Effects of Frequency Offset on MC/CDMA System Performance

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***Abstract*— In this letter, the effects of frequency offset on a multicarrier code-division multiple-access system are theoretically analyzed and verified by computer simulations for downlink channel. Both equal gain combining and maximal ratio combining are considered in combining subcarrier signals in the analysis.**

***Index Terms*—Multicarrier, code-division multiple access, frequency offset.**

I. Introduction

A multicarrier code-division multiple-access (MC/CDMA) system has been recently proposed for the implementation of high data rate system in the future [1], [2]. Like in OFDM systems, frequency offset between the transmitter and the receiver may cause severe performance degradation in the MC/CDMA system [1], [3].

Tomba and Krzymien evaluated the effects of frequency offset on the performance of MC/CDMA systems [3]. They used a computer simulation approach for a multiuser case, and an analytical approach for a single user case. In this letter, a multiuser case is analytically evaluated and verified by computer simulations. Two diversity combining techniques, equal gain combining (EGC) and maximal ratio combining (MRC), are considered for combining subcarrier signals.

II. System and Channel Model

In an MC/CDMA system, a single data bit is firstly replicated into N parallel copies. Each branch of the parallel stream is multiplied by an associated chip of a given spreading code and then modulated to the corresponding subcarrier [1]. We consider a downlink communication of an MC/CDMA system with M users where all users' signals experience the same fading channel characteristics and are synchronized.

In this letter, a frequency selective Rayleigh fading channel is considered, and the bit duration is assumed to be much longer than channel delay spread such that each subcarrier signal is assumed to undergo flat fading. We also assume independent fading for each subcarrier. In such a condition, the received signal may be represented as

$$r(t) = \sum_{k=-\infty}^{\infty}\sum_{m=0}^{M-1}\sum_{i=0}^{N-1}(a_m[k]c_m[i]p_{T_s}(t-kT_b)\rho_i(t) \cdot \cos[2\pi f_i t + \theta_i(t)]) + n(t) \tag{1}$$

where M is the number of users and N is the number of subcarriers which is the same as processing gain (PG). $a_m[k] \in \{+1,-1\}$ and $c_m[i] \in \{+1,-1\}$ denote the mth user's kth data bit and the ith chip of the mth user's spreading code, respectively. Note that Hadamard–Walsh codes are used for spreading codes. $f_i = f_c + i/T_b$ is the ith subcarrier frequency and T_b is the bit duration. $p_{T_s}(t)$ is the rectangular pulse defined on $[0, T_b]$. $\rho_i(t)$and $\theta_i(t)$ represent the magnitude and the phase of channel impulse response for the ith subcarrier. $n(t)$ denotes additive white Gaussian noise (AWGN) with zero-mean and double-sided power spectral density $N_0/2$. The mean power of the ith subcarrier signal is defined as $P_i = \frac{1}{2}E[\rho_i^2(t)]$. Assuming that the mean powers of all subcarrier signals are equal, the mean total power of a user signal is $P_{\text{Tot}} = NP_i$ and the bit energy is $E_b = P_{\text{Tot}}T_b$.

III. Interference Analysis for EGC

After demodulation and combining subcarrier signals, the decision variable for the zeroth user's kth data bit is obtained as

$$\begin{aligned}\nu_{0,k} &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\sum_{i=0}^{N-1}\Bigg(w_{0,n}a_m[k]c_m[i]\frac{1}{T_b}\int_{kT_b}^{(k+1)T_b}\rho_i(t) \\ &\quad \cdot \cos[2\pi(f_i - f_n - \Delta f)t + \theta_i(t) - \hat{\theta}_n(t)]dt\Bigg) + \eta \\ &= \eta + S + I_{MS} + I_{SO} + I_{MO}\end{aligned} \tag{2}$$

where Δf is frequency offset assumed constant, and $\hat{\theta}_n(t)$ represents a phase estimate for the nth subcarrier. The combining weight for the zeroth user's nth subcarrier is denoted by $w_{0,n}$ which is determined by a combining scheme. η denotes a noise term and S denotes a desired signal term. The interference terms I_{MS}, I_{SO}, and I_{MO} represent, respectively, multiuser interference from the same carriers, self-interference from the other carriers, and multiuser interference from the other carriers.

For EGC, $w_{0,n}$ is set to $c_0[n]$. In (2), is a Gaussian random variable (RV) with mean zero and variance $\sigma_\eta^2 = NN_0/T_b$. The following desired signal term S is obtained from (2) by

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setting $m = 0$ and $i = n$:

$$S = a_0[k]\frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\sum_{n=0}^{N-1}\rho_n\cos\left[-2\pi\left(k+\frac{1}{2}\right)N\epsilon+\theta_n-\hat{\theta}_n\right] = a_0[k]\frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\rho_{\text{Tot}} \quad (3)$$

where $\rho_{\text{Tot}} = \Sigma_{n=0}^{N-1}\ \rho_n$, and $\epsilon = \Delta f/BW = \Delta f T_b/N$ is frequency offset normalized by the total bandwidth. We assume the maximum Doppler frequency is much smaller than the bit rate such that the channel impulse response may be constant during one bit integration interval in (2). With this assumption, the magnitude and phase of the channel impulse response for the nth subcarrier are, respectively, represented by RV's ρ_n and θ_n in (3). The phase estimate $\hat{\theta}_n$ is assumed to be $\hat{\theta}_n = [-2\pi(k+\frac{1}{2})N\epsilon+\theta_n] \bmod (2\pi)$. This means that the subcarrier phase estimator determines the modulo (2π) value of the phase rotation due to frequency offset and fading at time $(k+\frac{1}{2})T_b$ which is the middle of the kth bit integration interval. Note that the value of S decreases as ϵ increases.

Three types of interference I_{MS}, I_{SO}, and I_{MO} may be assumed Gaussian RV's as follows:

A. Multiuser Interference from the Same Carriers: I_{MS}

I_{MS} is obtained from (2) with the condition $m \neq 0$ and $i = n$:

$$I_{MS} = \frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\sum_{m=1}^{M-1}\left(a_m[k]\sum_{n=0}^{N-1}(c_m[n]c_0[n]\rho_n)\right). \quad (4)$$

If we define $q_m[n] = c_m[n]c_0[n]$, then $\Sigma_{n=0}^{N-1}\ q_m[n] = 0$ for $m \neq 0$ because of the orthogonality of the spreading codes. This implies that a half of $q_m[n]$'s for $n = 0, 1, \cdots, N-1$ are $+1$'s and the other half are -1's. Let the index that yields $q_m[n] = +1$ be $\alpha_{m,j}$ and that yields $q_m[n] = -1$ be $\beta_{m,j}$ for $j = 0, 1, \cdots, N/2-1$. Using this notation, (4) may be reexpressed as

$$I_{MS} = \frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\sum_{m=1}^{M-1}\left(a_m[k]\sum_{j=0}^{N/2-1}(\rho_{\alpha_{m,j}}-\rho_{\beta_{m,j}})\right). \quad (5)$$

By the central limit theorem, the probability density function (pdf) of RV's $\Sigma_{j=0}^{N/2-1}\ (\rho_{\alpha_{m,j}}-\rho_{\beta_{m,j}})$ is approximated to be zero-mean Gaussian for large N. Since $a_m[k] \in \{+1, -1\}, I_{MS}$ may be viewed as a linear sum of M zero-mean Gaussian RV's. Hence it may be assumed to be a Gaussian RV with mean zero. After some calculation, the variance of I_{MS} is found to be

$$\sigma_{I_{MS}}^2 = (M-1)\left(\frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\right)^2\left(2-\frac{\pi}{2}\right)P_{\text{tot}}. \quad (6)$$

B. Self Interference from the Other Carriers: I_{SO}

I_{SO} with $m = 0$ and $i \neq n$ in (2) is given by

$$I_{SO} = a_0[k]\sum_{n=0}^{N-1}\sum_{i=0,i\neq n}^{N-1}\left(c_0[n]c_0[i]\frac{\sin(\pi N\epsilon)}{\pi(n-i+N\epsilon)} \cdot\rho_i\ \cos(\tilde{\theta}_{i,n})\right) \quad (7)$$

where $\tilde{\theta}_{i,n} = (\theta_i-\theta_n) \bmod (2\pi)$. Since θ_i and θ_n for $i \neq n$ are independent RV's that are uniformly distributed on $[0, 2\pi]$, $\tilde{\theta}_{i,n}$ is also an uniform RV on $[0, 2\pi]$. $\rho_i\ \cos(\tilde{\theta}_{i,n})$ is, therefore, a zero-mean Gaussian RV because the in-phase component of a Rayleigh RV is a zero-mean Gaussian RV. Hence I_{SO} is an weighted sum of $N(N-1)$ zero-mean Gaussian RV's, and may be assumed to be a zero-mean Gaussian RV with variance

$$\sigma_{I_{SO}}^2 = \frac{1}{N}P_{\text{Tot}}\sum_{n=0}^{N-1}\sum_{i=0,i\neq n}^{N-1}\left(\frac{\sin(\pi N\epsilon)}{\pi(n-i+N\epsilon)}\right)^2. \quad (8)$$

C. Multiuser Interference from the Other Carriers: I_{MO}

I_{MO} with $m \neq 0$ and $i \neq n$ in (2) is given by

$$I_{MO} = \sum_{n=0}^{N-1}\sum_{i=0,i\neq n}^{N-1}\left(B_i\rho_i\ \cos(\tilde{\theta}_{i,n})c_0[n]\frac{\sin(\pi N\epsilon)}{\pi(n-i+N\epsilon)}\right) \quad (9)$$

where $B_i = \Sigma_{m=1}^{M-1}\ a_m[k]c_m[i]$ is a binomial RV, and $\rho_i\ \cos(\tilde{\theta}_{i,n})$ is a Gaussian RV with mean zero. I_{MO} may be viewed as a sum of $N(N-1)$ independent RV's; each RV is a product of a binomial RV and a Gaussian RV. I_{MO} may be assumed to be a zero-mean Gaussian RV by the central limit theorem [4] with variance

$$\sigma_{I_{MO}}^2 = \frac{M-1}{N}P_{\text{Tot}}\sum_{n=0}^{N-1}\sum_{i=0,i\neq n}^{N-1}\left(\frac{\sin(\pi N\epsilon)}{\pi(n-i+N\epsilon)}\right)^2. \quad (10)$$

IV. Bit Error Rate Analysis for EGC

Since I_{MS}, I_{SO}, and I_{MO} are mutually uncorrelated, the variance of the total interference is given as

$$\sigma_{I_{\text{Tot}}}^2 = \sigma_{I_{MS}}^2+\sigma_{I_{SO}}^2+\sigma_{I_{MO}}^2. \quad (11)$$

From (3), (6), (8), (10), and (11), and assuming that a "+1" is transmitted, the conditional signal-to-interference-noise ratio given ρ_{Tot} for EGC may be defined as

$$\gamma = \frac{1}{2}\cdot\frac{S^2}{\sigma_{I_{\text{Tot}}}^2+\sigma_\eta^2} = \frac{1}{2}\left(\frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\right)^2\frac{\rho_{\text{Tot}}^2}{\sigma_{I_{\text{Tot}}}^2+\sigma_\eta^2}. \quad (12)$$

The average BER is given by

$$P_{b,\text{EGC}}(e) = \int_{-\infty}^{\infty}\frac{1}{2}\text{erfc}(\sqrt{\gamma})f(\rho_{\text{Tot}})\,d\rho_{\text{Tot}} \quad (13)$$

where $\text{erfc}(\cdot)$ is the complementary error function and $f(\rho_{\text{Tot}})$ is the pdf of ρ_{Tot}. For large N, the pdf of ρ_{Tot} may be assumed Gaussian by the central limit theorem because ρ_{Tot} is a sum of independent and identically distributed Rayleigh RV's. Its mean and variance are, respectively, $\mu_{\rho_{\text{Tot}}} = \sqrt{(\pi/2)NP_{\text{Tot}}}$ and $\sigma_{\rho_{\text{Tot}}}^2 = (2-(\pi/2))P_{\text{tot}}$ [5]. Using this assumption, the average BER (13) is simplified as

$$P_{b,\text{EGC}}(e) \approx \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{N_{\text{EGC}}}{D_{\text{EGC}}}}\right) \quad (14)$$

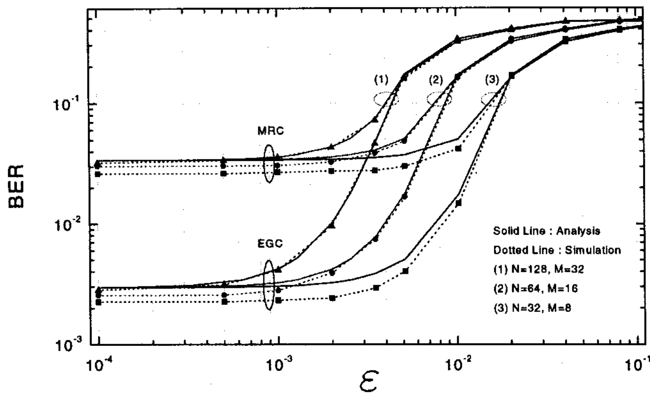

Fig. 1. BER versus frequency offset normalized by total bandwidth, $\varepsilon(E_b/N_0 = 10$ dB, $M/N = 0.25)$.

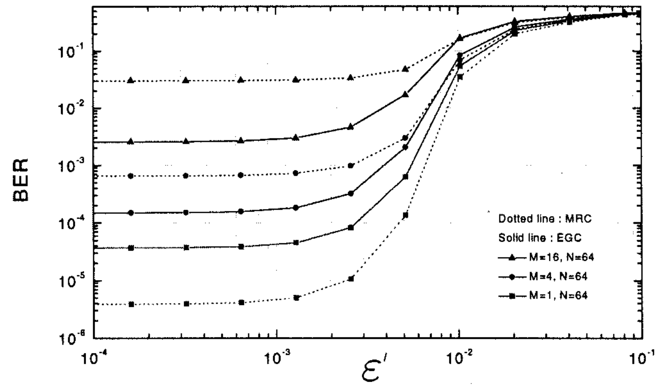

Fig. 2. BER versus frequency offset normalized by bit rate, $\varepsilon'(E_b/N_0 = 10$ dB).

where $N_{\rm EGC} = (\pi/4)(\sin(\pi N\epsilon)/\pi N\epsilon)^2$ and

$$D_{\rm EGC} = \frac{M}{N}\left(\frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\right)^2\left(2-\frac{\pi}{2}\right) + \frac{M}{N^2}\sum_{n=0}^{N-1}\sum_{i=0,i\neq n}^{N-1}\left(\frac{\sin(\pi N\epsilon)}{\pi(n-i+N\epsilon)}\right)^2 + \frac{N_0}{E_b}.$$

V. Bit Error Rate Analysis for MRC

For MRC, $w_{0,n}$ is set to $c_0[n]\rho_n \cdot I_{MS}, I_{SO}$, and I_{MO} for MRC may be calculated from (2), and then approximated as Gaussian RV's in the similar manner as the terms for EGC. Using the Gaussian approximation, the average BER for MRC is obtained as

$$P_{b,\rm MRC}(e) \approx \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{N_{\rm MRC}}{D_{\rm MRC}}}\right) \tag{15}$$

where $N_{\rm MRC} = (\sin(\pi N\epsilon)/\pi N\epsilon)^2$ and

$$D_{\rm MRC} = \frac{2M}{N}\left(\frac{\sin(\pi N\epsilon)}{\pi N\epsilon}\right)^2 + \frac{M}{N^2}\sum_{n=0}^{N-1}\sum_{i=0,i\neq n}^{N-1}\left(\frac{\sin(\pi N\epsilon)}{\pi(n-i+N\epsilon)}\right)^2 + \frac{N_0}{E_b}.$$

VI. Numerical Results

The relationship between the BER and the frequency offset normalized by the total bandwidth, $\epsilon = \Delta f/BW = \Delta f T_b/N$, is calculated using (14) and (15), and is shown in Fig. 1. Note that the ratio M/N is kept constant for fair comparison in investigating how the frequency offset effects vary with N. The BER performance degradation is found to increase with N for a given ϵ, regardless of a diversity combining technique. This means that when the bit rate varies as the PG varies for a given total bandwidth, a system with a large PG is more sensitive to frequency offset than that with a small PG. The reason is that the ratio of the frequency offset to the subcarrier spacing increases with N, when ϵ is fixed. The analysis results agree well with the simulation results when $N = 64$, 128. However, some difference is observed for $N = 32$. It is due to the less accurate Gaussian approximation for $N = 32$ than that for $N = 64$, 128.

Fig. 2 shows the relationship between the BER and the frequency offset normalized by the bit rate, $\epsilon' = \Delta f T_b = N\epsilon$. In contrast to the BER shown in Fig. 1, the BER is found to be unchanged with N for a given M/N and ϵ'. This means that the BER degradation due to frequency offset does not vary with N when the bit rate is fixed. The reason is that the ratio of the frequency offset to the subcarrier spacing does not vary with N, when ϵ' is fixed. As expected, the BER increases with M/N for a given ϵ'. This is because the multiuser interference increases as M/N increases.

In Figs. 1 and 2, the EGC is found to outperform the MRC in the presence of frequency offset. This means that the orthogonality loss for the EGC, that is caused by fading and frequency offset, is less serious than that for the MRC. Note that the BER equations for the EGC and MRC in [2] may be derived from (14) and (15) by setting frequency offset to zero. The BER equations in this letter are more general than those in [2].

VII. Conclusions

In this letter, the effects of frequency offset on the performance of an MC/CDMA system for a downlink channel are theoretically analyzed and verified by computer simulations. The BER performance is found to degrade significantly as the number of subcarriers increases, when the frequency offset normalized by the total bandwidth is fixed. On the other hand, it is found that the BER degradation due to frequency offset does not vary with the number of subcarriers, when the frequency offset normalized by the bit rate is fixed. In addition, the EGC is found to outperform the MRC.

References

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