A Bidding Mechanism to Resolve Asymmetry in Alternating Offer Bargaining

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It is well known that the unique P.E. of the alternating-offer bargaining games in Rubinstein (1982) suffers the first mover advantage problem arising from the artificial procedural asymmetry in dynamic strategic models. We introduce a bidding mechanism as a supergame or a mechanism to resolve the artificial procedural asymmetry in dynamic strategic models of complete information in Rubinstein (1982) in which the players bid for the right to choose a particular sequence of alternating game to play. We show that there exists a unique equilibrium bidding strategies of the players in this bidding mechanism in which the players submit equal bids. That is, the players are indifferent between winning and losing the bid. Correspondingly, there exists a unique P.E. in the bidding mechanism. In the unique P.E. shares of the bidding mechanism, the players with the same preferences and disagreement payoffs indeed share the pie equally by half and half so that the asymmetry or the first-mover advantage in the alternating-offer bargaining games disappears. Yet, our results also show the effect of the difference in time preferences on the P.E. outcome such that the more patient player gets more in the unique P.E. of the bidding mechanism.

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I. Introduction

Rubinstein (1982) shows that there exists a unique perfect equilibrium (P.E.) in the two person infinite horizon alternating-offer bargaining games of complete information by introducing the friction of time discount factors δ in the bargaining process. The unique P.E. is efficient in that the players reach the agreement on the P.E. partition in the first stage without delay with the share $1/1+\delta$ for the player making the first offer and $\delta/1+\delta$ for the other player. He also shows that the unique P.E. partitions in alternating-offer games converge to the axiomatic bargaining solution in Nash (1950, 1953), which is also known as the Nash Bargaining Solution, as the period between offers collapses to zero.

The dynamic strategic approach is based on the construction of noncooperative bargaining games that describe the bargaining process explicitly by sequences of moves. It also has an attraction as noted by Sutton (1986) that the study of games with alternative structure of moves also helps us understand the effects of changes in the bargaining environment. On the other hand, it also shows that the P.E. outcomes of dynamic strategic models are sensitive to the structure of moves, and there is an artificial asymmetry introduced by a particular sequence of offers and replies that gives one player advantage over the other such as the first-mover advantage in the P.E. in Rubinstein (1982) for example. We observe many instances in which bargaining occurs under symmetric procedure.[1] And in games of complete information and symmetric procedure, we except that player with the same preference[2] and the same disagreement payoffs will split the gains from trade equally. That is, the "symmetry" axiom in Nash (1950, 1953) holds.

Several ways to resolve the procedural asymmetry in dynamic strategic models of complete information have been suggested such as Binmore (1987a, 1987b) in discrete time and Perry and Reny (1993) and Sakovics (1993) in continuous time bargaining model.

Binmore (1987a, 1987b) suggests that we can eliminate the procedural asymmetry either by letting the time between successive offers in the alternating-offer bargaining games with discount factors in Rubinstein

[1] *i.e.*, the bargaining positions of the player do not differ except that their preference and disagreement payoffs may differ.

[2] *i.e.*, the players' attitudes toward risks in static axiomatic models and their time preferences and attitudes toward risks in dynamic strategic models.

(1982) go to zero or by letting the players determine the identity of the proposer, in each stage, by tossing an (unbiased) coin. Binmore (1985, 1987a, 1987b) and Binmore, Rubinstein, and Wolinsky (1986) argue that the unique P.E. partitions of the alternating-offer bargaining games with discount factors in Rubinstein (1982) converge to the asymmetric Nash Bargaining Solution in Roth (1977, 1985) as the time between successive offers goes to zero. The attempt to justify cooperative solutions by the solutions in noncooperative games is motivated by the view that regards noncooperative games as more fundamental than cooperative games.[3] And this line of approach to provide justification of the cooperative Nash Bargaining Solution by the noncooperative dynamic strategic game solutions is called as the Nash Program.

One way to avoid the artificial asymmetry introduced in the bargaining procedure of the alternating bargaining games in Rubinstein (1982) is to view the bargaining problem as a supergame or a mechanism. In this paper, we introduce an alternative bidding mechanism to resolve the artificial procedural asymmetry in dynamic strategic models of complete information in Rubinstein (1982). In the bidding mechanism, the players bid simultaneously for the right to choose a particular sequence of alternating game they are going to play in Rubinstein (1982). The player with high bid wins the right to choose a game, the losing bidder gets the amount of the high bid, and the players begin to make offers and replies on the remaining pie according to the sequence of the alternating game chosen by the winner.

We show that there exists a unique equilibrium bidding strategies of the players in which the players submit equal bids. That is, the players are indifferent between winning and losing the bid. Correspondingly, there exists a unique P.E. shares in the bidding mechanism. In the unique P.E. shares in the bidding mechanism, the players with the same preferences and disagreement payoffs indeed share the pie equally by half and half so that the asymmetry or the first-mover advantage in the alternating-offer bargaining games disappears. Yet, our results also show the effect of the difference in time preferences on the outcome such that the more patient player gets more in the unique P.E. of the bidding mechanism.

The paper is organized as follows. In Section II, we present the dynamic strategic model of Rubinstein (1982) with the corresponding existing results on the Nash Program. And in Section III, we introduce

[3] See Nash (1950, 1953) and Binmore (1985) for discussion on the subject.

a bidding mechanism to resolve the artificial procedural asymmetry in dynamic strategic models. We also compare the results on our bidding mechanism with the existing results. Finally, some brief remarks are given in Section IV. An alternative proof is provided in Appendix for reference.

II. Model and Existing Results

A. The Model and Perfect Equilibrium

Two players, player A and player B, set out to play a game to divide a pie of size 1 among them. If they reach an agreement, each player receives agreed share. And if they fail to reach an agreement, they all receive zero. We examine a discrete time bargaining model in which the game may continue indefinitely.

In each stage of the game, one player makes an offer on the division of the pie which the other player accept ("Y") or reject ("N"). If the offer is accepted, then the game ends and the players receive agreed shares; otherwise, the game continues to the next stage of the alternating offer game. We let t, $t=0, 1, 2, 3, \cdots$, represent stages of the game. And let (s, t), $s=<s_A, s_B>$, $0 \leq s_A, s_B \leq 1$, $s_A+s_B=1$, and $t<\infty$, represent the outcome of the game in which players reach an agreement on the division of the pie $s=<s_A, s_B>$ in stage t, where s_A and s_B represent the share of player A and the share of player B, respectively. For notational simplicity, we let $<0, \infty>$ represent the perpetual disagreement.

Each player prefers a pie today to a pie tomorrow and the degree of time preferences are represented by discount factors. To capture the asymmetry in time preferences between the players, we let δ_A represent the discount factor of player A and let δ_B represent the discount rate of player B, $\delta_A, \delta_B \in (0, 1)$, which are common knowledge. That is, if they reach an agreement in stage t on the division of the pie $s=(s_A, s_B)$, the payoffs are $\delta_A^t s_A$ for player A and $\delta_B^t s_B$ for player B.

The players alternate to make offers. We let game g represent the game in which player A makes the first offer and game g' represent the game in which player B makes the first offer in stage $t=0$. There is no restriction for the players to bind themselves to the past offers and replies.

Given the framework, we are ready to examine P.E. of the game.

The following theorem describes the results in Rubinstein (1982) on the unique and efficient P.E.

***Theorem* 1.** There exists unique P.E. in which the players reach an agreement in stage $t=0$ on the division of the pie s given as

$s(g)=<s_A, s_B>=<1-\delta_B/1-\delta_A\delta_B, \delta_B(1-\delta_A)/1-\delta_A\delta_B>$ for game g in which player A makes the first offer,
and

$s(g')=<s_A, s_B>=<\delta_A(1-\delta_B)/1-\delta_A\delta_B, 1-\delta_A/1-\delta_A\delta_B>$ for game g in which player B makes the first offer.

***Proof*:** See Rubinstein (1982) for the proof using the fixed point theorem. Also see Shaked and Sutton (1984) for an alternative method of proof using the structure of games.

Let $\delta_A=\delta_B=\delta$. Then we expect that the players with the same preferences and disagreement payoffs will share the pie equally in an equilibrium. On the other hand, we can easily see $1/1+\delta>\delta/1+\delta$ for any $\delta\in(0, 1)$ in Theorem 1 and there is a first-mover advantage in the alternating-offer bargaining games.

B. Existing Methods to Resolve Asymmetry

Two ways to eliminate the asymmetry in bargaining procedure are suggested in Binmore (1987a, 1987b).

One is to take the alternating-offer bargaining games in Rubinstein (1982) and letting the time between successive offers go to zero. This is the method used to eliminate asymmetry in bargaining procedure and the Nash Program in Rubinstein (1982). Suppose that the time between offers be Δ. Then the unique P.E. partitions of game g and game g' in Theorem 1 become

$$s(g)=\langle s_A, s_B\rangle=\langle\frac{1-\delta_B^{\Delta}}{1-\delta_A^{\Delta}\delta_B^{\Delta}}, \frac{\delta_B^{\Delta}(1-\delta_A^{\Delta})}{1-\delta_A^{\Delta}\delta_B^{\Delta}}\rangle$$

and

$$s(g')=\langle s_A, s_B\rangle=\langle\frac{\delta_A^{\Delta}(1-\delta_B^{\Delta})}{1-\delta_A^{\Delta}\delta_B^{\Delta}}, \frac{1-\delta_A^{\Delta}}{1-\delta_A^{\Delta}\delta_B^{\Delta}}\rangle.$$

As $\Delta \rightarrow 0$, applying L'Hopital's rule, we can show that the P.E. partitions of both game g and game g' converge to

$$\langle s_A,\ s_B \rangle = \langle \frac{\ln\delta_B}{\ln\delta_A + \ln\delta_B},\ \frac{\ln\delta_A}{\ln\delta_A + \ln\delta_B} \rangle. \tag{1}$$

And, in the case of $\delta_A = \delta_B = \delta$, this limiting partition becomes

$$\langle s_A,\ s_B \rangle = \langle \frac{1}{2},\ \frac{1}{2} \rangle$$

and the asymmetry arisen from bargaining procedure disappears. And it coincides with the Nash Bargaining Solution that satisfies "symmetry" axiom in Nash (1950, 1953).

Binmore (1985, 1987a, 1987b) and Binmore, Rubinstein, and Wolinsky (1986) argue that the above limiting P.E. partition is equivalent to the asymmetric Nash Bargaining Solution in Roth (1977, 1985) with the appropriate bargaining powers $\tau_A = (-\ln\delta_A)^{-1}$ for player A and $\tau_B = (-\ln\delta_B)^{-1}$ for player B. That is, the partition s that maximizes $s^{\tau_A}(1-s)^{\tau_B}$. However, for δ_A and δ_B with $e^{-1} < \delta_A$, $\delta_B < 1$, $\tau_A > 1$ and $\tau_B > 1$, the set of feasible payoffs in Nash bargaining problem is not convex, which violates the convexity assumption underlying in the Nash Bargaining Solution. With $\tau_A > 1$ and $\tau_B > 1$, the risk neutral players in Rubinstein (1982) become risk lovers in the asymmetric Nash Bargaining Solution and are likely to agree a lottery with fair gamble odds and we may not make much sense out of the Nash's product maximizing solution. Moreover, the difference in the limiting P.E. shares of the players comes from the difference in their time preferences, whereas the players get unequal shares in the above asymmetric Nash Bargaining Solution by the difference in their attitudes toward risks which are represented by τ_A and τ_B. In other words, this type of limiting approach for the Nash Program is misleading in that the source of the asymmetry in the players' shares in the limiting P.E. is not the difference in time preferences which we initially intended to consider, but is the difference in attitudes toward risks that are not described in the model.

An alternative way suggested in Binmore (1987a, 1987b) to resolve the asymmetry in bargaining procedure is to determine the identity of the proposer, in each stage, by tossing a unbiased coin. It is straightforward to show that the expected share of each player in this procedure equals the P.E. share he expects to get from the game in which each and every sequence offer and reply game is played with equal probability. However, the outcomes of games with the procedure depend more on luck than on the strategies of the players so that we may not draw meaningful implications about the bargaining behaviors of players from them. Moreover, it may not be applicable to the infinite horizon bargaining games examined in Rubinstein (1982) *etc*.

III. Bidding Mechanism

One way to avoid the artificial asymmetry introduced bargaining procedure of the alternating bargaining games in Rubinstein (1982) is to view the bargaining problem as a supergame or a mechanism in which the players bid simultaneously for the right to choose a particular sequence of game they are going to play.

Consider a bidding mechanism M in which the players bid for the right to choose a game between game g and game g' at time $t=0$. The player with high bid wins the right to choose a game, the losing bidder gets the fraction of the high bid, and the players begin to make offers and replies on the remaining pie at time $t=0$ according to the sequence of the moves chosen by the winner. If there is a tie in bids, the winner is determined by tossing a unbiased coin. There is no discounting between the bid and transfer and the first stage offer and reply. We let v_A and v_B represent the bids of player A and player B, respectively.

We can easily see that player A as a winner chooses game g and player B chooses game g' if he wins the bidding. That is, if $v_A \geq v_B$, player A wins chooses game g to play. And if $v_A < v_B$, player B wins chooses game g' to play. Thus, given the unique P.E. partitions of game g and game g' in Theorem 1, the players' shares in the case of $v_A \geq v_B$ in which player A wins the bid are given as

$$\langle s_A,\ s_B \rangle = \langle (1-v_A)\frac{1-\delta_B}{1-\delta_A\delta_B},\ v_A+(1-v_A)\frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} \rangle .$$

Similarly, the players' shares in the case of $v_A < v_B$ in which player B wins the bid are given as

$$\langle s_A,\ s_B \rangle = \langle v_B + (1-v_B)\frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B},\ (1-v_B)\frac{1-\delta_A}{1-\delta_A\delta_B} \rangle .$$

We let v_A^* and v_B^* represent the equilibrium bids of player A and player B, respectively. And let s_A^* and s_B^* represent the equilibrium shares of player A and player B, respectively. Theorem 2 describes the unique equilibrium bidding strategies and shares of the players in this bidding mechanism M.

***Theorem* 2.** The unique equilibrium bidding strategies of the players in M are to submit equal bids of

$$v^* = v_A^* = v_B^* = \frac{(1-\delta_A)(1-\delta_B)}{2-\delta_A-\delta_B}.$$

And the unique P.E. partition is given as

$$\langle s_A^*,\ s_B^* \rangle = \langle \frac{1-\delta_B}{2-\delta_A-\delta_B},\ \frac{1-\delta_A}{2-\delta_A-\delta_B} \rangle .$$

***Proof*:** We will not pursue a rigorous proof here. Instead, we will provide a sketch of proof that can illustrate the idea of the result.[4]

It can be easily shown that for any equilibrium bids of the players $v_A = v_B$, since otherwise the winning bidder can be strictly better off by slightly lowering the bid and still winning the bid. And it can also be shown that the players are indifferent between winning and losing in an equilibrium so that $v = v_A = v_B$ satisfy

$$(1-v)\frac{1-\delta_B}{1-\delta_A\delta_B} = v + (1-v)\frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B} \quad \text{for player A} \tag{2}$$

and

[4] A more rigorous and alternative proof can be provided upon request. Also refer Shin (1993).

$$(1-v)\frac{1-\delta_A}{1-\delta_A\delta_B}=v+(1-v)\frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} \text{ for player B.} \qquad (3)$$

Solving either Eq. (2) or Eq. (3), we get the equilibrium bids of the players

$$v^*=v_A^*=v_B^*=\frac{(1-\delta_A)(1-\delta_B)}{2-\delta_A-\delta_B}.$$

And by substituting v^* into Eq. (2) and Eq. (3), we get the unique P.E. shares in the bidding mechanism M

$$\langle s_A^*,\ s_B^*\rangle=\langle\frac{1-\delta_B}{2-\delta_A-\delta_B},\ \frac{1-\delta_A}{2-\delta_A-\delta_B}\rangle$$

in Theorem 2. □

In Theorem 2, we show that both parties submit the same bids

$$v^*=v_A^*=v_B^*=\frac{(1-\delta_A)(1-\delta_B)}{2-\delta_A-\delta_B}$$

in equilibrium irrespective of their discount factors. That is, they are indifferent between winning and losing in the equilibrium.

To show that the asymmetry or the first-mover advantage in the alternating-offer bargaining games disappears in this bidding mechanism M, let $\delta_A=\delta_B=\delta$ in Theorem 2. Then $\langle s_A^*,\ s_B^*\rangle=\langle 1/2,\ 1/2\rangle$ for any $\delta\in(0,1)$. And we can easily see that the asymmetry disappears and the players with the same preferences and disagreement payoffs indeed share the pie equally in the equilibrium.

Alternatively, let the time between successive offers Δ go to zero. Then the unique P.E. partition of the bidding mechanism M in Theorem 2 becomes

$$\langle s_A^*,\ s_B^*\rangle=\langle\frac{1-\delta_B^{\Delta}}{2-\delta_A^{\Delta}-\delta_B^{\Delta}},\ \frac{1-\delta_A^{\Delta}}{2-\delta_A^{\Delta}\delta_B^{\Delta}}\rangle$$

As $\Delta \to 0$, applying L'Hopital's rule, we can show that the P.E. partition of the bidding mechanism M becomes

$$\langle s_A,\ s_B \rangle = \langle \frac{\ln\delta_B}{\ln\delta_A + \ln\delta_B},\ \frac{\ln\delta_A}{\ln\delta_A + \ln\delta_B} \rangle,$$

which coincides with the limiting P.E. partitions of both game g and game g' in Eq. (1).

And, in the case of $\delta_A = \delta_B = \delta$, this limiting partition again becomes

$$\langle s_A,\ s_B \rangle = \langle \frac{1}{2},\ \frac{1}{2} \rangle$$

and the asymmetry arisen from bargaining procedure disappears. And it also coincides with the Nash Bargaining Solution that satisfies "symmetry" axiom in Nash (1950, 1953).

And our results also show the effect of the difference in time preferences on the outcome such that if $\delta_A > \delta_B$

$$s_A^* = \frac{1-\delta_B}{2-\delta_A-\delta_B} > s_B^* = \frac{1-\delta_A}{2-\delta_A-\delta_B}$$

and player A gets more in the unique P.E. of the bidding mechanism M, vice versa. That is, more patient player gets more than the other player in this unique P.E. partition.

On the other hand, it can be also noted that player A's share in the unique P.E. partition of the bidding mechanism M in Theorem 2 is strictly less than player A's share in the unique P.E. partition in game g and strictly greater than player A's share in the unique P.E. partition in game g' of alternating offers in Theorem 1 such that

$$\frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B} < s_A^* = \frac{1-\delta_B}{2-\delta_A-\delta_B} < \frac{1-\delta_B}{1-\delta_A\delta_B}.$$

And alternatively, player B's share in the unique P.E. partition of the bidding mechanism M in Theorem 2 is strictly greater than player B's share in the unique P.E. partition in game g and strictly less than player B's share in the unique P.E. partition in game g' of alternating offers in Theorem 1 such that

$$\frac{\delta_B(1-\delta_A)}{1-\delta_A\delta_B} < s_B^* = \frac{1-\delta_A}{2-\delta_A-\delta_B} < \frac{1-\delta_A}{1-\delta_A\delta_B}.$$

So the first-mover advantages in the alternating offer games of g and g' are disappeared or at least alleviated.

IV. Concluding Remarks

There is a first-mover advantage in the alternating-offer bargaining games in Rubinstein (1982).

One way to avoid the artificial asymmetry introduced in the bargaining procedure of the alternating bargaining games in Rubinstein (1982) is to view the bargaining problem as a supergame or a mechanism. We introduce a bidding mechanism to resolve the artificial procedural asymmetry in dynamic strategic models of complete information in Rubinstein (1982).

In the bidding mechanism, the players bid for the right to choose an alternating offer game in Rubinstein (1982). The player with high bid wins the right to choose a game, the losing bidder gets the amount of the high bid, and the players begin to make offers and replies on the remaining pie according to the particular alternating offer game chosen by the winner.

We show that there exists a unique equilibrium bidding strategies of the players in the bidding mechanism in which the players submit equal bids. That is, the players are indifferent between winning and losing the bid. Correspondingly, there exists a unique P.E. in the bidding mechanism. In the unique P.E. shares of the bidding mechanism, the players with the same preferences and disagreement payoffs indeed share the pie equally by half and half so that the asymmetry or the first-mover advantage in the alternating-offer bargaining games disappears. Yet, our results also show the effect of the difference in time preferences on the outcome such that the more patient player gets more in the unique P.E. of the bidding mechanism.

Shin (1993) examined an alternative way to resolve the asymmetry in bargaining procedure is to determine the identity of the proposer, in each stage, by bidding that can be applied to both finite horizon bargaining and infinite horizon bargaining games. And it can be noted that the unique P.E. shares of the bidding mechanism in this paper coincide with the unique P.E. shares of the mechanism for infinite horizon bargaining games in Shin (1993) in which the identity of the proposer is determined by bidding in each stage.

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