OPTIMIZING TRANSPORTATION PROBLEMS WITH MULTIPLE OBJECTIVES

Sang Moon Lee

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ABSTRACT

Virtually all models developed for transportation problems have focused upon the optimization of a single objective criterion, namely the minimization of total transportation costs. They have generally neglected or often ignored the multiple conflicting objectives involved in the problem, the priority structure of these objectives, various environmental constraints, unique organizational values of the firm, and bureaucratic decision structures. However, in reality these are important factors which greatly influence the decision process of transportation problems. In this study the GP approach is utilized because it allows the optimization of mutiple conflicting goals while permitting an explicit consideration of the existing decision evironment.

Author: Professor of Virginia Polytechnic Institute and State University

I. Introduction

The analysis of transportation problems has constituted one of the major areas of fruitful application for linear programming. Some of the initial work on transportation problems was done by F.L. Hitchcock (4) and T.C. Koopmans (6), in the 1940's. Subsequent to these pioneering works, many scholars have refined and extended the basic transportation model to include not only the determination of optimal shipping patterns, but also the analysis of production scheduling problems, transhipment problems, and assignment problems. Also, specialized algorithms have been developed for solving transportation and assignment models. The general mathematical model for the transportation problem is formulated in (3, p. 273).

In the general transportation problem, the objective is to minimize the total transportation (and possibly production) costs. Some studies have also included a penalty cost of supply shortage in the objective function (1). The basic assumption underlying the transportation model is that management is only concerned with cost minimization. However, as it is well documented in many studies (2, 5, 7, 8, 9), economic optimization (e.g., cost minimization) is not the only objective of practicing enterprises. In fact, business firms quite frequently place higher priorities on non-economic goals than on cost minimization. Or, firms seek cost minimization while pursuing other non-economic objectives. We have seen, for example, that firms are placing an increasing emphasis on public service, social responsibilities, industrial and labor relations, customer goodwill, etc. Whether such objectives are sought because of external pressure or voluntary management decisions, non-economic objectives do exist and they are gaining greater significance. This also applies to the transportation problem. In the transportation problem, there may be multiple objectives such as the fulfillment of transportation schedule contracts, the meeting of union contracts, providing for a stable employment level in various plants and transportation fleets, balancing work among the plants (sources), minimiznig transportation hazards (bad roads, bad weather, hyjacking, etc.), and of course cost minimization.

If we accept that most transportation problems involve multiple, and possibly conflicting, objectives, the operations research technique applied to the problem should be capable of treating multiple objectives in multiple dimensions. The objective function can no longer be restricted to a cardinal criterion (cost); rather it must involve several decision criteria. Certainly, costs will remain as an important model parameter. However, the location of cost in the model will be shifted from the objective function to a decision constraint.

The question still remains as to whether one should use the conventional linear programming approach to the transportation problem that involves multiple conflicting objectives. It should be pointed out that linear programming is still being used for contemporary multiple objective problems by estimating numerical measures of abstract objectve criteria in terms of convenient numerical values, i.e., utilities, profits, costs, etc. However, the process often results in a considerable degree of fabrication and distortion of information in order to express multiple objectives as a numerical criterion (7). Hence, the model solution may be of very little value to the decision maker. The only alternate method to the numerical approach of linear programming for problems involving multiple conflicting objectives is the ordinal solution approach. The purpose of this paper is to present a further extension in the analysis of transportation problems by introducing the goal programming approach. First, the goal programming approach will be discussed, followed by an example application of goal programming to a transportation problem.

II. The Goal Programming Approach

Goal programming (GP) is a special extension of linear programming (2, 5, 7, 9). This method is capable of handling decision problems which deal with a single goal with multiple subgoals (5). In the conventional linear programming (LP) method, the objective function is unidimensional --either to maximize profits (effectiveness) or to minimize costs (sacrifice). The GP model handles multiple goals in multiple dimensions. Therefore, there is no dimensional limitation to the objective function.

Often, goals set by the decision maker are achievable only at the expense of other goals. Furthermore, these goals are incommensurable. Thus, there is a need to establish a hierarchy of importance among these incompatible goals so that the lower order goals are considered only after the higher order goals are satisfied or have reached the point beyond which no further improve ments are desirable. If the decision maker can provide an ordinal ranking of goals in terms of their contributions or importance to the organization, the problem can be solved by GP.

In GP, instead of trying to maximize or minimize the objective criterion directly, the deviations between goals and what can be achieved within the given set of constraints are to be minimized. In the simplex algorithm of LP, such deviations are called "slack" variables. These deviational variables take on a new significance in GP. The deviational variable is represented in two dimensions, both positive and negative deviations from each subgoal or goal. Then, the objective function becomes the minimization of these deviations, based on the relative importance or preemptive priority weights assigned to them. The objective function, however, may also include real variables with ordinary or preemptive weights in addition to the deviational variables.

The primary characteristic of GP is that it allows for an ordinal solution. Stated differently, management may be unable to obtain information on the cost or value of agoal or a subgoal. Usually the manager has judgment to determine the priority of the desired attainment of each goal or subgoal and rank them in ordinal sequence. Economically speaking, the manager works with the problem of the allocation of scarce resources. Obviously, it is not always possible to achieve every goal to the extent desired by management. Thus, with or without GP the manager attaches a certain priority to the achievement of a certain goal. The true value of GP is, therefore, the solution of problems involving multiple, conflicting goals according to the manager's priority structure.

The general GP model can be mathematically expressed as (5):

$$\text{Minimize } Z=\sum_{i=1}^{m}(d_i^{+}+d_i^{-}) \qquad [1]$$

$$\text{subject to } Ax=Id^{+}+Id^{-}=b \qquad [2a]$$

$$x, d^{+},\ d^{-}\geq 0 \qquad [2b]$$

where m goals are expressed by an m component column vector $b(b_1, b_2, \ldots, b_m)$, A is an $m\times n$ matrix which expresses the relationship between goals and subgoals, x represents variables involved in the subgoals $(x_1, x_2, \ldots, x_n)$, d^{+} and d^{-} are m-component vectors for the variable representing deviations from goals, and I is an identity matrix in m dimensions.

The manager must analyze each one of the m goals considered in the model in terms of whether over or under achievement of the goal is satisfactory. If over-achievement is acceptable, d_i^{+}can be eliminated from the objective function. On the other hand, if underachievement is astisfactory, d_i^{-} should not be included in the objective function. If the exact achievement of the goal is desired, both d_i^{+} and d_i^{-} must be ranked according to their preemptive priority weights, from the most important to the least important. In this way the low order goals are considered only after the higher order goals are achieved as desired. If goals are classified in k ranks, the preemptive priority factor p_j $(j=1, 2, \ldots, k)$ should be assigned to the deviational variables, d_i^{+} and d_i^{-}. The priority factors have the relationship of $P_j >>> P_{j+1}$ $(j=1,2,\ldots,k)$, which implies that the multiplication of n, however large it may be, cannot make P_{j+1} greater than or equal to P_j. Of course, it is possible to refine goals even further by the means of decomposing the deviational variables. To do this, additional constraints and additional priority factors are required.

One more step in the procedure to be considered is the weighting of those deviational variables at the same priority level, i.e., variables with the same P_j coefficient. The criterion to be used here is the minimization of the opportunity cost or regret. This implies that the coefficient of regret σ_i, which is positive, must be assigned to the individual deviational variables on the same goal level. The coefficient σ_i simply represents the relative amount of unsatisfactory deviation from the goal.

III. The GP-Transporation Model

In order to demonstrate how GP canbe used in the analysis of a transportation problem that involves multiple conflicting goals, a simple example is presented. The hypothetical problem has been kept relatively simple in order that the GP application not be obscured by problem details and complexity.

We will assume that the company in question supplies a single product to four customers at various different locations from three warehouse locations. During the current planning period the company will be unable to meet its customer demands. However, the president of the company has decided that certain customer demands must be satisfied, at the expense of others. Also in order to avoid gross inequities management feels that it is mportant to balance the portion of diemand satisfied among certain customers. Also, due to union agreements the company must meet certain minimum shipment levels along certain routes. Finally, several of the routes over which the product might be shipped are characterized by such hazards as susceptibility to hijacking or poor road construction, and thus management feels that these routes should be avoided.

It has been the policy of the company in the past to solve their transportation problems by standard transportation algorithms (i.e., 3, pp. 291-314). However, in this case the company attempted to formulate the problem as a standard linear programming problem, with the primary goal of transportation cost minimization, and all other goals specified as constraints. Since this approach led to an infeasible solution, an alternate approach was required.

The alternative approach selected was goal programming. However, management did specify that the G.P. solution would be somewhat undesirable if the resultant transportation cost was more than 110% of the budgeted figure (which was determined by the standar dtransportation algorithm approach).

IV. Goals and Their Priorities

Of course, management wishes to fill as much of the customer demand as possible, while considering the above factors. Finally, management wishes to minmize the total shipping cost in meetiing the customer demand. Management has formulated the goals with their associated priority ranks (P_1=highest rank) as follows:

P_1: Meet entire demand of customer 4 (guaranteed delivery)

P_2: Ship at least 100 units over the route from warehouse 3 to customer 1 (union agreement)

P_3: Meet no less than 80% of the demand of each customer

P_4: Keep transportation cost to no more than 110% of budgeted figure ($2, 950, as determined by standard cost-minimization transportation method)

P_5: Minimize shipping over the route from warehouse 2 to customer 4 (hazards).

P_6: Balance the percentage of demand filled between customers 1 and 3.

P_7: Minimize the total transportation costs for goods shipped.

The transportation problem is summarized in Table 1, with shipping costs given in each cell (in parenthesis), demand and supply values given in the margins, and decision variables given by X_{ij}. The problem indicates that total demand exceeds total supply by 100 units.

Table 1.

Warehouses	Customers				Supply
	1	2	3	4	
1	$X_{11}(5)$	$X_{12}(2)$	$X_{13}(6)$	$X_{14}(7)$	300
2	$X_{21}(3)$	$X_{22}(5)$	$X_{23}(4)$	$X_{24}(6)$	200
3	$X_{31}(4)$	$X_{32}(5)$	$X_{33}(2)$	$X_{34}(3)$	400
Demand	200	100	450	250	1000/900

Variables

X_{ij}=amount to be transported from the i^{th} warehouse to the j^{th} customer.

d_i^-=underachievement of goals or constraints in the i^{th} equation.

d_i^+=overachievement of goals or constraints in the i^{th} equation.

Model Constraints

The GP model constraints for the preceeding transportation problem are formulated as follows:

(1) Supply

Supply is restricted to the maximum capacity of warehouses. Hence,

$$X_{11}+X_{12}+X_{13}+X_{14}+d_1^-=300 \qquad [3]$$

$$X_{21}+X_{22}+X_{23}+X_{24}+d_2^-=200 \qquad [4]$$

$$X_{31}+X_{32}+X_{33}+X_{34}+d_3^-=400 \qquad [5]$$

Since we assume that the right-hand-side (RHS) values indicate the maximum capacity of the warehouse, positive deviations can be excluded from the supply constraints.

(2) Demand

We assume that we will never wish to overfill a customer's demand, and therefore positive deviations can also be excluded from demand constraints However, since demand cannot be satisfied in all cases (total demand exceeds total supply), we must include negative deviations to identify the underachievement of demand goals.

$$X_{11}+X_{21}+X_{31}+d_4^-=200 \qquad [6]$$

$$X_{12}+X_{22}+X_{32}+d_5^-=100 \qquad [7]$$

$$X_{13}+X_{23}+X_{33}+d_6^-=450 \qquad [8]$$

$$X_{14}+X_{24}+X_{34}+d_7^-=250 \qquad [9]$$

(3) Union Agreement Goal

The firm has a union agreement which specifies that at least 100 units be transported from warehouse 3 to customer 1. The variable d_8^- represents negative deviation from the goal, while d_8^+ is the amount of overachievement of this goal.

$$X_{31}+d_8^- -d_8^+=100 \qquad [10]$$

(4) Minimum Satisfied Demand

Management wishes to avoid gross inequities of demand satisfaction among the various customers, and they have therefore set the goal of satisfying at least 80% of each customer's demand. The appropriate constraints, including deviational variables, are given as follows:

$$X_{11}+X_{21}+X_{31}+d_9^- -d_9^+=160 \qquad [11]$$

$$X_{12}+X_{22}+X_{32}+d_{10}^- -d_{10}^+=80 \qquad [12]$$

$$X_{13}+X_{23}+X_{33}+d_{11}^- -d_{11}^+=360 \qquad [13]$$

$$X_{14}+X_{24}+X_{34}+d_{12}^- -d_{12}^+=200 \qquad [14]$$

(5) Transportation Budget

The resnltant solution's total transportation cost should not exceed 110% of the budgeted figure of $2,950, which yields amaximum goal figure of $3,245.

$$\Sigma C_{ij}X_{ij}+d_{13}^- -d_{13}^+=3,245, \text{ all } i,j \qquad [15]$$

(6) Road Hazard Goal

The firm wishes to minimize transporting from warehouse 2 to customer 4, due to the high occurrence of hijackings along that route. Thus, the goal level for the constraint is set at zero, and we attempt to minimize d_{14}^+.

$$X_{24}-d_{14}^+=0 \qquad [16]$$

(7) Balance to Customers

It is desired to transport amounts to customers 1 and 3 such that an equal portion of demand for each is satisfied. This can be expressed as:

$$(X_{11}+X_{21}+X_{31})/200=(X_{13}+X_{23}+X_{33})/450 \qquad [17]$$

Thus, the goal constraint becomes:

$$X_{11}+X_{21}+X_{31}-.444(X_{13}+X_{23}+X_{33})+d_{15}^{-}-d_{15}^{+}=0 \qquad [18]$$

(8) Transportation Cost

If we denote C_{ij} as the unit transportation cost from the i^{th} warehouse to the j^{th} customer, the total transportation cost is given by $\Sigma C_{ij}X_{ij}$, for all i and j. Since we wish to minimize total transportation costs, we set a goal of zero cost and attempt to minimize the positive deviation from this goal figure.

$$\Sigma C_{ij}X_{ij}-d_{16}^{+}=0, \text{ all } i,j. \qquad [19]$$

The complete G.P. model is given as follows:

$$\text{Minimize } Z=P_1d_7^{-}+P_2d_8^{-}+P_3(d_9^{-}+d_{10}^{-}+d_{11}^{-}+d_{12}^{-})$$
$$+P_4d_{13}^{+}+P_5d_{14}^{+}+P_6(d_{15}^{-}+d_{15}^{+})+P_7d_{16}^{+}$$

subject to:

$$X_{11}+X_{12}+X_{13}+X_{14}+d_1^{-}=300$$
$$X_{21}+X_{22}+X_{23}+X_{24}+d_2^{-}=200$$
$$X_{31}+X_{32}+X_{33}+X_{34}+d_3^{-}=400$$
$$X_{11}+X_{21}+X_{31}+d_4^{-}=200$$
$$X_{12}+X_{22}+X_{32}+d_5^{-}=100$$
$$X_{13}+X_{23}+X_{33}+d_6^{-}=450$$
$$X_{14}+X_{34}+X_{34}+d_7^{-}=250$$
$$X_{31}+d_8^{-}-d_8^{+}=100$$
$$X_{11}+X_{21}+X_{31}+d_9^{-}=160$$
$$X_{12}+X_{22}+X_{32}+d_{10}^{-}-d_{10}^{+}=80$$
$$X_{13}+X_{23}+X_{33}+d_{11}^{-}-d_{11}^{+}=360$$
$$X_{14}+X_{24}+X_{34}+d_{12}^{-}-d_{12}^{+}=200$$
$$5X_{11}+2X_{12}+6X_{13}+7X_{14}+3X_{21}+5X_{22}+4X_{23}+6X_{24}+4X_{31}$$
$$+5X_{32}+2X_{33}+3X_{34}+d_{13}^{-}-d_{13}^{+}=3,245$$
$$X_{24}-d_{14}^{+}=0$$

$$X_{11}+X_{21}+X_{31}-.444X_{13}-.444X_{23}-444X_{33}+d_{15}^{-}-d_{15}^{+}=0$$
$$5X_{11}+2X_{12}+6X_{13}+7X_{14}+3X_{21}+5X_{22}+4X_{23}+6X_{24}+4X_{31}$$
$$+5X_{32}+2X_{33}+3X_{34}+d_{16}^{+}=0$$
$$X_{ij},\ d_i^{-}, d_i^{+}>0$$

V. Results and Discussion

The preceding G.P. transportation problem was solved using the G.P. modified simplex computer program (9, pp. 126-160), yielding the following results:

Real Variables:

$X_{12}=100$ units shipped from warehouse 1 to customer 2

$X_{14}=200$ units shipped from warehouse 1 to customer 4

$X_{21}=90$ units shipped from warehouse 2 to customer 1

$X_{23}=110$ units shipped from warehouse 2 to customer 3

$X_{31}=100$ units shipped from warehouse 3 to customer 1

$X_{33}=250$ units shipped from warehouse 3 to customer 3

$X_{34}=50$ units shipped from warehouse 3 to customer 4

Deviational Variables:

$d_4^{-}=10$ units underachievement of demand for customer 1

$d_6^{-}=90$ units underac hievement of demand for customer 3

$d_9^{+}=30$ units overachievement of 80% demand requirement for customer 1

$d_{10}^{+}=20$ units overachievement of 80% demand requirement for customer 2

$d_{12}^{+}=50$ units overachievement of 80% demand requirement for customer 4

$d_{13}^{+}=115$ dollars exceeded transportation budget goal constraint

$d_{15}^{+}=30$ units imbalance between customers 1 and 3

$d_{16}^{+}=3,360$ total transportation cost

All other real and deviational variables$=0$

The following observations can be made regarding the achievement or non-achievement of goals:

P_1 achieved: the fiirst goal of meeting the entire demand of customer 4 was completely achieved.($X_{14}+X_{24}+X_{34}=250$, and $d_7^-=0$).

P_2 achieved: the second goal to ship at least 100 units over the route from werehouse 3 to customer 1, due to union agreement, was completely attained. ($X_{31}=100$, and $d_8^-=0$).

P_3 achieved: the thirdgoal to meet no less than 80% of each customer's demand was fully met. Customer 1 received 190 units, which was 10 less than demand ($d_4^-=10$), but 30 units over the 80% goal ($d_9^+=30$). Customer 2 received 100% of demand ($d_5^-=0$), which was 20 over the 80% goal ($d_{10}^+=20$). Customer 3 received exactly 80% of demand (360 units), yielding $d_6^-=90$ and $d_{11}^+=0$. Customer 4 received, by the highest priority goal, 100% of demand(250 units), giving $d_7^-=0$ and $d_{12}^+=50$.

P_4 not achieved: the fourth goal to keep transportation cost to no more than 110% of the budgeted amount of 2,950, was not met. Total cost was 3,360, as given by d_{16}^+, which was **$115** over the goal (given by $d_{13}^+=115$). Thus, total transportation cost was 11.3% of budget. If this were considered unacceptable, then the G.P. problem would have to be reformulated, with the budget goal given a higher priority.

P_5 achieved: the fifth goal to minimize shipping over the route from warehouse 2 to customer 4, due to hazards, was fully met. ($X_{24}=0$, and $d_{14}^+=0$).

P_6 not achieved: the sixth goal to balance the portion of demand filled between customers 1 and 3 was not met, as indicated by the value for $d_{15}^+=30$, the imbalance between customers 1 and 3. ($X_{11}+X_{21}+X_{31}-.4444\ X_{13}+X_{23}+X_{33})=190—160=30$).

P_7 not achieved: the goal to minimize shipping cost to zero was, of course, impossible. However, this goal acted to minimize total shipping costs in the event they were less than the budget goal. It also provides as output the total shipping cost as the value of the deviational variable, d_{16}^+, which was 3,360. Since the total cost for the G.P. solution was 3,360 versus a cost of 2,950 for the standard transportation solution, the imputed price for achievement of higher priority goals is $410.

It is of interest to observe the redistribution of allocations achieved by the G.P. solution versus the standard transportation solution (cost minimization only). The standard transportation solution yielded alternate optimal solutions and is compared to the G.P. solution as follows:

Transportation Method Alternate Optimum

Routes	(1)	(2)	G.P. Optimum
X_{11}	200		
X_{12}	100	100	100
X_{13}		200	
X_{14}			200
X_{21}		200	90
X_{22}			
X_{23}	200		110
X_{24}			
X_{31}			100
X_{32}			
X_{33}	250	250	250
X_{34}	150	150	40

Both the alternate optimum solutions of the standard transportation approach violated the first three priority goals, as specified by management.

VI Conclusion

Virtually all models developed for transportation problems have focused upon the optimization of a single objective criterion, namely the minimization of total transportation costs. They have generally neglected or often ignored the multiple conflicting objectives involved in the problem, the priority structure of these objectives, various environmental constraints, unique organizational values of the firm, and bureaucratic decision structures. However, in reality these are important factors which greatly influence the decision process of transportation problems. In this study the GP approach is utilized because it allows the optimization of multiple conflicting goals while permitting an explicit consideration of the existing decision environment.

A simple illustration is presented in this study to demonstrate how goal programming may be applied to the transportation problem. Of course the transportation problem itself could be much more complex in a realistic situation, however, the basic approach to formulation of the GP model would be analagous. An extension to the basic transportation GP model could also be formulated for the case in which the sources were treated as manufacturing plants with regular time and overtime operations, and with associated production goals. Likewise, the transhipment problem in which sources and/or destinations may serve as intermediate shipping points could also be treated as a GP problem.

Developing and solving the GP model points out where some goals cannot be achieved under the desired policy and, hence, where tradeoffs must occur. Furthermore, the model allows the manager to review critically the priority structure for the goals in view of the solution derived by the model. Indeed, the most important property of the GP approach is its great flexibility which allows model experimentation with numerous variations of constraints and priortiy structure of goals.

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