Adaptive Expectations and Partial Adjustment: Estimation and Discrimination

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I. Introduction

When the "adaptive expectations" and "partial adjustment" models are combined into a single model, the expectations and adjustment parameters enter the final equation of the model symmetrically and pose a problem of identification.(1) Further, we cannot discriminate between the adaptive model and the adjustment model by regression analysis for their respective final equations contain the same observed variables.(2)

The purpose of this paper is, first, to present a model containing the concepts of both adaptive expectations and partial adjustment processes and to use the form of the disturbance terms of the final equation as a source of identifying parameters. We then present maximum likelihood (ML) methods of estimating the final equation under three alternative assumptions about the disturbance terms. The procedures are similar to the ML methods suggested by Dhrymes [1], Zellner and Geisel [7]. The estimation methods are then applied for illustration in the analysis of quarterly inve-

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(1) This identification problem is discussed in, e.g., Johnston[4, pp. 302-3] and Waud [6].

(2) Feige [3] and Waud [6] discuss the difficulties involved in interpreting distributed lag formulations and empirically discriminating between the expectations and adjustment models.

ntory data.

In Section 2 we describe a model embodying the conceptual ingredients of both partial adjustment and adaptive expectations models and introduce three alternative assumptions about error terms. Although the model is in the framework of inventory analysis, the estimation methods presented in Section 3 are more generally applicable. Section 3 present the proposed estimation methods. Section 4 is devoted to analyzing the behavior of quarterly inventory data, 1961:I—1973:III, of the Canadian manufacturing industry. The last section contains some concluding remarks.

II. The Model

Following the flexible accelerator approach, we assume that the firm has a desired level of finished goods inventories given by

$$(2.1) \qquad Y_t^* = \alpha + \beta X_{t+1}^*$$

where Y_t^* is the desired level of inventories at the end of period t and X_{t+1}^* the expectations on the sales volume for the following period held in period t. Both the desired level of inventories and the expected sales are theoretical magnitudes, which are not directly observable, and we introduce hypotheses about how these theoretical magnitudes are determined.

About the formation of the expectation we assume that the firm revises its expectations on sales each period by a fraction δ of the difference between the actual and expected sales:

$$(2.2) \qquad X_{t+1}^* - X_t^* = \delta(X_t - X_t^*), \quad 0 < \delta \leq 1$$

where the parameter δ is called the "coefficient of expectations" reflecting the proportion of the expectational error taken to be permanent. The desired level of inventories in (2.1) will not necessarily equal the actual level and the firm changes, we assume, its inventories according to the discrepancy. We postulate the partial adjustment function of the form

$$(2.3) \qquad Y_t - Y_{t-1} = \gamma(Y_t^* - Y_{t-1}) + u_t, \quad 0 < \gamma \leq 1$$

which asserts that in period t, a fraction γ of the gap between the desired end-of-period level and the actual starting level of inventories is filled on the average. The parameter γ is called the "coefficient of adjustment" reflecting the fact that there are limitations to the rate of inventory adjustment due to institutional rigidity or the increasing cost of instantaneous or rapid change in production. Although the firm plans to change inventories in proportion to the discrepancy between the desired and actual levels of inventories, its plans may be upset by the errors in sales forecasts; we introduce a random error term u_t in (2.3).

Combining (2.1), (2.2), and (2.3), we obtain a general model embodying the assumptions of both adaptive expectations and partial adjustment, and readily derive its final equation by the Koyck transformation as

$$(2.4)\qquad Y_t=\alpha\gamma\delta+\beta\gamma\delta X_t+[(1-\gamma)+(1-\delta)]Y_{t-1}-(1-\gamma)(1-\delta)Y_{t-2}+u_t-(1-\delta)u_{t-1}.$$

This is the basic equation we analyze. Notice that parameters γ and δ appear in the equation symmetrically except in the composite error term.

It is readily seen that the partial adjustment model is a special case of this general model with $\delta=1$, and the final equation (2.4) becomes

$$(2.5)\qquad Y_t=\alpha\gamma+\beta\gamma X_t+(1-\gamma)Y_{t-1}+u_t.$$

The expectations parameter δ being equal to 1 implies that the firm believes the sales in the current period to continue in the following period. On the other hand, the adaptive expectations model is another special case of the general model with $\gamma=1$, and the final equation (2.4) now becomes

$$(2.6)\qquad Y_t=\alpha\delta+\beta\delta X_t+(1-\delta)Y_{t-1}+u_t-(1-\delta)u_{t-1}$$

The adjustment parameter γ being equal to unity means that the firm closes the gap between the desired and the actual levels of inventories at the end of period t. The two final equations (2.5) and (2.6) contain exactly the same variables; the only difference is that the former has a simpler disturbance term than the latter. When both the adaptive expectations and partial adjustment processes are operative, the final equation (2.4) is relevant. When only one of the two processes is in operation, either (2.5) or (2.6) is relevant, but one cannot discriminate between the two processes on the basis of (2.5) and (2.6).

Using the final equation (2.4) we consider the problem of identifying and estimating structural parameters and examine how we can discriminate the adaptive expectations model from the partial adjustment model when only one of the two is operative. In analyzing (2.4) we assume that X_t is an exogenous variable. With regards to the error term in (2.4) we consider three alternative assumptions.

Assumption I: $u_t-(1-\delta)u_{t-1}=e_t$, and
$e_t \sim NID\ (0,\ \sigma_e^2)$.

This assumption is rather special because no economic theory suggests that the error term u_t's satisfy a first-order Markov process with parameter $(1-\delta)$, the same parameter appearing in the adaptive expectations hypothesis.

Assumption II: $u_t \sim NID\ (0,\ \sigma_u^2)$.

This assumption of serially independent u_t's may not be very plausible particularly when the data used pertain to periods of a short interval.

Assumption III: $u_t=\rho u_{t-1}+e_t$, $|\rho|<1$, and
$e_t \sim NID\ (0,\ \sigma_e^2)$.

This allows for the possibility that the u_t's may be autocorrelated. If $\rho=0$, Assumption III reduces to Asumption II; if $\rho=1-\delta$, it reduces to Assumption I.

III. Maximum Likelihood Estimation

Now we examine the ML procedures of estimating parameters of (2.4) under the three alternative assumptions about the error terms.

Case (a). $u_t-(1-\delta)u_{t-1}=e_t$, $e_t \sim NID(0,\ \sigma_e^2)$.

This is the simplest possible case. Reparameterizing as

$$(3.1)\qquad \beta_1=\alpha\gamma\delta, \qquad \beta_2=\beta\gamma\delta$$
$$\beta_3=(1-\gamma)+(1-\delta), \quad \beta_4=-(1-\gamma)(1-\delta),$$

we rewrite (2.4) as

$$(3.2)\qquad Y_t=\beta_1+\beta_2X_t+\beta_3Y_{t-1}+\beta_4Y_{t-2}+e_t.$$

If we assume that Y_0 and Y_{-1} are fixed, the logarithmic likelihood function for the parameters of (3.2) is given by

$$(3.3)\qquad L(\beta_1,\ \beta_2,\ \beta_3,\ \beta_4,\ \sigma_e^2|\text{data})=-\frac{T}{2}\ \ln\ (2\pi)-\frac{T}{2}\ \ln\ \sigma_e^2$$
$$-\frac{1}{2\sigma_e^2}\ \sum_{t=1}^{T}\ (Y_t-\beta_1-\beta_2X_t-\beta_3Y_{t-1}-\beta_4Y_{t-2})^2.$$

Thus ML estimators of the β_i's are identical with ordinary least squares estimators obtained by minimizing

$$(3.4) \qquad \sum_{t=1}^{T} (Y_t-\beta_1-\beta_2X_t-\beta_3Y_{t-1}-\beta_4Y_{t-2})^2.$$

So least squares applied to (3.2) will yield consistent estimates of β_i's under Assumption I. Then (3.1) can be employed to obtain consistent estimates of α and β; σ_e^2 can be estimated consistently by

$$(3.5) \qquad \hat{\sigma}_e^2=\frac{1}{T} \sum_{t=1}^{T} (Y_t-\hat{\beta}_1-\hat{\beta}_2X_t-\hat{\beta}_3Y_{t-1}-\hat{\beta}_4Y_{t-2})^2.$$

It is clear, however, that the symmetric appearance of γ and δ in (2.4) makes it impossible to estimate the two parameters separately. Only their function, $\gamma\delta$ and $\gamma+\delta$, can be consistently estimated from the relation, (3.1).[3] Further, if γ (or δ) equals one so that the partial adjustment (or adaptive expectations) model is the correct specification, it is not possible to distinguish empirically between the partial adjustment and adaptive expectation models on the basis of the least squares estimation.[4]

Case (b). $u_t \sim NID\ (0, \sigma_u^2)$.

On this assumption about the u_t's, the composite disturbance terms, $u_t-(1-\delta)u_{t-1}$, and the lagged values of Y on the right-hand side of (2.4) are correlated, and least squares applied to the equation will yield inconsistent estimators.

Following Zellner and Geisel [7], we define $\theta_t=Y_t-u_t$ and write θ_t from (2.4) as

$$(3.6) \qquad \theta_t=(1-\gamma)Y_{t-1}-(1-\gamma)(1-\delta)Y_{t-2}+\alpha\gamma\delta+\beta\gamma\delta X_t+(1-\delta)\theta_{t-1}.$$

By successive substitution for θ we have

$$(3.7) \qquad \theta_t=(1-\gamma)Z_{t1}-(1-\gamma)Z_{t2}+\alpha\gamma Z_{t3}+\beta\gamma Z_{t4}+\theta_o Z_{t5},$$

where

$$Z_{t1}=\sum_{i=1}^{t} (1-\delta)^{i-1}Y_{t-1},$$

$$Z_{t2}=\sum_{i=1}^{t} (1-\delta)^{i}Y_{t-i-1},$$

$$(3.8) \qquad Z_{t3}=\sum_{i=1}^{t} \delta(1-\delta)^{i-1},$$

(3) See Waud [6, p. 207, n. 2].

(4) See Waud [6, p. 206]. If the coefficient of Y_{t-2} is not significantly different from zero, all we can conclude is that (2.5) or (2.6) is the correct specification.

$$Z_{t4}=\sum_{i=1}^{t} \delta(1-\delta)^{i-1}X_{t-i+1},$$

$$Z_{t5}=(1-\delta)^{t}.$$

Therefore, we may write

$$(3.9) \qquad Y_t=(1-\gamma)(Z_{t1}-Z_{t2})+\alpha\gamma Z_{t3}+\beta\gamma Z_{t4}+\theta_0 Z_{t5}+u_t.$$

Suppose we are given initial values, Y_0 and Y_{-1}, of the dependent variable. We may also regard $\theta_0=Y_0-u_0$ as an unknown parameter. Since the u's are NID $(0, \sigma_u^2)$, it is clear from (3.9) that for fixed δ minimizing

$$(3.10) \qquad \sum_{t=1}^{T} u_t^2=\sum_{t=1}^{T}[Y_t-(1-\gamma)(Z_{t1}-Z_{t2})-\alpha\gamma Z_{t3}-\beta\gamma Z_{t4}-\theta_0 Z_{t5}]^2$$

with respect to γ, α, β and θ_0 will yield ML estimates. But δ is unknown, and for finding the global minimum of (3.10) we resort to a search procedure: we select a grid of δ value in the range of $0<\delta<1$, for each value of δ on the grid compute the data matrix from Y_t and X_t as described in (3.8), and apply least squares to (3.9). That value of δ and the associated estimates of α, β and γ are chosen which yield the minimum sum of squares. The estimate of σ_u^2 is the minimized sum of squares divided by sample size. The large sample standard errors of these estimates are obtained by taking the square roots of the main diagonal elements of the inverse of the information matrix evaluated at the given parameter estimates.[5]

Case (c). $u_t=\rho u_{t-1}+e_t$, $|\rho|<1$, $e_t\sim NID(0,\sigma_e^2)$.

As before we define $\theta_t=Y_t-u_t$ so that

$$(3.11) \qquad \theta_t-\rho\theta_{t-1}=Y_t-\rho Y_{t-1}-(u_t-\rho u_{t-1}).$$

From equation (2.4) we have

$$(3.12) \qquad \begin{aligned} Y_t(\rho)=&\alpha\gamma\delta(1-\rho)+\beta\gamma\delta X_t(\rho) \\ &+[(1-\gamma)+(1-\delta)]Y_{t-1}(\rho)-(1-\gamma)(1-\delta)Y_{t-2}(\rho) \\ &+u_t(\rho)-(1-\delta)u_{t-1}(\rho), \end{aligned}$$

that is,

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(5) As Dhrymes [2] noted, θ_0 cannot be consistently estimated because the variable which it corresponds, i.e., $Z_{t5}=(1-\delta)^t$ has the property that

$$\sum_{t=1}^{\infty} (1-\delta)^{2t}<\infty.$$

The parameter θ_0 represents the average inventory stock in the initial period, and we are seldom interested in its estimate. As the sample size increases, the observations on Z_{t5} approach zero, and this variable has been eliminated in forming the information matrix.

$$(3.13)\qquad \theta_t(\rho)=\alpha\gamma\delta(1-\rho)+\beta\gamma\delta X_t(\rho) \\ +(1-\gamma)Y_{t-1}(\rho)-(1-\gamma)(1-\delta)Y_{t-2}(\rho) \\ +(1-\delta)\theta_{t-1}(\rho),$$

where $Y_t(\rho)=Y_t-\rho Y_{t-1}$, $\theta_t(\rho)=\theta_t-\rho\theta_{t-1}$, etc.
By successive substitution in (3.13) we obtain

$$(3.14)\qquad \theta_t(\rho)=(1-\gamma)Z_{t1}(\rho)-(1-\gamma)Z_{t2}(\rho) \\ +\alpha\gamma Z_{t3}(\rho)+\beta\gamma Z_{t4}(\rho)+\theta_0(\rho)Z_{t5},$$

where Z_{t1} through Z_{t5} areas defined in (3.8) and $Z_{t1}(\rho)=Z_{t1}-\rho Z_{t-1,1}$ etc. Therefore, we may write

$$(3.15)\qquad Y_t(\rho)=\theta_t(\rho)+u_t(\rho) \\ =(1-\gamma)[Z_{t1}(\rho)-Z_{t2}(\rho)]+\alpha\gamma Z_{t3}(\rho) \\ +\beta\gamma Z_{t4}(\rho)+\theta_0(\rho)Z_{t5}+e_t$$

Since the e_t's are NID $(0,\ \sigma_e^2)$ by assumption, it is clear that for fixed δ and ρ minimizing

$$(3.16)\qquad \textstyle\sum_{t=1}^T e_t^2=\sum_{t=1}^T\{Y_t(\rho)-(1-\gamma)[Z_{t1}(\rho)-Z_{t2}(\rho)] \\ -\alpha\gamma Z_{t3}(\rho)-\beta\gamma Z_{t4}(\rho)-\theta_0(\rho)Z_{t5}\}^2$$

will yield ML estimates.(6) Again we rely on a search procedure: we minimize (3.16) over the region $0<\delta<1$ and $|\rho|<1$. We select a grid of δ, ρ values on the grid fit (3.16) by least squares and obtain the residual sum of squares. The desired estimate of the parameters are these which correspond to the grid point yielding the smallest residual sum of squares. The large sample standard errors of these estimates are estimated by the square roots of the main diagonal elements of the inverse of the information matrix evaluated at the given parameter estimates.

IV. An Example

We now turn to examine some preliminary empirical results of estimating the inventory equation (2.4) using quarterly Canadian data and ML methods described above. This section provides an illustration of estimating

(6) Refer to n. 5 above for the estimation of $\theta_0(\rho)$ and the formation of the information matrix.
(7) Unless otherwise noted, we use the 0.05 level of significance throughout this section.

and discriminating the adaptive expectations and partial adjustment models, and is not intended as a rigorous empirical study of inventory behavior. Quarterly data on sales (shipments) and finished-good inventories of total manufacturing industry have been obtained for the period of 1961:I-1973:III from deseasonalized monthly data published by Statistics Canada. Sources of data are given in the Appendix. Difficulties involved in deflating sales and inventories have led to use of the current dollar figures.

Results of estimation are presented in Table I. Taking the ML estimates under Assumption I first, we note from the column under heading I that all regression coefficients comply with a priori ranges and that all regression coefficients except $(1-\gamma)(1-\delta)$ are significantly different from zero at the 0.05 level.[7] The short-run marginal inventory/sales ratio ($\beta\gamma\delta$) is estimated to be 0.027 while the long-run marginal inventory/sales ratio

Table I. Maximum Likelihood Estimates of Parameters in (2.4) under Alternative Assumptions about Disturbance Terms[a]

Parameters	Assumptions		
	I	II	III
δ	—	.226 (.00799)	.260 (.00905)
γ	—	.242 (.0622)	.264 (.0685)
α	.542 (n.c.)	.535 (.125)	.519 (.130)
β	.210 (n.c.)	.212 (.012)	.212 (.013)
ρ	—	—	.10 (.995)
$\alpha\gamma\delta$	.0706 (.0311)	.0344 (n.c.)	.0356 (n.c.)
$\beta\gamma\delta$	.0274 (.0083)	.0137 (n.c.)	.0145 (n.c.)
$(1-\gamma)+(1-\delta)$	.9718 (.1516)	1.492 (n.c.)	1.476 (n.c.)
$(1-\gamma)(1-\delta)$	.1022 (.1488)	.5564 (n.c.)	.5446 (n.c.)
$\gamma\delta$	.1304 (n.c.)	.0644 (n.c.)	.0686 (n.c.)
$\gamma+\delta$	1.028 (n.c.)	.508 (n.c.)	.524 (n.c.)
σ^2	.0014	.001247	.001263

[a]The figures in parentheses below the coefficient estimates are the estimated large-sample standard errors. The symbol (n.c.) indicates not calculated.

(β) is 0.21. Point estimates of $\gamma\delta$ and $\gamma+\delta$ yield an average lag between changes in sales and inventory adjustment, $(1-\gamma)/\gamma+(1-\delta)/\delta$ of 5.88 quarters.

The coefficient $(1-\gamma)(1-\delta)$ of Y_{t-2} in equation (2.4) is not significantly different from zero, and we are led to conclude that either a partial adjustment model or an adaptive expectations model is the correct specification. As pointed out earlier, however, we are unable to decide which model is correct due to the fact that δ and γ cannot be estimated separately under Assumption I about the disturbance terms.

The parameter estimates under Assumption II are presented in the third column of Table I. Even if both δ and γ enter (2.4) symmetrically, the presence of only δ in the disturbance term is used in identifying parameters, and the ML estimates of δ and γ are, respectively, 0.266 and 0.242. All coefficient estimates fall within a priori ranges and that estimates of both δ and γ are significantly different from one. Thus the partial adjustment and adaptive expectations processes jointly explain the inventory stock behaviours.

The estimated long-run marginal inventory/sales ratio of 0.212 is almost identical with the estimate of 0.210 under Assumption I, yet the short-run marginal inventory/sales ratio of 0.0137 is only about one half of the corresponding figure obtained under Assumption I. Substantial differences in estimates appear between Assumptions I and II for $(1-\gamma)+(1-\delta)$ and $(1-\gamma)$ $(1-\delta)$ as well. The average lag in sales expectations, $(1-\delta)/\delta$, implied by the δ estimate is 2.76 quarters; the average lag in stock adjustment, $(1-\gamma)/\gamma$, is estimated to be 3.13 quarters; the combined average lag is 5.89 quarters, which is very close to the corresponding figure obtained under Assumption I.

Last, the results of estimating parameters in (2.4) under Assumption III are given in the last column of Table I. All estimates comply with a priori ranges, and both δ and γ are significantly different from one. The point estimate of autocorrelation parameter ρ is 0.10. This suggests that ρ may not indeed differ from zero. In this case Assumption III reduces to Assumptin II.

Point estimates of all parameters under Assumption III are almost identical with the corresponding estimates under Assumption II as we might expect in the case of no or weak autocorrelation of the u_t's. Furthermore, the point estimates of $1-\delta$ and ρ, 0.74 and 0.10 respectively, suggest that Assumption I may be false. Insignificant coefficient estimate of Y_{t-2} obtained under Assumption I appears to be the result of the error in the specification of the properties of disturbance terms.

V. Concluding Remarks

It has been the purpose of this paper to consider ML methods of estimating a general distributed lag model under three alternative assumptions about the disturbance terms. The model considered embodies the conceptual ingredients of the partial adjustment and adaptive expectations hypotheses. It is shown that proper specifications of the disturbance terms enable us to estimate the coefficients of adjustment and expectations separately and to distinguish between the partial adjustment and adaptive expectations model empirically.

The estimation methods have been applied for illustration in the analysis of the quarterly inventory data of Canadian manufacturing industry. It is also readily recognized that the estimation methods are more generally applicable. For example, they can also be applied in the analysis of consumer durable goods based on expected income hypothesis of consumption and the analysis of the monetary sector based on income expectations.

Appendix

Sources of Data

Statistics Canada, *Inventories, Shipments, and Orders in Manufacturing Industries.* Catalogue No. 31-001, monthly.

Years	Issues
1961—1964	July 1968
1965—1967	Annual Supplement 1970
1968—1969	October 1971
1970—1971	January 1973
1972—1973	December 1973

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