Inter-Sector Beamforming with MMSE Receiver in the Downlink of TDD Cellular Systems

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Abstract

The use of beamforming is effective for users in limited power environments. However, when it is applied to the downlink of a cellular system with universal frequency reuse, users near the sector boundary may experience significant interference from more than one sector. The use of a minimum mean square error (MMSE)-type receiver may not sufficiently cancel out the interference when a small number of receive antennas is used. In this paper, we consider the use of inter-sector beamforming that cooperates with a neighboring sector in the same cell to mitigate this interference problem in time-division duplex (TDD) environments. The proposed scheme can avoid interference from an adjacent sector in the same cell, while enhancing the transmit array gain by using the TDD reciprocity. The performance of the proposed scheme is analyzed in terms of the output signal-to-interference-plus-noise power ratio (SINR) with combined use of an MMSE receiver. The effectiveness of the proposed scheme is verified by computer simulation.

1. Introduction

Demand for higher throughput has motivated advanced wireless systems such as 3GPP LTE and mobile WiMAX to employ multi-cell configuration with universal frequency reuse [1], [2]. It has been reported that the use of beamforming is effective in the downlink when users are power limited (i.e., at medium to low signal-to-noise power ratio (SNR)) [3], [4]. In a time-division duplex (TDD) wireless system, base station (BS) can estimate channel state information (CSI) of the uplink using reference signals such as the channel sounding signal [5] and can utilize it for the transmit beamforming by means of channel response reciprocity [5]. However, the use of beamforming with universal frequency reuse may cause users near the cell boundary to experience serious inter-cell interference [1], [2]. In particular, when the cell is operated being divided into a number of sectors, users near the sector boundary may experience weak received signal strength (RSS) from the serving sector due to sector antenna pattern as well as strong interference from adjacent sectors [2], [6]. Advanced wireless systems consider the use of multiple receive antennas in the mobile station (MS) [2], [7], enabling the use of a minimum mean square error (MMSE)-type receiver to suppress interference, while reducing the fading effect [8]. However, when the number of strong interferers is larger than the number of receive antennas minus one, the output signal-to-interference-plus-noise power ratio (SINR) of the MMSE receiver significantly deteriorates [9].

To improve the performance of users near the sector/cell boundary, the use of BS coordination has recently been considered, where the BSs share the CSI and information streams to minimize the interference effect [10]. However, this BS cooperation may incur so-called signaling overhead increasing in exponentially proportional to the number of coordinating BSs and suffer from performance degradation due to the signaling delay. On the other hand, the use of softer handover, macro diversity handover and fast sector selection (FSS) with muting can be used to enhance the performance of users near the sector boundary without the exchange of inter-cell information [7]. However, these schemes do not consider the use of TDD reciprocity.

In this paper, to improve the performance of users near the sector boundary, we propose an inter-sector beamforming scheme that cooperates with an adjacent sector in the same cell. The proposed scheme can avoid interference from the adjacent sector in the same cell, while enhancing the transmit array gain using the TDD reciprocity. Moreover, it does not require additional signaling overhead compared to conventional beamforming (or single-sector beamforming) scheme. It is assumed that the MS transmits the reference signal using a single transmit antenna, as being considered in the IEEE 802.16e [2]. The average output SINR of the MMSE
receiver with the use of inter-sector beamforming are analyzed in the presence of correlation between the transmit antennas by using an approximation.

Following Introduction, Section 2 describes the system model in consideration. Section 3 presents the proposed inter-sector beamforming and analyzes its performance. The performance of the proposed scheme is verified by computer simulation in Section 4. Finally, conclusions are given in Section 5.

2. System Model

Consider the downlink of a cellular system that comprises $N$ hexagonal cells each of which comprises $S$ sectors (i.e., a total of $S \cdot N$ sectors). For ease of description, it is assumed that each sector uses $M$ transmit antennas and the MS uses two receive antennas. Assuming that the target user is under service from sector $m$, the received signal of the target user can be represented as

$$y = \alpha_m H_m w_m x_m + \sum_{i \in \Omega} \alpha_i H_i w_i x_i + n$$

(1)

where $x_i$ denotes the data of sector $i$ with unit average power, $\alpha_i$ denotes the received signal strength (RSS) from sector $i$, $H_i$ is the $(2 \times M)$ channel matrix from sector $i$, $w_i$ denotes an $(M \times 1)$ beam weight from sector $i$, $n$ is the $(2 \times 1)$ additive white Gaussian noise (AWGN) vector, and $\Omega$ denotes an active set comprising sectors defined by

$$\Omega = \left\{ k \left| \frac{\alpha_k^2}{\alpha_m^2} \geq \delta, \ 0 \leq k \leq S \cdot N - 1 \right. \right\}.$$  

(2)

Here $\delta$ denotes a threshold value for the active set, and $z$ denotes the $(2 \times 1)$ interference plus noise vector except the interference from sectors belonging to $\Omega$, which can be assumed to be zero-mean complex Gaussian with covariance $E\{zz^*\} = N_0 I_2$, where $I_2$ denotes a $(2 \times 2)$ identity matrix. The beam weight $w_i$ is determined by the eigenvector corresponding to the maximum eigenvalue of the channel covariance matrix. It is assumed that $\|w_i\| \leq 1$, where $\|$ denotes the Frobenius norm.

3. Proposed Inter-Sector Beamforming

Consider the use of inter-sector beamforming with cooperation between sectors in the same cell to enhance the performance near the sector boundary. The inter-sector beamforming configuration can be extended from $(M \times 2)$ to $(2M \times 2)$ antenna configuration by concatenating two adjacent sectors.

Assume that the BS transmits the target user signal through sector $m$ and $m'$. Then, the received signal can be represented as

$$y = \alpha_m H_m w_m x_m + \sum_{i \in \Omega} \alpha_i H_i w_i x_i + z$$

(3)

where $w_i = [w_i^T \ w_{m'}^T]^T$ and $H_i$ are respectively the $(2M \times 1)$ beamforming vector and $(2 \times 2M)$ channel matrix from sector $m$ and $m'$ to the target user, given by

$$H_i = [H_m \ H_{m'}] \Lambda_i.$$  

(4)

Here $\Lambda_i$ denotes the normalized RSS matrix represented in an $(2M \times 2M)$ diagonal matrix whose first and last $M$ diagonal elements are all one and $\alpha_{m'}/\alpha_m$, respectively, and the superscript $T$ denotes transpose. Note that the transmit power of $w_i$ is limited by

$$\|w_i\|^2 \leq \text{max} \left(\frac{\text{tr} \left[ \Lambda_i^2 \right]}{M} \right) = \text{max} \left(1 + \frac{\alpha_{m'}^2}{\alpha_m^2} \right) = 2$$  

(5)

where $\text{tr} \left[ \cdot \right]$ denotes the trace of a matrix. It can be seen that $\|w_i\|^2 = 2$ when the cooperating sectors have the same RSS (i.e., $\alpha_m = \alpha_{m'}$).

The channel covariance matrix can be represented as

$$R = E\{H_i^H H_i\}/2 = \Lambda_i \left(E\{[H_m \ H_{m'}][H_m^H \ H_{m'}^H]\}/2 \right) \Lambda_i$$

(6)

where $E\{\cdot\}$ denotes the expectation, the superscript $*$ denotes transpose conjugate. It can be seen that the RSS of two cooperating sectors changes the channel covariance. Since $R$ is Hermitian and positive definite, it can be decomposed as [11]

$$R = Q \Sigma Q^*$$

(7)

where $Q = [q_1 \ \cdots \ q_{2M}]$ is an $(2M \times 2M)$ unitary matrix whose columns $\{q_i\}$ are the eigenvectors of $R$, and $\Sigma^2$ is a $(2M \times 2M)$ diagonal matrix whose diagonal terms are descending-ordered eigenvalues (i.e., $\lambda_1 \geq \cdots \geq \lambda_{2M}$) of $R$. The channel matrix can be represented as [12]

$$H_D = H_n R^{1/2}$$

(8)

where $R^{1/2}$ denotes the square root of $R$ and $H_n$ is the $(2 \times 2M)$ spatially white complex Gaussian channel matrix [13].

Letting $h_i$ be the $k$-th row of channel matrix $H_D$, the coherent beam weight $w_D$ can be determined by

$$w_D = \sqrt{1 + \alpha_{m'}/\alpha_m} h_i / \|h_i\|.$$  

(9)

It can be seen that
\[ \mathbf{H}_D \mathbf{w}_D = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} \mathbf{w}_D = \left[ \sqrt{1 + \alpha_i^2} / \alpha_i \right] \mathbf{h}_i \] (10)

where
\[
\mathbf{h}_i \mathbf{w}_D = [\mathbf{H}_n]_i \mathbf{R}^{1/2} \mathbf{w}_D = [\sqrt{1 + \alpha_i^2} / \alpha_i] \mathbf{H}_n^{1/2} \mathbf{w}_D.
\] (11)

Here \([\mathbf{H}_n]_k\) denotes the \(k\)th row vector of \(\mathbf{H}_n\), and it can be shown that
\[
E\left[\mathbf{H}_n^{1/2} \Sigma^2 \mathbf{H}_n^{1/2} / \mathbf{h}_i \right] \approx \sum_{k=1}^{2M} \lambda_k^2 / \sum_{k=1}^{2M} \lambda_k = \varphi^2.
\] (12)

Thus, the signal of the second receive antenna can be rewritten as
\[
\mathbf{h}_i \mathbf{w}_D \approx \left[ \sqrt{1 + \alpha_i^2} / \alpha_i \right] \varphi \mathbf{h}_i.
\] (13)

where \(\mathbf{h}_i\) denotes a zero-mean Gaussian random variable with unit variance. The output SINR of an MMSE receiver can be represented by \([8]\)
\[
\gamma = \alpha_i^2 (\mathbf{H}_i \mathbf{w}_D) \mathbf{K}^{-1} (\mathbf{H}_i \mathbf{w}_D)
\] (14)

where \(\mathbf{K}\) denotes the covariance matrix of the interference plus noise, defined as
\[
\mathbf{K} = \sum_{m \in \Omega, m \neq m'} \alpha_{m-n}^2 (\mathbf{H}_m \mathbf{w}_m) (\mathbf{H}_m \mathbf{w}_m)^\dagger + \mathbf{N}_I.
\] (15)

The probability density function (pdf) of the output SINR \(\gamma\) can be represented as \([14]\)
\[
p(\gamma) = \frac{\gamma}{\sigma_i^2} \exp\left(-E\left[\sigma_i\right] \gamma\right)
\] (16)

where
\[
B_i = \frac{\gamma}{\sigma_i^2} \exp\left(-E\left[\sigma_i\right] \gamma\right) \left( \gamma - E\left[\sigma_i\right] \gamma\right)
\] (17)

and \(\sigma_i\) denotes the eigenvalue of \(\mathbf{G}^{-1}\). Here, \(\mathbf{G}\) denotes the covariance of the target signal, given by
\[
\mathbf{G} = \alpha_i^2 \mathbf{E} (\mathbf{H}_i \mathbf{w}_i) (\mathbf{H}_i \mathbf{w}_i)^\dagger = \begin{bmatrix} \alpha_i^2 + \varphi^2 & 0 \\ 0 & \varphi^2 \end{bmatrix}.
\] (18)

It can be seen from (16) that the pdf of the output SINR can be determined by the mean eigenvalue of \(\mathbf{G}^{-1}\). Since \(\mathbf{w}_i\) in (15) is an \(M\)-dimensional unit-norm vector and independent of \(\mathbf{H}_n\), the effective channel from sector \(i \in \Omega, m, m'\) can be assumed as
\[
\mathbf{H}_w = \mathbf{h}_{n,i}
\] (19)

where \(\mathbf{h}_{n,i}\) is a \((2 \times 1)\) spatially white Gaussian channel vector defined by \([13]\)
\[
E\left[\mathbf{h}_{n,i}^\dagger \mathbf{h}_{n,i}\right] = \begin{cases} 0, & k \neq i \\ 2, & k = i. \end{cases}
\] (20)

Thus, \(\mathbf{K}\) can be rewritten as
\[
\mathbf{K} = \mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^\dagger + \mathbf{N}_I.
\] (21)

In the presence of \(L\) interferences from sectors \(\{j \in \Omega; m, m', k = 1, \ldots, L\}\), \(\mathbf{C}_i\) is a \((2 \times L)\) random matrix comprising \(L\) interference vectors as columns, i.e.,
\[
\mathbf{C}_i = \begin{bmatrix} \mathbf{h}_{n,j_1} & \mathbf{h}_{n,j_2} & \cdots & \mathbf{h}_{n,j_L} \end{bmatrix}
\] (22)

and \(\mathbf{P}_i\) is an \((L \times L)\) diagonal matrix whose \(k\)th diagonal elements are \(\alpha_{n,i}^2\). Letting \(E[\beta_i]\) be the mean eigenvalue of \(\mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^\dagger\), it can be shown that \([15]\)
\[
E[\beta_i] = \frac{L(E^2 + 3)}{2} + \sum_{k=2}^{L} \frac{1}{2^{L-k} - 1} \left( 2L - k - 1 \right) \left( k \right).
\] (23)

Letting \(\mu_k\) be the eigenvalue of \(\mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^\dagger\) in (21) when the interferers have arbitrary power, it can be shown that
\[
\sum_{k=1}^{2} \mu_k = E[\beta_i] = E[\{\mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^\dagger\}]
\] (25)

\[
= \sum_{k=1}^{2} \alpha_{n,i}^2 E[\|\mathbf{h}_i\|^2] = \sum_{k=1}^{2} \alpha_{n,i}^2.
\] (26)

Since \([11]\)
\[
\prod_{k=1}^{2} \mu_k = \det[\mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^\dagger] = \det[\mathbf{C}_i] \det[\mathbf{P}_i] = \prod_{k=1}^{2} \beta_k \det[\mathbf{P}_i]
\] (27)

it can be approximated as
\[
\prod_{k=1}^{2} \mu_k \approx \prod_{k=1}^{2} E[\beta_k] \det[\mathbf{P}_i]
\] (28)

where \(\det[\cdot]\) denotes the determinant of a matrix. Similarly, it can be shown that
\[
\prod_{k=1}^{2} E[\sigma_i] = E[\{\mathbf{C}_i \mathbf{P}_i \mathbf{C}_i^\dagger\}]
\] (29)

\[
= \{ \sum_{k=1}^{2} \alpha_{n,i}^2 + \mathbf{N}_I \} \det[\mathbf{G}^{-1}].
\] (30)

It can be approximated as
\[
\prod_{k=1}^{2} E[\sigma_i] \approx \prod_{k=1}^{2} (E[\mu_k] + \mathbf{N}_I) \det[\mathbf{G}^{-1}].
\] (31)

Thus, the average output SINR can be approximated as
\[
E[\gamma] = \int_0^\infty \gamma p(\gamma) d\gamma \approx \frac{\prod_{k=1}^{2} E[\sigma_i]}{\prod_{k=1}^{2} (E[\mu_k] + \mathbf{N}_I) \det[\mathbf{G}^{-1}]}.
\] (32)
It can further be approximated using (25) and (27) as
\[
\sum_{i \in \Omega \setminus \{m,m'\}} \Gamma_i \cdot 1 + \left( E\{\theta_i\} E\{\theta_i / N_i^2\} \prod_{i \in \Omega \setminus \{m,m'\}} \alpha_i^2 e_i^2 + 2 \sum_{i \in \Omega \setminus \{m,m'\}} \Gamma_i \right) \cdot \gamma_{\text{target}} \tag{31}
\]
where \( \Gamma_i \) denotes the average SNR of sector \( i \) and \( \gamma_{\text{target}} \) is the average SNR of the target signal. The first term in the denominator of (31) represents the degradation due to the interference.

Assuming that users near the cell/sector boundary experience interference at most from two sources, the inter-sector beamforming can avoid interference from the adjacent sector, making users experience a single dominant interference (i.e., \( \Gamma_i \)). The average output SINR of the MMSE receiver is given by
\[
E\{\gamma\} \approx \frac{\Gamma_i^{+1} \gamma_{\text{target}}}{E\{\beta_i\} E\{\beta_i / N_i^2\} \Gamma_i + 2\Gamma_i^{+1}} \approx \frac{\gamma_{\text{target}}}{2} \tag{32}
\]
where \( E\{\beta_i\} = 2 \) and \( E\{\beta_i\} = 0 \). It can be shown that
\[
\gamma_{\text{target}} = \frac{\text{tr}\left( G^{-1} \right)}{N_i \cdot \text{det}\left( G^{-1} \right)} = \frac{(\alpha_m^2 + \alpha_m^2)(2M + \rho^2)}{N_i}. \tag{33}
\]

Thus, the use of inter-sector beamforming can achieve large transmit array gain and power gain as well as interference cancellation compared to the use of a single sector beamforming.

4. Performance Evaluation

The analytic design and performance of the proposed beamforming scheme are verified by computer simulation. The proposed scheme is applied to a \((2 \times 2)\) MIMO configuration in correlated MIMO fading channel with covariance matrix \( \Delta(\rho) \) given by
\[
\left[ \Delta(\rho) \right]_{p,q} = \rho^{p-q} \cdot \exp\left( j(p-q) \frac{\pi}{12} \right) \tag{34}
\]
where \( \rho \) denotes the magnitude of the correlation coefficient between two adjacent transmitter antennas, and \( \left[ \cdot \right]_{p,q} \) denotes the element of the \( p \)-th row and the \( q \)-th column. That is, \( E\left[ \left[ H_m H_m^H \right] \left[ H_m H_m^H \right] \right] / 2 = \Delta(\rho) \).

It is also assumed that sector 0 allocates the resource to the target user at each frame. The performance is evaluated in terms of the geometry defined by
\[
G = \sum_{i \in \Omega \setminus \{m,m'\}} \alpha_i^2 + \sum_{i \in \Omega \setminus \{m,m'\}} N_i \sum_{i \in \Omega \setminus \{m,m'\}} \alpha_i^2 + N_i \sum_{i \in \Omega \setminus \{m,m'\}} \Gamma_i \cdot 1 \tag{35}
\]

In the following figures, the legend ‘BF-MMSE’, ‘BF-MRC’, ‘ISBF-MMSE’, ‘SHO-MMSE’, ‘MD-MMSE’, ‘FSS-MMSE’ and ‘NullBF-MMSE’ denote the
single-sector beamforming with MRC, the inter-sector beamforming with MMSE receiver, the softer handover with MMSE receiver, the macro diversity handover with MMSE receiver, FSS with muting and MMSE receiver, and null beamforming with MMSE receiver [16], respectively.

Fig. 1 depicts the analytic and simulation results according to $\Gamma_0$, assuming a single interference from adjacent sector 1 with the same RSS as the serving sector (i.e., $\Omega = \{0,1\}$ and $\Gamma_1 = \Gamma_0$). In this case, the inter-sector beamforming can avoid the interference from the adjacent sector while increasing the transmit array gain and doubling the transmit power gain. It can be seen that the MMSE receiver increases the average output SINR in proportion to $\Gamma_0$, while the MRC does not mainly due to dominant interference. It can also be seen that the analytic results agree well with the simulation results. As $\rho$ increases, the performance improves due to the increase of the RSS of the second receiver antenna.

Fig. 2 depicts the analytic and the simulation results according to $\Gamma_0$ in the presence of two interferences; one from adjacent sector 1 and the other from sector 2 (i.e., $\Omega = \{0,1,2\}$, and $\Gamma_1 = \Gamma_0$ and $\Gamma_2 = \Gamma_0 + 2$ dB). It can be seen that the MMSE receiver with the single-sector beamforming suffers from performance degradation due to unremoved interference, while the MMSE receiver with the inter-sector beamforming works well by properly removing the dominant interference. It can be seen that the analytic results agree well with the simulation results. As $\rho$ increases, the performance improves due to the increase of the RSS of the second receiver antenna.

Fig. 3 depicts the performance in 19-cell environments (with $S = 3$), where the cell radius is 1km, the path loss follows $28.6 + 35 * \log_{10}(d)$, where $d$ is the distance (in meters) between the sector and the user, and the sector antenna pattern follows 70° (-3dB beamwidth) with a front-to-back ratio of 20dB [6]. It is assumed that sectors having an RSS larger than one half that of the serving sector belong to the active set (i.e., $\delta = -3$ dB), and that the target user is located near the boundary between sector 0 and 1 and experiences interference from sector 1 at the same time. Table 1 summarizes the active set and the corresponding SNR. It can be seen from Fig. 3 that the inter-sector beamforming outperforms the conventional schemes such as the FSS with muting and the macro diversity handover mainly due to the transmit array gain through beamforming.

5. Conclusions

We have considered the use of inter-sector beamforming for the service of users near the sector boundary in TDD based cellular systems. The performance of the proposed inter-sector beamforming scheme combined with MMSE receiver has been analyzed in terms of the average output SINR. The simulation results show that the inter-sector beamforming is very effective for users near the sector boundary, outperforming conventional schemes such as the null beamforming, softer handover, macro diversity handover and FSS with muting.

References