Alternative Methods of Cost Allocation for Regulatory Constraint and Their Effects on Output Choices of a Discriminating Firm

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It is widely observed that nearly all regulated industries, especially public utilities, engage in at least some simple forms of price discrimination among different classes of customers. Yet economic analysis of monopoly regulation has largely centered on simple monopoly, i.e., single-product single-price monopoly, as is exemplified by the Averch-Johnson type of models.(1) Therefore, these models deal mainly with input-choice effects of regulation on a simple monopoly. In this paper we will consider price discrimination explicitly and analyze output-choice effects of regulation on a discriminating monopoly.

A regulatory agency can regulate some or all of the submarkets of a discriminating firm and in practice it sometimes does one sometimes the other. We use the term "overall regulation" to describe the case where all submarkets of the firm are regulated(2) whereas partial regulation is that imposed only on some of its submarkets. Regulation of the inter state and inter state operations of telephone and electricity firms and regulation of intrastate gas utilities are of the overall type. An example of partial regulation is that of interstate gas transmitters. The Federal Power Commission (FPC) does not regulate direct sales of gas by the interstate pipelines to industrial consumers but does set profit limits on sales to gas utilities.

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(1) The seminal paper is Averch and Johnson (1962). Among numerous later works some of the more important ones are Baumol and Klevorick (1970) that clarifies a number of issues relating to the Averch-Johnson effect and Bailey (1973) that provides a broad analysis of regulatory models.

(2) Overall regulation should not be confused with separate regulation. The former covers all submarkets of the firm under a single jurisdiction whereas the latter refers to regulation of different submarkets by different jurisdictional regulatory agencies. An example of separate regulation is the separation of intra- and interstate regulation of the Bell System. We will not deal with this issue here, because our analysis can readily be applied to it.

A special issue with important public policy implication arises in the determination of the terms of partial regulation. When the rate base of a partial-regulatory constraint contains a common cost, some device for imputation of this cost is required, because the common cost, by definition, does not come to us separated between the regulated and unregulated outputs. The imputation of common cost is, however, an important yet controversial issues both theoretically and practically. As James MacKie (1970) aptly described the issue:

> Cost allocations are arbitrary, as everyone knows. They can be logical, appealing, and symmetrical, but still arbitrary, because they divide something that is in fact united. The common cost has to be allocated because of regulation, not because allocation reflects rational resource allocation. (p.12)

Without embarking on a futile search for a logical method of cost allocation, therefore, we will simply consider three operational conventions and study their effects on the firm's operation.

The method of cost imputation actually adopted by regulatory agencies, notably the FPC, seems to be based on ouput shares as in the Atlantic Seaboard Formula.(3) Under this method, the proportion of total cost imputed to regulated output depends on the share of total output that is accounted for by regulated sales. In addition to this we will consider two other allocation methods. One is the constant share method that was utilized by Paul MacAvoy and Roger Noll (1973). We will also examine an imputation method based on revenue shares, which seems to offer both practicality, feasibility and some other advantages over the other two. The implications of these three alternative methods wil be studied in the rest of this paper.

We will argue that the output decisions of a regulated discriminating firm are extremely sensitive to the method of cost allocation. Specifically, we will show that partial regulation by the revenue share method yields results identical to those of overall regulation, whereas partial regulation by the output share method usually has an effect opposite to that of partial regulation by the constant share method. Thus, overall regulation or partial

(3) A comprehensive review of the output share method is contained in Kahn (1971), p 150-8. The specifics of the Atlantic Seaboard Formula is found, for example, in Garfield and Lovejoy (1964), p 182-4.

regulation by the revenue share method is preferable to the other two forms of partial regulation from the viewpoint of consumers as a whole in that it has a balanced effect on individual submarkets whereas the latter two largely do not.

I. Model

In order to demonstrate our results, we will make use of a markup constraint rather than the more familiar one of a fair rate-of-return on capital base. The former method of regulation does not distort efficiency in input choices whereas the latter does, at least in theory.[4] As our purpose here is to analyze output allocation effects of regulation, our use of the markup constraint will simplify our argument and allow us to focus on this issue. This will not alter our results in any fundamental way as we will indicate later.[5]

The problem of a discriminating monopoly under markup regulation is to maximize:

$$(1)\quad \Pi=\sum_{i=1}^{n} R_i(q_i)-C_o(q_o)$$

subject to:

$$(2)\quad \sum_{i=1}^{h} R_i(q_i)\leq ma_jC_o(q_o)$$

where q_i is output for the i-th submarket, Π total profit, R_i revenue from the i-th submarket, C_o total cost, q_o total output, m the allowed markup, a_j the proportion of cost allocated to the regulated sectors under the j-th method of allocation, n total number of submarkets and h number of regulated submarkets. This formulation is sufficiently general for our purpose, since $n=1$ for a simple monopoly, $n>1$ for a discriminating monopoly, $h=n$ for overall regulation and $h<n$ for partial regulation.

The numerical value of a_j depends on the method of regulation, i.e.,

$$a_j=a_o=1 \text{ for } h=n.$$

and

$$0<a_j<1 \text{ for } h<n.$$

As mentioned before we will examine the following three methods of cost

(4) A general proof of this proposition has been furnished, e.g., by Bailey, *op. cit.*, p. 41–57.
(5) See footnote 13 below.

allocation for partial regulation:

$$(3)\qquad a_1=\sum_{i=1}^{h}R_i/\sum_{i=1}^{n}R_i \text{ (revenue share)},$$

$$(4)\qquad a_2=\sum_{i=1}^{h}q_i/\sum_{i=1}^{n}q_i \text{ (output share)},$$

and

$$(5)\qquad a=\text{constant (constant share)}.$$

The Lagrangian for (1) and (2) is:

$$(6)\qquad L(q_1\cdots q_n,\lambda_j)=\sum_{i=1}^{n}R_i(q_i)-C_o+\lambda_j[ma_jC_o,q_o)-\sum_{i=1}^{h}R_i(q_i)]$$

where λ_j is positive for an effective constraint and the index number of j varies for different methods of cost allocation, i.e., $j=0,1,2,3$. For positive values of q_i $(i=1,2,\cdots,n)$ and λ_j we derive the following Kuhn-Tucker conditions:(6)

for a regulated output

$$(7)\qquad (1-\lambda_j)MR_i=(1-\lambda_ja_jm)MC_o-\lambda_jmC_o\frac{\partial a_j}{\partial q_i}\ (i=1,2,\cdots h,)$$

and for an unregulated output

$$(8)\qquad MR_i=(1-\lambda_ja_jm)MC_o-\lambda_jmC_o\frac{\partial a_j}{\partial q_i}\ (i=h+1,\cdots n,)$$

where $MR_i=dR_i/dq_i$ and $MC_o=\partial C_o/\partial q_i$ for $i=1,2,\cdots,n$.(7)

Since the numbers are arbitrary, we will simplify our discussion by considering only a pair of markets one of which is regulated and one unregulated. For this purpose we will use subscripts u and r respectively for the regulated and an unregulated submarket. Of course, we do not need such a distinction in case of overall regulation. The superscripts r and * are employed to denote the regulated and unregulated discriminatory optimum

(6) It has been proved by many authors (e.g., Baumol and Klevorick, and Bailey) that in the model of regulated simple monopoly the value of the Lagrange multiplier for constrained profit maximization is bounded by zero and unity. However, we cannot derive the same general result for our present model. One intuitive reason for this may lie in that our model is basically that of partial regulation. If we interpret the Lagrange multiplier to measure the increment in the objective due to a marginal relaxation of the constraint, there seems to be no ***a priori*** reason why λ_j should always be less than unity in case of partial regulation That is, a unit relaxation in the constraint on regulated revenues may or may not contribute to the increase in total profit more than proportionately. Fortunately enough, our results are not qualified by the bound **on** λ_j when such bounds are not obvious

(7) To better characterize the discriminatory pricing behavior in the context of common cost we assume here that all of the cost be common and hence MC_0 remains unchanged whether the last unit of output is sold in one submarket or another. This will not harm our results, of course.

respectively. The major results that we will establish for this pair of submarkets are summarized in the following:[8]

	output choice	output results
overall regulation (a_0)	$MR^r_i = MR^r_j$	$q^r_i > q^*_i$, $q^r_j > q^*_j$,
partial regulation		
revenue share (a_1)	$MR^r_r = MR^r_u$	$q^r_r > q^*_r$, $q^r_u > q^*_u$,
output share (a_2)	$MR^r_r < MR^r_u$	$q^r_r \gtreqless q^*_r$, $q^r_u \gtreqless q^*_u$,
constant share (a_3)	$MR^r_r > MR^r_u$	$q^r_r \gtreqless q^*_r$, $q^r_u \gtreqless q^*_u$.

In other words, the revenue share method of partial regulation yields the same results as overall regulation while the results of the other methods do not, and involve considerable qualitative indeterminacies besides.

II. Overall Regulation

The output choice rule under overall regulation is derived from (7), i.e.,

$$(9)\quad MR^r_i = MR^r_j = \frac{1-\lambda_0 m}{1-\lambda_0} MC^o_r < MC^o_r \quad \text{for all } i,\ j=1,2,\cdots,n;\ i \neq j.^{(9)}$$

This result implies:

Proposition 1

Under overall regulation the output choice of a discriminating firm will satisfy the optimality conditions for the unregulated firm in the sense that marginal revenues are equal in all submarkets, and regulation will induce the firm to lower price and hence increase output in all submarkets.

We now discuss the first part of this proposition in more concrete terms with the aid of a graphical device. In Figure 1 the R^0_0 curve is the "optimal aggregated revenue curve"[10], a point on which represents maximal revenue hence maximal profit the firm can obtain by any allocation of a given quantity of total output among individual submarkets. Therefore, the R^0_0

(8) In the following table equalities under output results do not hold simultaneously, since otherwise regulation is not effective.

(9) The inequality is due to second-order condition for constrained profit maximization under overall regulation, which implies. $1-\lambda_0>0$. We can here find a definite bound on the Lagrange multiplier as in the model of regulated simple monopoly.

(10) The $R^0{}_0$ curve must be concave over the relevant range of output. If it has a valley before it reaches its maximum point, it means that by optimally allocating and selling more output the firm obtains less total revenue. This is a contradiction.

curve assumes that marginal revenues are equal in all submarkets for any level of total output.(11) The mC_o curve describes the constraint, which prohibits all points above that curve. That is, for the chosen level of q_0 the firm is allowed to earn total revenue which is not greater than the corresponding value of mC_0. As these two curves intersect at B″ and C″, we have two candidate solutions. Profit being larger for the solution C‴, i.e., C″ C‴> B′B″, however, the firm will produce total output OC. At this output level marginal revenues are equal in all submarkets because the R_0^0 curve is constructed to satisfy that condition.

Suppose the firm could find another solution by a nonoptimal output allocation. The firm will then choose solution on a lower aggregated revenue curve, say $R_o^{\#}$, because the $R_0{}^0$ curve is by definition the uppermost one. Since the C_0 curve and hence mC_o curve are not affected by different output allocations, the new solution will be D''. Profit is smaller here than for C'', i.e., $D'D''<C''C'''$ because the absolute difference between C_o and mC_o becomes smaller with smaller output. Hence, the optimal solution must lie on the R_o^o curve and consequently marginal revenues are equal in all submarkets.

The second part of Proposition 1 is already suggested by Figure 1, that is, we have $OC>OA$, and the necessity of this relationship can be verified immediately. If total output is to be decreased by regulation, the constraint curve must cut the R_o^o curve to the left of A‴, say E. This constraint curve, say $mC_o^{\#}$ will then increase faster than the R_o^o curve to the right of point E so that at output level OA it will lie above the point A''. In other words, in this case regulation does not preclude unconstrained profit maximization and hence it violates our assumption that the regulatory constraint is effective. If output is not altered by overall regulation, there is a similar contradiction. Therefore, the only consistent result is an increase in total output.

Having completed the proof of Proposition 1, we will briefly assess overall regulation. Overall regulation is symmetric with respect to the firm's choice of distribution of its outputs among its submarkets and hence the firm cannot lessen the burden of regulation by adjusting its output allocation

(11) This can be shown immediately by solving the problem maximize $\sum_{i=1}^{n} R_i$ subject to $\sum_{i=1}^{n} q_i=\bar{q}_0$

asymmetrically. This seems to be a trivial result but its significance will become obvious shortly as we compare it with those of other methods of regulation.

III. Partial Regulation by the Revenue Share Method

For the revenue share method of allocation of common cost the regulatory constraint (2) is written as:

$$(10) \qquad \sum_{i=1}^{h} R_i \leq \frac{\sum_{i=1}^{h} R_i}{\sum_{i=1}^{n} R_i} \cdot mC_o.$$

Substituting the partial derivative

$$\frac{\partial a_1}{\partial q_r} = \frac{\sum_{i=h+1}^{n} R_i}{\left(\sum_{i=1}^{n} R_i\right)^2} MR_r$$

into (7), we obtain:

$$(11) \qquad (1-\lambda_1)MR_r = \left(1-\lambda_1 m \frac{\sum_{i=1}^{h} R_i}{\sum_{i=1}^{n} R_i}\right)MC_o - \lambda_1 m C_o \frac{\sum_{i=h+1}^{n} R_i}{\left(\sum_{i=1}^{n} R_i\right)^2} MR_r.$$

Rearranging (11) and taking note of (10) and

$$\frac{\sum_{i=h+1}^{n} R_i}{\sum_{i=1}^{n} R_i} = 1 - \frac{\sum_{i=1}^{h} R_i}{\sum_{i=1}^{n} R_i},$$

we derive:

$$(12) \qquad \left(\sum_{i=1}^{n} R_i - \lambda_1 \sum_{i=1}^{h} R_i\right)MR_r = \left(\sum_{i=1}^{n} R_i - \lambda_1 m \sum_{i=1}^{h} R_i\right)MC_o.$$

Since

$$\sum_{i=1}^{n} R_i \neq \lambda_1 \sum_{i=1}^{h} R_i \text{[12]},$$

we rewrite (12) as:

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(12) If $\sum_{i=1}^{n} R_i = \lambda_1 \sum_{i=1}^{h} R_i$, then $\sum_{i=1}^{n} R_i < \lambda_1 m \sum_{i=1}^{h} R_i$ for $m>1$. Therefore, the lefthand side of (12) is zero whereas the righthand side of (12) is not. This is a contradiction.

$$(13)\quad MR_r^r = \frac{\sum_{i=1}^{n} R_i - \lambda_1 m \sum_{i=1}^{h} R_i}{\sum_{i=1}^{n} R_i - \lambda_1 \sum_{i=1}^{h} R_i} MC_o^r.$$

Applying a similar procedure to MR_u^r, we can show:

$$(14)\quad MR_r^r = MR_u^r,$$

This result shows that under partial regulation by the revenue share method marginal revenues are equal in both regulated and unregulated submarkets. We thus have the striking result that this method of partial regulation leads to the same effects as overall regulation.

Proposition 2

Partial regulation by the revenue share method will yield the sam eeffects as overall regulation. Therefore, marginal revenues are equal in all submarkets and the firm will increase output in all submarkets.

The equivalence of this method of partial regulation to overall regulation is easy to explain intuitively, since the constraint (10) can be rewritten directly as:

$$\sum_{i=1}^{n} R_i \leq mC_o.$$

which is nothing but the overall-regulatory constraint.[13] In other words. what is apparently partial regulation by this method turns out just to be a concealed form of overall regulation. Under this form of partial regulation, sales in any individual submarkets simply are not constrained because the constraint

$$\sum_{i=1}^{n} R_i \leq mC_o$$

treats sales in any markets interchangeably. Thus, the partial regulatory constraint transforms itself into one affecting only the firm as a whole. It

(13) The same result is obtained for rate-of-return constraint. We can always reduce the constraint:

$$\sum_{i=1}^{h} R_i - \frac{\sum_{i=1}^{h} R_i}{\sum_{i=1}^{n} R_i} wL \leq \frac{\sum_{i=1}^{h} R_i}{\sum_{i=1}^{n} R_i} sK$$

to:

$$\sum_{i=1}^{n} R_i \leq wL + sK,$$

which is nothing but the overall rate-of-return constraint, where s is the allowed rate-of-return.

therefore does not matter which markets are regulated. In other words, this method of partial regulation is completely substitutable for overall regulation. Because of its significance we will show more concretely why this must hold and then interpret some of its implications.

Consider Figure 1 again. The $(\sum_{i=1}^{h} R_i/\sum_{i=1}^{n} R_i)^o mC_o$ curve describes a partial regulatory constraint but it presupposes that the allocation of total output is optimal for the firm. The $\sum_{i=1}^{h} R_i^o$ curve represents the amount of revenue that the firm obtains from the regulated submarkets by allocating a given total output optimally. The intersection of these two curves, then, yields a candidate solution. This solution corresponds to the one for overall regulation, because

$$\sum_{i=1}^{r} R_i = mC_o,$$

for overall regulation and

$$\sum_{i=1}^{n} R_i^o \frac{\sum_{i=1}^{h} R_i^o}{\sum_{i=1}^{n} R_i^o} = \sum_{i=1}^{h} R_i^o = \frac{\sum_{i=1}^{h} R_i^o}{\sum_{i=1}^{n} R_i^o} mC_o,$$

i.e.,

$$CC' = CC''' \times \frac{CC'}{CC'''}$$

Let us now examine whether there can be a better solution for the firm by a non-optimal allocation. If there is such a solution that satisfies the constraint (10), then

$$\sum_{i=1}^{h} R_i^{\#} = (\sum_{i=1}^{h} R_i^{\#}/\sum_{i=1}^{n} R_i^{\#})\, mC_o^{\#}$$

which implies

$$\sum_{i=1}^{n} R_i^{\#} = mC_o^{\#}.$$

We showed in Section III that this entails a decrease in profit for the firm. As far as the objective of the firm is profit maximization and not a mere satisfaction of the constraint, therefore, this solution will be rejected.

In other words, the firm cannot obtain a higher profit by a non-optimal output allocation than by an optimal one. The first candidate solution turns out to be one desired by the firm and accordingly the revenue share method of partial regulation turns out to be identical with overall regulation. The essential point here is that the firm will adopt an output allocation which is optimal for itself in its attempt to alleviate the burden of partial regulation and at the same time all consumers of the firm will be benefited by this choice of the firm. As we shall show in the following sections, this does not necessarily hold for other methods of partial regulation and consequently the purpose of regulation may not be served.

IV. Partial Regulation by the Output Share Method

The partial markup constraint under the output share method is:

$$(15)\qquad \sum_{i=1}^{h} R_i \leq \frac{\sum_{i=1}^{h} q_i}{\sum_{i=1}^{n} q_i} mC_o,$$

i.e.,

$$(16)\qquad AR_r \leq mAC_o,$$

where

$$AR_r = \sum_{i=1}^{h} R_i / \sum_{i=1}^{h} q_i$$

and

$$AC_o = C_o / \sum_{i=1}^{n} q_i.$$

This constraint implies, therefore, that under effective regulation the average revenue from regulated submarkets will be equal to the allowed markup over the average total cost. If there is only one regulated submarket, it will serve as a direct price control in that market; if there is a number of regulated submarkets on the other hand, it imposes no restriction on prices themselves because for the same value of the average revenue there can be a multiplicity of different price sets. The most important characteristic of this method is, however, that it does not even constrain revenues from unregulated submarkets.

The reason partial regulation was adopted in gas regulation was believed at the time that regulation of industrial markets was redundant. As the FPC (1964) itself observed:

> It had been thought when the Natural Gas Act was adopted in 1938, that competition with other fuels, such as coal, would exert a self-regulatory effect and obviate the need for public regulation of the direct industrial sales of natural gas. (p. 8-9)

We will show that this argument does not hold where partial regulation relies on the output share method.

Output allocation under this method is derived by a procedure similar to the one used for the other methods. Without actually going through the procedure we simply state the result:

$$(17)\quad MR^r_r - MR^r_u = \lambda_2 (MR^r_r - AR^r_r).$$

Unless the regulated markets are perfectly competitive, MR_r is smaller than AR_r. Therefore,

$$(18)\quad MR^r_r < MR^r_u \text{ for } \lambda_2 \neq 0.$$

Proposition 3

> Under partial regulation by the output share method the discriminating firm will allocate more output to regulated submarkets and less to unregulated submarkets than it optimally should to maximize profit for the chosen level of total output.

To evaluate this output allocation result further we consider the effect of regulation on total output. Solving (7) and (8) for a_2 and rearranging terms, we obtain:

$$(19)\quad (1-\lambda_2) MR^r_r = (1-\lambda_2) MC^r_o + \lambda_2 m (q^r_u/q^r_o)(MC^r_o - AC^r_o) + (1-m) MC^r_o$$

and

$$(20)\quad MR^r_u = MC^r_o - \lambda_2 m (q^r_u/q^r_o)(MC^r_o - AC^r_o).$$

Combining (18), (19) and (20), we then derive the following relations: [14]

$$(21)\quad MR^r_r < MC^r_o = MR^r_u, \text{ if } MC^r_o = AC^r_o,$$

$$(22)\quad MR^r_r < MR^r_u < MC^r_o, \text{ if } MC^r_o > AC^r_o, \text{ and}$$

$$(23)\quad MR^r_r < MC^r_o < MR^r_u, \text{ if } MC^r_o < AC^r_o.$$

(14) If $MC^r_o \geq AC^r_o$, then $0 < \lambda_2 < 1$ from (18), (19) and (20). If $MC^r_o < AC^r_o$ on the other hand, the boundary of λ_2 is not determinate, and hence the relation among MR^r_r, MR^r_u and MC^r_o is not determinate. The relation (23) is valid if $0 < \lambda_2 < 1$. Yet this is not restrictive for our purpose since in any case (23) alone implies indeterminate output results.

which in turn imply:

(24) $q_o^r \gtrless q_o^*$, if $MC_o^r \geq AC_o^r$; and

(25) $q_o^r \lessgtr q_o^*$, if $MC_o^r < AC_o^r$.

Therefore, total output result is not determinate if the firm has a declining average cost curve. The more important point is, however, that total output results (24) and (25), combined with the output allocation rule (18), fail to produce determinate output results in individual submarkets. For example, even if total output is increased by regulation, output can actually be decreased in the unregulated submarkets. An increase in total output is a necessary and sufficient condition for output increases in regulated submarkets but merely a necessary one in unregulated submarkets.

In other words, under this method of partial regulation the increase in total output is merely a necessary but not a sufficient condition for overall gains to all consumers in all submarkets of the firm. If we interpret the principal purpose of partial regulation as the protection of consumers in the regulated submarkets without sacrifices on the part of those in the unregulated submarkets, we cannot make a strong case for the output share method. Under this method of partial regulation the firm will adjust its output in an asymmetric manner and consequently it is likely that consumers in the unregulated submarkets suffer losses. We showed in the preceding section that the revenue share method will not entail this asymmetric effect, since the firm cannot evade the burden of regulation through its adjustment in the unregulated submarkets.

The argument showing the preferability of the revenue share method over the output share method permits us an illuminating glance at a policy of the Federal Power Commission. In light of asymmetric effects resulting from its partial regulation by the output share method the overwhelming opinion of the FPC commissioners in the early sixties was that it was desirable to extend regulation to cover direct sales as well as sales to utilities.(15) The earlier recommendation of the FPC can be interpreted as an attempt to replace partial regulation by overall regulation. Our analysis suggests that the FPC could have done as well by the use of the revenue share method instead. It may be less difficult to replace one arbitrary

(15) FPC Annual Report (1964), p. 8-9.

method of cost allocation by another than to replace the regime of partial regulation by that of overall regulation.

V. Partial Regulation by the Constant Share Method

Though this method has not been utilized and does not have theoretical preferability over the output share method, we will consider it briefly to illustrate the sensitivity of the firm's adjustment to the method of regulation. When the common cost is divided between regulated and unregulated services in a constant ratio, the firm can be expected to respond to this sort of regulation in a way dramatically different from those we have shown for other methods of regulation.

While acknowledging that the output share method is a "more realistic" model describing pipeline regulation, Mac Avoy and Noll (1973) replace it by a constant share model in their analysis, since "in practice, the share has varied very little……, so that little is lost in descriptive content……by assuming this share to be fixed" (p. 21Cn). We will show that this assumption usually leads to a result reversed from that of the output share method, at least as far as output allocation is concerned.

For the constant share method $\partial a_3/\partial q_i$ vanishes in (7) and (8), so that:

$$(26)\qquad (1-\lambda_3)MR_r^r = MR_u^r = (1-\lambda_3 a_3 m)MC_o,$$ [16]

which implies:

$$(27)\qquad MR_r^r > MR_u^r,\ \text{unless } MR_r^r \leq 0.$$

Whether or not $MR_r^r \leq 0$ depends, among others, on the numerical values of λ_3, m and a_3. A further investigation of this issue is not our main purpose here, and we simply state:

Proposition 4

Under partial regulation by the constant share method the discriminating firm may allocate less output to regulated submarkets and more to the unregulated ones than it optimally should to maximize profit for the chosen level of total output.

From Propositions 3 and 4 it follows at once that the constant share method can produce effects opposite to that of the output share method

(16) $\lambda_3 \neq 1$ unless the bordered Hessian determinant that is relevant for regulated profit maximization under the constant share method vanishes.

thus illustrating the extreme sensitivity of the output choices of a discriminating firm to method of regulation.

VI. Summary

Overall regulation will induce the firm to increase output and decrease price in each submarket. The output choice of the firm under this type of regulation is optimal for the firm, because for the chosen level of total output marginal revenues are equal in all submarkets.

Second, partial regulation by the revenue share method yields effects identical with those of overall regulation. Therefore, the regulator has at least the appearance of a choice between these two forms of regulation. To achieve the same purpose, he can resort to overall regulation or partial regulation by the revenue share method.

Third, partial regulation by the output share method or the constant share method will yield asymmetric effects on output allocation. The former method tends to favor consumers in the regulated markets over those in the unregulated markets, whereas the latter has the opposite tendency. Though it may be expected that these two methods increase output in one group of submarkets it is not certain a priori that they will also increase output in other submarkets. If consumers in all submarkets of the firm are to achieve gains through regulation, therefore, these methods cannot be relied on.

Fourth, overall regulation or partial regulation by the revenue share method is preferable, therefore, if the regulator is to be sure of providing beneficial effects to all consumers of the regulated firm. The revenue share method may be easier to implement than overall regulation, if the regulator attempts to replace the output share method by a new one that would serve his purpose. The revenue share method is simply a substitute for an equally arbitrary method of cost allocation, i.e., the output share method, whereas overall regulation is a totally new system that extends control to the previously unregulated markets. Thus, practical feasibility would make the revenue share method an attractive alternative in considering the revision of partial regulation.

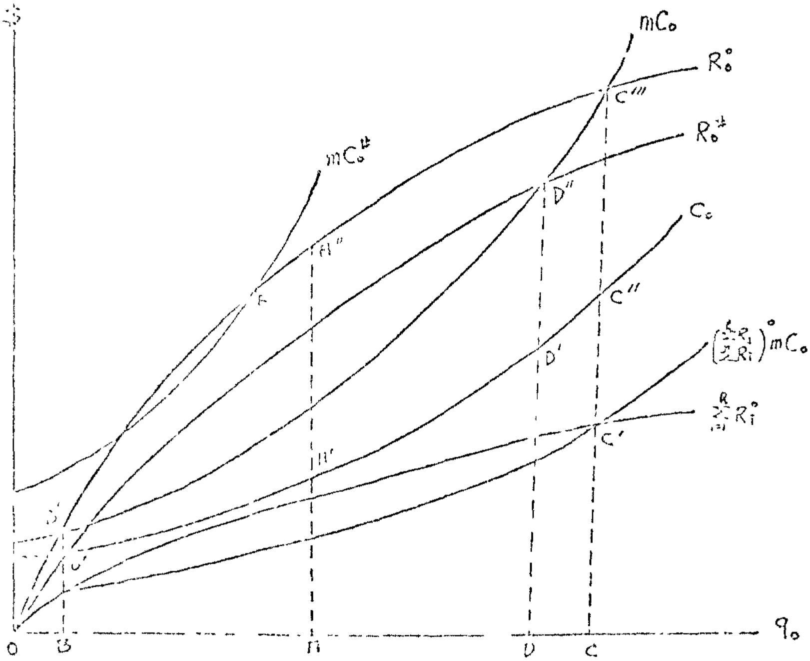

Figure 1

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