Wavelet Denoising Technique for Improvement of the Low Cost MEMS-GPS Integrated System

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Abstract
In this paper, wavelet signal processing technique is applied to improving navigation signals. Low cost MEMS signals can be distorted with conventional pre-filtering method such as low-pass filtering which causes unwanted smoothing on real signals. However, wavelet thresholding method does not distort the rapidly-changing signal but reduces signal noise. This paper has applied thresholding method to low cost MEMS/GPS integrated navigation system and verified the improvement of the navigation performance by the experiment.

Key words
low cost MEMS/GPS integrated system, Wavelet denoising, MEMS inertial sensor

1. Introduction
The integration of the inertial navigation system (INS) and the global positioning system (GPS) has been implemented for many years [1]. By recent development of micro electromechanical system (MEMS) technology, many applications of low cost MEMS-GPS integrated navigation system are popularly researched. These MEMS-based inertial sensors have been integrated with GPS to provide reliable positioning solutions in case of GPS outages that commonly occur in urban areas. Especially, low cost MEMS-GPS integrated navigation system is used for mobile robots, UAV (unmanned aerial vehicle) or MAV (micro-aerial vehicle) and PNS (pedestrian navigation system) [2, 4, 8]. In those systems, the GPS provides the position information which compensates the error of the INS signals. In case of GPS outages (signal blockages), the INS is used for positioning until the GPS signals are available again.

In the MEMS-GPS integrated system, inertial sensor data includes large signal noise and sensor bias. Among them, sensor biases make more significant error on position result because of integration. Sensor biases are well compensated on good observable trajectory in general integrated navigation system. But sensor biases cannot exactly be compensated on low cost system. If the noise component could be removed, the overall inertial navigation accuracy is expected to improve considerably. The resulting position errors are proportional to the existing sensor bias and sensor noise.

In this paper, the wavelet denoising technique is implemented to eliminate the sensor noise for improving the performance of inertial sensor signals [1, 2, 3, 4]. From the mid-1980s, wavelet techniques have been implemented in many applications such as: image processing, medical diagnostics, geophysical signal processing, pattern recognition, etc. The wavelet analysis has the advantage over other signal processing techniques in the capability of performing local analysis. It can decompose the signal to frequency component in local time. By using this characteristic, the wavelet denoising method shrinks the signal noise by eliminating the frequency component which contains only noise. Furthermore, the wavelet denoising method can also remove the signal bias by decomposing the signal and eliminating the low frequency component.

The wavelet thresholding method is verified using the collected real data from the field test. It is implemented to the MEMS/GPS integrated system, which will be shown in the conclusion.

This paper is organized as follows: Section 2 describes the basic information of wavelet denoising technique. Section 3 presents wavelet denoising method implemented to the MEMS/GPS integrated system. It contains the structure of MEMS/GPS integrated system and Kalman filter used in the integration of the system. The experimental result is analyzed in Section 4. Finally, conclusion is given in Section 5.

2. Wavelet Denoising Technique

2.1 Wavelet Transform
Wavelet transform is a signal transform technique popularly used in several areas such as image processing and audio signal processing. The comparison of the wavelet transform and Fourier transform is shown in Figure 1. As specified at the top left of Figure 1, the Fourier transform decomposes the signal into each frequency component over the entire time interval. It means that time domain information is lost in transforming to the frequency domain. When looking at a Fourier transform of a signal, it is impossible to recognize when a particular event take place. To overcome this drawback, the transform is adapted to analyze a window of the signal at a time. Furthermore, it is necessary to have multiple resolutions in time and frequency domain in order not to tradeoff corresponding to the choice of the window function’s width. The wavelet analysis is based on a windowing technique with variable-sized windows shown in Figure 1. The wavelet transform applies the wide window (long time intervals) to low frequency and the narrow window (short time intervals) to high frequency.
Fig. 1 Various Types of Time-Frequency Domain Sampling: Shannon, Fourier, Gabor, Wavelet[7]

Discrete time wavelet transform is executed as formula (1), (2) [10].

\[ \phi(t) = \sum_k g_0[k] \phi(2t-k) \]  
(1)

\[ \psi(t) = \sum_k g_1[k] \phi(2t-k) \]  
(2)

Where \( \phi \) is called the scale function and \( \psi \) is called the wavelet function. Each \( g_0 \) and \( g_1 \) refers to the wavelet coefficient.

Wavelet function can be any function that satisfies the relationship of formula (1) and (2). The scale function of upper level can be expressed as the convolution of the scale function and the wavelet function of lower level. It means that the low-frequency area can be decomposed to the high-frequency area and low-frequency area. Such relationship is shown in Figure 2.

Wavelet transform is progressed stage by stage. When the signal is divided into low-frequency waves, it requires twice the amount of data. In addition, the lowest possible decomposable frequency area matches the DC value of the Fourier transform calculated using the entire data [10].

2.2 Wavelet Thresholding Technique

Wavelet denoising technique was developed to reduce various noises included in the images. Among its variations, the most representative method is the wavelet shrinkage technique, which is one of the thresholding techniques. Wavelet thresholding technique is a signal estimation technique that exploits the capabilities of wavelet transform for signal denoising. It removes the noise by eliminating coefficients that are insignificant relative to some threshold. Researchers have developed various techniques for choosing denoising parameters and so far there is no best universal threshold determination technique. Wavelet thresholding technique assumes that the magnitude of the actual signal is greater than the noise level, and the noise is white noise.

General low-pass filter has the characteristics removing all frequencies over a certain threshold. If sensor data exist in the high-frequency component, low pass filter results in the loss of the sensor signal. For example, the accelerometer signal has sudden change when the vehicle accelerates. The low-pass filter distorts this change of the signal and turns it into a gradually changing signal. It is clearly shown in Figure 3. In Figure 3, the actual signal is expressed with the solid line. The result of the low-pass signal expressed with the 1-dot chain line is found to smoothly follow the signal change. For this reason, the low-pass filter cannot be used as the pre-processing filter in the inertial navigation system.

The wavelet thresholding technique, however, reduces the noise level with almost no distortion for the sudden change in signal, as shown in Figure 3. The result of the thresholding technique has almost no distortion and accurate, so that it sits right on the actual signal almost indistinguishably. Therefore, it can be used by pre-processing filter for the inertial sensor signal and overcomes the shortages of the existing low-pass filter.
The thresholding technique is classified into the hard thresholding and soft thresholding operation.

\[
T_{\text{hard}}^u = \begin{cases} u & \text{if } |u| \geq \lambda \\ 0 & \text{otherwise} \end{cases} \tag{3}
\]

\[
T_{\text{soft}}^u = \begin{cases} (u - \text{sign}(u)\lambda) & \text{if } |x| \geq \lambda \\ 0 & \text{otherwise} \end{cases} \tag{4}
\]

The formula (3) shows the hard thresholding function and the formula (4) shows the soft thresholding function, which are illustrated in Figure 4. \( u \) is the wavelet coefficient. The wavelet coefficient means the magnitude of a certain frequency component. Thus, wavelet coefficients in the band of noise become zero.

Among the two methods, the soft thresholding technique is known to have better performance than the hard thresholding technique, so the soft thresholding technique was used for the experiment [6, 8].

One of the important elements influencing the performance in thresholding technique is how the standard value of \( \lambda \) is set [2, 6]. Generally, it is determined by the formula (5). Here, \( \sigma \) is the standard deviation of the signal, and \( n \) is the number of signal samples.

\[
\lambda = \sqrt{2\log n} \sigma \tag{5}
\]

The value determined by the formula, however, does not lead to the optimal result, so the appropriate \( \lambda \) must be determined through experimentation.

The thresholding algorithm is executed as follows. First, the wavelet transform is executed for the signal to acquire the wavelet coefficients. Next, the thresholding operation is executed for each wavelet coefficient. Then, the original coefficients are replaced to the coefficients from the result of the thresholding operation. Finally, inverse transform is carried out [6, 8].

3. Wavelet Denoised MEMS-GPS Integrated Navigation System

The MEMS-GPS integrated navigation system used in the experiment is the general 15th order loosely coupled model [9]. The construction of the system is explained in Figure 5.

![Fig. 5 The Construction of MEMS-GPS integrated system.](image)

The state variables of the Kalman filter consist of position error, speed error, attitude error, accelerometer error, and gyro error. The measurement of the Kalman filter is the position information of the GPS updating every second. The inertial sensor signal using wavelet thresholding technique is updated through every 0.01 second and is used to calculate the integrated navigation result. If the low-pass filtering is applied to the GPS signal, it will yield better performance. However, the GPS signal was not filtered separately, because the objective of this research is to find out how much performance is improved the denoising of inertial sensor signal.

In this paper, the Daubechies wavelet was used for wavelet transform shown in Figure 6. The setting of wavelet does not have a big difference no matter which one is used. MEMS sensor signal includes the actual vehicle motion dynamics and the sensor noise as well as some other undesirable noise such as vehicle engine vibration. Therefore, the criterion for the selection of the...
appropriate wavelet level of decomposition (LOD) will be different from the stable motion case. Furthermore, the choice of threshold is crucial to maintain the quality of the denoising process and should be made carefully [2].

The wavelet transform was carried out to the 3\textsuperscript{rd} order, and the threshold value \( \lambda \) is selected from the formula (5) for the accelerometer and the gyro. These values are determined experimentally through multiple simulations.

The thresholding algorithm is implemented to the inertial sensor as follows. The wavelet transform is executed for the gyro and accelerometer signals of each axis, and the wavelet coefficients for total of six signals are acquired. Then, the original wavelet coefficients are replaced by the coefficients resulting from the thresholding operation and the inverse transform is carried out. The inverse transform signal is the noise-improved signal, which is used to execute the MEMS-GPS integrated navigation. The process is described briefly in the flowchart shown in Figure 7.

![Flowchart of the Wavelet Thresholding algorithm](image)

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**4. Experimental Result**

The trajectory of the experiment is the belt way of the Seoul National University campus shown in the Figure 8. It also shows the position data of the MEMS-GPS integrated system and the true trajectory is measured by DGPS for twenty minutes of travelling. In this test, a low grade MEMS INS (MTi-G) and GPS integrated system were used. The sensor specifications used in the test is shown in Table 1. The minimum number of available satellite was 8 and the average vehicle speed was 40km/h. All the analysis result in this paper was implemented using MATLAB computer-aided design software including the wavelet analysis.

<table>
<thead>
<tr>
<th>Table 1 Typical performance of the MTi-G</th>
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<tbody>
<tr>
<td>GPS Receiver</td>
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<tr>
<td>GPS Update rate 4Hz</td>
</tr>
<tr>
<td>Pos/Vel Update rate 120 Hz</td>
</tr>
<tr>
<td>Maximum Altitude 18km</td>
</tr>
<tr>
<td>Maximum Velocity 600m/s</td>
</tr>
<tr>
<td>Max dynamic GPS 4g</td>
</tr>
</tbody>
</table>

Figure 9 illustrates the experimental result of the MEMS output signal arranged x, y, and z axis in order. It shows original MEMS sensor signal and the signals applied to filtering methods. The comparison was executed between wavelet threshold method and the noise reduction method used in the research of [3]. In the research of [3], the method of controlling LOD eliminates the noise of INS signal. Its results are almost identical to that of the low-pass filter, because the frequency response of the scale function is similar to the low-pass filter. Thus, that result is marked as LPF in Figure 9, 10, 11, 12.

![Figure 8 The trajectory of the experiment](image)

![Figure 9(a) Accelerometer signal](image)

![Figure 9(b) Gyroscope signal](image)

![Figure 9 Comparison of the sensor data](image)
Figure 9 shows that result of the comparison between the wavelet thresholding technique and LPF. The original signal is shown in cyan line. The signal applied to wavelet thresholding technique and LPF is shown in red line and blue line, respectively.

In the stable state, both Wavelet thresholding technique and LPF do not distort of sensor data, but LPF is more efficient than Wavelet thresholding technique in terms of the signal denoising.

In the case of INS signals, passing through the low pass filter often causes the distortion of sensor signal shown in Figure 10 which shows the data influenced by the dynamic motion of the vehicle for one second periods. On the other hand, the signal applied to Wavelet thresholding technique maintains good performance during this period. Moreover, in the case of GPS signals shown Figure 11, using the low pass filter also results in deteriorated performance, while Wavelet thresholding technique has good performance at sudden change points.

Consequently, using the pre-filtering method such as the low pass filter to remove the noise of the MEMS signal results in decreased navigation performance.

Figure 12 shows the MEMS-GPS Navigation position output data and the navigation error is arranged latitude, longitude, and altitude in order. The data in the altitude position component applied to LPF is verified that LPF distorts the sudden change signal.

Table 3 shows the RMSE of the MEMS-GPS integrated system’s position output data based on DGPS position data. It compares the RMSE between wavelet thresholding method and LPF in the direction of north, east and down. The RMSE of wavelet thresholding technique is smaller than that of LPF, which verifies that the wavelet thresholding method has better performance than LPF.

Therefore, the chosen wavelet filter (Daubechies wavelet) and thresholding technique contributed to eliminate the undesired noise in the system and maintain better performance than the result used LPF.

Table 3 RMSE of the MEMS-GPS integrated system

<table>
<thead>
<tr>
<th></th>
<th>RMSE Original</th>
<th>Wavelet</th>
<th>LPF</th>
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<tbody>
<tr>
<td>North [m]</td>
<td>5.2748</td>
<td>4.7673</td>
<td>6.5268</td>
</tr>
<tr>
<td>East [m]</td>
<td>3.6503</td>
<td>3.4462</td>
<td>4.2447</td>
</tr>
<tr>
<td>Altitude[m]</td>
<td>7.3536</td>
<td>4.8587</td>
<td>5.5069</td>
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</table>
5. Conclusion

The low-pass filter used in the MEMS-GPS integrated navigation system has limitations such as signal distortion and signal delay. Such issues cause deterioration of navigation performance.

However, the noise reduction technique, using the wavelet transform like the recently developed thresholding technique, can reduce the signal distortion while improving the SNR of the signal. Those characteristics have advantages on MEMS INS signal which rapidly changes according to the motion of the vehicle.

By applying the technique in this paper, it was proved that the INS signal could be improved and the overall navigation performance was also enhanced.

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