Optimizing the Location-Allocation Problem with Multiple Objectives

Sang M. Lee · Jong-Hyun Baek***

≪Contents≫

I. INTRODUCTION

The location-allocation problem involves multiple shipping destinations, with known demands for a given product and known transportation costs from sources to destinations. The problem is to determine the number of facilities and their locations in order to best service the shipping destinations. Typically, the problem concerns itself with the tradeoff between the cost of building and operating facilities to meet product demand and the cost of transportation. The establishment of a large number of facilities would decrease transportation costs, while it would escalate fixed costs for the construction and operation of facilities. Conversely, the centralization of facilities will decrease the total fixed costs of the facilities, but the transportation costs would be at a high level.

While cost tradeoff has been and remains to be an essential consideration, the trend of the 1970's toward more socially conscious business activities reflects the increasing importance of noneconomic considerations in the location determination. We are all too familiar with the problems and issues surrounding recent location related news such as nuclear power plants, new international

---

Author: * Chairman and Professor, Department of Management, College of Business Administration, University of Nebraska-Lincoln

** Associate Professor, Department of Business Administration, Sogang University

airport, and the proposed new dams in our country.

A more comprehensive approach is required to analyze the ramifications of various multiple and often conflicting objectives in the location-allocation problem. The need for models of this type can be seen in the results of a locational survey of industrial development executives in the public utility industry. The 1967 survey revealed that the primary factors of site selection, in order of their importance, were labor availability, availability of site, and proximity to the market. The primary location considerations as reported in the same survey administered in 1972, however, were as follows: environmental considerations, labor quality and supply, and avilability of utilities (16). From these results it is clearly evident that pure cost minimization models are no longer as applicble in today's energy and environment conscious era.

Revelle *et al.* (19). point out that governmental regulations concerning pollution control, equal opportunity and affirmative action, and preservation of resources and aesthetics can also dictate a non-optimal site choice in terms of economic criteria. They also suggest that the stochastic nature of the supply and demand in regard to seasonal or periodic fluctuations, as well as changes in economic conditions and population patterns, should be considered in the facility location problem. Student(23) emphasizes the need to consider human values in addition to other criteria for determining plant location. These findings support studies of Simon(20) and Shubik(21) which indicate that the traditional economic motives are not the only consideration of real world managers.

Location analysis models can be classified into two structural categories: 1) location on a plane; and 2) location on a network(19). The first type models, location on a plane, are clearly based on a single objective. They assume total cost to be proportional to distance traveled, and thus attempt to determine some point or points which minimize the sum of the possible distances from sources to destinations. Location literature includes many examples and modifications of both rectilinear-distance and Euclidean-distance solutions to this problem(2,5,6,18,19).

The location of facilities as a point on a plane technique clearly does not treat many of today's realities. The location indicated may be in conflict with many corporate policies or legislated regulations. For example, an industrial site may be available, while the community may be opposed to that particular commercial activity, or the area may be environmentally stressed such that it would be unwise to locate there.

The second type of models based on the network location enumerate previously determined alternative facility locations and sites of demand as nodes on a network. Screening procedures can be used, as suggested by Nutt(17) to develop a list of alternative locations which would be acceptable choices. Although these initial checklist-type methods do consider noneconomic criteria and provide a list of satisfying alternatives, mechanisms for choosing among those alternatives revert to maximizing profit or minimizing costs associated with that location. Locational costs associated with the network models include total transportation costs associated with that site as well as the amortized facility cost or fixed cost associated with that facility. The problem then becomes a search for the optimal solution to the tradeoff between fixed costs and transportation costs.

Many authors have also considered the fixed charge solution approaches to the facility location problem(4,5,9). The primary focus of the research has been to develop more exact solution methods(3,22), or models which can efficiently handle larger sized problems(8,25). However, no consideration of multiple objective fixed charge facility location problems is apparent in the literature.

This paper presents an approach to the facility location problem which allows the analysis of multiple conflicting goals as an extension of previous solution approaches. Specifically, the paper applies the branch and bound integer goal programming approach to the location-allocation problem.

II. THE GOAL PROGRAMMING APPROACH

The concept of goal programming (GP) can be best described as an extension of linear programming which attempts to resolve the decision problem with multiple objectives. The concept was originally presented by Charnes and Cooper(1) and further refined by the work of Ijiri(7), Lee(10), and others. Lee developed the modified simplex method as a solution technique and many of his studies(11,12,13,14,15) demonstrated a variety of applications of the technique. Additionally, Lee and Morris [see 15] developed and tested integer goal programming algorithms based on the cutting plane, the branch and bound, and implicit enumeration methods.

The branch and bound goal programming method can be sumarized as follows:

Step 1: Solve the model by the ordinary modified simplex method of goal-programming(10).

Step 2: Examine the optimal solution. If the basic variables that have integer requirements are integer valued, the integer optimal solution is obtained-stop. If one or more basic variable do not satisfy integer requirements, go to step 3.

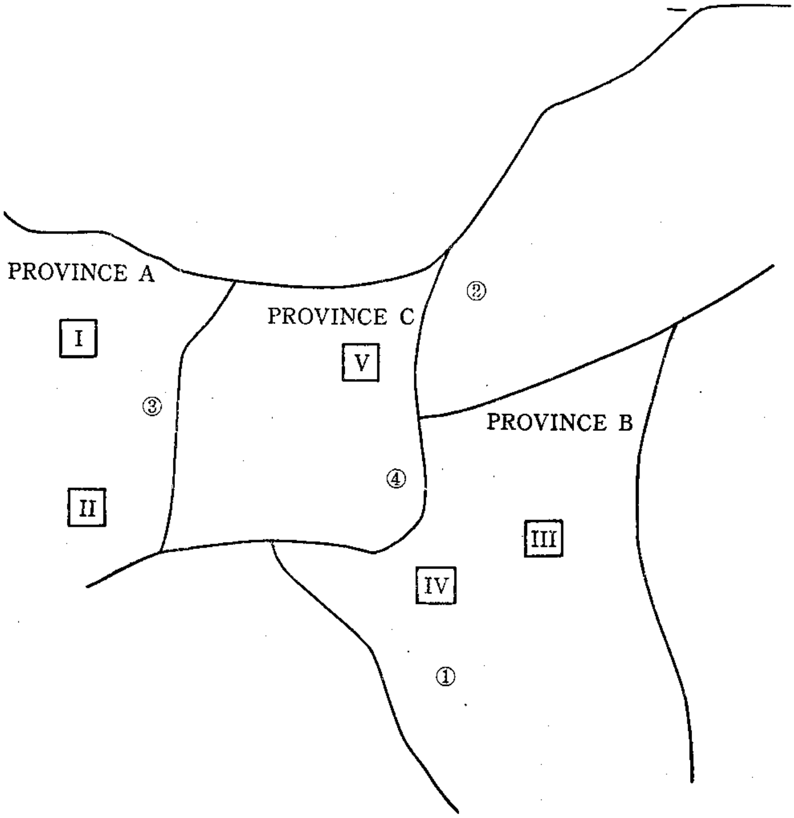
Step 3: The set of feasible solutions is branched into subsets (subproblems). The purpose of branching is to eliminate continuous solutions that do not satisfy the integer requirements of the problem. The branching is achieved by introducing mutually exclusive constraints that are necessary to satisfy integer requirements while making sure no feasible integer solution is excluded.

Step 4: For each subset, the optimal value of the objective function(degree of goal unattainment, U_k) is determined as the lower bound. The optimal U_k of a feasible solution which satisfies integer requirements becomes the upper bound. Those subsets having lower bounds that exceed the current upper bound must be excluded from further analysis. A feasible solution having U_k which is as good as or better than the lower bound for any subset is to be found. If there exists such a solution it is optimal-stop. If such a solution does not exist, a subset with the best lower bound is selected and go to step 3.

III. THE GP LOCATION-ALLOATION MODEL

In order to demonstrate the applicability of goal programming to facility location problems, a simple illustrative example is given. A manufacturing firm located on the west coast of Korea plans to construct a manufacturing facility or facilities which would subsequently service four major eastern distribution centers. For the sake of simplicity, it is assumed that a single product is produced. Preliminary screening of sites in the distribution sector has identified five potential communities for plant location which meet general production requirements such as available utilities, labor, access to transportation and raw materials. The first two sites are in the same province, and the next two sites

Fig. 1. Proposed Facility Sites and Existing Distribution Centers



Key: □ Potential Facility Sites ○ Existing Distribution Centers are in a second province(see figure 1). Because of differing construction and labor costs and varying levels of community development incentives such as free or partially free construction sites, the fixed costs vary from community to community. A list of five plant sites and associated annual fixed costs(amortized construction and operating costs) is given below:

Potential Sites	Fixed Cost(₩1,000)
Location I(Province A)	825
Location II(Province A)	750
Location III(Province B)	600
Location IV(Province B)	600
Location V(Province C)	650

The forecasted annual product demand for the four distribution centers and costs for shipping each case of goods are given in Table 1.

Table 1

		DIST CTR1	DIST CTR2	DIST CTR3	DIST CTR4
Transportaion Cost	Location I	200	110	40	90
	Location II	180	90	40	80
	Location III	50	200	225	25
	Location IV	35	160	250	35
	Location V	210	35	125	50
	DEMAND(In cases)	400	300	200	100

The firm's policy that its manufacturing process should not violate the air quality standards set by individual communities and by the provinces. It wishes to use the same equipment design as is used in the west coast plants, without modification if possible. The manufacturing process discharges 2.88 lb per hour particle emissions which is an acceptable amount in the three provinces under consideration, as long as the poundage of materials processed does not exceed the norms given in the province air quality standards for that rate(24). These standards vary among the provinces with province A having the most stringent standards. Given the two-shift day planned for the future plant, maximum yearly production level (cases of the product) that the firm will be allowed to produce by a single facility are as follows for each province:

Province A	600 cases per plant
Province B	480 cases per plant
Province C	800 cases per plant

Distribution center 3 is located in province A and is a particularly favored customer. In order to provide a high level of service to this account, at least one facility needs to be located in province A which is capable of providing a minimum of 50 cases of product to the center on short notice. Additionally, the firm has set 1.3 millon dollars as the ceiling for the total annual fixed costs for new facilities.

1. Goals and Priorities

With the above given information, as well as the company'sdesire to meet product demand to achieve the more traditional goals of keeping a balance between fixed costs of opening new plants and transportation costs, the following goals are established by the management of the company. They are listed in the order of their preemptive priorities $(P_k >>> P_{k+1})$, with P_1 indicating the most important goal.

P_1: Satisfy the product demands of the four distribution centers.

P_2: Limit fixed costs to ₩1,300,000 per year.

P_3: Keep the production level at each facility within acceptable air pollution emission standards for the given state.

P_4: Ensure that distribution center 3 would retain its favored customer status.

P_5: Minimize total costs (fixed costs plus transportation).

P_6: Minimize transportation costs.

2. Variables and Parameters

In order to formulate the goal programming model of the facility location problem, the following variables and parameters will be used:

X_{ij}=the amount of the product(in cases) to be transported from facility i to distribution center j.

Y_i=zero-one decision variables where $Y_i=1$ if facility i is opened, but $Y_i=0$ otherwise.

C_{ij}=transportion cost per case of the product from facility i to distribution center j.

d^-_k=underachievement of goals or constraints in the kth equation.

d^+_k=overachievement of goals or constraints in the kth equation.

3. Model Constraints

The constraints for the facility location model can be formulated as follows:

1. Product Demand Goal

In order to meet the demand of each distribution center the negative deviations should be minimized in the following goal constraints:

$$\sum_{i=1}^{5} X_{i1}+d^-{}_1-d^+{}_1-400 \tag{1}$$

$$\sum_{i=1}^{5} X_{i2}+d^-{}_2-d^+{}_2=300 \tag{2}$$

$$\sum_{i=1}^{5} X_{i3}+d^-{}_3-d^+{}_3=200 \tag{3}$$

$$\sum_{i=1}^{5} X_{i4}+d^-{}_4-d^+{}_4=100 \tag{4}$$

Minimize $d^-{}_i$ in each of the goal constraints.

2. Fixed Costs Limitation Goal (In ₩1, 000s)

$$825Y_1+750Y_2+600Y_3+600Y_4+600Y_5+d^-{}_5-d^+{}_5=1300 \tag{5}$$

Minimize $d^+{}_5$.

3. Air Standards Goal

To achieve the air standards goal, production must be limited to the following case amounts shown as the right-handside values for each new facility.

$$\sum_{j=1}^{4} X_{1j}+d^-{}_6-d^+{}_6=600 \tag{6}$$

$$\sum_{j=1}^{4} X_{2j}+d^{-}{}_{7}-d^{+}{}_{7}=600 \qquad (7)$$

$$\sum_{j=1}^{4} X_{3j}+d^{-}{}_{8}-d^{+}{}_{8}=480 \qquad (8)$$

$$\sum_{j=1}^{4} X_{4j}+d^{-}{}_{9}-d^{+}{}_{9}=480 \qquad (9)$$

$$\sum_{j=1}^{4} X_{5j}+d^{-}{}_{10}-d^{+}{}_{10}=800 \qquad (10)$$

Minimize positions deviations.

4. Favored Customer Service Goal

It is desired that distribution center 3 will receive good service by having a facility in the same general area.

$$\sum_{i=1}^{2} X_{i3}+d^{-}{}_{11}-d^{+}{}_{11}=50 \qquad (11)$$

Minimize $d^{-}{}_{11}$.

5. Total Cost Minimization Goal

It is desired to minimize total fixed and transportation costs.

$$825Y_1+750Y_2+600_3+600Y_4+650Y_5+\sum_{i=1}^{5}\sum_{j=1}^{4} C_{ij}X_{ij}+d^{-}{}_{12}-d^{+}{}_{12}=0 \qquad (12)$$

Minimize $d^{+}{}_{12}$.

6. Transportation Cost Minimization Goal

$$\sum_{i=1}^{5}\sum_{j=1}^{4} C_{ij}X_{ij}+d^{-}{}_{13}-d^{+}{}_{13}=0 \qquad (13)$$

Minimize $d^{+}{}_{13}$.

7. Zero-One System Constraints

Additionally, the following system constraints must be included for forcing Y_i to have a value of 1, thus incurring fixed costs, if any units are produced at plant i.

$$\sum_{j=1}^{4} X_{1j}-3000Y_1\leq 0$$

$$\sum_{j=1}^{4} X_{2j}-3000Y_2\leq 0$$

$$\sum_{j=1}^{4} X_{3jj}-3000Y_3\leq 0$$

$$\sum_{j=1}^{4} X_{4j}-3000Y_4\leq 0$$

$$\sum_{j=1}^{4} X_{5j}-3000Y_5\leq 0$$

where the Y_i coefficient of 3000 is chosen as a sufficiently large value to always force Y_i to be 1 if $\sum X_{ij}\mp 0$ for that facility. The following non-negativity constraints must also hold: $X_{ij}\leq 0$ and $Y_i\leq 1$.

4. The Objective Function

Given the above constraints and considering the priorities assigned to the firm's achievement of goals, the objection can be formulated as

$$\min\ Z=P_1\sum_{i=1}^{4}d^-{}_1+P_2d^+{}_5+P_3\sum_{i=6}^{10}d^+{}_i+P_4d^-{}_{11}+P_5d^+{}_{12}+P_6d^+{}_{13}$$

IV. RESULTS AND DISCUSSION

The preceding facility-allocation problem was solved using a computer program employing a branch and bound algorithm of integer goal programming. The solution (see figure 2) is as follows:

Real Variables

$X_{41}=400$ cases shipped from plant IV to distribution center 1

$X_{52}=300$ cases shipped from plant V to distribution center 2

$X_{53}=200$ cases shipped from plant V to distribution center 3

$X_{54}=100$ cases shipped from plant V to distribution center 4

Zero-one Variables

$Y_4=1$ A plant is opened at location IV

$Y_5=1$ A plant is opened at location V

Deviational Variables

$d^-{}_5=50,000$—underutilization of the allowed fixed costs

$d^-{}_6=600$

$d^-_7=600$ underachievement of production allowed under air quality standards for each facility location.

$d^-_8=480$

$d^-_9=80$

$d^-_{10}=200$

$d^-_{11}=50$—underachievement of minimal shipment to favored customers

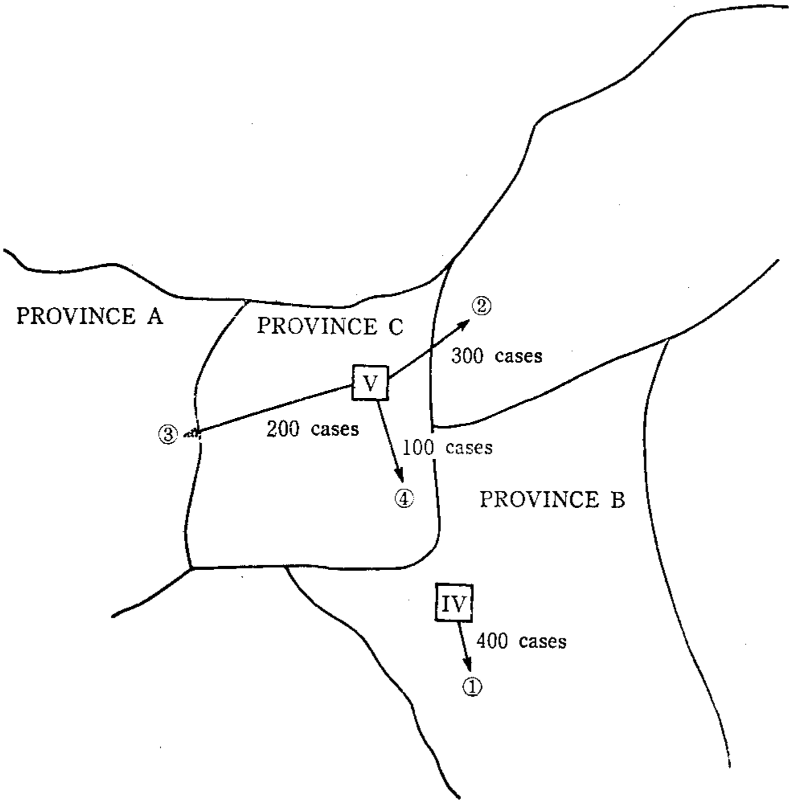
$d^+_{12}=1,302,000$—total cost

$d^+_{13}=52,000$—total transportation cost

All other real, zero-one and deviational variables$=0$.

Analysis of the goal achievement or underachievement indicates that two locations are selected for new manufacturing plants: location IV and location

Fig. 2. Solution to Facility Location Problem



Key: ☐ Facility Sites ○ Distribution Centers

V. The plant location IV will produce a total of 400 cases of the product for distribution center 1. The plant at location V will produce a total of 600 cases and they will be allocated to distribution center 2 for 300 cases, and to distribution 4 for 100 cases. The solution indicates that the goals regarding satisfaction of demand (P_1), fixed costs limitations (P_2), and anti-pollution requirements (P_3) were completely satisfied by this solution. The fourth goal, the delivering of preferred service to distribution center 3, could not be met. Additionally, the goals of minimizing total costs and transportation costs, goals 5 and 6, were not completely achieved. However, this is to be expected since the formulation attempts to minimize the deviation of costs to zero, which could never realistically be achieved.

Interpretation of the deviational variables associated with these goals show that this solution has a total yearly cost of ₩1,302,000 and a transportation cost of ₩52,000. Additionally, the firm can increase production at facility IV by 80 cases and at facility V by 200 cases without exceeding pollution emission limitations. This solution leaves the company ₩50,000 for investment in other projects which incur fixed costs.

V. CONCLUSION

Traditional models developed for facility location analysis have focused on the minimization of either transportation costs or a combination of fixed costs and transportation costs. The facility location problem, however, is complex, generally consisting of multiple conflicting goals. Few models have been able to consider a solution taking these factors into account.

This paper suggests the integer goal programming approach to facility location and demonstrates the application of the model to such a problem. Although the example presented is made simple purposely, the formulation and solution of more complex problems can be easily seen. The model strives to achieve as many of the management goals, in order of their priority, as is possible given the constraints of the situation. Further sensitivity analysis can be performed by restructuring priorities or constraints and by interpreting both the overachievement and underachievement of goals. An interactive mode based on theCMS (conversational monitor system) is available for this purpose(15).

The goal programming approach to the facility location problem offers a meaningful management science tool, which hopefully can provide more useful and practical information for decision making to management.

REFERENCE

1. Charnes, A., and Cooper, W.W. *Management Models and Industrial Applications of Linear Programming* (New York: John Wiley and Sons, 1961).
2. Cooper, Leon. "Location-Allocation Problems," *Operations Research*, 11, 3, (1963), pp. 331-343.
3. Cooper, Leon and Drebes, C. "An Approximate Solution Method for the Fixed Charge Problem," *Nav. Res. Log. Quarterly*, 14, 1, (1967), pp. 101-113.
4. Efroymson, M.A. and Ray, T.L. "A Branch and Bound Algorithm for Plant Location," *Operations Research*, 14, 3, (1966), pp. 361-368.
5. Ellwein, L.B. and Gray, P. "Solving Fixed Charge Location-Allocation Problems with Capacity and Configuration Constraints," *AIIE Transactions*, 3, 4, (1971), pp. 290-297.
6. Francis, R.L. and White, J.A. *Facility Layout and Location* (Englewood Cliffs: Prentice Hall, 1974).
7. Ijiri, Y. *Management Goals and Accounting for Control* (Amsterdam: North Holland, 1965).
8. Kennington, J. "The Fixed-Charge Transportation Problem: A Computational Study with a Branch-and-Bound Code," *AIIE Transactions*, 8, 2, (1976), pp. 241-247.
9. Kuehn, A.A. and Hamburger, M.J. "A Heuristic Program for Locating Warehouses," *Management Science*, 9, 4, (1963), 643-666.
10. Lee, S.M. *Goal Programming for Decision Analysis* (Philadelphia: Auerbach, 1972).
11. Lee, S.M. and Clayton, E. "A Goal Programming Model for Academic Resource Allo-

cation," *Management Science*, 18, 8, (1972), pp. 395-408.

12. Lee, S.M. and Lerro, A.J. "Optimizing the Portfolio Section for Mutual Funds," *Journal of Finance*, 28, 5, (1973), pp. 1087-1101.
13. Lee, S.M. and Nicely, R. "Goal Programming for Marketing: A Case Study," *Journal of Marketing*, 38, 1, (1974), pp. 24-32.
14. Lee, S.M. and Moore, L.J. "A Practical Approach to Production Scheduling," *Production and Inventory Management*, 15, 1, (1974), pp. 79-92.
15. Lee, S.M. *Goal Programming Methods for Multple Objective integer Programs*(Atlanta: American Institute of Industrial Engineers, forthcoming).
16. Lynch, Allen A. "Environment and Labor Quality Take Top Priority in Site Selection," *Industrial Development*, 142, 2, (1972), pp. 13-15.
17. Nutt, Merle C. *Functional Plant Planning*(New York: Exposition Press, 1970).
18. ReVelle, C.S. and Swain, R.W. "Central Facilities Location," *Geographical Analysis*, 2, 1, (1970), pp. 30-42.
19. ReVelle, C.S., Marks, D. and Liebman, J.C. "An Analysis of Private and Public Sector Location Models," *Management Science*, 16, 11, (1970), pp. 692-707.
20. Simon, H.A. "A Behavioral Model of Rational Choice," *Quarterly Journal of Economics*, 69, 1, (1955), pp. 99-118.
21. Shubik, M. "Approaches to the Study of Decision-Making Relevant to the Firm," in *The Making of Decisions*, edited by W.J. Gore and J.W. Dyson (London: Collier-Macmillan, Ltd., 1955).
22. Steinberg, D.I. "The Fixed Charge Problem," *Naval Research Logistics Quarterly*, 17, 2, (1970), pp. 217-235.
23. Student, K.R. "Cost vs. Human Values in Plant Location," *Buiness Horizons*, 19, 4, (1976), pp. 5-14.
24. "The Mid-Atlantic States: A Microcosm of the Nation's Pollution Problems," *Industrial Development*, 141, 1, (1973), pp. 9-29.
25. Walker, Warren E. "A Heuristic Adjacent Extreme Point Algorithm for the Fixed Charge Problem," *Management Science*, 22, 5, (1976), pp. 587-596.
26. 工業團地의 設計, 企銀調査部, 1965.

27. 國土計劃基本構想, 建設部, 1968.

28. 國土未來의 設計, 白永勳, 安元泰著, 1980.

29. 國土長期構想現況資料, 國土開發研究院, 1979.

30. 產業公害와 韓國經濟, 大韓商工會議所, 1980. 6.

31. 產業基地建設의 오늘과 來月, 產業基地開發公社, 1978.

32. 우리나라 工業立地政策의 方向, 大韓商工會議所, 1979.

33. 韓國의 綜合輸送體系, KDI, 1979.

34. 韓國의 國土・都市・環境, KDI, 1979. 9.

35. 韓國地方工業의 實態分析과 農村工業의 振興 및 工業分散에 관한 研究報告書, 韓國經濟開發協會, 1966.

한글요약

종래 설비입지를 위하여 개발하여온 모든 모형들은 단일목표의 최적화 기준 위에서 이뤄져 왔음을 파악할 수 있다. 즉 단일목표의 최적화란 수송비 또는 총입지비용의 최소화를 의미하고 있다. 따라서 입지에 관한 의사결정에서 반드시 고려해야만 할 요인들——환경적 책임, 피고용인들의 복지, 생산성, 그리고 노조의 활동——에 관해서는 전혀 고려하고 있지 않을 뿐만 아니라 이들 간의 상충(相衝)관계도 전혀 도외시 되고 있는 것이 문제가 되고 있다.

이 논문에서는 정수계획법(整數計劃法)과 목표계획법(目標計劃法)의 특징을 결합한 소위 "branch and bound integer goal programming"에 의하여 다수목표를 가지는 입지의사결정 문제의 해결을 시도하고 있다.