REFRACTION TOMOGRAPHY BY BLOCKY PARAMETERIZATION

CHANGSOO SHIN[1], JUNGHEE HA[1] and SANGYONG JEONG[2]

[1] *Seoul National University, School of Civil, Urban and Geosystem Engineering, San 56-1, Shillim-dong, Kwanak-gu, Seoul 151-742, Korea.*
[2] *Dong Ah Construction Industrial Co. Ltd., C.P.O. Box 383, Seoul 100-110, Korea.*

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ABSTRACT

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Refraction tomography can be used to interpret the velocity and geometry of the shallow subsurface for engineering surveys as well as for calculating static corrections in reflection data using refraction data. Our algorithm calculates head waves and partial derivatives by means of a wavefront method calculated on a triangular finite element mesh. We parameterize the subsurface model in blocky layers, such that we invert for the velocities and interface coordinates of the geologic model. We calibrate our technique by inverting synthetic data computed by the wavefront method, and then apply this method to the real seismic data we obtained. The velocity and depths are consistent with those obtained by the more commonly used Coherency Inversion Method.

KEY WORDS: refraction tomography, partial derivatives, wavefront method,
triangular finite element, blocky layers.

INTRODUCTION

Seismic refraction techniques have been widely used to map and delineate shallow subsurface. In particular, refraction data inversion is routinely used to calculate static corrections (Russell, 1989) and to estimate the shallow velocity-depth model (Hampson and Russell, 1984; Docherty, 1992; Landa et al., 1995) prior to seismic imaging. Seismic traveltime inversion can be classified into two methods depending upon the choice of the objective function.

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The more common objective is to minimize the error (or difference) between the field data and the synthetic seismograms. Another, less commonly used objective is to maximize the cross-correlation between the synthetics and the observations. Full waveform inversion can be broadly divided into the direct inversion methods (Clarke, 1984; Yagle and Levy, 1985) and iterative, non-linear least-squares inversion methods (Tarantola, 1987; Pan et al., 1988). Ray tracing is one of the more efficient tools used to calculate the forward seismogram used in seismic inversion. Traveltimes can be obtained by using three types of ray tracing methods.

In shooting methods (Cassell, 1982; Červený et al., 1977), a fan of rays is shot from a source point at equal angles to the medium of interest. These rays are propagated until they intercept the earth's surface somewhere near the receiver. A more accurate estimates of the path and traveltime between the source and receiver may then be approached by successively subdividing the shooting angle. When the structure is complex or the receivers are arranged in a line, the shooting method can be very efficient in calculating the traveltime.

An alternative and a more expensive bending method (Um and Thurber, 1987; Pereyra, 1988) starts with an initial, usually incorrect, guess for the ray path connecting the source and receiver. The ray path is bent by a perturbation method until it satisfies a minimum traveltime criterion. When the interface and velocity variation can be represented by analytic functions, the bending method is simple and efficient.

A third method exploits the finite-difference solution to the eikonal equations (Vidale, 1988). Unfortunately, it is difficult to explicitly express the traveltime as a function of velocity and interface coordinates.

In this paper, we assume that every geologic model can be subdivided into a finite number of arbitrary shaped blocky regions, each having a different constant velocity. Instead of using the shooting method, we use wavefront algorithm developed by Coultrip (1993) which handles complex geologic models with irregular topography and for which we may analytically calculate the partial derivative of the traveltime. In Coultrip's (1993) method, the geologic model is subdivided into a triangular finite element mesh. The traveltime between the source and receiver is calculated from the raypath segment length and velocity of each element, such that we may easily calculate the partial derivatives of the traveltime with respect to the velocities and interface coordinates. While Coultrip (1993) tessellated the arbitrarily shaped geologic model with small cells, each having different velocity, we choose to fit the data by moving the nodes and changing the velocity of each blocky region. Although we may allow the velocity of each element to be arbitrarily, it is more geologically reasonable (and mathematically stable!) to constrain them to have some simple functional interrelationship. For simplicity, we will restrict the elements in each layer to have the same, yet to be determined, velocity. Such a blocky parameterization greatly reduces the number of unknowns compared to the cell based refraction tomography. Since layer boundary nodes position and elemental velocities are measured with different units, we use a logarithmic change of variables (Madden, 1972). In the next section, we will show how we will calculate traveltimes and partial derivatives. We will then show how we invert our traveltime data using a damped least-squares inversion method. Finally, we will calibrate this inversion method for both synthetic and real data.

BLOCKY PARAMETERIZATION

Small cell refraction tomography has a tendency to be non-unique because there are so many parameters compared to the number of observed traveltime measurements. To reduce this non-uniqueness, we assume that the subsurface can be divided into a small number of discrete layers or blocky regions delineated by straight-line segments. Head waves satisfy Fermat's least-time principle, as shown in Fig. 1. Coultrip's (1993) ray tracing can be easily applied to our geologic model subdivided into blocky regions having abrupt changes of velocity in the lateral direction. We tessellate our irregular shaped geologic model by means of Delauney (1934) triangles using an algorithm derived by Shewchuck (1996). After choosing the ray path which is nearest to the source and receiver, calculating the traveltime corresponding to those ray paths, we obtain the traveltime of the head wave from the source to receiver to be:

$$T = \sum_{k=1}^{K} (r_k/v_{i(k)}) \ , \tag{1}$$

where k is the number of ray path segments, $v_{i(k)}$ is the velocity of the i-th blocky region through which the k-th ray passes,

$$r_k = \sqrt{[(x_{k+1}-x_k)^2 + (z_{k+1}-z_k)^2]} \quad k = 1,2,3,\ldots,K \ ,$$

and x_k and z_k are the coordinates of the points indicated in Fig. 1. When applying the Gauss-Newton method using blocky parameterization, we need to take the partial derivatives of equation (1) with respect to the velocity of blocky region and interface coordinates. The partial derivatives of equation (1) with respect to the velocity of each blocky region, v_i (for example v_1, shown in Fig. 1) is given by:

$$\partial T/\partial v_1 = -(1/v_1^2)(r_1 + r_5) \ . \tag{2}$$

The key point to note is that when we take the derivative of the traveltime with respect to the velocity of i-th blocky region, the distance associated with the velocity of i-th blocky region only appears (see the Appendix).

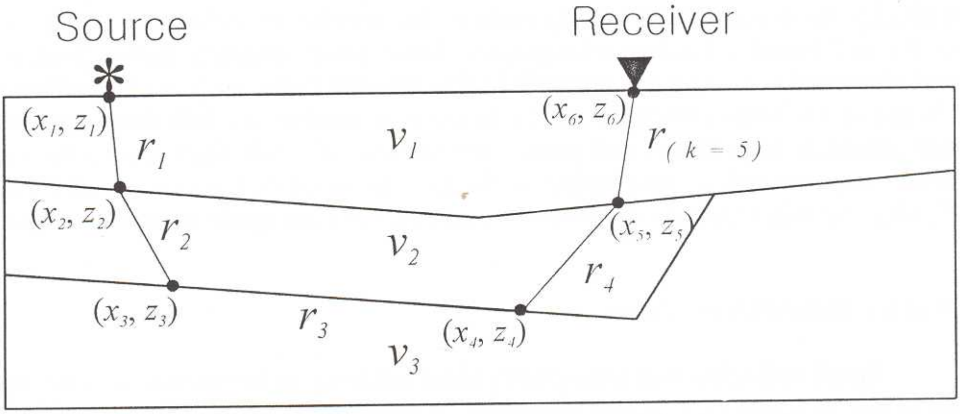


Fig. 1. Ray path in a model with irregular interfaces. Interfaces are assumed to consist of straight line segments.

The partial derivatives of equation (1) with respect to the interface coordinates are given by:

$$\partial T/\partial x_k = [(x_k - x_{k-1})/v_{i-1(k-1)}r_{k-1}] - [(x_{k+1} - x_k)/v_{i(k)}r_k] \quad , \tag{3}$$

and

$$\partial T/\partial z_k = [(z_k - z_{k-1})/v_{i-1(k-1)}r_{k-1}] - [(z_{k+1} - z_k)/v_{i(k)}r_k] \quad . \tag{4}$$

To illustrate the change in traveltime with respect to an interface node (X_m,Z_m) shown in Fig. 2, we need to identify the nodes to the left (X_l,X_l) and right (X_r,Z_r), along the same interface. Since we assume that the interface of the blocky region is constrained to move in a straight line when a model parameter (X_m,Z_m) is perturbed, we define:

$$z_k = [(Z_m - Z_l)/(X_m - X_l)]x_k + Z_m - [(Z_m - Z_l)/(X_m - X_l)]X_m \quad , \tag{5a}$$

and

$$z_j = [(Z_m - Z_r)/(X_m - X_r)]x_j + Z_m - [(Z_m - Z_r)/(X_m - X_r)]X_m \quad . \tag{5b}$$

Taking partial derivatives of the dependent variables z_k and z_j with respect to the independent variable Z_m leads to

$$\partial z_k/\partial Z_m = 1 - |(x_k - X_m)/(X_m - X_l)| \quad , \tag{6a}$$

and

$$\partial z_j/\partial Z_m = 1 - |(x_j - X_m)/(X_m - X_r)| \quad . \tag{6b}$$

Similarly, taking partial derivatives of the dependent variables x_i and x_j with respect to the independent variable X_m leads to

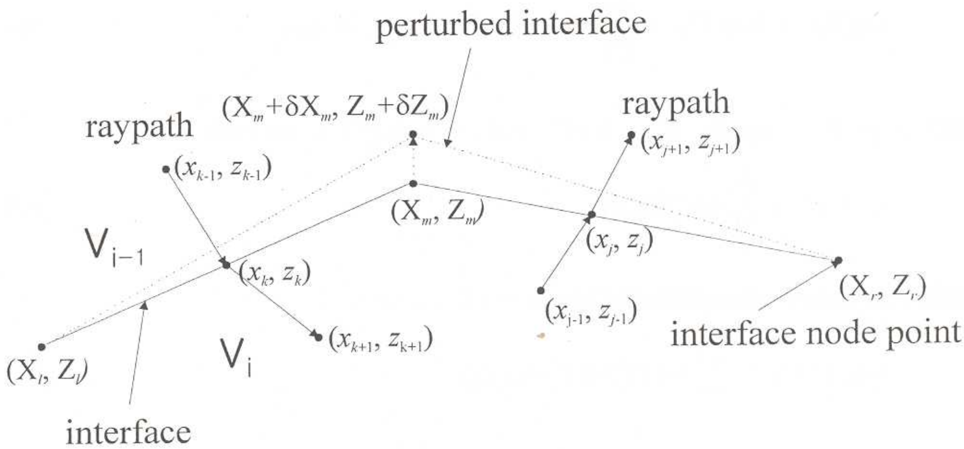


Fig. 2. Shape of interface where the coordinate (X_m,Z_m) is perturbed into vertical direction.

$$\partial x_k/\partial X_m = 1 - |(z_k - Z_m)/(Z_m - Z_l)| \quad , \tag{6c}$$

and

$$\partial x_j/\partial X_m = 1 - |(z_j - Z_m)/(Z_m - Z_r)| \quad . \tag{6d}$$

The partial derivative of the traveltime of ray path shown in Fig. 2 then becomes:

$$\partial T/\partial X_m = (\partial x_k/\partial X_m)(\partial T/\partial x_k) + (\partial x_j/\partial X_m)(\partial T/\partial x_j) \quad , \tag{7a}$$

and

$$\partial T/\partial Z_m = (\partial z_k/\partial Z_m)(\partial T/\partial z_k) + (\partial z_j/\partial Z_m)(\partial T/\partial z_j) \quad . \tag{7b}$$

THE DAMPED LEAST-SQUARES METHOD

Since the first breaks of head waves are functions of both the velocity and the coordinates of the interfaces, we need to perform simultaneous inversion. Unfortunately, solutions may become very unstable because the partial derivative of the traveltime with respect to layer velocity and interface coordinates are measured with different units. To avoid this scale problem of parameters, we employed the logarithmic variation method developed by Madden (1972). This change of variables could also be interpreted as a means of mapping our strictly positive model and measurement variables X_m, Z_m, V_l, and T_i as having a Poisson, rather than a Gaussian, distribution (Tarantola, 1987). Taking the logarithmic variation of the Taylor series expansion of the i-th traveltime, T_i, about an a priori model estimate, m_j, $j = 1,2,\ldots,J$,

$$\log(T_i) = \log(T_i^0) + \sum_{j=1}^{J} [\partial\log(T_i^0)/\partial\log(m_j)]\ \delta\log m_j \quad . \tag{8}$$

Defining: $p_j = \log m_j$, $\tau_i^0 = \log T_i^0$ and $\tau_i = \log(T_i)$, we state

$$\tau_i = \tau_i^0 + \sum_{j=1}^{J} (\partial\tau_i^0/\partial p_j)\delta p_j \quad . \tag{9}$$

After algebraic manipulation, equation (8) becomes:

$$\log(T_i/T_i^0) \cong \sum_{j=1}^{J} (m_j/T_i^0)(\partial T_i^0/\partial m_j)\delta p_j \quad . \tag{10}$$

Defining the matrix $\mathbf{S}$ as $S_{ij} = (m_j/T_i)(\partial T_i/\partial m_j)$, and the vector $\delta\tau$ as:

$$\delta\tau = \log T_i - \log T_i^0 = \log(T_i/T_i^0) \quad .$$

We apply a standard damped least squares method to equation (10) to obtain:

$$(S^T S + \boldsymbol{\beta}\mathbf{I})\delta\mathbf{p} = S^T\delta\tau \quad , \tag{12}$$

where $\mathbf{I}$ is the identify matrix, and $\boldsymbol{\beta}$ is a damping vector. By solving equation (12) and updating the parameter space by a general iterative rule, we can find the optimum parameters which minimize the residual between the picked traveltime and the model response.

SYNTHETIC DATA EXAMPLES

Fig. 3 shows the inversion result for a 3-layer model having irregular topography. Input data consisting of six picked common shot records with one hundred receivers spaced three meter. As an initial starting model, we define horizontal interfaces, as shown in Fig. 3(b). The unknown parameters for each layer are the velocity and the twenty interface coordinates. After the eighth iteration, the velocity-depth model has converged to the true model. Fig. 3(c) shows the inverted interface of the model. There are two possible ways to employ the blocky parameterization for the inversion of head waves. One is the pure blocky parameterization which requires a priori information of the geologic model. The other is the mimicking of small cell tomography by subdividing the geologic model into big cells and allowing the change of the velocity of the block and the movement of the interface of the block. We employed two different approaches to attack the traveltime inversion of a complex geologic model.

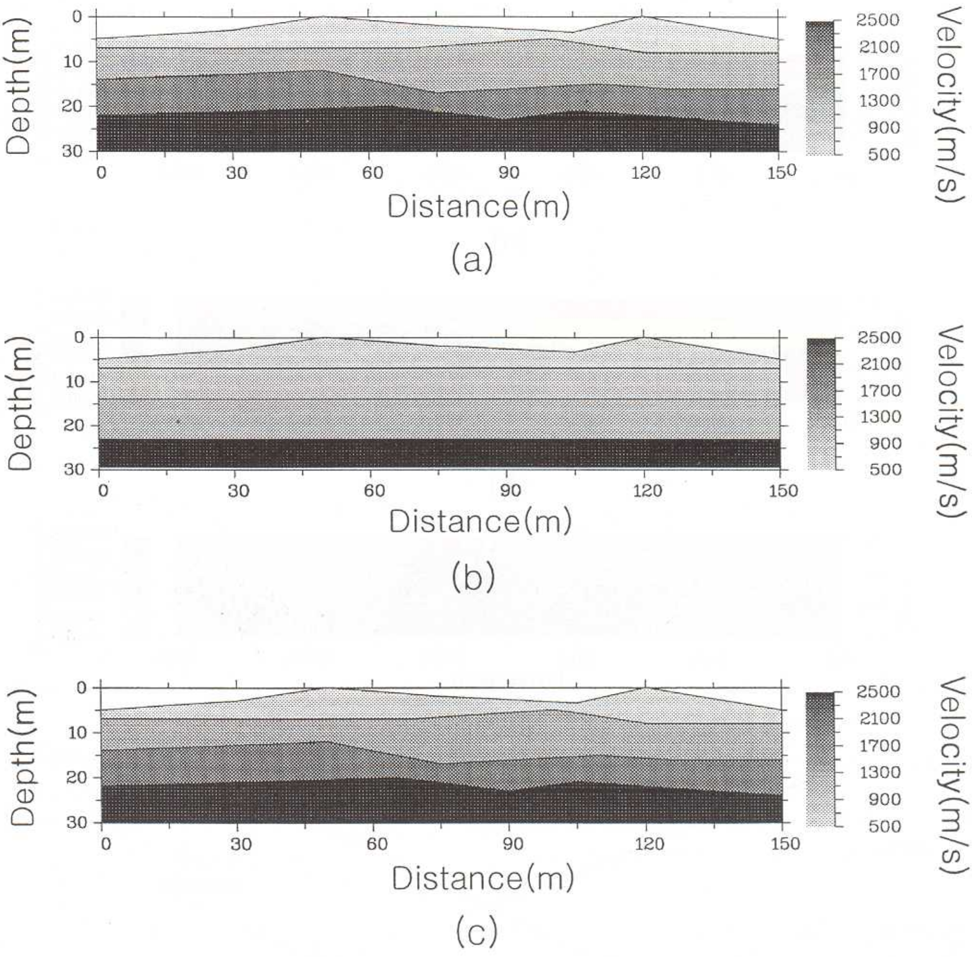
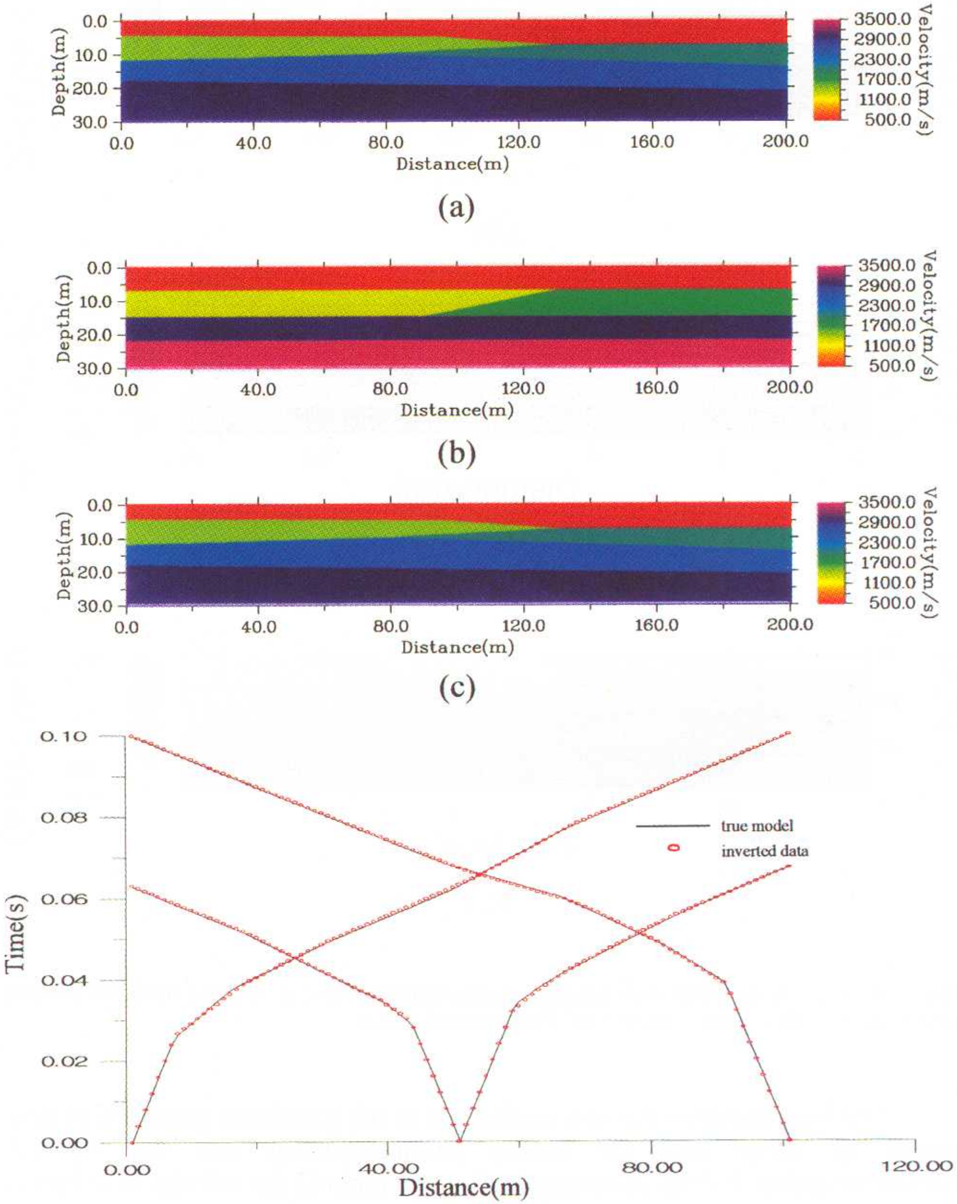


Fig. 3. A three-layered model with irregular surface boundaries and its inverted result. (a) The true model, (b) the initial model, and (c) the final inverted model.

We now illustrate the non-uniqueness of the traveltime inversion of head waves by using both the blocky parameterization and the big cell parameterization. In both parameterization, we allowed the velocity of the block and the interface of the blocks to change, and used a synthetic data generated for pinch out model shown in Fig. 4(a). As an initial guess for blocky parameterization, we used the model shown in Fig. 4(b). After the eleventh iteration, the inversion result converges to the true model, as shown in Fig. 4(c). The traveltime curves calculated for the models shown in Fig. 4(a) and 4(c) match perfectly and do not show any difference, as shown in Fig. 4(d).



(d)

Fig. 4. A pinch out model with horizontal surface boundaries and its inverted result. (a) The true model, (b) the initial model (c) the final inverted model, and (d) traveltime curve for the true model and the final inverted model. The dotted line indicates the traveltime curve for the true model. The dashed line indicates the traveltime curve of the final inverted model.

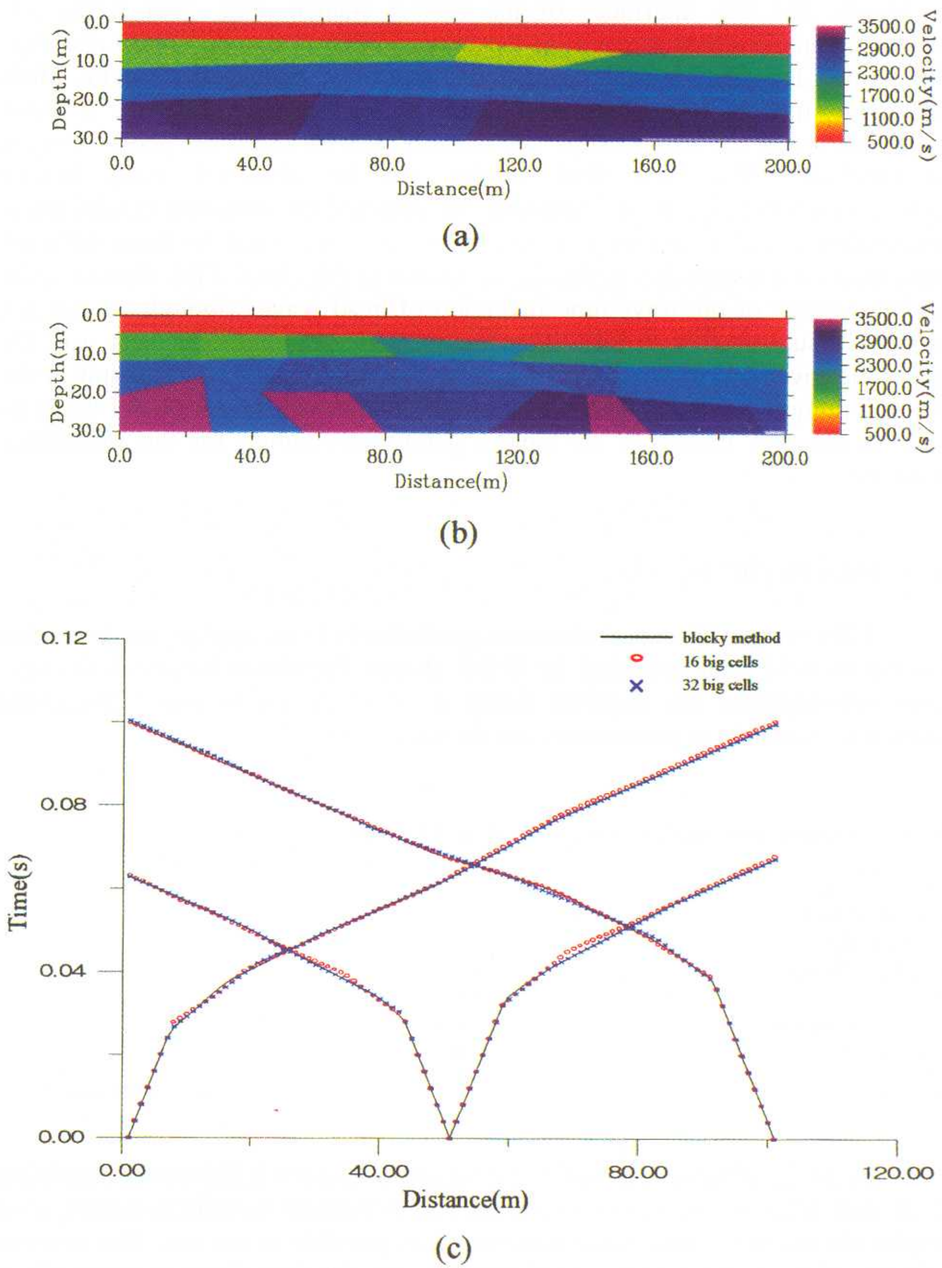


Fig. 5. Non-uniqueness of traveltime inversion of head waves of (a) final inverted result when sixteen blocks are used, (b) of final inverted model when thirty two blocks are used, and (c) of dotted line indicates the traveltime curve when sixteen blocks are used to invert the traveltime. The dashed line indicates the traveltime curve when thirty two blocks are used to invert the traveltime.

Unlike the blocky parameterization, we take the sixteen and the thirty-two rectangular blocks as initial guesses. In both cases, we allowed the velocities of the blocks and the interfaces of the blocks change. Fig. 5(a) shows the converged model constructed from the initial guess of sixteen blocks and Fig. 5(b) shows the final model constructed from the thirty two blocks. Both converged models using many blocks as initial guess shows a poor reconstruction of the true model. The more blocks we used, the poorer images we obtained. The best final model can be obtained using blocky parameterization [Fig. 4(c)]. Although we obtained the different results using three different initial guesses, the traveltime curve calculated for three different converged models matches perfectly, as shown in Fig. 5(c). This illustrates the non-uniqueness of the traveltime inversion of head waves, though we did not show the usual small cell refraction tomography example. The best way for traveltime inversion of head waves is to use large blocks , which result in the less non-unique solutions. The best one is to choose the initial model being as close as the true model, if we have a priori information for the subsurface structure.

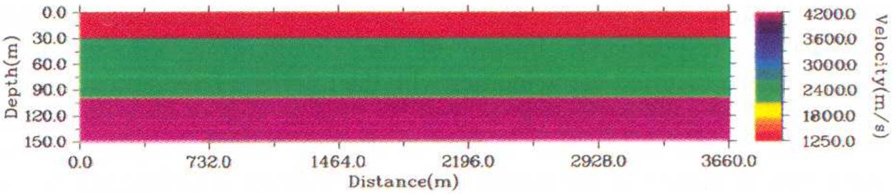
REAL DATA EXAMPLE

After successful experiments on synthetic data, we applied this inversion scheme to field data provided by IPRG (Israel Petroleum Research Group). Field seismograms are acquired using a vibroseis source and 120-channel receivers. Acquisition parameters are shown in Table 1.

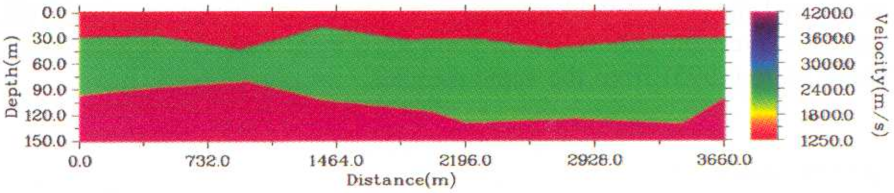
Table 1. Acquisition parameters of the IPRG (Israel) field data.

Number of Shots	21
Record Length	1 s
Sampling Interval	2 ms
Offset	100 m
Number of Groups	120 ch
Group Interval	20 m

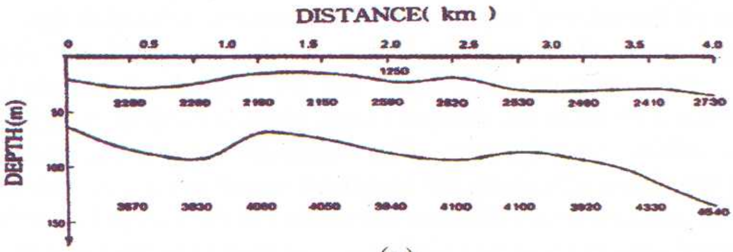
By rough picking and muting the refraction data of 2,520 traces consisting of 21 shot records, we can preserve the real refraction waveform events, tried to pick the accurate head wave traveltimes as possible as we can. The average velocity of the surface layer is calculated using the time-distance curve of the direct wave. The velocity of the first layer is fixed at 1,250 m/s. The Initial values of the velocities in the second and the third layers are 2,500 m/s and 4,500 m/s, respectively, which are based on the average of velocities calculated from the slope of the time-distance curve. The initial depths of the interfaces of



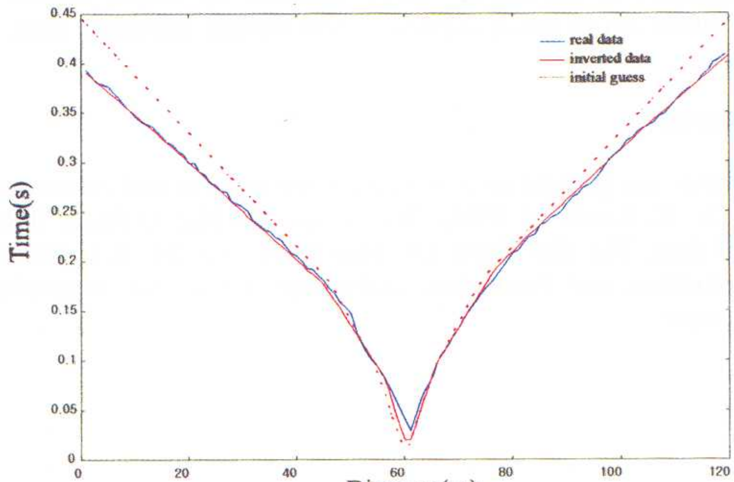
(a)



(b)



(c)



(d)

Fig. 6. Shows (a) an initial velocity model taken for the inversion of field refraction data, (b) inverted velocity model obtained by blocky parameterization technique, (c) velocity model obtained by Landa's (1995) cross coherence technique, and (d) comparison of the picked traveltimes and the first arrival time generated from the final updated model.

the second and third layers, which are composed of eight segments, are 30 m and 80 m, respectively. Instead of the simultaneous velocity and depth inversion, we inverted the velocities with the interface fixed and obtained 2,400 m/s and 4,200 m/s. Fixing the velocities of the layered model, we inverted the interfaces of the horizontal layers so that we could roughly estimate the geometric structure of the model. After 4 iterations, we obtained the velocity model shown in Fig. 6(b) when three flat layered model shown in Fig. 6(a) was used as an initial guess. Fig. 6(c) shows the traveltimes of the final inverted model and the picked field seismogram. These traveltimes matched well. Note that our inversion results are in good agreement with the results obtained by Landa et al.(1995) from the same data set.

CONCLUSIONS

In this paper, we have proposed a new topographic method for the determination of the velocity-depth model using seismic refraction data. One of the advantages over other methods is that, unlike the small cell refraction tomography, the inversion technique can be performed in the blocky parameterization way using the damped Gauss-Newton method, which result in reducing the number of unknowns. The wavefront ray tracing allows us to calculate the traveltime efficiently for complex layered structures with irregular topography. In addition to this efficient calculation of the forward modeling, the analytical partial derivative of the traveltime with respect to the velocity and the interface coordinates of the model can be obtained by simple algebra. The proposed algorithm can be easily applied to real seismic refraction data.

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APPENDIX

THE PARTIAL DERIVATIVE OF TRAVELTIME USING FERMAT'S LEAST-TIME PRINCIPLE

To show that when taking a derivative of the traveltime with respect to v_n, i.e., the distance associated with v_n only appears,we can express a simple traveltime for the ray paths shown in Fig. 1 as

$$T = s_1r_1 + s_2r_2 + s_3r_3 + s_2r_4 + s_1r_5 \ , \tag{A-1}$$

where (s_k, k = 1,...,5) is the slowness of the blocky region the ray passes through. For example, taking derivative of equation (A-1) with respect to s_1 gives

$$\begin{aligned} dT/ds_1 = & \ r_1 + r_5 + s_1(dr_1/ds_1) + s_2(dr_2/ds_1) + s_3(dr_3/ds_1) \\ & + s_2(dr_4/ds_1) + s_1(dr_5/ds_1) \ . \end{aligned} \tag{A-2}$$

Fermat's least-time principle requires for traveltime T to be minimum, so that $dT/dr_1 = 0$.

Hence the following equation is obtained:

$$dT/dr_1 = s_1 + s_2(dr_2/dr_1) + s_3(dr_3/dr_1) + s_2(dr_4/dr_1) + s_1(dr_5/dr_1) = 0 \ . \quad \text{(A-3)}$$

From equation (A-3), s_1 can be given as

$$s_1 = -s_2(dr_2/dr_1) - s_3(dr_3/dr_1) - s_2(dr_4/dr_1) - s_1(dr_5/dr_1) \ . \quad \text{(A-4)}$$

Substitution of equation (A-4) into equation (A-2) yields

$$dT/ds_1 = r_1 + r_5 \ . \quad \text{(A-5)}$$

Thus in a similar manner to the above example, when taking the partial derivative of the traveltime (t) with respect to v_n, the term associated with v_n only appears.