Can Counterfactuals Be Accounted for as Tense and Modality Combined?

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Kim, Jeongyoon. 2005. Can Counterfactuals Be Accounted for as Tense and Modality Combined?. SNU Working Papers in English Linguistics and Language 4, 41-58. Linguistic expressions of counterfactually usually accompany tense and modal morphemes, which opens up the question whether they can be reduced into logical operations pertinent to the two. An attempting trial in this approach appears to be successful, but can not satisfactorily explicate failures of strengthened antecedent and transitivity, cruxes ingenious to counterfactuals. Davidian possible world semantics provides a reasonable account for why the apparent success cannot be extended further by always accompanying incompatible worlds for each stage of the conjunct counterfactuals. (Seoul National University)

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1. Conceptualizing counterfactuals

Counterfactual constructions are unique in that while indicating apparent impossibility of their antecedent, they are expressed in linguistic forms that combine tense and modal elements, as seen by (1)

(1) If I had gone, I would have found happiness.

I did not in fact go, but I might have. What is unique here is that by asserting explicit falsehood of the antecedent the counterfactual is not asserting that any consequent can be inferred (as an ordinary material implication goes) but that in a hypothetical world similar to the actual world as far as it can be where the antecedents hold then the consequent must be true. A feature like this seems to be caught by corresponding linguistic expression of modal items ('would') adjoined with pluperfect constructions ('had') that insinuate the truth condition should be reconstructed in a
context other than the actual, present one. Then there would be some worth in attempting to cover counterfactual constructions with formal apparatuses of past tense and modality, as tried by Gamut (1991).

Former studies on this line of thought include: Prior (1961) where tense was understood as logical operator that has a proposition as argument and its tensed interpretation as value, which forms the base for past/pluperfect construction in sections below; Lewis (1969, revised 1973) that provided a classic analysis on counterfactuals and longtime controversy on irreducibility of these constructions into strict conditionals; Von Fintel (1999) that contradicted this claim by appealing to a specific type of entailment (Strawson entailment) supplemented with dynamically expansionable context; and finally, Gamut (1998) that invited a combined notion of tense and modality into evaluating the constructions at stake.

This paper will come to its end by evincing the tense modal combining approach will not resolve the counterfactuals, at least in its standard form. An approach like this gains some relevance in interpreting individual constructions, but never is able to maneuver through basic constraints on counterfactuals, namely those of strengthened antecedents and failure of transitivity. But let the attempt start from a brief refreshing on Prior’s tense logic system and standard modal logic.

2. Prerequisites: Prior tense system & standard modal logic digested

Prior tense logic (Prior 1967) consists of modeling on time flow and logical operator that illustrates tense semantics. The first assumption is that we view times as instants, that is, as objects which have no duration, which do not take time. This correlates with the idea that tense is not discrete, but continuous: There is no ‘gaps’ between moments in time. Thus Time flow is viewed as a set of time points, or instants that resemble that of radical numbers in nature. Furthermore, we distinguish between past, present (‘now’) and future, and assume that time has a specific directionality.

The intuition of the flow of time is formally captured by the
introduction of a precedence relation on the set of times. This guarantees that past times are earlier that future times. The set T is thus ordered by a reflexive, transitive, and anti symmetric realation $\prec$, which makes $(T, \prec)$ a partial order:

(i) $\prec$ is reflexive; $\forall t (t \prec t)$
   (every time instant is earlier than itself)
(ii) $\prec$ is transitive: $\forall t_1 \forall t_2 \forall t_3 ((t_1 \prec t_2 \& t_2 \prec t_3) \rightarrow t_1 \prec t_3)$
   (If $t_1$ is earlier than $t_2$ and $t_2$ is earlier than $t_3$, then $t_1$ is earlier than $t_3$)
(iii) $\prec$ is anti symmetric: $\forall t_1 \forall t_2 ((t_1 \prec t_2 \& t_2 \prec t_1) \rightarrow t_1=t_2)$
   (If $t_1$ is earlier than $t_2$ and $t_2$ is earlier than $t_3$, then $t_1$ is earlier than $t_3$)

Now that we have added a set of times to our semantics, we can start interpreting expressions that are dependent on reference to time. We start with the interpreting expressions that are dependent on reference to time. We start with the interpretation of tense morphemes. The past tense of sentence conveys the information that the situation holds at some time earlier that the speech time. The future tense requires the proposition toe be true at some time later than the speech time. One way of evaluating tensed sentences is to interpret tenses in natural language as operators on formulas (Prior 1967). Symbolized as $P$ for 'past' and $F$ for 'future', these operators quantify over the times $t$ with respect to which a proposition is evaluated. These propositional operators can be prefixed to any well formed formula to yield a new well formed formula. This gives us a legitimate extension of first order propositional logic, namely that if $\Phi$ is a well formed formula then $P\Phi$ and $F\Phi$ are also well formed formulae.

Intuitively, the operators $P$ and $F$ can be read as 'It was the case at some time in the past that $\Phi$' and 'It will be the case at some time in the future that $\Phi$' and 'It will be the case at some time in the future that $\Phi$.' We can illustrate how these operators can actually be put into use in translations below:

(2) (Bill gave Susan a book for her birthday.
     
     Translated. $P(Bill$ give Susan a book for her birthday)
(3) Susan will become a great linguist.

*Translated.* F(Susan become a great linguist)

With the help of our extended interpretation function we can make the semantics of P and F more precise:1)

(4) $[P\phi]^{M,t,g} = 1$ iff there exists $t' \in T$ such that $t' < t$ and $[\phi]^{M,t',g} = 1$

(5) $[F\phi]^{M,t,g} = 1$ iff there exists $t' \in T$ such that $t < t'$ and $[\phi]^{M,t',g} = 1$

That is, any expression constructed with operators of P and F gains its truth value contingent upon whether there exists time instance that satisfies given sentential restriction as compared to t, the reference time, respectively. In general (though not always), sentences like those in (2) are evaluated with respect to the time of speech situation. In such cases, the index t on the interpretation function is identified with the utterance time. The dependence on the time of the speech situation implies tense is an essentially deictic category. We often give the speech time a special index, e.g. $t_0$, to highlight the indexical nature of the tense system of natural language. Then the meaning of examples of (2) and (3) is spelled out in (6) and (7)

(6) $[P(Bill give Susan a book for her birthday)]^{M,t',g} = 1$ iff there exists a $t \in T$ such that $t < t_0$ and $[Bill give Susan a book for her birthday]^{M,t',g} = 1$

(7) $[F(Bill give Susan a book for her birthday)]^{M,t',g} = 1$ iff there exists a $t \in T$ such that $t_0 < t$ and $[Bill give Susan a book for her birthday]^{M,t',g} = 1$

The interpretations in (3) capture the intuition that sentences in the

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1) M stands for model, g for *variable assignment function* within the given language. A Model designates characteristic function between logical signs and entities given in a domain (world), while variable assignment function does the same work between unbound variable and logical signs. Detailed explanation upon the concepts goes beyond the scope of this article. See Cann(1993) for instance.
past or future tense refer to propositions which are true with respect to some time t prior or antecedent to the moment of speech to.

Modality in a linguistic construction can also be captured in a way similar to the tense one, this time the set of possible world W2) substituting for T. Modal expressions - allegedly including counterfactuals - presuppose some possible worlds where the concerned proposition retains simple truth values. Necessity holds when the proposition is true of all accessible possible worlds, while possibility holds if any of the worlds permit it to be true. Then modality also can be covered with logical operators quantifying over possible worlds that vary in range of worlds assigning truth to the proposition in concern. This line of characteristic can be captured with an expansion of first predicate logic (w stands for the world given)

(8) [□ϕ]_{M,w,g} =1 iff for all w' ∈ W such that w' is accessible from w, [ϕ]_{M,w,g} =1

(9) [ ◊ ϕ]_{M,w,g} =1 iff for at least one w' ∈ W such that w' is accessible from w, [ϕ]_{M,w,g} =1

One particular case demanding our attention is strict conditional. Simply speaking, a strict conditional is a material conditional preceded by some sort of necessity operator, i.e. □(ϕ→ψ). Given with the set of possible world accessible to the actual world i, we can infer from the formulation above that the strict conditional is true and only if and if only ψ is true at all worlds where ϕ (abbreviated as ϕ-world below) holds true. (Fig.1); there are no inaccessible ϕ worlds to be left out of consideration.

2) A ‘world’ can be roughly captured as a set of logically compatible propositions. Worlds can demonstrate a ‘degree’ of similarities between them contingent upon the number of propositions (and their concomitantly derivable results) they share. This notion helps modal thinking through allowing modal aspects viewed in extensions. See Cann (1993).
This can be paraphrased as:

\[(\Box(\phi \rightarrow \psi))^{M,w,g} = 1 \text{ iff for all } w' \in W \text{ such that } w' \text{ is accessible from } w, [\phi]^{M,w',g} = 1; [\psi]^{M,w,g} = 1. \text{ (strict conditional)}\]

In other words: \[(\Box(\phi \rightarrow \psi))^{M,w,g} = 1 \text{ iff } \psi \text{ is true at every } \phi \text{ world in } S_i \text{ at } M \text{ and } g. \text{ The formulation turns important when we come to analyze counterfactuals as modal variation.}\]

3. What characterizes counterfactual constructions

Counterfactuals are known for their hypothetical world additive capability and (somewhat weaker) concessive implication. Examples below show both of the insinuations:

(11) If Boris had gone to the party, Olga would still have gone.
(12) If J. Edgar Hoover had been born a Russian, then he would have been a Communist.

Boris would necessarily be accompanied by Olga in a state of affair-world-where he goes to the party. (But in fact he did not.) Likewise, Hoover would necessarily by a communist in a state of affair where he was born as a Russian. (But in fact he was not.) A counterfactual is a conditional the antecedent of which describes something which is definitely not true in the actual world, but the
consequent can still be true. Of course it is a natural outcome of a material implication that false antecedent can make any conditional true. But what counterfactuals assert goes much farther than that, by placing us to evaluate the antecedent in a context in which the antecedent is assumed to be true, with everything else kept intact as close as it can be to the actual world. (It is of no use in strongly proclaiming if Boris had gone to the party, Olga would have gone in a world where Olga went to the party anyway.) We can formalize this by saying that:

Any counterfactual is true in a world w if an only if all the worlds that most closely resemble w and are such that the antecedent is true and the consequent is also true (Lewis 1973)

Two of the features that distinguish counterfactuals from other ordinary if conditionals should be noted. One of which is that strengthened antecedent does not give license to the same consequent before the modification, as can be shown at the example below:

(13) Failure of strengthening the antecedent

If Boris had gone to the party, Olga would still have gone
does not entail:

If Boris had gone to the party and Olga had died, Olga would still have gone.

Let '□→' be a symbol marking the counterfactual relationship (whatever its logical structure might be) following Lewis(1973), then this can be summarized as:

(14) formulation on failure of strengthened antecedent

\[ \phi \square \rightarrow \psi \]

\[ \sim (\phi \land x \square \rightarrow \psi) \]

Another point worthy of our notice is a phenomena known as failure of (syllogistic) transitivity: whereas inferential rules of first order predicate logic guarantee ordinary conditionals to be transitive in syllogism (p→q & q→r then p→r in other words), an example
like below and many others negate such qualification onto counterfactual ones.

(15) Failure of syllogistic transitivity:
If Hoover had been a communist, he would have been a traitor. &
If Hoover had been born in Russia, he would have been a communist does not entail:
If Hoover had been born in Russia, he would have been a traitor.

Summarized as:

(16) Formulation on the failure of transitivity

\[
\phi \Box \rightarrow \psi \\
\chi \Box \rightarrow \phi \\
\sim (\chi \Box \rightarrow \psi)
\]

The two constraints will arouse a main crux for the attempt to be made upon counterfactuals.

4. Structuring counterfactual construction with modal logic combined with Prior tense system, and how it fails

Apparatuses on modal and tense intentions illustrated above seem to provide a concise way to grasp semantics of counterfactual constructions. Their hypothesis assuming sense will be covered by modal operators, while concessive (blurred to a degree, though) implicature fall unto workings of tense operators that the antecedent was not the case. Then an example like (6) which in turn will be primitively paraphrased as in (17), which in turn can be formulated in a format of (18).

(11) If Boris had gone to the party, Olga would still have gone.
(17) Boris if fact did not go. But it was at the time a necessary fact that he went, Olga was sure to go to the party, too.
(18) P (¬(Boris goes to the party) & (Boris goes to the party → F(Olga goes to the party))
There are some obstacles to be coped with, however, in this approach. As tense functions are defined at temporal instance set T and modal ones at the set of worlds, W, that are accessible\(^3\) from the actual worlds, the two notions are not interoperable as to guarantee satisfactory interplay between them as is required to cover a formula like (18). Reflecting on the formal formulation on past tense and modality of (4) and (8) above, There is something to be added for allowing the accessibility of worlds to each other change from time to time. \(\phi\) then must be revised as to be true in \(w\) at \(t\) iff \(\phi\) is true at \(t\) in each world \(w'\) accessible from \(w\). It presupposes the accessibility of worlds to each other to change from time. Intuitively, this can be achieved by providing \(R\), the function denoting accessibility relation, with a temporal parameter, thus:

\[
(8') \text{ (reformulated with the } R \text{ relation)} \\
\square_{M.w.g} = 1 \text{ iff for all } w' \in W \iff \text{ for all } w' \text{ such that } wRw': \square_{M.w'.g} = 1''
\]

\[
(19) \square_{M.w.g.t} = 1 \iff \text{ for all } w' \text{ such that } wR^t w': \square_{M.w'.g.t} = 1
\]

Paraphrasing, \(\phi\) is true in \(w\) at time \(t\) iff \(\phi\) is true at \(t\) in each world \(w'\) which is accessible from \(w\) at the time \(t\). Let \(R_t\) be defined in the following manner, \(wR_t^t w'\) holds just in case \(w\) and \(w'\) have the same history up until \(t\), then \(\square_{M.w.g.t} = 1\) if and in only any branching futures stemming from a given time point \(t\) result in developments that share identical situational results where \(\phi\) holds true.

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3) *accessibility* roughly stands for relevance of worlds evaluated against a given world, usually the current world as we are within. For example, any world where a statement 'water is H\(_2\)O' hold is not accessible to any world where water is not existent or H\(_2\)O stands for completely different chemical compose than it is in the current world. But the question of accessibility is a very intricate question, because the definition of different accessibility relations leads to different inference patterns on modal statements. In depth introduction to modal logic can be found in Hughes and Cresswell (1968).
Let us apply this revision of modal relationship to the time-related necessary conditional of (17). Let the antecedent be represented by $\phi$ and the consequent by $\psi$, then we gain a formulation of $P (\neg \phi \& \Box (\phi \rightarrow F \psi))$. This naturally leads to a structure of bifurcating (or multi-furcating) futures like Fig. 2 below.

![Diagram](image)

Fig. 2 $P (\neg \phi \& \Box (\phi \rightarrow F \psi))$ (bold lines depicting actual development of affairs)

Does this picture represent legitimately what the formula of (19) actually signifies? It apparently seems to be so. In order for (19) to be true, there must be some temporal instance earlier than 'now' where $\neg \phi \& \Box (\phi \rightarrow F \psi)$ holds true, pursuant to the premise (6). As $\neg \phi$ holds true at all worlds at instances from the t0 up until now, every instances in between satisfies the requirement, thus leaving $\neg \phi$ condition fulfilled within the range. $\Box (\phi \rightarrow F \psi)$ requires that any world temporally accessible to the one where $\phi$ holds true should also have $F \psi$ hold true, according to (10) and (19). Partitioned temporal ranges in the picture above help us to pass through the crux.

Any $\phi$-worlds at temporal points between t0 and t1 are accessible to each other in that they share the identical history up to the point. And they are all true of $F \psi$ in that any future branches stemming from the point result in $\psi$-world. World bifurcations between t1 and t2 are not accessible to each other without identical history between them, thus irrelevant to coverage of (19). This account is also effective for branchings between t1 and now. And
any world that shares 'same history' at each branch is allowing both \( \phi \) and \( F\Psi \), in penultimate line with the logic as of the temporal (world-) partition between \( t_0 \) and \( t_1 \). In sum, any hypothetical world accessible to each other within the range from \( t_0 \) and now are true of \( \phi \rightarrow F\Psi \), thus allowing \( (\phi \rightarrow F\Psi) \) to be true through the temporal backgrounds given here. Thus the entire formula \( P(\neg \phi \& \Box (\phi \rightarrow F\Psi)) \) also comes to be attributed of truth with both of its propositions satisfied. In summary, any strict conditionals defined over the standard \( T \) are legitimately accepted with all of its hypothetical branching futures satisfying the required consequent.

This appears quite satisfactory, and we see no reason why to consider tense and modal modeling like (19) should not be represent counterfactual constructions. Only that all future branchings in sum are given in hypothetical conclusion of \( \Psi \), (19) corresponds well to what the linguistic conclusions signify. But there still remain other points to be testified with characteristics of counterfactuals as well.

It is well known that \( ((\phi \& x) \rightarrow \Psi) \) always follows from \( (\phi \rightarrow \Psi) \) for if any world satisfying \( \phi \) allows \( \Psi \) to hold true as well, then any set of worlds that are more restricted than \( \phi \) world - that is, \( \phi \& x \)-world, necessarily permit \( \Psi \) to be true. Thus \( P(\neg \phi \& \Box (\phi \rightarrow F\Psi)) \) should entail \( P(\neg \phi \& \Box (\phi \& x) \rightarrow F\Psi)) \) by virtue of the strict conditional. Figure 3 ascertains this:

\[ P(\neg \phi \& \Box (\phi \rightarrow F\Psi)) \] is entailed by \( P(\neg \phi \& \Box (\phi \& x) \rightarrow F\Psi)) \)

\( (\neg \phi \& \Box (\phi \rightarrow F\Psi)) \) holds regardless of whether the antecedent is strengthened by \( x \), so branching after \( \phi \) took place are necessarily
true of $\Psi$ regardless of (hypothetical) factuality of $x$. But it is also true that branchings that both license $\phi$ and $x$ are also lead up to a development where $\Psi$ holds true. Thus it is safe to say that ($\neg \phi \& \Box(\phi \rightarrow F\Psi)$) entails P ($\neg \phi \& \Box((\phi \& x) \rightarrow F\Psi)$).

But this is not the case as of the linguistic counterfactuals, as was shown at the section on their characteristics. Even though if it is true that if Boris had gone to the party($\phi$) Olga would have gone to the party ($\Psi$) it does not induce if Boris had gone to the party ($\phi$) and Olga had died($x$), then Olga would have gone to the party ($\Psi$), for it is prima facie wrong to assert this. But it is what the strict conditional interpretation argues for. This lightens a discrepancy between the verbal expression and its formulation of (15), and it is the verbal expression that we are to account for primarily.

Another point to take notice is syllogism from two counterfactuals that share mediating premises. Under the strict conditional approach it is a given fact that ($\phi \rightarrow x$) always follows from ($\phi \rightarrow \Psi$) & ($\Psi \rightarrow x$), for any world permitting $\phi$ is $\Psi$ world and any world permitting $\Psi$ is $x$-world, then any world permitting $\phi$ has no alternative but to license $x$ as well. Thus P ($\neg \phi \& \neg \Psi \& \Box(\phi \rightarrow Fx)$) should follow from P ($\neg \phi \& \Box(\phi \rightarrow F\Psi)$) juxtaposed with P ($\neg \Psi \& \Box(\Psi \rightarrow Fx)$). This is illustrated in the figure 4:

![Figure 4](image)

**Fig. 4.** P ($\neg \phi \& \neg \Psi \& \Box(\phi \rightarrow Fx)$) from P ($\neg \phi \& \Box(\phi \rightarrow F\Psi)$) & P ($\neg \Psi \& \Box(\Psi \rightarrow Fx)$)

Any branching that select $\phi$ should develop into $\Psi$ according to
the first conditional and any branching stemming from $\Psi$ should result in $x$ world at any temporal points, according to the second conditional. Therefore any branch that selects $\phi$ necessarily comes up with $x$, which perfectly matches what strict conditional brings to the realm as a result of syllogism. But this is in mismatch with followings from linguistic expressions, as even if If Hoover had been a communist, he would have been a traitor and If Hoover had been born in Russia, he would have been a communist at the same time, these would not justify If Hoover had been born in Russia, he would have been a traitor at all, as in (15). And this time again, what primarily counts is the linguistic practice. Therefore there arises a concern for the reason why counterfactuals cannot be reduced into a combination of tense and modal operators -tensed strict conditional, namely- at least in its standard form.

5. Discussion: why the modal logic combined with tense system fails at explaining counterfactuals.

Remember that the frameworks above are depending upon strict conditional as well as Prior tense system. For $P (\neg \phi \& \Box(\phi \rightarrow \Psi))$, $[\Box(\phi \rightarrow \Psi)] M.w.g =1$ iff for all $W'$ such that $W'$ is accessible from $w$, $[\phi] M.w'.g =1$: $[\Psi] M.w'.g =1$. This requires, as in Fig. 1, set membership relation between the worlds that allow $\phi$ to be true and those allowing $\Psi$ true. But it should be noted that there was no additional concern for the similarity of the possible worlds that satisfy the requirement, for it was enough to find out any set of worlds within a certain range of similarity from the actual world that licenses both $\phi$ and $\Psi$. That is, it was rather unitary relation that concerned only about the truth conditional relation between the two predicates. But things can be different when we introduce another predicate, for example $x$, that is hypothetically independent from the two in respect of the closeness of the world to the actual one which licenses $x$. And this is the key in approaches to the cruxes of the failures at strengthened antecedent and syllogistic transitivity.

Let us first handle the strengthened antecedent problem. From the example above we can get $\phi$: "Boris goes to the party", $\Psi$: "Olga
goes to the party" and x: "Olga dies." And what should not be overlooked is that counterfactuals always tend to be interpreted at a possible world (or a set of them) as close as to the actual world. Also must be noticed is any addition of antecedents to the counterfactual ends up with expanded range of the set of world that licenses them as compared to the actual world, for any further modification to the antecedent would mean as much divergence from the actual world. That is, The set of worlds where Boris goes to the party and Olga dies is naturally less similar to the actual one than the set of worlds where Boris goes, ceteris paribus, to the party, given that neither Boris attend the banquet nor Olga dies in the actual world. These remarks can be summarized in the figure 5. (Lewis 1973):

![Diagram](image)

**Fig. 5**

Let $S_i$'s be sets of worlds that are on the same degree of similarity to the actual worlds, $i$. Their diameters represent different extent of closeness they are of to $i$. As counterfactuals tend to be quantified in a world as close as possible, $\phi$ firsthandedly finds its niche at $S_i^1$, which portion of the world also licenses $\psi$. But it also follows from the above that the $S_i$ in which both $\phi$ and $x$ hold are less similar to $i$ than the previous one, resulting in the intersection of diagrams representing $x$ and $S_i^2$. Given with eccentric semantic relationship between the world and the predicate $\psi$ - namely the
relationship between the world where Boris goes to the party and Olga dies and the worlds where Olga goes to the party. The portion of world does not permit \( \Psi \) to be true. And this negates a development of affairs as seen on Fig. 2.

Why does it happen in this way? It is because the \( \phi \) \& \( x \) worlds are so different in similarity from \( \phi \) only world that they have no common identical world to license both \( \phi \) and \( (\phi \& x) \). The set of worlds that allows both \( \phi \) \& \( x \) hold true include such worlds that are less similar to the actual world than those in the set of the worlds that permits only \( \phi \) true, for it must satisfy more mutatis mutandis conditions that are not true of I.

And this feature also holds for the set of \( (\phi \& x \& \Pi) \) worlds, \( (\phi \& x \& \Pi \& \xi) \) worlds..., progressively, rendering \( \Box((\phi\&x) \rightarrow \Box\Psi) \) and \( \Box(\phi \rightarrow \Box\Psi) \) not feasible simultaneously. Therefore if \( P(\neg\phi \& \Box(\phi \rightarrow \Box\Psi)) \) is true then \( P(\neg\phi \& \Box((\phi\&x) \rightarrow \Box\Psi)) \) cannot be true in the same context, and vice versa. It does not negate that either \( P(\neg\phi \& \Box(\phi \rightarrow \Box\Psi)) \) can be true or \( P(\neg\phi \& \Box((\phi\&x) \rightarrow \Box\Psi)) \) can be so, but renounces a possibility that both of them can hold at the same time. This entails the counterfactuals before and after the strengthening cannot be maintained in the same context, for \( x \) branchings at Fig.3 nullifies the worlds represented by Fig.2. Analyses depending upon strict conditional fail because of its disregard for difference in closeness of worlds to I.

Similar things can be said of the failure of transitivity. Drawing from the example (11) let each sentence be assigned with relevant symbols, i.e. \( \phi \): Hoover is a communist, \( \Psi \): Hoover is a traitor, and \( x \): Hoover is born in Russia. Principles on strict conditionals mandate \( \Box(x \rightarrow \Psi) \) to be inferable from \( \Box(\phi \rightarrow \Psi) \& \Box(x \rightarrow \Psi) \). But \( \phi \) and \( x \) quantified over worlds of different range of similarity with \( i \) can result in incompatibility of their worlds of play, as can be shown by the figure 6.:
Mutually independent antecedents of $\phi$ and $x$ presuppose worlds of different extent of similarity as was the case above. Let $\phi$ be able to be quantified in a more similar world to $i$ than $x$ is. (It is of no significance if the reverse is true of the two predicates). With the tendency of counterfactuals to be attributed with truth in a maximally similar world, ceteris paribus, $\phi$ finds its place in $S_{i1}$ while ($\phi \& x$) reserves its one at $S_{i2}$. Given with a predicate of consequent of unique set of worlds - for the purpose of argument that demarcates its sphere of truth, $\Psi$, then $\phi$ worlds also fall into $\Psi$ worlds while $x$ worlds do not, without any revisions added to the antecedent. A description like this is enough to have both ($\phi \rightarrow \Psi$) and ($x \rightarrow \phi$) true.

Their apparently transitive result of ($x \rightarrow \Psi$), consistent with interpretation of strict conditional, however, does not follow from their combination, for (i) the conjoint itself cannot take place in one, unified, identical world and (ii) any $\phi \& x$ licensing worlds (called for by the seconds premise) are not permissive of $\Psi$ licensing worlds. Then Fig 2. and Fig. 4 are also incompatible in tandem with any $x$ branchings at Fig. 4 shifting the world in concern to a completely different one. The analysis is structurally isomorphic to those for that of strengthened antecedent, with its world differentiating antecedents and a favoritism of the consequent for either of the worlds, but not both.
6. Summary and prospects

So far for counterfactuals viewed from tensed modal construction, or strict conditional analysis. Their apparent plausibility was given with some figures that ended up with any future bifurcations resulting in consequent, laying ground for a form like П (~Ф & □(Ф →Ψ)). But it soon met obstacles in failures of strengthened antecedent and failure of transitivity because of its disregard for similarity relationship of each world licensing the antecedents. In order for the second counterfactuals of the two respective cases to hold true, we had to expand the sphere of accessibility to reach some worlds that allows them to be so, and it falsified the first counterfactuals. Then it may be that for every stage of the sequence, there is a choice of set of maximally similar worlds. But the choice that works at every make false (i.e. shares no common world with) all the counterfactuals at previous stages, contingent upon semantic specification of the consequents. If counterfactuals are strict conditionals in its standard sense, we have no hope of deciding, once and for all, what exact context range of worlds they should have.

Is there any simple resolution that puts the stalemate into settlement? One approach is to denounce conditional approach and substitute it for a sui generis one. (Lewis 1973). Conditionals mandate one single context for interpreting any sentence including those with strengthened antecedent or with another syllogistic argument, which made them inherently indecisive as to what exact context they should be interpreted in. If counterfactuals are conceived as capable of being quantified over any of the related world, then the crux melts down and ‘all stages coexist in peace (ibid.).’ Another approach is to retain intuition on conditionals but legitimize instead expansion of contexts, e.g. from Φ world into Χ-world, by an operation of dynamic semantics (Von Fintel 1999), with de facto similar effect with the former attempt. Either way is beyond the coverage of this paper, but clearly neither is admitting standard tensed modal logic into counterfactuals. And this is what this paper targeted to convey.
References


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