ABSTRACT: Head wave stacking and velocity analysis are used to image the shallow subsurface, while CMP stacking and velocity analysis are used to image deep structures of the earth. I relate these concepts to partial derivative seismograms, which gives stacking straight line of head waves. The stacking straight line can be described kinematically by partial derivative seismograms, resulting in an interesting seismic imaging relationship.

Key words: Head wave, Velocity analysis, Partial derivative seismogram, Head wave stacking, image.

INTRODUCTION

Since imaging subsurface has been an important tool to geophysicists, there have been extensive studies about imaging the subsurface. Some of the studies are based on the fact that kinematics of a scattered or a Born perturbation wavefield can be represented by the sum of travel time from a source to a receiver through a scatterer (Apostoiu-Marin and Ehinger, 1997; Bleistein et al., 1985; Gardner et al., 1974). Geophysicists have imaged the subsurface by summing seismic signals along a hyperbola representing the kinematics of the Born perturbation seismogram for any source and receiver configuration. Shin and Chung (1999) showed that the CMP stacking hyperbola, obtained in velocity analysis, can be obtained by a finite-difference or a finite-element function. The discretized form of the scalar wave equations is given as

\[ \frac{\partial^2}{\partial t^2} - \frac{1}{K} \left( \frac{\partial}{\partial x} \right)^2 = f(x, z, t) \]  

where \( x \) and \( z \) are the horizontal and vertical distances, \( t \) is the time, \( d(x, z, t) \) is the seismic data, \( \rho(x, z) \) is the density, \( k(x, z) \) is the bulk modulus and \( f(x, z, t) \) is the source function. The discretized form of the scalar wave equations can be obtained by a finite-difference or a finite-element approach as

\[ Md + Kd = f \]  

where \( M \) is the mass matrix, \( K \) is the stiffness matrix, \( f \) is the source vector, and \( d \) is the discretized seismic data vector.
At the finite-difference grid set shown in Fig. 1, if we identify a velocity and a density at a node \( i \) as \( v_i \) and \( \rho_i \), we can define our model parameter vector, \( \mathbf{p} \), to be simply \( \mathbf{p} = [\rho_1, v_1, \rho_2, v_2, \ldots, \rho_n, v_n]^T \). By taking the partial derivative of equation (2) with respect to density or velocity at the interface coordinate and then by rearranging the order of the derivatives to preserve the original discretized wave equation, we can obtain

\[
M \frac{\partial d}{\partial \mathbf{p}} + K \frac{\partial d}{\partial \mathbf{p}} = f^* \tag{3}
\]

where \( \frac{\partial d}{\partial \mathbf{p}} \) is the partial derivative seismogram and \( f^* \) is the virtual source vector defined as

\[
f^* = -\frac{\partial M}{\partial \mathbf{p}} \frac{\partial d}{\partial \mathbf{p}} = f^* \tag{4}
\]

To synthesize a partial derivative seismogram recorded at the surface for a source located at the top of the grid set in Fig. 1, we use equation (3). In order to solve equation (3), we need the virtual source \( f^* \) in addition to the forward wavefield at each grid point. The partial derivative seismograms at the surface nodes computed by the virtual sources make up a huge Jacobian matrix \( J \). A component of Jacobian matrix \( J_{ij} \) corresponds to the partial derivative wavefield due to the change of velocity or density at the \( j \)th node for the \( i \)th source gather. In Fig. 3, we display the virtual sources required to perturb the velocities or densities at the \( k \)th, \( l \)th, and \( m \)th nodal point in Fig. 2. The physical properties of the three layers in Fig. 2 are shown in Table 1. Figs. 4a, 4b, and 4c show the partial derivative seismograms computed by using the virtual sources in Fig. 3. We note that the first arrival events of the partial derivative seismogram at the shallow \( k \)th and \( l \)th nodal point resemble a straight line or can be approximated as a straight line (e.g., Figs. 4a and 4b); those at the deeper \( m \)th nodal point change into a hyperbola (see Fig. 4c). From these Figures, we demonstrate that the first arrival events of the partial derivative seismogram generated by the virtual sources located in the shallow subsurface can be represented as a straight line.

**Tables 1.** The physical properties of the three layers in Fig. 2

<table>
<thead>
<tr>
<th>Layer</th>
<th>Velocity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000 m/s</td>
<td>1.0 g/</td>
</tr>
<tr>
<td>2</td>
<td>2000 m/s</td>
<td>2.0 g/</td>
</tr>
<tr>
<td>3</td>
<td>3000 m/s</td>
<td>3.0 g/</td>
</tr>
</tbody>
</table>

**MATHEMATICS OF STACKING SEISMOGRAM**

How much the field seismogram is responsive to a certain interface segment can be estimated by taking a zero-lag crosscorrelation between the field seismogram and the partial derivative seismogram with respect to material parameter such as velocity or density at the interface (Shin and Chung, 1999). By measuring and displaying the zero-lag crosscorrelation at each nodal point, one can image the subsurface structure (Shin and Chung, 1999). This technique has often been used in prestack Kirchhoff
Headwave Stacking in Terms of Partial Derivative Wavefield

The zero-lag crosscorrelation of field seismogram $d$ and the partial derivative seismogram $J$ is expressed by

$$ r = J^T d $$  \hspace{1cm} (5)

where the Jacobian matrix $J$, approximated kinematically by the asymptotic ray theory in the prestack Kirchhoff depth migration, could be calculated via equation (3). We recognize the vector $r$ as an unscaled image or stacked value of the measured seismic data. Since the partial derivative seismogram in Fig. 4c have different amplitudes and different phases along the hyperbola (Shin and Chung, 1999), we can identify the hyperbola of the partial derivative seismogram in equation (5) as a weighted

Fig. 3. Virtual sources calculated by a finite-difference method when velocities or densities are perturbed at the (a) $\theta$, (b) $\theta$ and (c) $\theta$ grid point shown in Fig. 2.

Fig. 4. Partial derivative seismograms computed by finite-difference method for the virtual source at the (a) $\theta$, (b) $\theta$ and (c) $\theta$ nodal point shown in Fig. 3.
Kirchhoff hyperbola. Equation (5) can be written in an integral form as

$$r_i = \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{0}^{t_{\text{max}}} \frac{\partial u(x,t)}{\partial p_i} d(x,t) \, dx \, dt$$

(6)

where $r_i$ is the $i$th element of the unscaled image vector $r$, $(\partial u(x,t)/\partial p_i)$ is the partial derivative seismogram with respect to the velocity or the density of the $i$th nodal point in the subsurface, $x$ is the horizontal offset, $t$ is the time and $d(x,t)$ is the field seismogram.

In the usual prestack Kirchhoff migration, the kinematics of the partial derivative seismogram is only considered. If we consider a summing wavefield along the hyperbola representing the kinematics of the partial derivative seismogram shown in Fig. 4c, stacking seismograms can be represented as

$$r_i = \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{0}^{t_{\text{max}}} d(x,t) \delta(t - t_0 - x/v) \, dx \, dt$$

(7)

where $t_0$ is the minimum travel time from a source to a receiver through a scatterer, $v$ is the arbitrary velocity which is yet to be determined and $\delta$ is the delta function. In the prestack Kirchhoff migration, we estimate the kinematics of the partial derivative seismogram using a ray tracing technique.

We already showed that the first arrival events of the partial derivative seismogram shown in Fig. 4a and 4b can be approximated as a straight line. An integral expression for stacking the headwaves along the straight line (corresponding to the kinematics of the partial derivative seismogram) is given as

$$r_i = \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{0}^{t_{\text{max}}} d(x,t) \delta(t - t_0 - x/v) \, dx \, dt$$

(8)

where $r_i$ is the $i$th element of the unscaled image vector $r$, $d(x,t)$ is the field seismogram, $\delta$ is the delta function, $x$ is the horizontal offset, $t$ is the time, $t_0$ is the intercept time, and $v$ is the stacking velocity (for refracted waves) where we have the maximum velocity spectrum. The $\delta$ function expresses the kinematics of the partial derivative seismogram with respect to the velocity or the density at the $i$th nodal point in the shallow subsurface. In order to obtain the stacking velocity for the refracted waves, Landa et al. (1995) performed a velocity analysis as in the conventional reflection data processing. The velocity analysis applied by Landa et al. (1995) is different from that of conventional reflection data processing in measuring the semblance not along the trial hyperbolas but along the trial straight lines.

**NUMERICAL EXAMPLES**

We stacked field seimograms along straight lines which approximate partial derivative seismogram kinematically for a horizontal layer and a dipping layer model. We also carried out velocity analysis along a number of straight lines for obtaining stacking velocity as Landa et al. (1995) did.

Fig. 5 shows the horizontal model composed of two horizontal layers whose velocities are 1000 and 3000 m/s. The velocity spectrum calculated for selecting the optimal stacking velocity is displayed in Fig. 6. Since the velocity spectrum is obtained by computing coherence along a number of straight lines for each time window, it is natural that the velocity spectrum has larger coherence when $v=1000$ m/s at $t=0$ (for the direct wave) and when $v=1000$ m/s at the intercept time, as shown in Fig. 6. The subsurface image results from stacking the synthetic seismogram with the optimal stacking velocities. Fig. 7 shows the subsurface image for the horizontal model.

As a second numerical example, we take the dipping layer model depicted in Fig. 8. Figs. 9 and 10 show the velocity spectrum and the stacked image obtained for the dipping layer model. In Fig. 9, we can see that larger
coherence appear when \( v = 1500 \text{ m/s} \) at \( t = 0 \) and when \( v = 3800 \text{ m/s} \) at the intercept time. These velocities correspond to the first and second layer velocities. The stacked image in Fig. 10 is compatible with the dipping layer model in Fig. 8.

**CONCLUSION**

By analyzing the kinematics of the partial derivative seismograms, we showed that the straight line used by Landa *et al.* (1995) for the headwave-stacking technique is a substitution for the stacking hyperbola of the reflection seismogram in the prestack Kirchhoff migration. Our analysis indicates that the straight line used in stacking the refracted waves is the kinematics of partial derivative seismogram with respect to the velocity or the density at a depth point located in the shallow subsurface. As shown by Landa *et al.* (1995), the headwave-stacking method can be used to stack or migrate the shallow subsurface seismic data.
REFERENCES


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