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Timing of Demand Realization in an LP and Operations Strategies*

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Abstract

In a linear programming (LP), market demand is assumed to be constant, but the demand is often random variable which is to be realized as time lapses. We refined the previous work of Higle and Wallace (2003) who studied an LP depending on the timing of market demand realization. The motivation of this research is the need to consider three operational strategies used for dealing with demand uncertainty. We improved the previous work in the following aspects. These are the strategy of speed, the strategy of forecasting, and the strategy of outsourcing. Nine distinct LP examples are studied depending on the velocity of supply chain process and the type of operations strategy.

Keywords: linear programming, stochastic demand, quick response, forecasting, outsourcing

INTRODUCTION

Sensitivity analysis is often used for dealing with the variability in a linear programming (LP). But there is much stochastic uncertainty which sensitivity analysis can not deal with. One of such typical cases is the LP with stochastic demand. In an LP we usually take the market demand for our products as a constant and use it as a right hand side for a constraint. However the

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demand is often a random variable which is to be realized as time lapses. Higle and Wallace (2003) dealt with three cases in an LP depending on the timing of the market demand realization. In this paper, we analyze a similar LP problem and study three operations strategies. In Sen and Higle (1999), more complete introduction on stochastic linear programming was given and outsourcing was partially mentioned.

Compared with Higle and Wallace (2003), this paper has several distinctions. First, we categorize the constraints into two sets: short-term and long-term constraints. Short-term constraints are the ones which are affected by the decision making itself in an LP. And long-term constraints are the ones which the decision maker should take as given for modeling purpose. According to our definition, all the constraints in Higle and Wallace (2003) are shot-term constraints. Second, Higle and Wallace (2003) develop three cases of LP depending on the timing of the demand realization. But we offer the interpretation of the timing in a more proactive perspective. That is, we introduce the speed in operations strategy in dealing with the three cases. The speediest producer could get the parts and produce the goods after watching the realized market demand. The second speediest producer could manufacture the products after buying the required parts and watching the market demand. This would be the case such as overseas sourcing where the lead-time for parts is fairly long. The least speedy producer should source the parts and manufacture the goods before knowing the actual market demand. We can interpret the three cases in Higle and Wallace (2003) depending on the lead-times of sourcing and manufacturing. Third, we introduce the operations strategy of demand forecasting. By using a forecasting, we can refine the prior probability on the market demand and improve our decision making output. Fourth, we deal with the operations strategy of outsourcing. We consider the option of outsourcing which relaxes a long-term constraint. Therefore, in analyzing an LP with stochastic demand, we deal with the three operational strategies (quick response, forecasting, and outsourcing) which are considered to be important in production and operations management area.

SAMPLE LINEAR PROGRAM

In this paper we introduce a sample linear program for comparison purpose. Firm S manufactures two kinds of products ($i = 1, 2$), and the market prices are $10 and $20 respectively. We can think of product $i = 2$ as high-end goods and $i = 1$ as economy version of 2. The market demand is assumed to be stochastic depending on the market condition ($j = 1, 2, 3$), and the prior probability distribution of the market condition is as follows:

Table 1.

j	1	2	3
$\{D_1, D_2\}$	(20,10)	(25,20)	(30,28)
$P(j)$	.3	.5	.2

Firm S needs two kinds of resources in manufacturing the goods: parts and machines. Parts can be procured without limit from the market at $4/unit. Available machine hours are assumed to be fixed since machines are capital assets. These two resources define the constraints for firm S. The constraint for the parts is a short-term constraint, and the machine time availability offers a long-term constraint. Denoting r_1 and m_i as the amount of parts procured and quantity of product i manufactured respectively, we have the following two constraints:

$$m_1 + 2m_2 \leq r_1$$
$$3m_1 + 4m_2 \leq 120$$

We should note that the first constraint is a short-term constraint and the second one is a long-term constraint. We denote s_i and D_i (random variable) as the sales amount and demand of product i, and then the optimization problem can be described as follows.

$$\text{Max}\{10s_1 + 10s_2 - 4r_1\}$$

s.t.

$m_1 + 2m_2 \le r_1$

$3m_1 + 4m_2 \le 120$

$s_1 \le m_1$

$s_1 \le D_1$

$s_2 \le m_2$

$s_2 \le D_2$

Three stages in the supply chain for firm S are 'parts procurement → manufacturing → sales'. As in Higle and Wallace (2003), we can consider three cases of this problem depending on when $\{D_i\}$ are realized and known to firm S. Case 1 (after manufacturing) is the one where firm S should procure the parts and manufacture the goods before knowing the realized demand. Case 2 (between procurement and manufacturing) is the one where the firm should procure the parts first but can manufacture the goods after watching the realized demand. Case 3 (before procurement) is the one where the firm can procure the parts and manufacture the goods after getting the realized demand information. Firm S is assumed to be risk-neutral, and thus it considers the expected value in maximizing its profit. We now deal with each of the operations strategy for the three cases.

STRATEGY OF VELOCITY

Case 1

In this case the decision variable set is to be $\{s_{ij}, m_i, r_1\}$, and we should note that the decision variables of $\{r_1, m_1, m_2\}$ ought to be defined independent of market condition j.

The optimal objective function value is \$270 and the optimal solutions are

$s_{11} = 20$, $s_{12} = 20$, $s_{13} = 20$, $s_{21} = 10$, $s_{22} = 15$, $s_{23} = 15$, $m_1 = 20$, $m_2 = 15$, $r_1 = 50$.

Case 2

In this case we should decide the procurement of parts before the demand realization and can manufacture and sell products afterwards. Thus the decision variables are $\{m_{ij}, s_{ij}, r_1\}$. We note that the manufacturing and selling amount can depend on the market condition of j. Expected value of the objective function in our example becomes

$$z_2 = .3(10s_{11} + 20s_{21}) + .5(10s_{12} + 20s_{22}) + .2(10s_{13} + 20s_{23}) - 4r_1.$$

The optimal objective function value is \$280 and the optimal solutions are

$$s_{11} = 20,\ s_{12} = 13.33,\ s_{13} = 0,\ s_{21} = 10,\ s_{22} = 20,\ s_{23} = 26.67,\ m_{11} = 20,\ m_{12} = 13.33,\ m_{13} = 0,\ m_{21} = 15,\ m_{22} = 20,\ m_{23} = 26.67,\ m_1 = 53.33.$$

In interpreting the optimal solution of case 2, we should note that only four variables of $\{s_1, s_{2j}, m_{1j}, m_{2j}\}$ from $\{s_{ij}, m_{ij}\}$ are implemented according to the ex-post market condition j.

Case 3

Here we can solve three separate LPs depending on the market condition j. For $j = 1$ as an example, we substitute $D_1 = 20$, $D_2 = 10$ in the optimization model and solve the corresponding LP. The optimal objective function value and the optimal solutions are $\pi(1) = 240$, $s_1 = 20$, $s_2 - m_1 = 20$, $m_2 = 10$, $r_1 = 40$.

Likewise we can derive the optimal objective function values for $j = 2$ and $j = 3$ as $\pi(2) = 320$, $\pi(3) = 352$. Thus the expected value of the objective functions in case 3 is $\pi_3 = P(j = 1)\pi(1) + P(j = 2)\pi(2) + P(j = 3)\pi(3) = .3*240 + .5*320 + .2*352 = 302.4$.

In summary, we get the following table of the optimal objective

Table 2.

Speed	Case 1	Case 2	Case 3
Base	270	280	302.4

Table 3.

j \ e	1	2	3
1	.8	.1	.1
2	.1	.8	.1
3	.1	.1	.8

function values. We can see that the values increase as we go from left to right as expected. This is because the firm has faster lead-time in procurement and manufacturing as it goes to the right. The difference in values represents the value of faster lead-time in production process.

STRATEGY OF FORECASTING

In this section, we consider the option of using a marketing research firm which offers complementary demand forecasts. The estimate information on the market condition offered by the research firm is denoted as e. Analysis of the historical data reveals the conditional probability of $P(e|j)$ as follows.

Using the Bayes' theorem with the conditional probability of $P(e|j)$ and the prior probability of $P(j)$, we can derive the refined probability of $P(e|j)$. For example,

$$P(j=1 \mid e=1) = \frac{P(j=1, e=1)}{P(e=1)} = \frac{P(e=1, j=1)P(j=1)}{\sum_j P(e=1 \mid j)P(j)}$$

$$= \frac{.8*.3}{.8*.3+.1*.5+.1*.2} = \frac{24}{31}.$$

We can thus construct the table for $P(e|j)$ as follows:

In deriving this probability, we also get the following

Table 4.

j \ e	1	2	3
1	24/31	5/31	2/31
2	3/45	40/45	2/45
3	3/24	5/24	16/24

Table 5.

e	1	2	3
$P(e)$	.31	.45	.24

probability of $P(e)$.

For case 3, we do not need to use the research firm since firm S has the perfect information on demand before solving an LP. However for cases 1 and 2, firm S can refine the prior probability on demand and improve the LP formulation using the estimate e from the research firm.

Case 1

In this case, we should solve an LP for each value of e. For an example, the expected value of the objective function for $e = 2$ is

$$z(e = 2) = \frac{3}{45}(10s_{11} + 20s_{21}) + \frac{40}{45}(10s_{12} + 20s_{22}) + \frac{2}{45}(10s_{13} + 20s_{23}) - 4r_1.$$

As an example, the LP for $e = 2$ gives us the optimal objective function value and the optimal solutions as follows.

$$\pi(e = 2) = 306.67,\ s_{11} = 13.33,\ s_{12} = 13.33,\ s_{13} = 13.33,\ s_{21} = 10,\ s_{22} = 20.\ s_{23} = 20,\ m_1 = 13.33,\ m_2 = 20,\ r_1 = 53.3$$

Depending on the estimate e from the research firm, we should construct and solve the corresponding LP. The optimal objective function values for each e are $\pi(e = 1) = 240$, $\pi = 306.67$, $\pi(e = 3) = 295$. Thus the expected value of the optimal objective function is

$$\sum_{k=1}^{3} P(e = k)\pi(e = k) = .31 * 240 + .45 * 306.67 + .24 * 295 = 283.$$

Case 2

Following the same logic, we note that an LP for $e = 1$ in case 2 gives us the optimal objective function value and the optimal solutions as follows.

$\pi(e = 1) = 240$, $s_{11} = 20$, $s_{12} = 0$, $s_{13} = 0$, $s_{21} = 10$, $s_{22} = 20$. $s_{23} = 20$, $m_{11} = 20$, $m_{12} = 0$, $m_{13} = 0$, $m_{21} = 10$, $m_{22} = 20$, $m_{23} = 20$, $r_1 = 40$.

Likewise, we get $\pi(e = 2) = 311.11$. and $\pi(e = 3) = 317.55$. Therefore, the expected value of the optimal objective functions for case 2 is

$$\sum_{k=1}^{3} P(e = k)\pi(e = k) = .31 * 240 + .45 * 311.11 + .24 * 317.55 = 290.6$$

Summarizing the expected value of the optimal objective functions for each case, we get the following table.

Table 6.

	Case 1	Case 2	Case 3
Forecasting	283.2	290.6	302.4

STRATEGY OF OUTSOURCING

In this section, we deal with the strategy of outsourcing machine hours. This means that we convert the long-term constraint into a short-term constraint. By incorporating the available machine hours into the decision domain, we can improve the objective function. In our example, we denote r_2 as the machine hours outsourced and the cost is assumed to be \$0.5/hour. Then the optimization model becomes:

$\text{Max}\{10s_1 + 20s_2 - 4r_1 - 0.5r_2\}$
s.t.
$m_1 + 2m_2 \le r_1$
$3m_1 + 4m_2 \le 120 + r_2$

$s_1 \le m_1$
$s_1 \le D_1$
$s_2 \le m_2$
$s_2 \le D_2$

Case 1

The objective function should be the expected value using the prior probability.

$$z = .3(10s_{11} + 20s_{21}) + .5(10s_{12} + 20s_{22}) + .2(10s_{13} + 20s_{23}) - 4r_1 - .5r_2.$$

From the appropriate LP, we get the optimal objective function value and the optimal solutions as follows.

$\pi = 297.5$, $s_{11} = 20$, $s_{12} = 25$, $s_{13} = 25$, $s_{21} = 10$, $s_{22} = 20$, $s_{23} = 20$, $m_1 = 25$, $m_2 = 20$, $r_1 = 65$, $r_2 = 35$.

Case 2

Likewise, we get the optimal objective function value and the optimal solutions as follows.

$\pi = 297.5$, $s_{11} = 20$, $s_{12} = 25$, $s_{13} = 9$, $s_{21} = 10$, $s_{22} = 20$, $s_{23} = 28$, $m_{11} = 20$, $m_{12} = 25$, $m_{13} = 9$, $m_{21} = 22.5$, $m_{22} = 20$, $m_{23} = 28$, $r_1 = 65$, $r_2 = 35$.

Case 3

When the market condition is $j = 1$, we use $D_1 = 20$, $D_2 = 10$ in the LP and get the optimal amount of outsourcing of zero and $\pi(1) = 240$. Likewise when $j = 2$, $r_2^* = 35$ and $\pi(2) = 372.5$. When $j = 3$, $r_2^* = 82$ and $\pi(3) = 475$. Therefore, the expected value of the optimal objective function is

$$= \pi_1 = P(j = 1)\pi(1) + P(j = 2)\pi(2) + P(j = 3)\pi(3) = .3*240 + .5*372.5 + .2*475 = 353.25.$$

We should note that the objective function values for both case

Table 7.

	Case 1	Case 2	Case 3
Outsourcing	297.5	297.5	353.25

1 and 2 are the same. This is not the case in general, and the demand uncertainty didn't affect the final objective values by purchasing the same amount of outsourcing in our specific example.

CONCLUSION

In this paper we dealt with an LP problem with stochastic demand. We considered nine distinct LPs depending upon the velocity of supply chain process and the type of operations strategy. Taking an LP as an example, we can summarize the expected value of optimal objective function for each LP as follows.

The comparison between columns gives us the value of supply chain velocity. And the comparison between rows represents the effect of each operations strategy of utilizing forecasting and outsourcing. Obviously the specific numbers depend on the example and the parameters of each model. We intended to convey the idea of manipulating LP depending on the timing of demand realization and the operations strategy adopted.

Table 8.

Speed	Case 1(C1)	Case 2(C2)	Case 3(C3)	C2 – C1	C3 – C2
Base(R1)	270	280	302.4	10	22.4
Forecasting(R2)	283.2	290.6	302.4	7.4	11.8
Outsourcing(R3)	297.5	297.5	353.25	0	55.75
R2 – R1	13.2	10.6	0		
R3 – R1	27.5	17.5	50.85		

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