\right)^{\frac{1}{2+\theta}}\left(\frac{a\dfrac{N_H}{N}}{(a\dfrac{N_H}{N}+\dfrac{N_L}{N})}\,\frac{2(1+\theta)}{2+\theta}\right)^{\frac{\theta}{2+\theta}} \geq 1 \text{, or getting } a \text{ by itself,} \tag{A.7}$$

$$a = \frac{w_H}{w_L} \geq \frac{N_L / N_H}{\left(\dfrac{2}{2+\theta}\right)^{\frac{2+\theta}{\theta}}(1+\theta)^{\frac{1+\theta}{\theta}} - 1}. \tag{A.8}$$

To show that the right hand side of (A.8) is falling in θ, define $z = \left(\dfrac{2}{2+\theta}\right)^{\frac{2+\theta}{\theta}}(1+\theta)^{\frac{1+\theta}{\theta}}$. We show that a monotonic transformation, ln z, is rising in θ.

$$\frac{d\ln z}{d\theta} = \frac{1}{\theta^2}\left[2\ln(2+\theta) - \ln(1+\theta) - \ln 4\right]. \tag{A.9}$$

We consider the sign of (A.9) when $\theta \to 0$, $0 < \theta < 2$, and $2 \leq \theta$.

As $\theta \to 0$, by L'hopital's rule,

$$\frac{d\ln z}{d\theta} = \left.\frac{f'(\theta)}{g'(\theta)}\right|_{\theta=0} = \frac{\dfrac{2}{2+\theta} - \dfrac{1}{1+\theta}}{2\theta} = \frac{0}{0} \text{ at } \theta = 0. \tag{A.10}$$

Reapplying L'hopital's rule, $\left.\dfrac{f''(\theta)}{g''(\theta)}\right|_{\theta=0} = \dfrac{\dfrac{-2}{(2+\theta)^2} + \dfrac{1}{(1+\theta)^2}}{2} > 0$ at $\theta = 0$. (A.11)

For $0 < \theta < 2$, from (A.9) $\ln(2+\theta) - \ln(1+\theta) > \ln(2+\theta) - \ln(4)$, so $\dfrac{d\ln z}{d\theta} > 0$.

For $2 \leq \theta$, from (A.9) $\ln(2+\theta) - \ln(1+\theta) > 0$ and $\ln(2+\theta) - \ln(4) \geq 0$,

so $\dfrac{d\ln z}{d\theta} > 0$.

Appendix 5: Derivation of the welfare comparison between differential pricing and uniform provision without private purchase when preferences are heterogeneous.

In the special case where $\rho = 0$, and the distribution of preferences is identical across income groups, optimal prices adjust under differential pricing such that $g^*_{R,L} = g^*_{R,H}$ and $g^*_{S,L} = g^*_{S,H}$. Then, just as under uniform provision,

$$V_{R,H} = a^{\frac{1}{2+\theta_R}} V_{R,L} \text{ and } V_{S,H} = a^{\frac{1}{2+\theta_S}} V_{S,L}$$

where $a = w_H / w_L$. Social welfare under either policy can then be expressed as

$$\sum_i \sum_j V_{i,j} = (1-s)(N_L + a^{\frac{1}{2+\theta_R}} N_H) V_{R,L} + s(N_L + a^{\frac{1}{2+\theta_S}} N_H) V_{S,L} \tag{A.12}$$

Social welfare will be higher under differential pricing than uniform provision if and only if

$$\begin{aligned}&(1-s)(N_L + a^{\frac{1}{2+\theta_R}} N_H)(V_{R,L}(P^*_{g,L}) - V_{R,L}(\bar{g}^*, t^*)) \\ &+ s(N_L + a^{\frac{1}{2+\theta_S}} N_H)(V_{S,L}(P^*_{g,L}) - V_{S,L}(\bar{g}^*, t^*)) \geq 0\end{aligned} \tag{A.13}$$

We proceed by showing that the left hand side of (A.13) will be higher than the equivalent difference between mechanisms under homogenous preferences, $(N_L + a^{\frac{1}{2+\theta}} N_H)(V_{\theta,L}(P^*_{g,L}) - V_{\theta,L}(\bar{g}^*, t^*))$. In other words we will show that $(N_L + a^{\frac{1}{2+\theta}} N_H)(V_{\theta,L}(P^*_{g,L}) - V_{\theta,L}(\bar{g}^*, t^*))$ is increasing as preferences become heterogeneous. To do so, we consider a mean preserving spread (θ_R, θ_S) around homogenous preferences θ. This can be defined as $(1-s)\theta_R + s\theta_S = \theta$, where $\theta_R \equiv \theta - \varepsilon$, and $\theta_S \equiv \frac{\theta - (1-s)\theta_R}{s} \equiv \theta + \frac{(1-s)}{s}\varepsilon$. With this mean preserving spread, we can introduce heterogeneity while holding constant the (average) level of θ.

As shown in Appendix 4, for a given ratio wages, differential pricing will perform relatively better than uniform provision, or $V_{\theta,L}(P^*_{g,L}) / V_{\theta,L}(\bar{g}^*, t^*)$ is increasing in θ. We define Φ as the total marginal effect on $V_{\theta,L}(P^*_{g,L}) / V_{\theta,L}(\bar{g}^*, t^*)$ of increasing θ for the population, or

$$\Phi = \frac{\partial (N_L + a^{\frac{1}{2+\theta}} N_H)(V_{\theta,L}(P^*_{g,L}) - V_{\theta,L}(\bar{g}^*, t^*))}{\partial \theta} \tag{A.14}$$

Conversely, differential pricing will perform relatively worse than uniform provision as θ falls, or $V_{\theta,L}(P^*_{g,L})/V_{\theta,L}(\bar{g}^*, t^*)$ is decreasing in θ. The absolute value of the total (negative) marginal effect on $V_{\theta,L}(P^*_{g,L})/V_{\theta,L}(\bar{g}^*, t^*)$ of a decrease in θ is also Φ. By using total differentials, the total effect on social welfare of an increase in θ is

$$d(N_L + a^{\frac{1}{2+\theta}} N_H)(V_{\theta,L}(P^*_{g,L}) - V_{\theta,L}(\bar{g}^*, t^*)) = \Phi d\theta \tag{A.15}$$

Next, we assume hypothetically that a policy maker could identify people's preference type θ_R and θ_S so that he could provide a different level of g for each type even under a tax mechanism. The policy maker would apply $(\bar{g}^*, t^*)$ for the s proportion of population with θ_S and $(\bar{g}^{**}, t^{**})$ for the $(1 - s)$ proportion with θ_R. Under this hypothetical scenario, a mean preserving spread away from θ would have zero net effect on the difference in social welfare between differential pricing and tax-funded provision:

$$s\Phi d\theta\,|_{\theta \to \theta_S} + (1-s)\Phi d\theta\,|_{\theta \to \theta_R} = s\Phi((1-s)/s)\varepsilon - (1-s)\Phi\varepsilon = 0 \tag{A.16}$$

where $d\theta|_{\theta \to \theta_R} = \theta_R - \theta = -\varepsilon$ and $d\theta|_{\theta \to \theta_S} = \theta_S - \theta = ((1 - s)/s)\varepsilon$. Thus, a mean preserving spread (θ_R, θ_S) from θ will not change the relative performance between differential pricing and this hypothetical uniform provision.

However, we also know that social welfare under this hypothetical tax-funded provision with two tax and provision rates $(\bar{g}^*, t^*)|_{\theta=\theta_S}$ and $(\bar{g}^{**}, t^{**})|_{\theta=\theta_R}$ would be as much or higher than under our actual uniform provision mechanism $(\bar{g}^{**} = \bar{g}^*, t^{**} = t^*)$ since it would be the solution to a less constrained problem. Thus, a mean preserving spread (θ_R, θ_S) around any θ will increase $(N_L + a^{\frac{1}{2+\theta}} N_H)(V_{\theta,L}(P^*_{g,L}) - V_{\theta,L}(\bar{g}^*, t^*))$. It will increase the relative efficiency of differential pricing to uniform provision.

(Submitted Oct 23 2023, Accepted Nov 3 2023)

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