Macro Cross-Elasticities and Analysis of Intermarket Pressures

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I. Introduction

The classical general equilibrium model assumes that the price variables, such as the commodity price, the rate of interest, and the wage rate, have an infinite speed of adjustment toward a new equilibrium whenever exogenous disturbances are introduced into the system. In reality, however, we can rather easily observe the cases in which these variables are not so flexible and there exists disequilibrium in the market for a considerable length of time.

The theoretical development regarding the possibility of the existence of disequilibrium and its effect on the whole economic system has been made by several economists.\(^1\) One of the most interesting and theoretically important propositions is that the adjustment direction of price variables derived under the neoclassical models, such as Patinkin's,\(^2\) may not be

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(2) Patinkin (1965).
ied to the case where there exist intermarket pressures during the ailing disequilibrium period.

This paper we shall build a model of the general disequilibrium, studying the intermarket pressures (or spillover forces), and investigate reticently the sign properties of the dynamic base matrix. For this purpose shall introduce three fundamental hypotheses which characterize the prove of the spillover forces. In the empirical part we shall compute ral key indicators in order to find out the most probable sign property one ambiguous elements in the base matrix. Finally, the discrepancies een the adjustment of price variables in the general equilibrium model in the disequilibrium model will be examined.

II. Analytical Framework

The methodology of this study is based, in the main, upon Samuelson’s correspondence Principle. That is, if “the dynamical properties of the systems specified, and the hypothesis is made that the system is in stable librium,”(3) then definite operationally meaningful theorems, such as properties of variables, can be derived.

First, we present a model of the general disequilibrium which includes intermarket variables (i.e., spillover variables) among the system’s genous forces. These spillover variables are defined by the demand for \(=1, \cdots, n, \) which is spilled over from the \(X_j \) market \(j=1, \cdots, n, j \neq i \) to disequilibrium in the latter which confines the transactions of indi- ids and firms within the limit of their effective income.(4) We use the wing aggregate equation system. Let:

- \(p_t\) = commodity price
- \(p_b\) = bond price, or inverse of the interest rate
- \(p_r\) = the real wage rate
- \(M^0_b\) = the initial money holdings of the household sector
- \(M^0_s\) = the initial money holdings of the production sector

---

Samuelson (1947), p. 5.

We assume here that Hicksian false tradings are allowed to occur in the state of disequilibrium, and thus, there exists a gap between the planned and the realized transactions. The effective income refers to the receipts from the realized transactions.
\[ M_0 = M_0^h + M_0^e \]
\[ \frac{M_0^h}{p_c}, \quad \frac{M_0^e}{p_c} = M_0^h \text{ and } M_0^e \text{ in real terms} \]

\[ C^d, B^d, N^d = \text{notional demand for commodities, bonds, and labor} \]
\[ C^s, B^s, N^s = \text{notional supply of commodities, bonds, and labor} \]
\[ c, b, N = \text{actual quantities demanded of commodities, bonds, and labor} \]
\[ b^s, b = B^d, B^s, B \text{ in real terms (i.e., deflated by } r_p) \]
\[ y = \text{real income} \]

The predetermined variables are real money \( \frac{M_0}{p_c} \), \( \frac{M_0^h}{p_c} \), and \( \frac{M_0^e}{p_c} \), and real income. Money is assumed to be inside money so that the quantities of real money can be treated as constant even if price changes.\(^{(5)}\) As the endogenous forces, the intermarket variables (i.e., spillover variables) are introduced into the following equation system:

\[(1) \quad ED_c = C^d(p_c, p_b, p_N; \frac{M_0^h}{p_c}, y) - C^s(p_c, p_b, p_N; \frac{M_0^e}{p_c}, y) + C^d* + S^c, \]
\[(2) \quad ED_b = b^d(p_c, p_b, p_N; \frac{M_0^h}{p_c}, y) - b^s(p_c, p_b, p_N; \frac{M_0^e}{p_c}, y) + b^d* + S^b, \]
\[(3) \quad ED_N = N^d(p_c, p_b, p_N; \frac{M_0^h}{p_c}, y) - N^s(p_c, p_b, p_N; \frac{M_0^e}{p_c}, y) + N^d* + S^n, \]

where \( C^d* \) denotes the demand for commodities spilled over from either or both the bond market and the labor market due to disequilibrium in those markets. Similarly, \( b^d* \) and \( N^d* \) denote the same kind of spillover demands in the bond and labor market. \( S^c, S^b \) and \( S^n \) are the exogenous shocks which cause the excess demand existing in the jth market.\(^{(6)}\) The system has a dynamic property such that:

\[(4) \quad \frac{dp_i}{dt} = h_i ED_i, \quad i = C, b, N, \]

where \( h_i \)'s are positive coefficients appropriate to the ith market representing the speed of adjustments of \( p_i \)'s. Assuming, without loss of generality, that the \( h_i \)'s are equal to one,\(^{(7)}\) equation (4) can be written in terms of \( p_c \).
aylor’s series expansion, such that:

\[
(5) \quad p_i = \sum_j q_{ij} (p_i - p_j^e) + \cdots ,
\]

re \( p_j^e \) denotes the equilibrium set of prices. Let matrix \( Q \) denote \([q_{ij}]\) re, then the solution of equation (5) can be written:

\[
(6) \quad p_i(t) = p_i^e + \sum_j k_{ij} p_j^e(t)
\]

re \( \lambda_1, \cdots , \lambda_n \) are characteristic roots of \( (Q - \lambda I) \), and \( k_{ij} \) is a polynomial of degree of at most one less than the number of times the \( j \)th root repeated. In order for the system of equations—therefore equation (1), and (3) — to be stable, the sign property of matrix \( Q \) must satisfy stability conditions.\(^8\)

III. Fundamental Hypotheses

The spillover variables are the functions of the exchange rates in other markets. It is:

\[
(7) \quad C^{*} = C^{*}(p_b, p_n),
\]

\[
(8) \quad B^{*} = B^{*}(p_e, p_b),
\]

\[
(9) \quad N^{*} = N^{*}(p_e, p_n).
\]

Intermarket spillover forces occur due to an improper set of prices in system. Thus, how far the prices are from the equilibrium position may be the most important factor influencing the magnitude of the spillo-

This assumption is made for simplicity, but the sign property of the characteristic roots in equation (6) below remains the same. This assumption is also used by Samuelson (1947), p.271, and Patinkin (1952), p.39.

For a differential equation system to be stable, the characteristic equation of coefficients is:

\[
f(\lambda) = \begin{vmatrix} q_{11} - \lambda & q_{12} & \cdots & q_{1n} \\
q_{21} & q_{22} - \lambda & \cdots & q_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1} & q_{n2} & \cdots & q_{nn} - \lambda \end{vmatrix} = |Q - \lambda I| = 0,
\]

and the real part of \( \lambda \)'s are all negative. Specifically, for a \( 3 \times 3 \) matrix, the system is stable if and only if one of the following conditions is satisfied: (1) \( Q \) has all diagonal elements negative; (2) \( Q \) has exactly two negative diagonal elements, and there exists a term in the expansion of \( |Q| \) of negative sign; (3) \( Q \) has exactly one negative diagonal element \( q_{11} \), and either (3a) or (3b) is satisfied; (3a) \( q_{j1}q_{j1} < 0 \) for some \( j = 2,3 \), and there exists a term in the expansion of \( |Q| \) of negative sign; (3b) \( q_{23}q_{32} < 0 \), and there exists a term in the expansion of \( |Q| \) of positive sign. For details, see Quirk (1968), Lancaster (1968), and Samuelson (1947), p.271.
over forces. Since $C^{**}$, for example, is defined as the transferred demand from the bond market and the labor market, it should be a function of the bond price, $p_b$, and the wage rate, $p_N$.

2. The spillover portion of demand has the sign property such that:

$$\frac{\partial C^{**}}{\partial p_N}, \leq 0, \quad \frac{\partial C^{**}}{\partial p_b}, \leq 0,$$

$$\frac{\partial b^{**}}{\partial p_c}, \leq 0, \quad \frac{\partial b^{**}}{\partial p_N}, \leq 0,$$

$$\frac{\partial N^{**}}{\partial p_c}, \leq 0, \quad \frac{\partial N^{**}}{\partial p_b}, \leq 0,$$

the equality sign denoting the possibility of one-to-one spillover—the excess demand or supply in one market affects only one of the other two markets.

3. The effect of the change in the $i$th market price on the demand and supply in the $j$th market can be decomposed into two parts: substitution effect and spillover effect. Total cross-elasticity should, therefore, consist of the two effects. Let $T_{ji}$ be the total cross-elasticity, $e_{ji}^d$ be the price cross-elasticity of demand, $e_{ji}^s$ be the price cross-elasticity of supply and $s_{ji}$ be:

$$s_{ji} = -\frac{\partial x_{ji}^{**}/x_{ji}^{**}}{\partial p_i/p_i}$$

denoting the rate of spillover from the $i$th market to the $j$th market due to the change in the $i$th price. Then:

$$T_{ji} = e_{ji}^d - e_{ji}^s + s_{ji}$$

or

$$\frac{dx_j/x_j}{dp_i/p_i} = \frac{\partial \phi_j/\phi_j}{\partial p_i/p_i} - \frac{\partial \pi_j/\pi_j}{\partial p_i/p_i} + \frac{\partial x_{ji}^{**}/x_{ji}^{**}}{\partial p_i/p_i},$$

where:

$$\phi_j = \phi_j(p_i, \ldots, p_n, Y),$$

$$\pi_j = \pi_j(p_i, \ldots, p_n, R),$$

representing the demand and supply for the $j$th good. $Y$ and $R$ denote income and firms revenue respectively. In general, $(e_{ji}^d - e_{ji}^s)$ has a positive sign if $X_j$ and $X_i$ are substitutes, and $s_{ji}$ has a negative sign whether or not they are substitutes.

IV. Theoretical Sign Properties

Under the first and second fundamental hypothesis we have the following partial derivatives from equations (1), (2), and (3):
\[ \begin{align*}
  a_{11} &= \frac{\partial C_d^d}{\partial p_c} - \frac{\partial C_s^d}{\partial p_c} < 0, \\
  a_{12} &= \frac{\partial C_d^d}{\partial p_b} - \frac{\partial C_s^a}{\partial p_b} + \frac{\partial C_s^d}{\partial p_b} =?, \\
  a_{15} &= \frac{\partial C_d^d}{\partial p_N} - \frac{\partial C_s^a}{\partial p_N} + \frac{\partial C_s^d}{\partial p_N} =?, \\
  a_{21} &= -\frac{\partial b_d^d}{\partial p_c} + \frac{\partial b_s^s}{\partial p_c} + \frac{\partial b_s^d}{\partial p_c} < 0, \\
  a_{22} &= -\frac{\partial b_d^d}{\partial p_b} - \frac{\partial b_s^s}{\partial p_b} < 0, \\
  a_{23} &= -\frac{\partial b_d^d}{\partial p_N} - \frac{\partial b_s^s}{\partial p_N} + \frac{\partial b_s^d}{\partial p_N} < 0, \\
  a_{31} &= -\frac{\partial N_d^d}{\partial p_c} + \frac{\partial N_s^s}{\partial p_c} =?, \\
  a_{32} &= -\frac{\partial N_d^d}{\partial p_b} + \frac{\partial N_s^s}{\partial p_b} + \frac{\partial N_s^d}{\partial p_b} < 0, \\
  a_{33} &= -\frac{\partial N_d^d}{\partial p_N} - \frac{\partial N_s^s}{\partial p_N} < 0.
\end{align*} \]

The signs of \(a_{12}, a_{13}, \) and \(a_{31}\) are ambiguous. The first two terms in the first line of each of these give us a positive sign, but the last term, spillover demand, is negative according to the first hypothesis. Consequently, we have an incomplete sign system of matrix \(A\), where \(A\) denotes the matrix in (13) above; that is:

\[ A = \begin{bmatrix}
  - & ? & ? \\
  - & ? & - \\
  ? & - & - 
\end{bmatrix} \]

is crucial to determine the correct sign of those three ambiguous terms. Once they are determined, the cofactor of the matrix \(A\) can be obtained, and we can derive a certain useful theorem which explains the relationships among the price variables in various disequilibrium states. In the following section we shall investigate the most probable sign of \(a_{12}, a_{13}, \) and \(a_{31}\) by using some actual data.

V. Empirical Sign-Investigation

From the third hypothesis, we have the following relationship:

\[ \frac{d x_j}{d p_i / p_i} = \frac{\partial \phi_j / \phi_j}{\partial p_i / p_i} - \frac{\partial \pi_j / \pi_j}{\partial p_i / p_i} + \frac{\partial x_j^* / x_j^*}{\partial p_i / p_i} \]

It is safe enough to say that the sign of \(a_{12}\), for example, should be the
same as the sign of:

\[
\frac{dx_i}{x_i} = \frac{\partial p_i}{\partial p_j}.
\]

Therefore, it depends upon the three terms on the right hand side of (12). The first and the second terms on the right hand side are the price cross-elasticity of demand and supply, respectively. It is difficult to measure the exact magnitude of these elasticities. There is, however, an approximating method used by Ragnar Frisch.\(^9\) His method was originally designed for the analysis of micro-behavior, but mutatis mutandis we can apply it to our macro-theory. According to Frisch \(e^p_{ij}\) can be computed by the following formula:

\[
(15) \quad e^p_{ij} = \left(-E_i \alpha^p_{ij} \right) \left( \frac{1 + e^d_{ij}}{1 - \alpha^d_{ij} E_i} \right), \quad j \neq i,
\]

where:

- \(E_i\) = Engel elasticity of demand for \(X_i\),
- \(e^d_{ij}\) = Direct price elasticity of demand for \(X_i\),
- \(\alpha^d_{ij}\) = Budget proportion of \(X_i\).

For the cross-elasticities of supply we use:

\[
(16) \quad e^p_{ij} = \left(R_i \alpha^s_{ij} \right) \left( \frac{1 + e^s_{ij}}{1 - \alpha^s_{ij} R_i} \right), \quad j \neq i,
\]

where:

- \(R_i\) = Revenue elasticity of supply of \(X_i\),\(^{10}\)
- \(e^s_{ij}\) = Direct elasticity of supply of \(X_i\),
- \(\alpha^s_{ij}\) = Relative importance of \(X_i\).\(^{11}\)

To make use of (15) and (16) for our purpose, we collect the time series (quarterly) data of the United States during the period of 1960–1972\(^{12}\) and regress the demand and supply data onto each price variable and income, lagged by one quarter, in the following way:

\(^{(9)}\) Frisch (1959).
\(^{(10)}\) Firms’ revenue here represents firms’ budget for production together with profits occurring through their sales activities.
\(^{(11)}\) \(\alpha^s_{ij}\)'s are calculated by dividing the value of annual output, bonds and securities, and labor supply by total value of all of these. The data used here is for the period 1960–1972.
\(^{(12)}\) For the commodity supply, we use the actual output data, and for the commodity demand the value of final sales is used. In the category of commodity, the following items are included: automobiles and parts, other durables, non-durables, services, plant and equipment, houses, inventory of durables and non-durables. By bonds we mean the aggregation of stocks, bonds, and various securities. As the labor supply we use 96% of total labor force.
\( \ln x_i = \beta_0 + \rho_i \ln p_{i-1} + E_i \ln Y_{i-1} + \epsilon_i, \)
\( \ln x_i = \gamma_i + \rho_i \ln p_{i-1} + R_i \ln R_{i-1} + \delta_i \)

\((i=\text{commodity, bond, or labor}).\)

Coefficients of these regressions represent the direct elasticities and Engel (Revenue) elasticities. The regression results are shown in Table I.\(^{(13)}\)

<table>
<thead>
<tr>
<th></th>
<th>Demand</th>
<th></th>
<th>Supply</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\alpha_i^d)</td>
<td>(E_i)</td>
<td>(e_i^d)</td>
<td>(\alpha_i^s)</td>
</tr>
<tr>
<td>modities</td>
<td>0.326</td>
<td>0.701</td>
<td>-1.186</td>
<td>0.366</td>
</tr>
<tr>
<td>ds</td>
<td>0.154</td>
<td>1.056</td>
<td>-2.355</td>
<td>0.191</td>
</tr>
<tr>
<td>or</td>
<td>0.412</td>
<td>0.523</td>
<td>-1.930</td>
<td>0.420</td>
</tr>
</tbody>
</table>

To obtain the ratios and elasticities above, we calculate the following \(e_i^d\)'s and \(e_i^s\)'s, according to formula (15) and (16):

\[ e_{12}^d = 0.1748 \]
\[ e_{13}^d = 0.3424 \]
\[ e_{14}^d = 0.0411 \]
\[ e_{12}^s = -0.0418 \]
\[ e_{13}^s = -0.0692 \]
\[ e_{14}^s = -0.1470 \]

Next, we should estimate the spillover elasticities, i.e., \(s_{12}, s_{13}, \) and \(s_{14}\). This is the most difficult part of our empirical investigation because we do not know in reality the quantity actually transferred among the markets. For example, suppose the commodity is in excess demand. Households will transfer their unspent income to the bond market, purchasing more bonds, or to the labor market, raising more leisure (i.e., working less). How much of the unspent income will be transferred to the bond market and to the labor market? We propose three possible cases, as shown in Table II. Even though probabilistic conjecture is only a rough and naive idea, this will give considerably better information than we would have obtained by merely guessing in the dark.

As indicated in footnote (11), total outlay of the economy as a whole consists of households' and firms' outlay. Likewise, total income consists of firms' revenue and households' labor income. Therefore, the last row of the table shows the firms' behavior in the aggregate demand for labor at the first column and the households' behavior in the aggregate supply of labor at the last column. Thus, labor is treated as a commodity in our aggregate system.
Table II.

<table>
<thead>
<tr>
<th>Spillover From</th>
<th>Spillover To</th>
<th>Commodity Market</th>
<th>Bond Market</th>
<th>Labor Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity Market</td>
<td></td>
<td></td>
<td>(1) 25%</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2) 50%</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3) 75%</td>
<td>25%</td>
</tr>
<tr>
<td>Bond Market</td>
<td></td>
<td>(1) 25%</td>
<td></td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) 50%</td>
<td></td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) 75%</td>
<td></td>
<td>25%</td>
</tr>
<tr>
<td>Labor Market</td>
<td></td>
<td>(1) 25%</td>
<td>75%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) 50%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) 75%</td>
<td>25%</td>
<td></td>
</tr>
</tbody>
</table>

Using the same data of demand and supply in each market used for (17) above, we calculate the three probable quantities of spillover from one market to other markets. To determine how much would spill over to each market, we do need a strong theory, or empirical evidence, in order to choose one of the above proportion combinations. Assuming here that the intermarket forces between the commodity and bond markets are more sensitive than those between the labor and other markets, we may choose the third combination. To find \( s_{12} \), \( s_{13} \) and \( s_{31} \) we use the following simple regressions:

\[
\ln x_{12}^* = \beta_{10} + \beta_{12} \ln p_{t-1} + E_1, \\
\ln x_{13}^* = \beta_{20} + \beta_{13} \ln p_{t-1} + E_2, \\
\ln x_{31}^* = \beta_{30} + \beta_{21} \ln p_{t-1} + E_3.
\]

\( \beta_{12} \), \( \beta_{13} \), and \( \beta_{31} \) represent \( s_{12} \), \( s_{13} \), and \( s_{31} \) respectively. The regression results are shown in Table III. The values in parentheses are student “\( t \)” values.

Table III

<table>
<thead>
<tr>
<th></th>
<th>( \beta_{10} )</th>
<th>( \beta_{12} )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st eq.</td>
<td>4.146</td>
<td>-0.3042</td>
<td>0.867</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.0413)</td>
<td></td>
</tr>
<tr>
<td>2nd eq.</td>
<td>3.293</td>
<td>-0.2917</td>
<td>0.924</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-7.4926)</td>
<td></td>
</tr>
<tr>
<td>3rd eq.</td>
<td>9.498</td>
<td>-0.2049</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.6923)</td>
<td></td>
</tr>
</tbody>
</table>

(14) This is not merely an assumption. There are several well-known theories supporting the supposition that the intermarket adjustment speed between the labor and other markets is considerably slower due to the reservation wage rate and the illiquidity of labor. See Leijonhufvud (1968), p.79, and Alchian (1969).
summarize what we computed in (18) and Table III above, as shown
the following table.

<table>
<thead>
<tr>
<th></th>
<th>$e_{ij}'$</th>
<th>$e_{ij}^*$</th>
<th>$s_{ij}$</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i=2, j=1$</td>
<td>0.1748</td>
<td>-0.0418</td>
<td>-0.3042</td>
<td>-0.0876</td>
</tr>
<tr>
<td>$i=3, j=1$</td>
<td>0.3424</td>
<td>-0.0692</td>
<td>-0.2917</td>
<td>0.2975</td>
</tr>
<tr>
<td>$i=1, j=3$</td>
<td>0.0411</td>
<td>-0.1470</td>
<td>-0.2049</td>
<td>0.0168</td>
</tr>
</tbody>
</table>

values in the last column (balance) represent total elasticities defined by
). Consequently, we end up with the base matrix $A$, the sign of each
ent being:

$$A = \begin{bmatrix}
+ & - & - \\
- & + & - \\
- & - & + \\
\end{bmatrix}$$

the signs of its cofactor turn out to be:\(^{(15)}\)

$$\begin{bmatrix}
+ & - & - \\
- & + & - \\
- & - & + \\
\end{bmatrix}$$

difference-equation system is, therefore:

$$\begin{bmatrix}
\frac{dp_c}{dp_b} \\
\frac{dp_b}{dp_s} \\
\frac{dp_s}{dp_n} \\
\end{bmatrix} = \frac{1}{|A|} \begin{bmatrix}
d_{ij} \\
- & - & S \\
- & S & - \\
- & - & S \\
\end{bmatrix}$$

ere $|A|$ is the determinant of $A$. Since we are interested in the case
ere the system is stable,\(^{(16)}\) determinant of $A$ must be negative. Thus, sign system should be:

$$\begin{bmatrix}
\frac{dp_c}{dp_b} \\
\frac{dp_b}{dp_s} \\
\frac{dp_s}{dp_n} \\
\end{bmatrix} = \begin{bmatrix}
+ & - & - \\
- & + & - \\
- & - & + \\
\end{bmatrix} \begin{bmatrix}
- & - & S \\
- & S & - \\
- & - & S \\
\end{bmatrix}$$

VI. Economic Implications

0 far we have presented a macro equation-system with the spillover

As a matter of fact, signs of diagonal elements, $d_{ii}, i=1,2,3,$ are somewhat ambiguous in
this case. But they should be positive because they will eventually be divided by the inverse
of the determinant of $A$ which is negative, so that the Walrasian stability condition can be
maintained. Therefore, their signs are mathematically ambiguous, but theoretically obvious,
as shown by (23) below.

1) Refer to the Correspondence Principle.
forces and investigated either theoretically or empirically the sign property of each element in matrix $A$ which is the base matrix in the difference equation system of our model. We are now ready to utilize what we have found. That is, we can test various cases of disequilibrium by using (23) above.

Let us first consider $ESC$ (excess supply of commodities) together with $ESL$ (excess supply of labor). This is the case where Patinkin tries to project the notion of involuntary unemployment into his disequilibrium theory. The sign of the exogenous shock, $-S$, should be positive because $S^e$ itself represents the excess demand. The sign of $-S^w$ should also be positive. Replacing them in (23), we have:

\[
\begin{bmatrix}
\frac{dp_c}{dp_h} \\
\frac{dp_h}{dp_p} \\
\frac{dp_p}{dp_n}
\end{bmatrix} = \begin{bmatrix}
- & + & - \\
+ & - & + \\
- & + & -
\end{bmatrix} \begin{bmatrix}
+ \\
0 \\
+
\end{bmatrix} = \begin{bmatrix}
- \\
+ \\
-
\end{bmatrix}
\]

Thus, due to $ESC$ and $ESL$, the commodity price and the interest rate fall (or $p_h$ rises), and obviously, the wage rate also falls. This is precisely what Patinkin tries to analyze in his *Money, Interest, and Prices* (Chapter XIII).

Next, as a contrasting case with Patinkin’s let us illustrate the disequilibrium with $ESC$ (excess supply of commodities) and $ESB$ (excess supply of bonds). Since $-S^c > 0$, and $-S^w > 0$, we get:

\[
\begin{bmatrix}
\frac{dp_c}{dp_h} \\
\frac{dp_h}{dp_p} \\
\frac{dp_p}{dp_n}
\end{bmatrix} = \begin{bmatrix}
- & + & - \\
+ & - & + \\
- & + & -
\end{bmatrix} \begin{bmatrix}
+ \\
0 \\
+
\end{bmatrix} = \begin{bmatrix}
-(-?) \\
-(-?) \\
-(-?)
\end{bmatrix}
\]

None of the signs of $dp_c$, $dp_h$, and $dp_p$ is unambiguous. Assuming that the price variable in each market is affected most by its own excess quantity, we may conjecture that $dp_c < 0$ and $dp_h < 0$. But even under this assumption, the sign of $dp_n$ is unknown. Let us compare the above results to Patinkin’s analysis. In Chapter X of the same book Patinkin simply assumes that this $ESC-ESB$ type of disequilibrium will cause both the commodity price and the interest rate to fall.\(^{(17)}\) According to our analysis, however, this is only a special case when the price variables are affected most by its own excess quantity, spillover forces from other markets being minor in their strength. Patinkin seems somewhat too optimistic on this point regarding

\(^{(17)}\) Patinkin (1965). See particularly pp. 324 and 326.
adjustment mechanism of the rate of interest. This is because he looks a firm's reaction in such a case where, because of excess supply, planned budgets cannot be supported if they do not issue additional s to finance it. Our signs of $dp_e/dS^e$ represent these reacting forces of which causes a fall in bond price (i.e., increase in the rate of est) in our system. If Patinkin had realized this, he would have ed the possibility of an increase in the rate of interest.

the same fashion we can check the direction of $dp_e$, $dp_i$, and $dp_N$ in us cases of disequilibrium. Table V shows the similarities and differen direction between the general equilibrium method and our dis- librium method.

<table>
<thead>
<tr>
<th>of Disequilibrium</th>
<th>Patinkin's</th>
<th>Ours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESC &amp; EDB</td>
<td>$dp_e&lt;0$,  $dp_i&gt;0$</td>
<td>$dp_e&lt;0$,  $dp_i&gt;0$,  $dp_N&lt;0$</td>
</tr>
<tr>
<td>EDC &amp; EDB</td>
<td>$dp_i&gt;0$,  $dp_e&gt;0$</td>
<td>$dp_i&gt;0$,  $dp_e&gt;0$,  $dp_N&gt;0$</td>
</tr>
<tr>
<td>ESC &amp; ESB</td>
<td>$dp_e&lt;0$,  $dp_i&lt;0$</td>
<td>$dp_e&lt;0$,  $dp_i&lt;0$,  $dp_N&lt;0$</td>
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<tr>
<td>EDC &amp; ESB</td>
<td>$dp_i&gt;0$,  $dp_e&lt;0$</td>
<td>$dp_i&gt;0$,  $dp_e&lt;0$,  $dp_N&gt;0$</td>
</tr>
</tbody>
</table>

hown in the table, our method provides us with more information: direction of change in the wage rate can be obtained unambiguously be first and last case. In the second and third cases, the sign of change rice variables may be anything depending upon the strength of the over forces relative to that of the intramarket forces. Finally, it is not ssary to assume, as Patinkin does, that the labor market is cleared e we deal with the case of ESC & EDB or EDC & ESB. Due to the over forces, it would not remain cleared although it may be so at the aning. That is, the labor market is continuously disturbed so that the rate adjusts itself according to the exogenous forces.

VII. Concluding Remarks

this paper we developed a model for general disequilibrium, introdu- the spillover forces into each equation. Our concept of the spillover s is the demand for $X_i$ which is spilled over from the $X_i$ market ($i \neq j$)
due to disequilibrium in the $X_i$ market where either households or firms are not satisfied with their transactions actually made. Under the stability assumption we investigated the dynamic sign properties of the system, i.e., the direction of change in each price variable when an initial disturbance is introduced into the market. In the matrix of the difference-equation system there were several elements whose sign properties were ambiguous. As an attempt to dispel this ambiguity, we used the price cross-elasticities and the spillover elasticities which constitute total cross-elasticities.

According to our findings both the commodity price and the interest rate fall when the system is in excess supply of commodities and excess demand for bonds. But this is not the whole story, as the conventional equilibrium usually argues. Due to the spillover effect the labor market will most likely experience a fall in the wage rate even though it is initially in equilibrium. Similarly, in the case of excess demand for commodities with excess supply of bonds, the wage rate should rise along with the commodity price and the interest rate. On the other hand, the direction of the change in the price variables is not clear when the commodity market and the bond market are in the same situation. The classical theory in this case is applicable only to the system where the intermarket spillover forces are relatively weak compared to the intramarket forces.

This study contains in itself several limitations. First, we used Frisch's method for computing the cross-elasticities which may be too abstracted to be applicable to macro-analysis. Second, we chose a particular probability combination for the distribution of spillover forces assuming that the commodity market is more influential to the bond market than to the labor market. Third, but not necessarily least significant, we assigned six signs in matrix $A$ in accordance with theory, assigning the other three ambiguous terms by means of empirical data. Thus, the sign properties in matrix $A$ were not uniformly detected. The removal of these limitations will be the objective of future studies.

References

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