Performance Analysis of Low Processing Gain DS/CDMA Systems with Random Spreading Sequences

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Abstract—**The performance analysis of a RAKE receiver for low processing gain direct sequence code-division multiple access (DS/CDMA) systems is presented in this letter. The multipath interference effects, that have typically been assumed negligible or approximated as Gaussian noise, become significant for low processing gain systems. In this letter, a scheme to accurately evaluate the multipath interference effects is proposed.**

Index Terms—**DS/CDMA system, performance, RAKE receiver.**

I. Introduction

NEXT GENERATION mobile communication systems such as IMT-2000 are being developed around the world [1], [2]. A wide-band direct-sequence code-division multiple access (DS/CDMA) system, where the RAKE receiver is employed to achieve diversity gain by combining multipath signal components, is a strong candidate for next generation systems. Some of these systems are planned to support various data rate services including high data rate service by varying a processing gain (PG) factor [1], [2]. As the PG becomes small for a high data rate service, the multipath interference (MPI) effects become significant and should accurately be included in performance analysis. In the analysis of bit-error rate (BER) performance, the effects of MPI have typically been neglected [3] or approximated as Gaussian noise [4], [5]. Recently, Noneaker and Pursley [6] included MPI effects in evaluating the effects of spreading sequence on DS spread spectrum systems using a characteristic function approach. However, they did not investigate in what manner the performance changes due to MPI effects as the processing gain varies.

In this letter, a scheme to evaluate the MPI effects accurately is proposed, and by using this scheme the performance of low processing gain DS/CDMA systems with random spreading sequences is investigated. Furthermore, these investigation results are compared with performance analysis results with the Gaussian approximation of the MPI effects.

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II. System and Channel Models

In our analysis, the transmitted signal under consideration is a binary phase-shift keying (BPSK) data signal with BPSK spreading. The received complex baseband equivalent signal may be expressed as

$$r(t) = \sqrt{2P} \sum_{l=1}^{L} \alpha_l e^{j\theta_l} \cdot b(t-\tau_l) \cdot a(t-\tau_l) + \eta(t) \qquad (1)$$

where P is the average transmitted power, α_l's are the slowly varying independent Rayleigh-distributed channel amplitude responses, θ_l's are the independent channel phase responses which are uniformly distributed on $[0, 2\pi)$, and τ_l's are the relative delays of the L separable multipath components. $b(t)$ is the unit amplitude rectangular pulse waveform representing a random binary data sequence. The spreading waveform may be expressed as $a(t) = \sum_{i=-\infty}^{\infty} a_i \psi(t - iT_c)$, where a_i is the nonperiodic random binary sequence that has unit amplitude, and T_c is the chip duration. The chip waveform $\psi(t)$ has duration T_c and normalized energy T_c. $\eta(t)$ is a zero-mean complex Gaussian random process representing additive white Gaussian noise (AWGN) with two-sided power spectral density η_0.

The receiver is assumed to have perfect knowledge of the amplitude, phase and delay of each path signal that is selected for combining. The RAKE receiver output after diversity combining, V, may be expressed as $V = \mathrm{Re}\{\sum_{m=1}^{M}(w_m \cdot \int_{\tau_m}^{\tau_m+T_b} r(t) \cdot a(t-\tau_m)\, dt)\}$, where M is the number of fingers, w_m is the combining weight, and T_b is the data bit duration.

III. Performance Analysis

A. Maximal Ratio Combining (MRC)

The combining weight for maximal ratio combining (MRC) may be expressed as $w_m = \sqrt{2P}\alpha_m e^{-j\theta_m}$ [5]. The output of the RAKE receiver with MRC, V, may be expressed as $V = V_s + V_{mp}^c + V_{mp}^u + V_{mp}^{na} + V_\eta$. The desired signal component, V_s, may be expressed as $V_s = b_0 \cdot 2NE_c \sum_{m=1}^{M} \alpha_m^2$, where b_0 is the desired bit to be determined, $N = T_b/T_c$ is the processing gain, and $E_c = PT_c$ is the transmitted energy per chip. The term V_{mp}^c denotes correlated multipath interference between the paths assigned to fingers [4]. It may be expressed as $V_{mp}^c = 4P \sum_{m=1}^{M-1} \sum_{n=m+1}^{M} Y_{m,n}^c \alpha_m \alpha_n \cos(\theta_n - \theta_m)$, where $Y^c_{m,n}(=\int_{\text{overlapping}} b(t-\tau_n)a(t-\tau_n)\cdot a(t-\tau_m)\,dt)$ is determined by the data sequence, the spreading sequence, and τ_l's. Since the data sequence and spreading sequence are assumed to be random binary sequences, the term $Y^c_{m,n}$, given τ_l's, may be expressed as

$$Y^c_{m,n} = C_{m,n}R_\psi(s_{m,n}) + C'_{m,n}R_\psi(T_c - s_{m,n}) \qquad (2)$$

where $s_{m,n} = \tau_{m,n} - \gamma_{m,n}\cdot T_c$, $\tau_{m,n} = |\tau_m - \tau_n|$ and $\gamma_{m,n} = \lfloor \tau_{m,n}/T_c \rfloor$. The continuous time partial autocorrelation function of the spreading waveform may be expressed as $R_\psi(s) = \int_s^{T_c} \psi(t)\psi(t-s)\,dt$, $0 \le s < T_c$. The random variables, $C_{m,n}$ and $C'_{m,n}$, are related to discrete autocorrelation of spreading sequence and may be modeled as binomial random variables, i.e., $C_{m,n} \sim B(N - \gamma^{\min}_{m,n})$ and $C'_{m,n} \sim B(N - 1 - \gamma^{\min}_{m,n})$, where $\gamma^{\min}_{m,n} = \min(N, \gamma_{m,n})$ and $B(z)$ denotes the binomial distribution with probability density function: $f(x) = \sum_{i=0}^{z} \binom{z}{i} 2^{-z}\delta(x+z-2i)u(z)$. Note that the approach used in expressing the term $Y^c_{m,n}$ is similar to the one used in evaluating the effects of multiple access interference in [7] and [8].

The uncorrelated multipath interference between the paths assigned to fingers [4] is denoted by V^u_{mp}, and may be expressed as

$$V^u_{mp} = 2P \sum_{m=1}^{M} \sum_{\substack{n=1\\ n\neq m}}^{M} Y^u_{m,n}\alpha_m\alpha_n \cos(\theta_n - \theta_m).$$

Like $Y^c_{m,n}$, the term $Y^u_{m,n}(=\int_{\text{nonoverlapping}} b(t-\tau_n)a(t-\tau_n)\cdot a(t-\tau_m)\,dt)$, given τ_l's, may be expressed as

$$Y^u_{m,n} = U_{m,n}R_\psi(s_{m,n}) + U'_{m,n}R_\psi(T_c - s_{m,n}) \qquad (3)$$

where $U_{m,n}$ and $U'_{m,n}$ may be modeled as binomial random variables, i.e., $U_{m,n} \sim B(\gamma^{\min}_{m,n})$ and $U'_{m,n} \sim B(1 + \gamma^{\min}_{m,n})$. The term V^{na}_{mp} denotes multipath interference by the paths not assigned to fingers [4], and may be expressed as $V^{na}_{mp} = 2P\sum_{m=1}^{M}\sum_{n=M+1}^{L} Y^{na}_{m,n}\alpha_m\alpha_n\cos(\theta_n - \theta_m)$. Similar to $Y^c_{m,n}$, the term $Y^{na}_{m,n}(=\int_{\tau_m}^{\tau_m+T_b} b(t-\tau_n)a(t-\tau_n)\cdot a(t-\tau_m)\,dt)$, given τ_l's, may be expressed as

$$Y^{na}_{m,n} = X_{m,n}R_\psi(s_{m,n}) + X'_{m,n}R_\psi(T_c - s_{m,n}) \qquad (4)$$

where $X_{m,n}$ and $X'_{m,n}$ may be modeled as binomial random variables, i.e., $X_{m,n} \sim B(N)$ and $X'_{m,n} \sim B(N)$. Note that since the spreading sequence is assumed as a nonperiodic random binary sequence, the random variables $C_{m,n}$, $C'_{m,n}$, $U_{m,n}$, $U'_{m,n}$, $X_{m,n}$ and $X'_{m,n}$, are mutually independent. The term V_η, which is due to AWGN, is a zero-mean Gaussian random variable.

The decision statistic, V, may be considered as a Gaussian random variable with mean $S = V_s + V^c_{mp} + V^u_{mp} + V^{na}_{mp}$ and variance $2\eta_0 NE_c\sum_{m=1}^{M}\alpha_m^2$, when $\underline{\alpha} = \{\alpha_l\}$, $\underline{\theta} = \{\theta_l\}$ and $\underline{A} = \{\{C_{m,n}\}, \{C'_{m,n}\}, \{U_{m,n}\}, \{U'_{m,n}\}, \{X_{m,n}\}, \{X'_{m,n}\}, \{s_{m,n}\}\}$ are given. Since b_0 is equally 1 or -1 in probability, the bit error probability for $b_0 = 1$ is only considered. The conditional bit error probability for MRC, given $\underline{\alpha}$, $\underline{\theta}$ and $\underline{A}$, is

$$P_b^{\text{MRC}}(\underline{\alpha}, \underline{\theta}, \underline{A}) = \frac{1}{2}\,\text{erfc}\left(\frac{S}{\sqrt{4\eta_0 NE_c\sum_{m=1}^{M}\alpha_m^2}}\right). \qquad (5)$$

The average bit error probability, $\overline{P}_b$, is calculated from the conditional error probability by averaging over $\underline{\alpha}$, $\underline{\theta}$ and $\underline{A}$, and may be expressed as

$$\overline{P}_b = \int\cdots\int P_b^{\text{MRC}}(\underline{\alpha}, \underline{\theta}, \underline{A})\cdot f(\underline{\alpha})\cdot f(\underline{\theta})\cdot f(\underline{A})\,d\underline{\alpha}\,d\underline{\theta}\,d\underline{A}. \qquad (6)$$

Note that the multiple access interference (MAI) may be considered as additional multipath components that are not assigned to fingers. Hence, the scheme used in evaluating the MPI effects in this letter, may be applied to the MAI effects. For a large number of users, MAI may be modeled as Gaussian noise [5].

B. Equal Gain Combining (EGC)

The RAKE receiver with equal gain combining (EGC) is similar to the receiver with MRC, except for the combining weight which is modified to $w_m = \sqrt{2P}e^{-j\theta_m}$. The terms, V_s, V^c_{mp}, V^u_{mp} and V^{na}_{mp}, are changed accordingly, and the conditional variance of V_η is modified to $2\eta_0 NE_c M$. Hence, the conditional bit error probability for RAKE receiver with EGC, given $\underline{\alpha}$, $\underline{\theta}$ and $\underline{A}$, becomes

$$P_b^{\text{EGC}}(\underline{\alpha}, \underline{\theta}, \underline{A}) = \frac{1}{2}\,\text{erfc}\left(\frac{S}{\sqrt{4\eta_0 NE_c M}}\right). \qquad (7)$$

The average bit error probability is calculated by averaging the conditional bit error probability, as in the MRC case.

IV. Numerical Results

In this section, we present performance analysis results for low processing gain DS/CDMA systems based on (5)–(7), and compare these results with performance analysis results with the Gaussian approximation of MPI effects [4, eqs. 45, 50]. Averaging operations required for performing (6) and analysis with Gaussian approximation are carried out by Monte Carlo technique [9]. We assume that the chip waveform is a rectangular pulse. We also assume that the average power of each path is equal. The average bit error probabilities are plotted against the total average signal-to-noise ratio (SNR).

Fig. 1 depicts the average bit error probabilities of a RAKE receiver with MRC for two path delay cases, (i) and (ii). In case (i), the differences between successive path delays, $\tau_{l+1,l}$'s, are independent and uniformly distributed on $[T_c, 2T_c)$. In case (ii), $\tau_{l+1,l}$'s are independent and uniformly distributed on $[T_b, T_b + T_c)$. Note that the correlated part of the MPI, V^c_{mp}, is not zero in case (i), whereas V^c_{mp} is zero in case (ii). In this figure, $L = M = 2$ case is considered, and the processing gain (PG) is varied. The $PG = \infty$ case, where the MPI effects are absent, is also included for reference. Fig. 1 indicates that as the processing gain decreases, the MPI effects become significant and the Gaussian approximation becomes

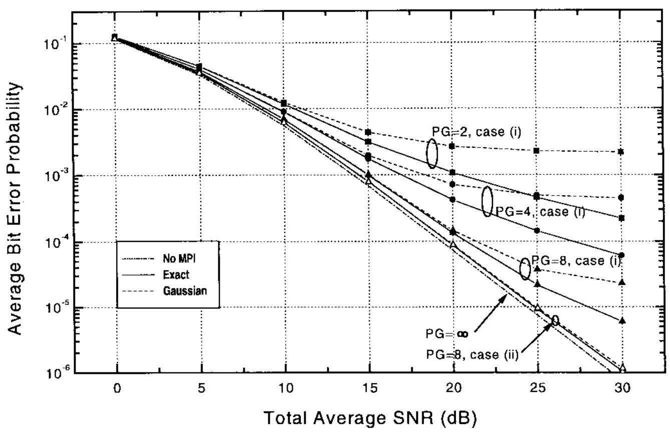

Fig. 1. $\overline{P}_b$ with $L = 2$, $M = 2$, MRC.

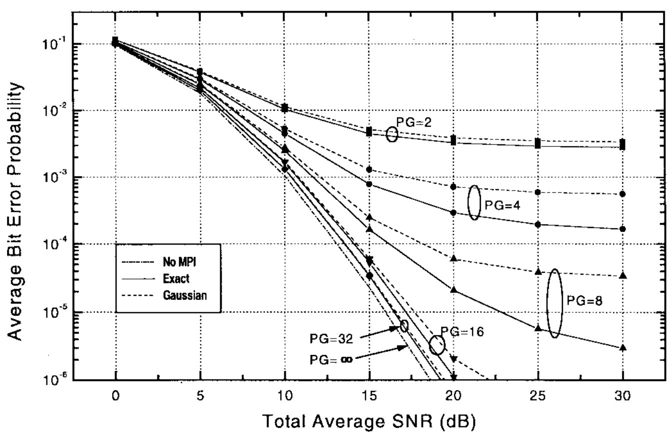

Fig. 2. $\overline{P}_b$ with $L = 4$, $M = 4$, MRC, case (i).

inaccurate. Due to the MPI effects, the SNR required for a BER of 10^{-3} increases from 14.1 dB for a $PG = \infty$ case to 16.8 dB for a $PG = 4$, case (i). The difference is 2.7 dB. Fig. 1 indicates that the MPI effects are more significant for the case (i) than for the case (ii). The reason is that there is no correlated MPI component in the case (ii). Performance results for $L = M = 4$ are given in Fig. 2. Like Fig. 1, this shows that as the processing gain decreases, the MPI effects become significant and the Gaussian approximation accuracy decreases. The SNR required for a BER of 10^{-3} increases from 10.0 dB for a $PG = \infty$ case to 14.1 dB for a $PG = 4$. The difference is 4.1 dB. Note that this difference is larger than that for an $L = M = 2$ case, since there is more MPI for an $L = M = 4$ case.

Fig. 3 depicts the average bit error probabilities of a RAKE receiver with EGC for $L = M = 2$. Like Fig. 1, this indicates that as the processing gain decreases, the MPI effects

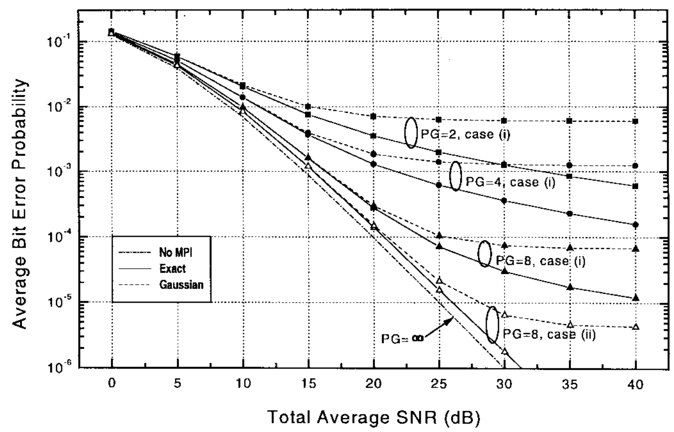

Fig. 3. $\overline{P}_b$ with $L = 2$, $M = 2$, EGC.

become significant and the Gaussian approximation accuracy decreases.

V. Conclusions

The performance of a RAKE receiver for low processing gain DS/CDMA systems has been evaluated with the aid of the proposed scheme to evaluate multipath interference effects. It was found that as the processing gain decreases, the multipath interference effects become significant, and the Gaussian approximation of multipath interference effects becomes inaccurate.

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