Profit Taxation,  
Monopolistic Competition and  
International Relocation of Firms

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This paper presents a two-country monopolistic competition trade model to analyze how the profit taxation determines the location of firms and national welfare. Profit tax cuts may increase or decrease the number of firms in a country, depending on the elasticities of substitution between domestic and foreign goods and between goods produced in the same country. Accordingly, profit tax cuts may increase or decrease domestic consumption and welfare, depending on these elasticities. The paper provides parameter conditions under which a decrease in the domestic profit tax attracts foreign firms and increases domestic welfare.

Keywords: Profit tax, Location, Monopolistic competition, Two country model

JEL Classification: F2, H2

I. Introduction

In the last two decades, profit tax competition among OECD countries has increased. This is because under greater international firm mobility, lowering profit tax attracts foreign firms, creates new businesses, and thereby increases national income. Accordingly, OECD countries have increasingly lowered profit taxes to attract foreign firms (see, e.g., Haufler 1999; Fuest and Huber 2002). The purpose of this paper is to
investigate the effects on welfare of a reduction in the profit tax in a world in which production is globalized so that firms can relocate easily, using a two-country monopolistic competition trade model.

The relationship between profit taxation and firm location (or foreign direct investment) has been studied extensively at a game theoretic level (Janeba 1995; Konan 1997; Haufler and Wooton 1999; Haufler and Schjelderup 2000; Kind, Midelfart, and Schjelderup 2000, 2005; Fuest and Huber 2002; Huizinga and Nielsen 2002). Of particular interest is the issue whether or not each country will levy positive profit taxes from the viewpoint of household welfare. For example, Janeba (1995) shows how the location of foreign direct investment and national incomes are influenced by noncooperative profit taxation policies under capital mobility using a game theoretic tax competition model where two governments compete by strategically setting profit tax rates to attract new foreign capital. Janeba (1995) found that the equilibrium national income pair is independent of profit taxation policies. As a result, optimal tax rates under the exemption and the credit method are both zero, while under the deduction method the positive profit tax rate does not impact upon the capital location. As mentioned above, over the past few decades, in the game theoretic tax competition literature, numerous attempts have been made by researchers to show that tax competition leads to a 'race to the bottom.'

In contrast, Baldwin and Krugman (2004) extend the tax competition analysis to economic geography models and consider how agglomeration externalities which induce mobile firms to prefer to stay together create a 'race to the top' in capital taxes. In their core-periphery model with increasing returns to scale and iceberg trade costs, they show that greater integration may lead to a 'race to the top' in taxes in the presence of agglomeration in the core region. This is because in the presence of agglomeration, mobile firms can earn more agglomeration rent in the core region and therefore the government in the core region can attract mobile firms to the domestic country, even if the profit tax is set at a high level. In addition, using an economic geography model, Borck and Pflüger (2006) show the 'race to the top' in capital taxes

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1 See Wilson (1999) for a comprehensive survey of the tax competition literature.

2 See Krugman (1991) and Krugman and Venables (1995), who show how trade integration (falling transport costs) may produce a concentration of firms in a monopolistic competition model with increasing returns to scale and transport costs.
generalizes to a framework with partial agglomeration. Thus, in contrast to the results in the tax competition literature, the above studies in the new economic geography reveal how the significance of agglomeration leads to a ‘race to the top.’

Although a large number of studies have been made on the effects of capital mobility on tax competition based on a noncooperative game theoretic approach, little is known about the effects of changing an exogenous profit tax rate on the welfare of each country under free movement of firms based on a two-country monopolistic competition trade model. The exception is Johdo and Hashimoto (2005), who investigate the welfare effects of a profit tax in a world in which firms can relocate easily across countries and the terms of trade effect is considered. Johdo and Hashimoto (2005) showed that the welfare impacts of a profit tax increase can be positive or negative, depending on the relative share of ownership of firms between the two countries. However, the following question remains unresolved: how does the relationship between profit taxation and firms’ choice of location change when we take into account two types of elasticity of substitution: that is, ‘the elasticity of substitution between home and foreign goods’ and ‘the elasticity of substitution between goods produced in the same country’? Further, how do changes in the profit tax in one country affect another’s welfare when we take into account the two types of elasticity of substitution? We emphasize that none of the existing literature focuses on the profit tax, firms’ location and the two types of elasticity of substitution, nor how the interactions between these affect welfare at home and abroad. In order to address these issues, we propose a two-country monopolistic trade model, and examine in detail the relationship between profit taxes, firms’ location and the two types of elasticity of substitution. The results indicate that the linkage between

3 Tille (2001) defined the former as ‘the cross-country substitutability’ and the latter as ‘the within-country substitutability.’

4 One difference between the profit tax competition literature and the current analysis is whether the profit tax rate is optimized or fixed. This paper attempts to examine the welfare effect of a reduction in a fixed profit tax rate considering both firms’ location and the two types of elasticity of substitution.

5 A number of other factors affecting firms’ location choices other than profit taxation have also been examined in the literature. These include: commodity taxes (Haufler and Pflüger 2004); emission taxes (Pflüger 2001); firm specific fixed (sunk) costs (Hosoe and Sugeta 1995); trade liberalization in intermediate inputs (Wang 1994); public infrastructure (Martin and Rogers 1995); and wage taxes (Pflüger 2004). See Ricci (1999) for an extensive survey of location theories.
firms’ location and the two types of elasticity of substitution can play an important role in determining how profit taxes affect welfare in each country.\textsuperscript{6}

Two interesting results arise from this analysis: 1) when the elasticity of substitution between home and foreign goods is relatively small, a decrease in domestic profit tax rate can decrease domestic welfare and raise foreign welfare; and 2) when the elasticity of substitution between ‘home and foreign goods’ and between ‘goods produced in the same country’ are both large, and a large proportion of the firm profits accrue to domestic residents, then a decrease in the domestic profit tax rate will effectively increase domestic welfare and decrease foreign welfare.

The remainder of this paper is structured as follows. Section II outlines the features of the model. Section III describes the equilibrium. In Section IV, we examine the impact of a profit tax reduction on the spatial distribution of firms across the two countries, the terms of trade, wage rates, consumption and welfare in each country. The final section summarizes the findings and concludes the paper.

\section*{II. The Model}

We assume a two-country world economy, with a home and a foreign country, in which the overall number of firms is exogenously given, but firms can relocate freely and without any cost between two countries. Monopolistically competitive firms exist continuously in the world in the \([0, 1]\) range. Firms in the interval \([0, n]\) locate in the home country, and the remaining \((n, 1]\) firms locate in the foreign country, where \(n\) is endogenous. Firms charge mark-up prices based on product differentiation, each producing a unique variety in a single location to serve world demand. Labor is the only input with constant marginal productivity and no fixed costs are required. There is free trade between two countries that share identical preferences, and have a predetermined size in terms of labor endowment. Departing from the conventional free entry set-up, profits are not wiped out in equilibrium. Instead, the key ad-

\textsuperscript{6}This paper is also related to work by Melitz (2003) who investigated the welfare effects of trade liberalization under monopolistic competition with heterogeneous firms. The paper differs in their analysis of government instruments, but shares many of the concerns about the welfare consequences via general equilibrium effects.
justment is a relocation of firms between the two countries driven by
the equilibrium condition that profits are equalized across countries.
This analysis further assumes that firms are mobile internationally,
but their owners are not. Hence, all profit flows are distributed to the
immobile owners according to the respective holding shares. Finally, we
assume that the home government imposes a profit tax rate $\tau$ on the
profits of domestically located firms, and all tax revenue are shared
equally by corresponding resident households in a lump sum fashion.

A. Households

The size of the world population is normalized to unity. We assume
that the shares of households in the home and foreign locations are $s$
and $s'(=1-s)$, respectively. Each household is endowed with one unit
of labor. Every household supplies one unit of labor to domestic firms
at the real domestic wage, and receives profits from the internationally
mobile firms. The households in each country consume a group of dif-
ferentiated goods. The model’s central assumption is that domestic and
foreign goods affect consumer welfare in a different way although firms
are perfectly mobile across countries. This implicitly assumes that house-
holds receive a different level of utility from a product depending on
whether it is produced in the home country or in the foreign country
(e.g., wine from France, whiskey from the United Kingdom, rice from
Japan). The utility maximization problem in a typical household in the
home country is then:

\[
\begin{align*}
\max_{c_h, c_f} U &= \log C, \\
\text{subject to } &\int_0^n \pi_j ((1-\sigma) dj + \int_n^1 \pi^*_j d\eta)/s + w + z = C. \quad (1)
\end{align*}
\]

where

\[
C = (C_h^{(\sigma-1)/\sigma} + C_f^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}, \quad \sigma > 0 \quad (2)
\]

\[
C_h = (\int_0^n c_{hj}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}, \quad C_f = (\int_n^1 c_{fj}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}, \quad \theta > 1 \quad (3)
\]

In (1), the consumption index, $C$, is a constant-elasticity-of-substitution
(CES) function composed of two types of goods, home and foreign, and
$\sigma$ is the elasticity of substitution between the home and the foreign

\[
7\text{Foreign-country variables are indicated by an asterisk.}
\]
The second equation in (1) is the household’s budget constraint per capita, and \( \alpha (1 - \alpha) \) denotes the share of the total profit flows of firms repatriated to the home (foreign) agents.\(^8\) Throughout the paper, we also use the index \( j \in [0, 1] \) to refer to the product of firm \( j \). Therefore, \( \int_0^n \pi_j \, dj (\int_n^1 \pi_j \, dj) \) represents the total profit flows of home- (foreign-) located firms. In addition, in (1), \( w(=W/P) \) denotes the real wage rate in terms of the consumption index where \( W \) is the home country’s nominal wage rate and \( P \) is the price index of the home country corresponding to \( C \), and \( z \) is the lump-sum transfer per capita. In (2), \( C_h \) and \( C_f \) represent the consumption of the home and foreign goods, respectively, and are defined by a CES function across goods in the same country as defined by (3). In (3), \( \theta \) is the elasticity of substitution among goods produced in the same country. \( c_{hj} \) and \( c_{fj} \) represent the consumption of a particular good \( j \) produced in the home and the foreign country, respectively.

We divide household decisions into two stages: first, consumers allocate their consumption between two types of goods, \( C_h \) and \( C_f \); next they allocate their consumption across various goods, \( c_{hj} \) and \( c_{fj} \), within each type. In the first stage, households solve the following problem:

\[
\max_{C_h, C_f} U = \log C = \log \left( \frac{C_h^{(\sigma - 1)/\sigma} + C_f^{(\sigma - 1)/\sigma}}{\sigma/(\sigma - 1)} \right)
\]

subject to \( E = P_h C_h + P_f^* C_f \),

where \( P_h = \left( \int_0^n P_{hj} \frac{1}{\theta} dj \right)^{1/(1 - \theta)} \) and \( P_f^* = \left( \int_n^1 P_{fj}^{*1-\theta} dj \right)^{1/(1 - \theta)} \) are the price indexes of the home and foreign goods, respectively, and \( E \) is the total consumption expenditure. We then obtain the following demand functions:

\[
C_h = \left( P_h / P \right)^{-\sigma} C, \quad C_f = \left( P_f^* / P \right)^{-\sigma} C \tag{4}
\]

where \( P = [P_h^{1-\sigma} + P_f^{1-\sigma}]^{1/(1-\sigma)} \) is the price index corresponding to the consumption index \( C \). In the second stage, households solve the following two problems:

\(\text{In what follows, we mainly focus on the description of the home country because the foreign country is described analogously.}\)

\(\text{In other words, } \alpha \text{ denotes the extent to which firms are domestically owned. Huizinga and Nielsen (1997, 2002) and Fuest and Huber (2002) studied the feasibility of profit taxation in the presence of foreign ownership of the domestic firm.}\)
\[
\max_{C_h} C_h = \left( \int_0^{n} c_{by}^{(\theta-1)/\theta} \, dj \right)^{\theta/(\theta-1)}, \quad \text{subject to } P_h C_h = \int_0^{n} p_{bj} \, c_{by} \, dj
\]
\[
\max_{C_f} C_f = \left( \int_1^{n} c_{fy}^{(\theta-1)/\theta} \, dj \right)^{\theta/(\theta-1)}, \quad \text{subject to } P_f^* C_f = \int_1^{n} p_{fj}^* \, c_{fy} \, dj
\]

where \( p_{bj} \) and \( p_{fj}^* \) are the prices of good \( j \) manufactured in the home and foreign countries. From these, we obtain the following demand functions of the home country:

\[
c_{by} = \left( \frac{p_{by}}{P_h} \right)^{-\theta} C_h, \quad c_{fy} = \left( \frac{p_{fy}^*}{P_f^*} \right)^{-\theta} C_f
\]  

(5)

Combining (4) and (5) yields the following demand functions of the home country:

\[
c_{by} = \left( \frac{p_{by}}{P_h} \right)^{-\theta} (P_h/P)^{-\sigma} C, \quad c_{fy} = \left( \frac{p_{fy}^*}{P_f^*} \right)^{-\theta} (P_f^*/P)^{-\sigma} C
\]  

(6)

Similarly, the demand functions of the foreign country are:

\[
c_{by}^* = \left( \frac{p_{by}}{P_h} \right)^{-\theta} (P_h/P^*)^{-\sigma} C^*, \quad c_{fy}^* = \left( \frac{p_{fy}^*}{P_f^*} \right)^{-\theta} (P_f^*/P^*)^{-\sigma} C^*
\]  

(7)

where \( P^* = \left( P_h^1 - \sigma + P_f^* - \sigma \right)^{1/(1-\sigma)} \) is the price index corresponding to the consumption index \( C^* \).

Substituting (6) into \( C \), and (7) into \( C^* = (C_h^\sigma + C_f^\sigma) \), respectively, yields \( C = e \) and \( C^* = e^* \), where \( e = E/P \) and \( e^* = E^*/P^* \) represent the home and foreign household’s expenditure in terms of the consumption index. In a symmetric equilibrium, the price indexes, \( P = P^* = \left( P_h^1 - \sigma + P_f^* - \sigma \right)^{1/(1-\sigma)} \), are rewritten as:

\[
P_h/P = P_h/P^* = \omega^{-1} [a^{\sigma-1} + 1]^{-1/(1-\sigma)}, \quad P_f^*/P = P_f^*/P^* = [a^{\sigma-1} + 1]^{-1/(1-\sigma)}
\]  

(8)

where \( \omega = P_f^*/P_h \) is the relative price index between home and foreign countries. Accordingly, the terms of trade (the price of home goods relative to foreign goods) are \( 1/\omega \) for the home country, and \( \omega \) for the foreign country. In other words, the terms of trade of home (foreign) country is defined by the relative price of home (foreign) exports to home (foreign) imports. Furthermore, from \( P_h = \left[ \int_0^{n} p_{by}^{(\theta-1)/\theta} \, dj \right]^{1/(1-\theta)} \) and \( P_f^* = \left[ \int_1^{n} p_{fj}^{\* (\theta-1)/\theta} \, dj \right]^{1/(1-\theta)} \), we obtain the following symmetric price ratios:

\[
p_{by}/P_h = n^{-1/(1-\theta)}, \quad p_{fy}^*/P_f^* = (1-n)^{-1/(1-\theta)}
\]  

(9)
Summing the demand functions (6) and (7) across all households, and equating the resulting equation to the output of good $j$ produced in the home country, $y_j$, yields the following market clearing condition for any product $j$:

$$y_j = s c_{hj} + s^* c_{j}^* = (p_{hj}/P_h)^{-\theta}(P_h/P)^{-\sigma}s_{w}, \quad j \in [0, n] \quad (10)$$

where $s c_{hj}$ ($s^* c_{hj}^*$) is aggregate home (foreign) consumption demand for product $j$ and $s_{w} = (se + s^* e^*)$ is global consumption expenditure index. Similarly, the market clearing condition for any good $j$ produced in the foreign country is $y^*_j = s c^*_{fj} + s^* c^*_j = (p^*_{fj}/P_f)^{-\theta}(P_f/P)^{-\sigma}s_{w}, \quad j \in [n, 1]$.

**B. Firms**

We assume that any monopolistically competitive firm that operates in either of the two countries employs the same production technology. These firms use constant returns-to-scale technology to produce the differentiated consumption products, according to $y_j = l_j$, where $l_j$ represents labor input. Since the home-located firm $j$ hires labor domestically, given $W, P_h, P$ and $s_{w}$, and subject to (10), the home-located firm $j$ faces the following profit-maximization problem:

$$\max_{p_{hj}} \Pi_j = (p_{hj} - W)y_j, \quad \text{subject to } y_j = s c_{j} + s^* c_{j}^* = (p_{hj}/P_h)^{-\theta}(P_h/P)^{-\sigma}s_{w} \quad (11)$$

Given the above, the price mark-up is chosen according to:

$$p_{hj} = (\theta/(\theta - 1))W \quad (12)$$

Since $W$ is given, (12) yields $p_{hj} = p_h, j \in [0, n]$. These relationships imply that each home-located firm supplies the same quantity of goods. Similarly, the price mark-ups of foreign-located firms are identical, since $p^*_{fj} = p^*_f, j \in [n, 1]$. Dropping the firm index because of symmetry and denoting the maximized profit flows of the home- and foreign-located firms in terms of the consumption index, respectively, by $\pi_h(=\Pi_h/P)$ and $\pi^*_f(=\Pi^*_f/P)$, and substituting (10) and (12) into $\Pi_j$ of (11) yields:

$$\pi_h = (1/\theta)(p_{hj}/P_h)^{1-\theta}(P_h/P)^{1-\sigma}s_{w}, \quad \pi^*_f = (1/\theta)(p^*_{fj}/P_f)^{1-\theta}(P_f/P)^{1-\sigma}s_{w} \quad (13)$$
III. Equilibrium

A. Profit-equalization Condition

We assume that firms do not face any relocation costs such that it does not take any time to relocate to another country. For a firm to be indifferent between home and foreign locations after location arbitrage, the returns from the two locations must be equalized. Hence, for an equilibrium where monopolistic firms are located in both countries, the following profit-equalization condition must be satisfied.10

\[(1 - \tau)\pi_h = \pi_f^*\]  \hspace{1cm} (14)

This condition enables us to determine the equilibrium spatial distribution of firms across the two countries.

B. Labor Market Clearing Conditions

The equilibrium conditions for the labor market of each country are, respectively, given by:

\[n_l = s, \quad (1 - n)l_j^* = s^*\]  \hspace{1cm} (15)

In (15), the left-hand sides denote total labor demand and the right-hand sides denote total labor supply, respectively.

C. Equilibrium Values

Using the price mark-up, the profit-equalization condition, product and labor market-clearing conditions for domestic and foreign countries yield the equilibrium relative price, the distribution of firms and real consumptions of both countries, \(\omega_c, n_c, e_c,\) and \(e^*_c\).

a) The Distribution of Firms

Equations (8), (9), and (13) can be rewritten as:

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10 In the direct foreign investment literature (e.g., Janeba 1995; Konan 1997; Rhee 1998; Huizinga and Nielsen 2002), locations take the form of financial portfolio investment. The distribution of foreign direct locations is then determined so as to equalize the net return on domestic and foreign investments for capital owners.
\[\pi_h = \left(\frac{1}{\theta}\right)n^{-1}\omega^{\sigma-1}[\omega^{\sigma-1}+1]^{-1}S_w, \quad \pi_f^* = \left(\frac{1}{\theta}(1-n)^{-1}\right)[\omega^{\sigma-1}+1]^{-1}S_w \quad (16)\]

Substituting (16) into (14) yields the following relationship between \(\omega\) and \(n\) in location-equilibrium:

\[\omega = \frac{n}{(1-n)(1-\tau)} \left[\omega^{\sigma-1}+1\right]^{1/(\sigma-1)} \quad (17)\]

From (17), when \(0 < \sigma \leq 1\), we obtain \(dn/d\omega < 0\), which indicates that the domestic share of mobile firms is inversely related to the relative price \(\omega\). We intuitively explain this relationship as follows: when \(0 < \sigma \leq 1\) is assumed, an exogenous rise in \(\omega\) decreases \(\pi_h\) and increases \(\pi_f^*\). since \(0 < \sigma \leq 1\) implies that the cross-country price elasticity for each product is smaller than unity. Therefore, some firms relocate to the foreign country from the home country. Also, \(dn/d\omega > 0\) holds when \(\sigma > 1\). This is because when \(\sigma > 1\) it implies that the cross-country price elasticity for each product is larger than unity, so the inverse mechanism occurs in (17).

By substituting (8) and (9) into (10), and using \(y_j = l_j\) and (15) yields:

\[n^{1/(1-\theta)}\omega^{\sigma}[\omega^{\sigma-1}+1]^{\sigma/(1-\sigma)}S_w = s \quad (18)\]

The government budget constraint in the home country is

\[\tau n \pi_h = sz \quad (19)\]

From (8), (9), and (12), real wages are

\[w = \left(\frac{W}{p_{j_0}/P_h}\right)\left(\frac{p_{j_0}}{P_h}\right) = \left((\theta - 1)/\theta\right)n^{-1/(1-\theta)}\omega^{\sigma-1}[\omega^{\sigma-1}+1]^{-1/(1-\sigma)} \quad (20)\]

\[w^* = \left(\frac{W^*}{p_{j_0}/P_f}\right)\left(\frac{p_{j_0}/P_f}{P_f}\right) = \left((\theta - 1)/\theta\right)(1-n)^{-1/(1-\theta)}[\omega^{\sigma-1}+1]^{-1/(1-\sigma)} \quad (21)\]

From (1) and (16), combining the budget constraints of each country yields: 

\[s_w = s^e + s^* = s n^{-1/(1-\theta)}\omega^{\sigma-1}[\omega^{\sigma-1}+1]^{-1/(1-\sigma)} + s(1-n)^{-1/(1-\theta)}[\omega^{\sigma-1}+1]^{-1/(1-\sigma)} \quad (22)\]

Substituting this into (18) yields:

\[11 \text{ See the Appendix for derivation of } s_w.\]
\[ s \omega^{\sigma-1}(\omega^{\sigma-1}+1)^{-1} + s^{*}(n/(1-n))^{1/(1-\theta)} \omega^{\sigma}(\omega^{\sigma-1}+1)^{-1} = s \]  

(22)

From (22) and \( \bar{n} = n/(1-n) \), we obtain \( d\bar{n}/d\omega = \sigma(\theta-1)(\bar{n}/\omega) > 0 \), which indicates that the domestic share of mobile firms \( n \) is positively related to the relative price \( \omega \) (dn/d\omega > 0).

From (17) and (22), we obtain the equilibrium distribution of firms:

\[ n_e = [1 + (s/s^*)]^{[1-\theta(\sigma-1)]/[1-\sigma(2-\theta)]} [1-\tau]^{\theta/\sigma} \]  

(23)

From (23), when either \( 0 < \sigma \leq 1 \), or \( 1 > \sigma(2-\theta) \) and \( \sigma > 1 \), the profit tax decrease \( (d\tau < 0) \) will lead firms to relocate into the home country, i.e., \( dn_e/d\tau < 0 \). In contrast, when \( 1 < \sigma(2-\theta) \) and \( \sigma > 1 \), the relocation effect of decreasing the profit tax rate in the home country is \( dn_e/d\tau > 0 \). The mechanism of \( dn_e/d\tau > 0 \) is explained as follows. First, from (17) and (22), a decrease in the profit tax rate reduces the relative price \( \omega \) for a given \( n \), or increases \( n \) for a given \( \omega \) because the decrease in \( \tau \) leads to \( (1-\tau)\pi_h > \pi_f \), and thereby induces some firms to relocate into the home country (hereafter we call this the ‘first relocation effect’). However, the reduction in \( \omega \) in turn stimulates firm relocation from the home to the foreign country because \( dn/d\omega > 0 \) holds from (22) (hereafter we call this the ‘second relocation effect’). Hence, the net outcome of the spatial distribution of firms by decreasing the profit tax rate depends on the relative strength of these first and second pressures. In order to establish the net outcome, it is worthwhile to note that the positive relationship between \( n \) and \( \omega \) (dn/d\omega > 0) is increasing in the size of \( \sigma \) because \( d\bar{n}/d\omega = \sigma(\theta-1)(\bar{n}/\omega) > 0(\bar{n} = n/(1-n)) \) holds from (22). This implies that a change in \( \omega \) have larger (smaller) effect on \( n \) the larger (smaller) is \( \sigma \). Hence, the larger (smaller) is \( \sigma \), the larger (smaller) is \( dn/d\omega > 0 \) and the larger (smaller) is the ‘second relocation effect.’ In the latter condition, \( 1 < \sigma(2-\theta) \) and \( \sigma > 1 \); here, \( \theta \) is restricted to be relatively small, but \( \sigma \) is restricted to be large. This reinforces the second relocation effect. Thus, in the case of \( 1 < \sigma(2-\theta) \) and \( \sigma > 1 \), the second relocation effect dominates the first relocation effect, and thereby makes firms relocate to the foreign country, that is, \( dn_e/d\tau > 0 \). Similarly, we can also consider the case of \( 1 > \sigma(2-\theta) \) and \( \sigma > 1 \), that is, \( \sigma \) is relatively small. As shown in the above, this reduces the second relocation effect, and hence, the second relocation effect is dominated by the first relocation effect, and thereby makes firms relocate to the home country, i.e., \( dn_e/d\tau < 0 \). In addition, from (17), when \( 0 < \sigma \leq 1 \), a decrease in the profit tax rate raises the relative price \( \omega \) for a given \( n \),
or increases $n$ for a given $\omega$ (the first relocation effect). The rise in $\omega$ stimulates firm relocation from the foreign to the home country because $dn/d\omega > 0$ holds from (22) (the second relocation effect). Therefore, in this case, the second relocation effect reinforces the first relocation effect, and hence, $dne/d\tau < 0$.

b) The Equilibrium Relative Price

Next, substituting (23) into (17) yields the following equilibrium relative price:

$$\omega_e = \left(\frac{s}{s^*}\right)^{1-\theta/(1-\sigma(2-\theta))} \left(1-\tau\right)^{1/(1-\sigma(2-\theta))}$$

From (24), $d\omega_e/d\tau < 0$ if either $0 < \sigma \leq 1$, or $1 > \sigma(2-\theta)$ and $\sigma > 1$ are satisfied. Thus, a decrease in the home country’s profit tax rate increases the equilibrium relative price, $P_f^*/P_h$, and hence the terms-of-trade for the home country decreases for the change $d\tau < 0$. By contrast, if $1 < \sigma(2-\theta)$ and $\sigma > 1$, $d\omega_e/d\tau > 0$. In sum, from (23) and (24), we obtain the following relationships:

$$dn_e/d\tau < 0, \ d\omega_e/d\tau < 0 \text{ when } 0 < \sigma \leq 1 \quad (25)$$

$$dn_e/d\tau > 0, \ d\omega_e/d\tau > 0 \text{ when } 1 < \sigma(2-\theta) \text{ and } \sigma > 1 \quad (26)$$

$$dn_e/d\tau < 0, \ d\omega_e/d\tau < 0 \text{ when } 1 > \sigma(2-\theta) \text{ and } \sigma > 1 \quad (27)$$

c) Real Consumptions and the Rent Redistribution Effect

From (23) and (24), the symmetric equilibrium pair, $n_e=1/2$ and $\omega_e=1$, are always a solution when the population size of home and foreign countries is equal, i.e., $s=s^*$ and $\tau=0$. Finally, substituting (8), (9) and (12) first into (13), and then along with (14) and (19) into (1) and its foreign counterpart, respectively, and solving for the equilibrium levels of consumptions of both countries gives:

$$e_e = (\alpha/(\theta-1))(w+w^*) + w + (1-\alpha)\tau(\omega^{\sigma-1}/(\omega^{\sigma-1}+1))(1/(\theta-1))(w+w^*) $$

$$e_e^* = ((1-\alpha)/(\theta-1))(w+w^*) + w^* - (1-\alpha)\tau(\omega^{\sigma-1}/(\omega^{\sigma-1}+1))$$

$$1/(\theta-1))(w+w^*) $$

where $w = ((\theta-1)/\theta)n_e^{-1-\theta}(\omega^{\sigma-1}+1)^{-1/(1-\sigma)}$ and $w^* = ((\theta-1)/\theta)(1-n_e)^{-1/(1-\theta)}(\omega^{\sigma-1}+1)^{-1/(1-\sigma)}$ from (20) and (21). The first terms in the
above equations are the rent income. The second term is the labor income. The third term denotes the tax transfer from the foreign country to the domestic country. Therefore, the tax decrease leads the domestic government to shift part of the tax revenue from home country to foreign country as a rent income repatriation to foreign households (hereafter we call this the ‘rent redistribution effect’). Therefore, we obtain the following lemma.

Lemma 1: The ‘rent redistribution effect’ is always negative for the domestic real consumption, and positive for the foreign real consumption.

In what follows, we assume that the numeraire is the labour of home-located households making $W=1$.

IV. The Impacts of the Profit Tax Rate

A. Wage Effects, the Labor Shifting Effect, and the Terms-of-trade Effect

In this section, we investigate the impact on the wages of both countries of a decrease in the home country’s profit tax rate ($dτ<0$). From (20) and (21), the changes in home and foreign wages are given by differentiating $w$ and $w^*$ with respect to $τ$:

$$\frac{dw}{dτ}=w\{\frac{1}{(θ-1)}n^{-1}\frac{dn}{dτ}-(ω^{-1}/(ωσ^{-1}+1))dω/dτ\} \quad (30)$$

$$\frac{dw^*}{dτ}=-w^*\{(1/(θ-1))(1-n)^{-1}\frac{dn}{dτ}-(ωσ^{-2}/(ωσ^{-1}+1))dω/dτ\} \quad (31)$$

From (23) and (24), the effect on $n_c$ and $ω_e$ of $τ$ are obtained explicitly as

$$\frac{dn_c}{dτ}=n_c^2\frac{(s/s^*)^{(1-θ)}\{1-(σ^{-1}+1)/(1-σ(2-θ))\}[σ(1-θ)/(1-σ(2-θ))]\{1-τ\}^{\sigma(1-θ)/[1-σ(2-θ)]-1}}{1-τ^{-1}} \quad (32)$$

$$\frac{dω_e}{dτ}=-ω_e[1/\{1-σ(2-θ)\}]\{1-τ\}^{-1} \quad (33)$$

From (25), (27), (30), and (31), if either $0<σ≤1$ or $1>σ(2-θ)$ and $σ>1$, the impacts of the profit tax have two opposing effects on each country’s wage income. On the one hand, from $dn_c/dτ<0$, a decrease in the profit tax rate brings more differentiated products produced in
the home country due to relocation from the foreign country. This then leads to a shift in labor demand away from the foreign country towards the home country, thereby increasing $w$ and decreasing $w^*$ (hereafter we call this the ‘labor demand shifting effect’). Therefore, we obtain the following lemma.

Lemma 2: The ‘labor demand shifting effect’ raises (lowers) domestic real wage, and lowers (raises) foreign real wage when $dω_e/dτ<0$. On the other hand, from $dω_e/dτ<0$, a decrease in the profit tax rate leads to an increase in the $ω$ required for the after-tax profits to be equalized between the two countries, and this induces a negative (positive) wage response in the home (foreign) country (hereafter we call this the ‘terms-of-trade effect’). Therefore, we obtain the following lemma.

Lemma 3: The ‘terms-of-trade effect’ lowers (raises) domestic real wage, and raises (lowers) foreign real wage when $dω_e/dτ<0$. Thus, as stated in (30), the first element in the brace is the positive effect of a tax decrease on domestic wage and the second element is the negative effect due to deterioration of terms of trade. Also, from (26), (30), and (31), when $1<σ(2−θ)$ and $σ>1$, the opposite causal relationships in both firm location and the terms of trade materialize. Substituting (32) and (33) into (30) and (31), respectively, then yields:

\[
dw/dτ = -\left[ω(1−τ)^{-1}/[1−σ(2−θ)]\right] \\
\quad \times [n_s s_1 σ(1−τ)^{[σ(1−θ)/1−σ(2−θ)]}−(ω^{σ−1}−1)]
\]

\[
dw^*/dτ = [ω^*(1−τ)^{-1}ω^{σ−1}/[1−σ(2−θ)]] \\
\quad \times [n_s s_1 σ(1−τ)^{[σ(1−θ)/1−σ(2−θ)]}+(1/(ω^{σ−1}−1))]
\]

where $s_i = (s/s^*)^{[1−θ/(σ−1)]/[1−σ(2−θ)]}$. Here we can simplify the notation without affecting any of our main results by assuming that the two countries are identical in labor endowment such that $s=s^*$, and therefore $s_i=1$ holds. Evaluating the signs of $dw/dτ$ and $dw^*/dτ$ at $τ=0$ yields:

\[
dw/dτ|_{τ=0} = (ω/2)\left[(1−σ)/[1−σ(2−θ)]\right], \quad (34)
\]

\[
dw^*/dτ|_{τ=0} = -(ω^*/2)\left[(1−σ)/[1−σ(2−θ)]\right]. \quad (35)
\]
where \( w = w^* = ((\theta - 1)/\theta)^{2(\theta^\sigma)/(1-\theta(1-\sigma))} > 0 \). In sum, from (34) and (35), we obtain the following relationship:

\[
\begin{align*}
\frac{dw}{d\tau} \big|_{\tau=0} &> 0, \quad \frac{dw^*}{d\tau} \big|_{\tau=0} < 0 \quad \text{when } 0 < \sigma \leq 1 \\
\frac{dw}{d\tau} \big|_{\tau=0} &> 0, \quad \frac{dw^*}{d\tau} \big|_{\tau=0} < 0 \quad \text{when } 1 < \sigma(2-\theta) \text{ and } \sigma > 1 \\
\frac{dw}{d\tau} \big|_{\tau=0} &< 0, \quad \frac{dw^*}{d\tau} \big|_{\tau=0} > 0 \quad \text{when } 1 > \sigma(2-\theta) \text{ and } \sigma > 1 \\
\frac{dw}{d\tau} \big|_{\tau=0} + \frac{dw^*}{d\tau} \big|_{\tau=0} &= 0
\end{align*}
\]

From (39), the wage effects of a small decrease in the profit tax rate are exactly offset between the two countries.

**B. Consumption Effects**

We now examine the impact of a reduction in the home country’s profit tax on the consumption levels of both countries. In what follows, we divide the cross-country price elasticity into two cases, the case of \( 0 < \sigma \leq 1 \) and the case of \( \sigma > 1 \).

a) The Case of \( 0 < \sigma \leq 1 \)

Here, we consider the case of \( 0 < \sigma \leq 1 \), where the cross-country price elasticity is relatively small. From (28), (29), (36), and (39), evaluating the signs of \( de/d\tau \) and \( de^*/d\tau \) at \( \tau = 0 \) yields:

\[
\begin{align*}
\frac{de_c}{d\tau} \big|_{\tau=0} &= (w/2)(1-\sigma)/(1-\sigma(2-\theta)) + 2(1-\alpha)/(\theta - 1) > 0 \\
\frac{de^*_c}{d\tau} \big|_{\tau=0} &= -(w^*/2)(1-\sigma)/(1-\sigma(2-\theta)) + 2(1-\alpha)/(\theta - 1) < 0
\end{align*}
\]

where \( 1 > \sigma(2-\theta) \) holds from \( 0 < \sigma \leq 1 \) and \( \theta > 1 \). To explain the above, we examine the two terms collected in the brackets in (40). From (39), \( \frac{dw}{d\tau} \big|_{\tau=0} + \frac{dw^*}{d\tau} \big|_{\tau=0} = 0 \). Therefore, the first term in the brackets in (40) is derived by differentiating the second term in Equation (28) with respect to \( \tau \) and then considering (30), (32), and (33). As stated in (30), from (25), the impacts of the profit tax have two opposing effects on the domestic real wage, \( w \). On the one hand, from \( dw_c/d\tau < 0 \), the tax decrease leads to a shift in labor demand away from the foreign country.

\(^{12}\) See the Appendix for derivation of (40) and (41).
towards the home country, thereby increasing $w$ and decreasing $w^*$ (see Lemma 2). On the other hand, from $\frac{d\omega_e}{d\tau} < 0$, the tax decrease induces a negative wage response in the home country, and a positive wage response in the foreign country (see Lemma 3). Hence, the first term in the brackets in (40) denotes the composition of the positive ‘labor demand shifting effect’ and the negative ‘terms-of-trade effect.’ In addition, the second term in the brackets in (40), which is derived by differentiating the third term in Equation (28) with respect to $\tau$, denotes the ‘rent redistribution effect’ of the domestic profit tax decrease. The tax decrease leads the domestic government to shift part of the tax revenue from home country to foreign country as a rent income repatriation to foreign households. Hence, the ‘rent redistribution effect’ is negative for the home country, and positive for the foreign country (see Lemma 1). In sum, the negative effect of a marginal decrease in the domestic profit tax rate is the sum of the ‘terms-of-trade effect’ and the ‘rent redistribution effect,’ while the benefit of the domestic profit tax decrease is the ‘labor demand shifting effect.’ Therefore, the net effect depends on the relative strength of these pressures. However, if $0 < \sigma \leq 1$, the former effects always dominate the latter, so we obtain the following proposition.

**Proposition 1:** When $0 < \sigma \leq 1$ is assumed, a decrease in the home country’s profit tax rate lowers home country consumption $C = e$. Conversely, the tax decrease raises foreign country consumption $C^* = e^*$.

The above mechanism can be intuitively explained as follows: A decrease in $\tau$ leads to $(1 - \tau)\pi_h > \pi_f^*$ and thereby induces some firms to relocate into the home country. Since $0 < \sigma \leq 1$, this implies that the cross-country price elasticity for each product is smaller than unity. Thus, a decrease in the profit tax rate leads to an increase in the $\omega$ required for the after-tax profits to be equalized between the two countries from $\frac{d\omega_e}{d\tau} < 0$ (see Equation (25)). Since an increase in $\omega$ implies fall in the home country’s terms of trade, this has a negative effect for home consumption and a positive effect for foreign consumption. In addition, the reduction in the profit tax rate raises the wage rate of the home country because of relocation of firms away from the foreign country toward the home country from $\frac{dn_e}{d\tau} < 0$ (the ‘labor demand shifting effect’ in (25)). Furthermore, the profit tax decrease shifts partial rent incomes from the home to the foreign owners (the ‘rent redistribution effect’). Thus, we obtain ambiguous effects of a marginal profit
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tax reduction on consumption in each country. However, $0 < \sigma \leq 1$ implies that the available goods are less substitutable for each other globally, so that a relatively large increase in $\omega$ is required for the after-tax profits to be equalized following the profit tax reduction. This is why the negative terms-of-trade effect dominates the positive labor demand shifting effect in the home country under $0 < \sigma \leq 1$ (see Equations (30) and (36)). Thus, from (40) and (41), a reduction in the profit tax rate lowers home country consumption and raises foreign country consumption under $0 < \sigma \leq 1$.

b) The Case of $\sigma > 1$

Next, we consider the case of $\sigma > 1$, where the cross-country price elasticity is relatively large. In addition, this case is divided into two cases by noting the size of $\theta$; $1 < \sigma(2 - \theta)$ and $\sigma > (2 - \theta)$.

In the case of $\sigma > 1$ and $1 < \sigma(2 - \theta)$ where the cross-country price elasticity is large and the within-country price elasticity is small, from (40) and (41), the consumption impacts of the profit tax reduction are the same as the results in the Proposition 1. Therefore, we obtain the following proposition.

**Proposition 2:** When $1 < \sigma(2 - \theta)$ and $\sigma > 1$ are assumed, a decrease in the home country’s profit tax rate lowers home country consumption $C = e$. Conversely, the tax decrease raises foreign country consumption $C^* = e^*$.

This result is explained intuitively as follows. In the home country, when $1 < \sigma(2 - \theta)$ and $\sigma > 1$, the ‘terms-of-trade effect’ is positive from $d\omega_e/d\tau > 0$, while the ‘labor demand shifting effect’ is negative from $d\omega_e/d\tau > 0$ (see Equation (26)). In addition, in this case, from (30) and (37), the negative ‘labor demand shifting effect’ always dominates the positive ‘terms-of-trade effect.’ This is because the condition, $1 < \sigma(2 - \theta)$ and $\sigma > 1$, implies that $\sigma$ is restricted to be relatively large, so that available goods are required to be more substitutable for each other globally. Therefore, the condition, $1 < \sigma(2 - \theta)$ and $\sigma > 1$, reduces the impact of the positive ‘terms-of-trade effect,’ because a relatively small decrease in $\omega$ is required for the after-tax profits to be equalized following the profit tax reduction. This is why the negative ‘labor demand shifting effect’ always dominates the positive ‘terms-of-trade effect’ under $1 < \sigma(2 - \theta)$ and $\sigma > 1$. In addition, recall that a decrease in the profit tax rate redistributes rent incomes partially from the home to the foreign country, i.e., the ‘rent redistribution effect.’ This leads to a reduction in
total income in the home country, thereby reducing home country consumption. Therefore, this effect reinforces the negative 'labor demand shifting effect,' and hence, $de/d\tau|_{r=0}>0$. The opposite mechanism is valid for the foreign country, so that $de'|d\tau|_{r=0}<0$.

In contrast, when $1>\sigma(2-\theta)$ and $\sigma>1$, for the home country, the 'terms-of-trade effect' is negative from $d\omega/e/d\tau<0$, while the 'labor demand shifting effect' is positive from $dn_e/d\tau<0$ (see Equation (27)). In this case, from (30) and (38), the positive 'labor demand shifting effect' dominates the negative 'terms-of-trade effect.' This is because the condition, $1>\sigma(2-\theta)$ and $\sigma>1$, implies that $\sigma$ and $\theta$ are both large, so that available goods are required to be more substitutable for each other globally. Therefore, the condition, $1>\sigma(2-\theta)$ and $\sigma>1$, reduces the impact of the negative 'terms-of-trade effect.' because a relatively small decrease in $\omega$ is required for the after-tax profits to be equalized between the two countries. This is why the positive 'labor demand shifting effect' always dominates the negative 'terms-of-trade effect' under $1>\sigma(2-\theta)$ and $\sigma>1$. However, there is yet another effect, the 'rent redistribution effect,' as an additional negative effect. Therefore, we obtain ambiguous effects of the marginal profit tax reduction on consumption of each country. However, $de/e/d\tau|_{r=0}<0$ and $de'|d\tau|_{r=0}>0$ are obtained if $\alpha$ is close to 1. This is because if $\alpha$ is close to 1, the negative 'rent redistribution effect' approaches zero. Thus, when the cross-country price elasticity and the within-country price elasticity are both large, and a large proportion of the firm profits accrue to domestic residents, then a decrease in the domestic profit tax rate will effectively increase domestic real consumption and decrease foreign consumption. Hence, we obtain the following proposition.

**Proposition 3:** When $1>\sigma(2-\theta)$ and $\sigma>1$ are assumed and when $\alpha$ is sufficiently large (close to 1), a decrease in the home country’s profit tax rate raises home country consumption $C=e$. Conversely, the tax decrease lowers foreign country consumption $C^*=e^*$. If $\alpha$ is relatively small, we may get the opposite results.

This proposition shows that when $\theta$ and $\sigma$ are both large such that when $1>\sigma(2-\theta)$ and $\sigma>1$ and $\alpha$ is close to 1, the consumption impacts of the profit tax reduction of one country are counter to the results of Proposition 1 and 2.
C. Welfare Effects

In the previous subsection, it was shown how a marginal profit tax decrease affects consumption. We now consider the impact of a decrease in the home country’s profit tax rate on the welfare of both countries. The utility of a home household is $U = \log e_e$. Differentiating this with respect to $\tau$ and evaluating the resulting equation at $\tau = 0$ yields:

$$
\frac{dU}{d\tau} |_{\tau=0} = (\frac{\partial U}{\partial e_e})(\frac{de_e}{d\tau}) |_{\tau=0} = e_e^{-1}(\frac{de_e}{d\tau}) |_{\tau=0}
$$

Similarly, the marginal impact of a decrease in the corporation tax rate on the foreign household’s utility is $\frac{dU^*}{d\tau} |_{\tau=0} = e_e^*-1(\frac{de_e^*}{d\tau}) |_{\tau=0}$. As shown, the sign depends solely on the sign of $\frac{de_e}{d\tau}$ and $\frac{de_e^*}{d\tau}$, because $U$ and $U^*$ are strictly increasing functions of $e_e$ and $e_e^*$, respectively. Hence, we obtain the following

**Proposition 4:**

- $\frac{dU}{d\tau} |_{\tau=0} > 0$ and $\frac{dU^*}{d\tau} |_{\tau=0} < 0$ when $0 < \sigma \leq 1$
- $\frac{dU}{d\tau} |_{\tau=0} > 0$ and $\frac{dU^*}{d\tau} |_{\tau=0} < 0$ when $1 < \sigma(2-\theta)$ and $\sigma > 1$
- $\frac{dU}{d\tau} |_{\tau=0} < 0$ and $\frac{dU^*}{d\tau} |_{\tau=0} > 0$ when $1 > \sigma(2-\theta)$, $\sigma > 1$
- and $\alpha$ closes to 1

The above results are similar to those of the propositions obtained in the previous subsection. In particular, the third results in Proposition 4 imply that the home country will have an incentive to enhance firm relocation into the home country by reducing the domestic profit tax rate. Thus, it is better if a reduction in the profit tax rate takes place in the home country as it leads to an increase in domestic welfare when the cross-country price elasticity and the within-country price elasticity are both large, and most firms are domestically owned.

V. Conclusions

This paper has presented the impact of changing profit tax rate on international firms’ locations and on countries’ welfare using a two-country monopolistic competition trade model. In such a model, it was found that two types of elasticity of substitution offer the key to understanding the potential impacts of the profit tax: 1) when the cross-country price elasticity is relatively small, or when the within-country price elasticity is small and the cross-country price elasticity is large, a
A decrease in domestic profit tax rate can decrease domestic welfare and raise foreign welfare; and 2) when the within-country price elasticity and the cross-country price elasticity are together large, and a large proportion of the firm profits accrue to domestic residents, then a decrease in the domestic profit tax rate will effectively increase domestic welfare and decrease foreign welfare. The results then indicate the following policy implication: if the aim of profit tax policy is to attract foreign firms to the domestic country and increase domestic welfare, then taxes must be reduced if the cross-country price elasticity and the within-country price elasticity are both large and if most firms are domestically owned.

The model developed here is rather simple in a number of respects. This suggests many directions for future research. Firstly, this paper focused on analyzing the effects of changing profit tax rate on several key variables under the assumption that the countries in the model are of the same size. Analyzing the effects of different country size might be important. Secondly, the firms’ decision to relocate is rather simplistic in this framework as it postulated that firm relocation depends on cross-country profit differences. This formulation may be unrealistic because the international relocation of firms is also determined by many other factors besides the relative price and the profit tax rate. Incorporating other factors affecting relocation (wage tax, consumption tax, transport costs, tariff, and public goods) may be important. Thirdly, this paper assumed implicitly that each household exogenously owns an equity portfolio that is perfectly diversified across all firms. However, this assumption may be unrealistic because real-world portfolios exhibit home bias, as the home households invest most of their wealth in local firms. Therefore, incorporating the home bias issue in the analysis might be interesting. Furthermore, as the main purpose of this paper is to analyze the effects of a decrease in an exogenously fixed profit tax, interactions between the two governments in setting optimal profit taxes are not considered in the model. Therefore, extending the present model to a noncooperative game theoretic analysis and taking the profit tax as a strategic variable may be interesting. Finally, this paper has attempted to shed light on the theoretical aspects of the effects of profit taxes under perfect mobility of firms. Therefore, whether the results of this paper are consistent with empirical evidence is the question that we must evaluate.

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13 For a discussion on the puzzle of home bias in equity portfolios, see Obstfeld and Rogoff (2000).
consider next. These issues remain for future research.

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Appendix

Derivation of \( s_{w} \): Substituting \( C=e \) into the household’s budget constraint in the home country (1) yields

\[
\alpha((1 - r)\int_0^\tau \pi_j dj + \int_{\tau}^1 \pi_j^j dj) + sw + sz = se. \tag{A.1}
\]

Similarly, in the case of the foreign country,

\[
(1 - \alpha)((1 - r)\int_0^\tau \pi_j dj + \int_{\tau}^1 \pi_j^j dj) + s^* w^* = s^* e^*. \tag{A.2}
\]

Combining (A.1) and (A.2) yields

\[
se + s^* e^* = (1 - r)\int_0^\tau \pi_j dj + \int_{\tau}^1 \pi_j^j dj + sw + s^* w^* + sz. \tag{A.3}
\]

Substituting the government budget constraint, \( \tau \int_0^\tau \pi_j dj = sz \), into (A.3) yields

\[
s_{w} = se + s^* e^* = \int_0^\tau \pi_j dj + \int_{\tau}^1 \pi_j^j dj + sw + s^* w^*. \tag{A.4}
\]

Furthermore, substituting (16) into (A.4) yields

\[
s_{w} = (\theta/ (\theta - 1))(sw + s^* w^*). \tag{A.5}
\]

Finally, substituting (20) and (21) into (A.5) yields

\[
s_{w} = sn^{-1/(1 - \theta)} \omega^{-1}[\omega^{\sigma - 1} + 1]^{-1/(1 - \sigma)} + s^* (1 - n)^{-1/(1 - \theta)} [\omega^{\sigma - 1} + 1]^{-1/(1 - \sigma)}. \tag{A.6}
\]

Derivation of (40) and (41): From the household’s budget constraint in the home country (1) and the government budget constraint, \( \tau n_x = \)}
we obtain
\[ e = \left( \frac{\alpha}{s} \right) (n\pi_n + (1-n)\pi_f) + w + (1-\alpha)(\pi_n/s). \]  
\tag{A.7}

Similarly, in the case of the foreign country,
\[ e^* = \left( \frac{(1-\alpha)}{s^*} \right) (n\pi_n + (1-n)\pi_f^*) + w^* - (1-\alpha)(\pi_n/s^*). \]  
\tag{A.8}

Summing the real profit flows (16) across all firms yields
\[ n\pi_n + (1-n)\pi_f^* = \left( 1/\theta \right) s. \]  
\tag{A.9}

In addition, substituting (A.5) into (A.9) yields
\[ n\pi_n + (1-n)\pi_f^* = \left( 1/(\theta - 1) \right) (sw + s^* w^*). \]  
\tag{A.10}

Substituting (A.5) into \( \pi_n \) in (16) yields
\[ \pi_n = n^{-1} \omega^{(\sigma - 1)} [\omega^{\sigma - 1} + 1]^{-1} (sw + s^* w^*). \]  
\tag{A.11}

Then, substituting (A.10) and (A.11) into (A.7) and (A.8), respectively, yields
\[ e = \left( \frac{\alpha}{s} \right) (1/\theta - 1) [sw + s^* w^*] + w + ((1-\alpha)/s)\tau\omega^{(\sigma - 1)} [\omega^{\sigma - 1} + 1]^{-1} (sw + s^* w^*). \]  
\tag{A.12}

\[ e^* = \left( \frac{(1-\alpha)}{s^*} \right) (1/\theta - 1) [sw + s^* w^*] + w^* - (1-\alpha)(s^*/s)\tau\omega^{(\sigma - 1)} [\omega^{\sigma - 1} + 1]^{-1} (sw + s^* w^*). \]  
\tag{A.13}

When \( s = s^* \), (A.12) and (A.13) can be rewritten as
\[ e = \left( \frac{\alpha}{\theta - 1} \right) (w + w^*) + w + ((1-\alpha)/(\theta - 1))\tau\omega^{(\sigma - 1)} [\omega^{\sigma - 1} + 1]^{-1} (w + w^*). \]  
\tag{A.14}

\[ e^* = \left( \frac{(1-\alpha)}{\theta - 1} \right) (w + w^*) + w^* - (1-\alpha)(\theta - 1))\tau\omega^{(\sigma - 1)} [\omega^{\sigma - 1} + 1]^{-1} (w + w^*). \]  
\tag{A.15}

Differentiating (A.14) with respect to \( \tau \) and evaluating the resulting at \( \tau = 0 \) and then considering (24), (34), and (39) yields
\[ de_e/d\tau \bigg|_{\tau=0} = (w/2)\left[ (1-\sigma)/(1-\sigma(2-\theta)) + 2(1-\alpha)/(\theta - 1) \right]. \]  
\tag{A.16}
where \( w^* = w^* = ((\theta - 1)/\theta)^{2(\theta - \sigma)/[1 - \theta(1 - \sigma)]} > 0 \). Similarly, differentiating (A.15) with respect to \( \tau \) and evaluating the resulting at \( \tau = 0 \) and then considering (24), (35), and (39) yields

\[
\frac{d e^*_\tau}{d \tau} |_{\tau = 0} = -\left( w^*/2 \right) \left[ (1 - \sigma)/[1 - \sigma(2 - \theta)] + 2(1 - \alpha)/\theta \right]. \tag{A.17}
\]

(A.16) and (A.17) are equivalent to (40) and (41).

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