LETTER

An Iterative Sequence Estimator for QAM-OFDM Signals

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SUMMARY In this letter, iterative sequence estimation technique based on expectation-maximization (EM) algorithm is considered for quadrature amplitude modulation (QAM)-orthogonal frequency division multiplexing (OFDM) signals. For QAM-OFDM signaling, the optimal EM algorithm requires high computational complexity due to the inversion of complex matrix executed at each iteration. To avoid this problem, we propose a sub-optimal iterative sequence estimation algorithm with some approximations, which results in reduced computational complexity for QAM-OFDM signals. Moreover, we use two different approaches to obtain initial estimate for beginning iteration of proposed algorithm. One is for less time-dispersive but fast fading channel and the other is for highly time-dispersive but relatively slow fading channel. The bit error rate (BER) performances of the proposed algorithm are evaluated using computer simulations. The results show that the proposed algorithm performs nearly as well as the optimal EM algorithm.

key words: *iterative sequence estimation, EM algorithm, QAM, OFDM*

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is well known to be a useful technique for high data rate transmissions over a frequency selective fading channel. However, for coherent detection of OFDM signals, fading compensation techniques are required to mitigate amplitude and phase distortions due to the multipath fading channel. Fading compensation technique is getting more crucial when quadrature amplitude modulation (QAM) is used for the modulation of each OFDM subcarrier in order to increase spectral efficiency.

A channel estimation technique using periodically inserted pilot symbols in the data stream is well known to provide a reliable way to mitigate the distortions. Many channel estimation schemes for the coherent detection of OFDM signals are reported in the technical literatures [1]–[4]. Especially in [4], two different estimators using pilot symbols are investigated. One is the deterministic maximum likelihood (ML) estimator assuming unknown but deterministic channel impulse response (CIR). The other is the minimum mean square error (MMSE) estimator assuming unknown and random CIR. The interesting problem in the above consideration is how to obtain the ML estimate with unknown and random CIR.

In order to obtain the ML estimate over random channels, the expectation-maximization (EM) algorithm is introduced [5]. The application of the EM algorithm to a fading channel is known to provide ML estimate under some conditions [6]. In addition, the applications of the EM algorithm are studied to OFDM signals and to space-time coded OFDM signals in [7] and in [8], [9], respectively. However, these studies are limited to constant envelope modulations such as phase shift keying (PSK). Theses applications of the optimal EM algorithm mentioned above are found to have too much computational complexity for QAM signaling. Therefore, we propose a sub-optimal iterative sequence estimation technique based on the EM algorithm, which is shown to have significantly reduced computational complexity but to provide nearly optimal performance for QAM-OFDM signals. Moreover, for initial estimate to begin iteration of proposed algorithm, we suggest two different methods depending on whether channel is less time-dispersive but fast fading or highly time-dispersive but relatively slow fading.

This letter is composed in the following order. In Sect. 2, we describe OFDM system to be considered. In Sect. 3, we evaluate the computational complexity of optimal EM algorithm for QAM signals and suggest a sub-optimal EM-based sequence estimation algorithm. Section 4 represents the initialization of proposed algorithm to begin iteration and shows BER performances of the proposed algorithm obtained using computer simulations. Finally, the conclusion is followed in Sect. 5.

2. System Model

Let N be the number of subcarriers and $K = 2N_\alpha+1$ be the number of parallel data symbols to be transmitted. Then, we can index each subcarrier by numbers from $-N_\alpha$ to N_α. After serial to parallel conversion, data symbols transmitted at each subcarrier are to be the entries of data symbol sequence vector $\mathbf{s}$ shown as

$$\mathbf{s} = [s(-N_\alpha), \cdots, s(0), \cdots, s(N_\alpha)]^T. \tag{1}$$

Note that $N - K$ subcarriers at the edges of the spec-

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trum are not used. Then, the received signals are given as

$$\mathbf{y} = \mathbf{SFh} + \mathbf{n} \tag{2}$$

where $\mathbf{h} = [h(1), h(2), \cdots, h(L)]^T$ denotes the channel impulse response and

$$\mathbf{S} = diag[s(-N_\alpha), \cdots, s(0), \cdots, s(N_\alpha)] \tag{3}$$

is a diagonal symbol matrix. Moreover, the channel response in the frequency domain $\mathbf{H}$ can be given as

$$\begin{aligned} \mathbf{H} &= [H(-N_\alpha), \cdots, H(0), \cdots, H(N_\alpha)]^T \\ &= \mathbf{Fh}. \end{aligned} \tag{4}$$

The additive white Gaussian noise vector $\mathbf{n}$ in (2) has zero mean and the covariance matrix of $\sigma_n^2\mathbf{I}$. The entries of discrete Fourier transform (DFT) matrix $\mathbf{F}$ in (2) and (4) are given by

$$[\mathbf{F}]_{k,l} = e^{-j\frac{2\pi k(l-1)}{N}} \tag{5}$$

where $|k| \le N_\alpha$ and $1 \le l \le L$.

3. Iterative Sequence Estimation

In this section, we evaluate the computational complexity of optimal EM algorithm for QAM-OFDM signals. Then, we propose a sub-optimal iterative sequence estimation algorithm with some approximations, which results in much reduced computational complexity.

3.1 Optimal EM-Based Sequence Estimation

The EM algorithm is an iterative two-step algorithm that includes the expectation step and maximization step. The EM algorithm iterates until the estimate converges [5].

The optimal application of EM algorithm for OFDM signals is studied in [7]–[9]. In the expectation step, the log-likelihood function derived in [7] can be evaluated with the following equation

$$\begin{aligned} Q(\mathbf{s} \mid \mathbf{s}^i) = \sum_{k=-N_\alpha}^{N_\alpha} \Bigg\{ &Re\left[y^*(k)s(k)\sum_{l=1}^{L}[\mathbf{F}]_{k,l}m_1^i(l)\right] \\ &-\frac{1}{2}|s(k)|^2\sum_{l=1}^{L}\sum_{n=1}^{L}[\mathbf{F}]_{k,l}[\mathbf{F}]_{k,n}^*m_2^i(l,n)\Bigg\} \end{aligned} \tag{6}$$

where

$$\begin{aligned} \mathbf{m}_1^i &= [m_1^i(1), m_1^i(2), \cdots, m_1^i(L)]^T \\ &= E[\mathbf{h} \mid \mathbf{y}, \mathbf{s}^i] \end{aligned} \tag{7}$$

and

$$\begin{aligned} \mathbf{m}_2^i &= \begin{bmatrix} m_2^i(1,1) & m_2^i(1,2) & \cdots & m_2^i(1,L) \\ m_2^i(2,1) & m_2^i(2,2) & \cdots & m_2^i(2,L) \\ & \vdots & \ddots & \vdots \\ m_2^i(L,1) & m_2^i(L,2) & \cdots & m_2^i(L,L) \end{bmatrix} \\ &= E[\mathbf{hh}^\dagger \mid \mathbf{y}, \mathbf{s}^i]. \end{aligned} \tag{8}$$

In order to obtain (7) and (8), we require the calculation of the $L \times L$ covariance matrix of CIR given the received signal $\mathbf{y}$ and the i-th sequence estimate $\mathbf{s}^i$ shown as

$$\mathbf{R}^i = \left[\sigma_n^2\mathbf{R}_h^{-1} + \mathbf{F}^\dagger\left(\mathbf{S}^i\right)^*\left(\mathbf{S}^i\right)\mathbf{F}\right]^{-1} \tag{9}$$

where $\mathbf{R}_h = E[\mathbf{hh}^\dagger]$ (see Eqs. (15)–(17) in [7]). For PSK signals, the covariance matrix $\mathbf{R}^i$ in (9) is always constant because the signal envelope is constant. For QAM signals, however, the covariance matrix is variable according to the change of sequence estimate and also complex. Therefore, the optimal EM algorithm for QAM-OFDM signals requires the inversion of complex matrix at each iteration corresponding to the additional computation of $O((2L)^3)$ in order to compute covariance matrix. In the next subsection, we propose a sub-optimal EM based iterative sequence estimation algorithm for QAM-OFDM signals, which does not need the inversion of complex matrix at each iteration.

3.2 Sub-optimal Iterative Sequence Estimation

Given i-th sequence estimate $\mathbf{s}^i$, which is assumed to be the same as the transmitted data symbols, we define the normalized received signal vector $\mathbf{y}'$, which is given by

$$\mathbf{y}' = \left(\mathbf{S}^i\right)^{-1}\mathbf{y} = \mathbf{Fh} + \mathbf{n}' \tag{10}$$

where $\mathbf{n}' = \left(\mathbf{S}^i\right)^{-1}\mathbf{n}$.

As shown in Fig. 1, we assume that, for M-ary QAM scheme, $\mathbf{n}'$ in (10) is an additive white Gaussian noise vector which has the scaled variance of

$$\sigma_{n'}^2 = \frac{1}{M}\sum_{m=1}^{M}\frac{\sigma_n^2}{|s_m|^2} = \beta\sigma_n^2 \tag{11}$$

where s_m is the m-th possible symbol and β is defined as variance scaling factor. Variance scaling factor can

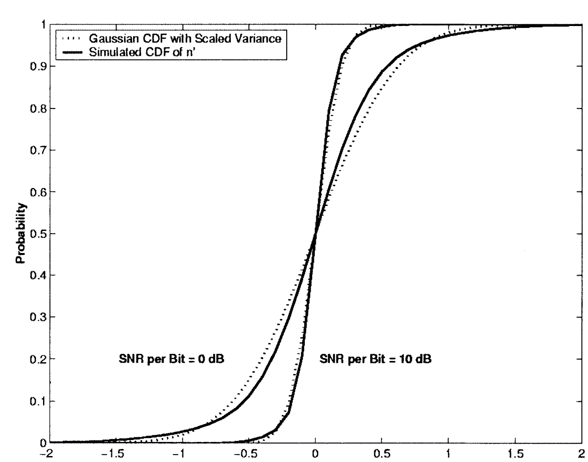

Fig. 1 Comparison of cumulative distribution functions (CDF) for the assumption in (11).

be shown to have values of $\beta = 1.8889$ for 16-QAM signaling when the average symbol energy is normalized to unity [10]. Using this approximation, $\mathbf{R}^i$ in (9) can be substituted by $\mathbf{R}'$ as given in the following equation

$$\mathbf{R}' = \left[\sigma_{n'}^2 \mathbf{R}_h^{-1} + \mathbf{F}^\dagger \mathbf{F}\right]^{-1}. \tag{12}$$

Note that the matrix inversion in (12) can be computed only once in advance because it does not depend on the i-th data symbol sequence estimate $\mathbf{s}^i$ by the approximation in (11). Therefore, we can avoid the inversion of the complex matrix in (9) and does not need the computation of $O((2L)^3)$ at each iteration, which makes its implementation practical.

The conditional moment $\mathbf{m}_1^i$ and $\mathbf{m}_2^i$ at the i-th iteration for the sub-optimal algorithm can be evaluated by following the derivation for the optimal EM algorithm given in [7] with $\mathbf{y}'$ in (10) and $\mathbf{R}'$ in (12) shown as

$$\mathbf{m}_1^i = \mathbf{R}' \mathbf{F}^\dagger \mathbf{y}' \tag{13}$$

$$\mathbf{m}_2^i = \sigma_{n'}^2 \mathbf{R}' + \mathbf{m}_1^i \left(\mathbf{m}_1^i\right)^\dagger. \tag{14}$$

The maximization step, which is used to generate the i+1-th data symbol sequence estimate, can be represented by

$$\mathbf{s}^{i+1} = \arg\max_s Q(\mathbf{s} \mid \mathbf{s}^i). \tag{15}$$

The iteration continues until the data symbol sequence estimate converges.

4. Simulation

4.1 Simulation Environments

The system parameters of the simulation environments correspond to the IEEE 802.11a physical layer standard, which is summarized as follows [11].

- The DFT size N is 64.
- The number of modulated subcarriers equals to 52, that is, $N_\alpha = 26$. The subcarrier number is $0 < |k| \le 26$ except the 0-th subcarrier.
- The number of pilot subcarriers embedded in OFDM symbol is equal to 4.
- Each pilot subcarrier is located at $k = -21, -7, 7$ and 21.
- The OFDM subcarriers are modulated using 16-QAM.

It is assumed that the channel impulse response has L taps and the amplitude of each path varies independently according to Rayleigh distribution with exponentially decaying power delay profile, that is,

$$E[|h(l)|^2] = \exp\left(-\frac{(l-1)}{5}\right) \tag{16}$$

where $l = 1, 2, \cdots, L$. It is also assumed that the guard time is large enough to eliminate intersymbol interferences.

4.2 Initialization

In order to begin the iteration of proposed algorithm, we have to obtain the initial estimate, $\mathbf{m}_1^0$ and $\mathbf{m}_2^0$, which affects the convergence properties of the EM-based iterative algorithm. Obtaining initial estimate, we considered two different scenarios. Scenario 1 is that the number of taps of channel impulse response is less than or equal to the number of pilot subcarriers embedded in each OFDM symbol, that is $L \le 4$. On the other hand, scenario 2 is the case of $L > 4$.

In scenario 1, we use the conventional deterministic ML estimator and pilot subcarriers [4] to obtain the initial estimate of the proposed algorithm. In scenario 2, however, the simulation results revealed that the conventional MMSE or deterministic ML estimator using pilot subcarriers show relatively poor performance to be used for obtaining initial estimate. In this case, we assume that the time variance between adjacent OFDM symbols is ignorable and use the channel estimate of expectation step after I iterations for previous OFDM symbol, $\mathbf{m}_1^{I-1}$ and $\mathbf{m}_2^{I-1}$, to obtain initial sequence estimate for present OFDM symbol. For the initial OFDM data symbol, the preamble provides the channel estimate.

In scenario 1, we can expect that the proposed algorithm is robust against fast fading since it uses pilot subcarriers embedded in each OFDM symbol to obtain initial estimate, but it is not efficient for highly time-dispersive channel, i.e. $L > 4$. The proposed algorithm in scenario 2 shows good performances for $L > 4$, but it is not robust for fast fading channel due to the assumption of ignoring time variance between adjacent OFDM symbols. In our simulations, we have chosen $L = 4$ and the normalized Doppler spread $BT = 0.03$ for scenario 1. For scenario 2, we assumed that $L = 8$, $BT = 0.005$ and preamble symbols are inserted at every 50-th OFDM symbol.

4.3 Performance Evaluation

Figure 2 compares the convergence properties of the proposed algorithm and optimal EM algorithm for both scenarios described in the above subsection. We counted the process to obtain initial estimate as one iteration. It is shown that the convergence rate of the proposed algorithm is almost equal to that of optimal EM algorithm above the SNR of 15 dB.

In scenario 1, we observed that two iterations are enough to improve the BER performance of the proposed algorithm from the simulation results. Figure 3 compares BER performances of the proposed algorithm with 2 iterations, the MMSE estimator [4], the optimal EM algorithm and the ideal channel information (ICI) case for 16-QAM. It is seen that the proposed algorithm outperforms MMSE estimator and performs as well as

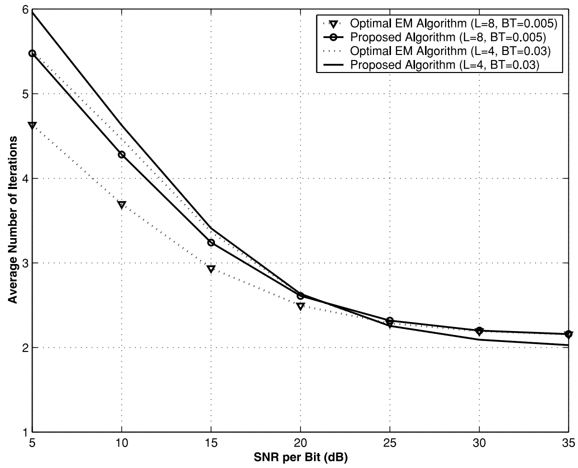

Fig. 2 Comparison of convergence properties for 16-QAM.

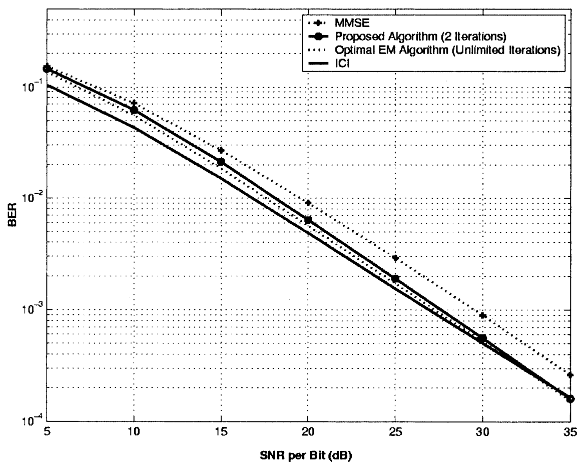

Fig. 3 Comparison of BER performances for 16-QAM ($L = 4$, $BT = 0.03$).

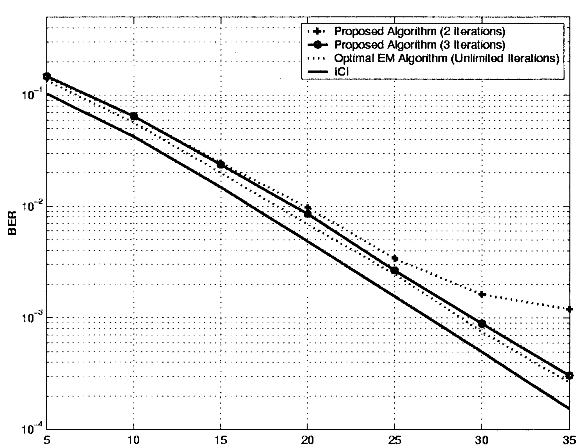

Fig. 4 Comparison of BER performances for 16-QAM ($L = 8$, $BT = 0.005$).

the optimal EM algorithm with unlimited iterations for sequence estimate converges.

In Fig. 4, BER performances of the proposed algorithm are compared with that of optimal EM algorithm in scenario 2. We can see that the proposed algorithm with 2 iterations shows error floor in high SNR, but it performs near optimally with 3 iterations.

5. Conclusion

In this letter, we evaluated the computational complexity for the optimal EM algorithm for QAM-OFDM signals. Then, using some approximation, we proposed a sub-optimal iterative sequence estimation algorithm with reduced computational complexity. The proposed algorithm starts iteration after obtaining the initial estimate. The simulation results indicate that the proposed algorithm shows near optimal performances and outperforms the conventional algorithm.

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