RISK-POOLING EFFECTS OF DIRECT SHIPPING IN A ONE-WAREHOUSE N-RETAILER DISTRIBUTION SYSTEM

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〈目 次〉

Ⅰ. Introduction

The other day I visited one of the discount stores in my area, Service Merchandise to buy a humidifier. I decided to buy one of the models exhibited in the store, filled out an order form, and submitted it to a cashier. But the cashier told me that the model I chose was out of stock. After checking its availability at other places (i.e., the company distribution center and other stores), she suggested that she could ship what I wanted to me directly without additional shipping and handling charge. I did so and the humidifier arrived in about a week. Through this experience, I got to know that this type of direct shipping from a distribution center or a retail store with available inventory is very common in the U.S retail industry. This paper models direct shipping in a multi-echelon distribution system.

This paper examines the multi-echelon distribution system that consists of one warehouse and multiple retailers and allows the direct shipments from the warehouse or the retailers to an individual customer at the end of each replenishment cycle. In particular, part of each system-replenishment quantity can be kept at the warehouse for direct shipments. If at the end of cycle a retailer is out of stock and the warehouse has available stock, demand is met from the warehouse inventory. If the warehouse can't satisfy all the backorders, the retailers with available inventory at the end of cycle ship their inventory directly to customers in need. In case when system-wide demand during a replenishment cycle is greater than initial system-wide inventory, the difference is backordered. The system with direct shipments requires fewer inventory to attain a specified service level (= portion of demand met from inventory on-hand both at the retailers and at the warehouse) than the equivalent inventory system without them. But it incurs additional shipping cost. The optimal replenishment policy balances reduced inventory-holding and backorder costs with additional shipping cost, i.e., minimizes the sum of expected inventory-holding, backorders, and shipping costs.

We want to identify the form of the optimal replenishment policy in a one-warehouse N-retailer distribution system and interpret it in the Newsboy problem context. In addition, we want to determine the effects of direct shipping on the optimal system-replenishment quantity.

Ⅱ. Model and Assumptions

This paper studies a one-warehouse N-retailer system facing stochastic demand and operating in a periodic-review mode. In the specific system examined, the warehouse places a system-replenishment order every period, and receives it after a fixed leadtime L. At that time the warehouse makes allocation decision; that is, the warehouse retains part of the system-replenishment quantity, and allocates the rest to the N retailers. The allocations to the retailers are delivered after a fixed leadtime l.

At the end of each cycle, retailer backorders are met with the warehouse inventory or available inventory at other retailers through direct shipments to customers. If total retailer backorders at the end of cycle is greater than the warehouse inventory plus total excess inventory at the retailers, the difference is backordered. The objective of the paper is to develop a model that identifies the form of the optimal system-replenishment policy. The additional assumptions are as follows:

(i) Period demand is i.i.d. across periods and among the retailers.

(ii) Inventory-holding and backorder costs are a linear function of net inventory. Unit inventory-holding and backorders cost per period are h and p, respectively.

(iii) Unit shipping costs to an individual customer (t) are identical whether shipping from the warehouse or from any of the retailers, and are a linear function of quantities shipped.

(iv) That is, direct shipping is economical. If at the end of cycle, a retailer experiences backorders and there are available inventory at the warehouse or at any other retailer, it is economical to ship available units to customers in need.

(v) Equal allocation is optimal: that is, at the time of allocation, we replenish every retailer up to the same amount. This is true if we relax the non-negativity constraints on retailer allocations, which is called the "allocation assumption" (Eppen & Schrage, 1981).

Ⅲ. Previous Study

There are many articles on replenishment and allocation policies of a one-warehouse N-retailer system. Both Schwarz et al. (1985) and Badinelli and Schwarz (1988) investigate the so-called "portfolio" motive for holding warehouse safety-stock inventory in a continuous-review system operating under a (Q,R) inventory-replenishment policy. The results of these two papers indicate that the value of using warehouse inventory to rebalance retailer inventories between system replenishments is very small. This observation has many things to do with their service rule, FCFS (first come, first served).

Schwarz (1989) examines the value of warehouse risk-pooling over outside-supplier leadtimes in a periodic-review system in which the warehouse holds no inventory and "static" allocations (= allocations to all the retailers are made at the same time) are made to all the retailers. He assumes that having the warehouse between the supplier and the retailers increases supplier- to-retailer leadtime. He extracts an explicit "price" of risk-pooling; i.e., extra pipeline inventory-holding cost from increased leadtime.

Jnsson and Silver (1987) analyze a periodic-review system with total redistribution of inventory among retailers one period before the end of the order cycle, and compare the expected backorders of this system with that of the system without redistribution. Computational tests show that the system with redistribution can provide the same service level (as the system without redistribution) with a considerably reduced inventory investment. McGavin, Schwarz, and Ward (1993) develop an infinite-retailer model and use it to determine two-interval allocation heuristics for N-retailer systems. Simulation tests suggest that the infinite-retailer heuristic policies are near-optimal for as few as two retailers, and that the risk-pooling benefits of allocation policies with two well-chosen intervals are comparable to those of base-stock policies with four equal intervals. Kumar, Schwarz, and Ward (1995) study the risk-pooling effect of a "dynamic" inventory-allocation policy (= allocations are made sequentially at each retailer) in a periodic-review system with one warehouse and N retailers. In their model a delivery vehicle visits retailers along a fixed route. They compare static and dynamic allocation policies. Through simulation experiments they conclude that dynamic allocations yield significantly lower holding and backordering costs per replenishment cycle than static allocations.

IV. The Model

In this section, we formulate the system-replenishment problem that minimizes the sum of expected inventory-holding, backorders, and shipping costs in a single cycle. We use the following notation for the formulation.

Notation

p = backorders cost per unit per period

h = holding cost per unit per period

t = shipping cost per unit

L = outside-supplier's leadtime

l = leadtime between the warehouse and any retailer

y = order-up-to level at the time of system-replenishment order

$s = \frac{y}{N}$, allocation to each retailer

α = portion of s assigned to each retailer at the time of allocation

αs = allocation to each retailer

$(1-\alpha)s = \frac{\text{stock kept at the warehouseat the time of allocation}}{N}$

δ_i = random period demand at retailer i, $i=1,\ldots,N$, with p.d.f. $\phi(.)$and c.d.f. $\Phi(.)$

$\delta_i^n = \sum_{i=1}^{N}\delta_i$, with p.d.f. $\phi^N(.)$ and c.d.f. $\Phi^N(.)$

Under the allocation assumption, it is easily shown to be true that a myopic policy is optimal in the direct-shipping case. See Federgruen and Zipkin (1984) and Kumar et al. (1995). Henceforce, we focus our attention on the myopic problem. When direct shipments are allowed, system backorders each cycle depend on system-wide available inventory (warehouse inventory plus excess inventory at the retailers) and total retailer backorders at the end of cycle.

Assuming that it costs the same whether we directly ship from the warehouse to an individual customer or from any of the retailers, it is optimal that the warehouse allocates all units to the retailers at the beginning of each cycle. We present the following lemma without the proof.

(*Lemma 1*) If it costs the same whether we directly ship from the warehouse to an individual customer or from any of the retailers, the optimal α is 1.

For simplicity of the presentation, without loss of generality, we assume that the leadtimes (L and l) are both zero. If s is assigned to each retailer at the time of allocation, total expected costs per cycle is

$$\begin{aligned} TC &= p\int_{Ns}^{\infty}(\delta - Ns)\phi^{N}(\delta)d\delta + h\int_{0}^{Ns}(Ns-\delta)\phi^{N}(\delta)d\delta \\ &\quad + t(N\int_{s}^{\infty}(\delta - s)\phi(\delta)d\delta - \int_{Ns}^{\infty}(\delta - Ns)\phi^{N}(\delta)d\delta) \end{aligned} \tag{1}$$

In (1), the first and second terms represent expected system backorders and inventory-holding cost after direct shipments, respectively, and the third expected system direct-shipping cost. When system-wide cycle demand exceeds initial system inventory, we can't satisfy all of retailer backorders through direct shipping at the end of cycle. This adjustment is shown in the third term in (1).

Given (1), we can prove the following lemma.

(Lemma 2) When direct shipments are allowed at the end of each cycle, the optimal system-replenishment policy is a base-stock policy.

Proof: By taking the second derivative of (1) with respect to s, we get

$$\frac{\partial^2 TC}{(\partial s)^2} = (p+h-t)N^2\phi^N(Ns) + tN\phi(s) \tag{2}$$

It is clear that (2)>0 for $\forall s$ since we assume that $p+h-t>0$. The optimal system-replenishment policy is a base-stock policy, since (1) is a convex function of s. []

Let x to be the system inventory position at the beginning of cycle. According to (Lemma 2), we order up to Ns^* if $x<Ns^*$, order nothing if $x \geq Ns^*$, where s^* minimizes (1).

(Lemma 2) says that a base-stock policy is optimal. The following lemma presents the necessary and sufficient condition of the optimal s.

(*Lemma 3*) s is optima if and only if s satisfies

$$\Phi(s)=\frac{p(1-\Phi^N(Ns))+(t-h)\Phi^N(Ns)}{t} \tag{3}$$

<u>Proof</u>: By taking the first derivative of (1), we get

$$\frac{\partial TC}{\partial s}=-(p-t)N(1-\Phi^N(Ns))+hN\Phi^N(Ns)-tN(1-\Phi(s)) \tag{4}$$

By setting (4) to be zero and solving for , we can get (3). []

We can interpret (3) in the Newsboy problem context. According to (3), the cost of under-stocking and that of over-stocking are $p(1-\Phi^N(Ns))+(t-h)\Phi^N(Ns)$ and $(t-p)(1-\Phi^N(Ns))+h\Phi^N(Ns)$, respectively. These costs are conditional expectations on if system inventory can meet system demand. In particular, the cost under-stocking is p if system hasn't enough inventory to meet all the demand since the unit should be backordered, and $(t-h)$ if it has enough inventory since the unit will be shipped from any retailer with available inventory to an individual customer in need. The same interpretation is possible for the cost of over-stocking; i.e., it is $(t-p)$ if system hasn't enough inventory to meet all the demand, h if it has.

It is also interesting to compare the optimal s in the direct-shipping case with that of the no direct-shipping case. The following lemma identifies the effect of direct shipping on the magnitude of system-replenishment quantity.

(*Lemma 4*) When direct shipping is allowed, the warehouse orders less from the supplier.

<u>Proof</u>: Without direct shipping, the optimal s satisfies

$$\Phi(s) = \frac{p}{p+h} \tag{5}$$

Subtracting the right-hand side of (5) from that of (3), we get

$$\frac{(p+h-t)(p(1-\Phi^N(Ns^*)) - h\Phi^N(Ns^*))}{t(p+h)} \tag{6}$$

The sign of (6) is determined by that of since . Rearranging (3), we get

$$\Phi(s^*) = \Phi^N(ns^*) + \frac{p(1-\Phi^N(Ns^*)) - h\Phi^N(Ns^*)}{t} \tag{7}$$

Since $\Phi(s^*) < \Phi^N(Ns^*)$, $\frac{p(1-\Phi^N(Ns^*)) - h\Phi^N(Ns^*)}{t}$ should be negative. Therefore, (6) should be negative. This proves the lemma. []

Ⅴ. Concluding remarks

This paper analyzes the two-echelon distribution system with direct shipping to identify the form of optimal system-replenishment policy. We found that the optimal system-replenishment policy is a base-stock policy. We also presented the necessary and sufficient condition for the optimal system-replenishment quantity. Finally, we proved that the warehouse orders less with direct shipping.

The possible extensions of this paper would include incorporating different shipping costs for the warehouse and each retailer and prohibiting direct shipping from the retailers, both of which will give more reality to the model but complicate the analysis.

References

Badinelli, R. D. & Schwarz, Leroy B. (1988), "Backorders Optimization in a One-Warehouse N-Identical Retailer Distribution System," *Naval Research Logistics*, 35:5, pp. 427-440.

Clark, A. J. & Scarf, H. E. (1960), "Optimal Policies for a Multi-Echelon Inventory Problem," *Management Science*, 6:4, pp. 475-490.

Eppen, G. D. & Schrage, L. (1981), "Centralized Ordering Policies in a Multi-Warehouse System with Leadtimes and Random Demand," Multi-Level Production/Inventory Control Systems: Theory and Practice, pp. 51-67, edited by L. B. Schwarz, North-Holland.

Federgruen, A., & Zipkin, P. (1984), "Approximations of Dynamic Multilocation Production and Inventory Problems," *Management Science*, 30, pp. 69-84

Hadley, G. & Whitin, T. M. (1963), *Analysis of Inventory Systems*, Englewood Cliffs, Prentice Hall.

Jnsson, H. & Silver, E. A. (1987), "Analysis of a Two-Echelon Inventory Control System with Complete Redistribution," *Management Science*, 33, pp. 215-227.

Kumar, A., Schwarz, L. B., & Ward, J. (1995), "Risk-Pooling along a Fixed Delivery Route Using a Dynamic Inventory-Allocation Policy," *Management Science*, 41, pp. 344-362.

McGavin, E. J., Schwarz, L. B., & Ward, J. E. (1993), "Two-Interval Inventory Allocation Policies in a One-Warehouse N-Identical Retailer Distribution System," *Management Science*, 39, pp. 1092-1107

Schwarz, Leroy B. (1981), "Physical Distribution: The Analysis of Inventory and Location," *AIIE Transactions*, 13, pp. 138-150.

Schwarz, Leroy B. (1989), "A Model for Assessing the Value of Warehouse Risk-Pooling: Risk-Pooling over Outside Supplier Leadtimes," *Management Science*, 35:7, pp. 828-842.

Schwarz, L. B., Deuermeyer, & Badinelli, R. D. (1985), "Fill-Rate Optimization in a One-Warehouse N-Identical Retailer Distribution System," *Management Science*, 31:4, pp. 488-498.