Adaptive Precoding for Wireless MIMO Broadcast Channels with Limited Feedback*

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SUMMARY Recently, a number of techniques have been introduced to exploit multiuser diversity of a wireless multiple-input multiple-output (MIMO) broadcast channel (BC) that consists of a base station with \( t \) transmit antennas and \( K \) users with multiple antennas. However, prior works have ignored the rate overhead associated with feedback of MIMO BC channel state information at transmitter (CSIT), which is roughly \( K \) times larger than single-user MIMO CSIT (i.e., it is \( \mathcal{O}(r) \)) where \( r = \sum_{k=1}^{K} r_k \) and \( r_k \) is the number of antennas at the \( k \)th user. Considering the amount of feedback signaling, quantization is a necessity for effective feedback transmission as a form of partial CSIT. In this paper, we propose the greedy multi-channel selection diversity (greedy MCSD) scheme based on block MMSE QR decomposition with dirty paper coding (block MMSE-DP), where partial CSIT is almost sufficient. The sum-rate performance of our scheme approaches extremely close to the sum capacity of MIMO BC as the number of users increases, whereas the feedback overhead is reduced by a factor of \( 2^{2 L} / (L^{2} - 1) \), in which \( L \) is the number of active channel vectors. Simulation results validate the expectation from the analysis. In addition, the proposed scheme is shown to be appropriate for reconfigurable implementation.

key words: MIMO, broadcast channel (BC), adaptive precoding, SDR implementation

1. Introduction

Multiple-input multiple-output (MIMO) systems have been one of key techniques to achieve high rate and high reliability over wireless broadcast channels [2]–[6]. The investigation of the capacity region has been of concern in an MIMO broadcast channel (BC), where the base station has multiple transmit antennas and each user has possibly multiple receive antennas [7]–[13]. In [8], it was shown that an achievable rate region for the multiple-input single-output (MISO) BC is obtained by applying dirty paper coding (DPC), or known interference cancellation, at the transmitter [14]. Sum-power iterative water-filling (SP-IWF) provides the optimum transmission policies, whereas reducing the computational complexity and feedback overhead is still an ongoing research area [13], [15]. As a low-computational complexity approach, the greedy-type user selection [16] and the joint-channel decomposition [9], [17], [18] are utilized instead of the optimal power allocation policy, respectively. For feedback reduction, the random beamforming technique, or opportunistic beamforming [19], for MIMO BC is introduced in [20], where a significant number of users is, however, required to achieve sum capacity compared to the case of deterministic beamforming. As another approach for feedback reduction, the efficient vector quantization [21] is developed to represent the partial channel state information at the transmitter (CSIT) [22], where, however, most attention has been focused on single-user MIMO systems. That is, prior works on sum-rate near-optimal transmit schemes do not take into account the feedback overhead, to our best knowledge.

In this paper, we propose the cost-effective scheme in terms of complexity and feedback overhead that obtains near the maximum sum-rate of the wireless MIMO BC. As a solution to this problem, we propose the greedy multi-channel selection diversity (greedy MCSD) scheme based on novel block MMSE QR decomposition with DPC (block MMSE-DP) where DPC is an abbreviation for dirty paper coding. Simulation results indicate that the sum-rate performance of our scheme approaches extremely close to the sum capacity of MIMO BC with only a few users (e.g., a gap of 0.4 bps/Hz from SP-IWF), whereas the feedback overhead is significantly reduced.

The remainder of this paper is organized as follows. In Sect. 2, the system model is introduced. In Sect. 3, SP-IWF, greedy-type user selection, and time-division multiple-access MIMO (TDMA-MIMO) are analyzed. Our novel scheme is presented in Sect. 4. Section 5 provides the analysis of sum-rate performance. Numerical results are presented in Sect. 6 and reconfigurable implementation using the software defined radio technology is discussed in Sect. 7. Finally, we conclude in Sect. 8.

Notation: the superscripts \(^T\) and \(^H\) stand for transpose, conjugate transpose, respectively. The cardinality of the set \( S \) is notated as \(|S|\).

2. System Model

Consider a \( K \) user wireless communication system with multiple transmit antennas at the base station and multiple receive antennas for each user, as shown in Fig. 1. We assume that the base station has \( t \) transmit antennas, and each user \( k \) has \( r_k \) receive antennas, and the number of all receive antennas in the system is \( r = \sum_{k=1}^{K} r_k \). Also, we model the channel as a frequency-flat block fading channel. Interference from
For the comparison purpose, we present the algorithm of greedy MMSE-DP for MIMO, which is the updated version of greedy (ZF) QR decomposition with DPC (ZF-DP) for MISO in [16] so as to apply MMSE filtering and to consider the receivers equipped with multiple receive antennas. In this case, the receive array gain is, however, not exploited (see Lemma 2). The algorithm for greedy MMSE-DP is outlined in Appendix.

4. Greedy MCSD with Block MMSE-DP

We propose a multiuser MIMO scheme, i.e., greedy MCSD with block MMSE-DP, on the basis of the transmit structure in per-user unitary FB/BF and rate control (PU2RC) described in [24]. FB and BF abbreviate feedback and beamforming, respectively. The transmitter structure is shown in Fig. 2, where the precoding block, consisting of DPC and beamforming, is employed at the transmitter. Beamforming is performed using unitary matrix $W_k$ that is a function of unitary matrix $V_k$ and diagonal matrix $D_k$ for all $k$, which are obtained by singular value decomposition (SVD) such that $H_k = U_k D_k V_k^H$. Using space division multiple access (SDMA) approach, the data stream of each user can be allocated to each beam vector of the unitary transform matrix $W_k$, depending on channel conditions. Also, the transmitter adjusts the data rate per-stream independently. Note that by this structure, the proposed scheme uses spatial multiplexing to transmit simultaneously to multiple users, and multiple streams are transmitted to multiple users.

In [24], it was proposed that PU2RC performs unitary beamforming with a finite set in a predetermined way, where combining with the user and beam selection leads to additional throughput improvement due to interference reduction between users. The amount of feedback information can be reduced by using a predetermined finite set, e.g., applying a codebook design such as Grassmannian line packing to the finite set [25]. However, performance is severely degraded when there are smaller number of receive antennas than transmit antennas. To mitigate the performance degradation for such a case, in this paper we propose the enhanced PU2RC scheme, i.e., greedy MCSD with Block MMSE-DP, that is effectively combined with DPC. A detailed discussion of this scheme will be found in the following subsections. In brief, our proposed scheme uses known interference cancellation and non-predetermined beamforming at the transmitter.

4.1 Block MMSE-DP

This section discusses block MMSE QR decomposition, noting that the combination of block MMSE QR decomposition with known interference cancellation, or DPC, is denoted as block MMSE-DP throughout this paper. As the first stage of block QR decomposition, the channel is rotated using the left unitary matrix $U_i$ obtained by SVD of the each user channel (i.e. $U_i = U_i D_i V_i^H$). This is equivalent to the process that sets the receiver spatial filtering as $W_k = U_k^H$.

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Fig. 1  Greedy MCSD with block MMSE-DP in a MIMO broadcast channel.

neighboring cells is modeled as additive Gaussian noise, as we concentrate on the single cell model. The received signal of user $k$ is expressed as

$$y_k = H_k x + z_k$$

where $x$ is the $t \times 1$ sum transmit signal vector, i.e., $x = \sum_{k=1}^{K} x_k$, in which $x_k$ is the transmit signal vector intended for user $k$. The total sum transmit power of all users is constrained by $P$, i.e., $\text{tr}(\sum_{k=1}^{K} \Sigma_k) \leq P$ where $\Sigma_k = E[x_k x_k^H]$ is the transmit covariance matrix of user $k$. The $r_k \times 1$ vector $z_k$ represents the random additive noise for user $k$ where $z_k \sim \mathcal{CN}(0, I)$. The channel $H_k$ is an $r_k \times t$ matrix, whose entries are assumed to be independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero-mean and unit variance. Also, $H_k$ is independent of $H_j$ for all $j \neq k$.

In general, it is difficult for the base station to have the perfect knowledge of CSIT because the feedback link has delayed lossy feedback characteristics. Hence, the problem at hand is to find the transmit and receive structure that minimizes the feedback rate subject to the performance constraint such that the data throughput is kept as close as possible to the sum capacity.

3. SP-IWF, TDMA-MIMO, and MMSE-DP

In this section, we describe the following approaches: SP-IWF [13], TDMA-MIMO [23], and greedy MMSE-DP. The sum-rate maximization can be solved by using SP-IWF, which achieves the sum capacity of an MIMO BC at the cost of higher complexity. In the case of TDMA-MIMO, the base station transmits to only a single-user at a time by using all transmit antennas, which is a suboptimal solution when the base station has multiple transmit antennas. So the optimality holds if and only if the number of transmit antennas is equal to one. It is shown that the maximum sum-rate of TDMA-MIMO is the largest single-user capacity of the $K$ users, which is given by

$$C_{TD} = \max_{i=1,...,K} C(H_i, P)$$

where $C(H_i, P)$ denotes the single-user capacity of the $i$th user subject to power constraint $P$. 

![Diagram](attachment:image.png)
(see Lemma 1). By feeding back $F_k$ instead of $H_k$ to the base
station, the feedback overhead can be reduced by a factor of
$2r^2/(r^2 - r)$ as described in [22], where the rotated channel
matrix $F_k$ is given by

$$F_k = U_k^H H_k = D_k V_k^H.$$  (3)

It can be seen that rotating with the left unitary matrix de-
composes MIMO channels into multiple MISO channels
$F^H = [F_1^H, \ldots, F_K^H] = [f_1, \ldots, f_K]$, which is referred to as
the rotated channel matrix. We also denote each row of $F_k$ as
the rotated channel vector. That is, since $F^H$ is the combi-
nation of the rotated channel vectors $f_i \in C^{r \times 1}$ for
$i = 1, \ldots, r$, $F$ can be regarded as the multiple MISO cha-
nels.

For the second stage, controlled beamforming, which is
implemented by applying MMSE QR decomposition to
the combination of the rotated channels, is employed at the
base station. As in the algorithm of [16] for MISO, the QR
decomposition is obtained using the Gram-Schmidt orthog-
normalization procedure (e.g. to the rows of $F$ in this case).
Hence, the block MMSE QR decomposition consists of two
consecutive procedures such that geometrical projection is
performed to obtain $F$ using SVD decomposition of $H_k$ for
all $k$ and then the finite dimensional subspace is determined
by QR process with $F$. Mathematically, MMSE QR decom-
position of $F$ leads to the solution for transmit beamforming
satisfying the constraints in Theorem 1. The solution is
$W_k = [w_1, w_2, \ldots, w_r]$ where $w_j$ for $j = 1, \ldots, r$ is given by

$$w_j = \frac{\bar{w}_j}{||\bar{w}_j||}, \quad \bar{w}_j = \left( \sum_{l=1}^{j-1} f_l f_l^H + \frac{r}{P} I \right)^{-1} f_j$$  (4)

when equal power allocation and no ordering are assumed for
different $f_i$. The transmitted signal is represented as

$$r = \hat{s} + z = F W_k \hat{s} + z$$  (5)

where $\hat{s}$ is the DPC encoded signal vector from the original
signal vector $s$, $z = [z'_1, \ldots, z'_r]^T$ and $r = [r'_1, \ldots, r'_r]^T$, in
which $r_k = U_k^H y_k$. The optimality of block QR decom-
position and the actual implementation of this technique, i.e.,
determining the optimal strategy for power allocation and
ordering, are treated in the following two subsections, re-
spectively.

4.2 Optimality of Block QR Decomposition

In order to derive the procedure employing known inter-
ference cancellation, the congregate interfering channel matrix
is defined as

$$H_k = \begin{bmatrix} H_1^T & H_2^T & \cdots & H_{k-1}^T \end{bmatrix}^T.$$  (6)

Theorem 1: The objective of the transmit covariance ma-
trix design for block MMSE-DP is to find a covariance ma-
trix set that maximizes the system throughput, subject to
the unknown-interference free constraint and the sum power
constraint. Note that the unknown-interference free con-
straint leads to the block MMSE-DP solution, which reduces
complexity and feedback overhead at the expense of negli-
gible throughput performance degradation. The transmit
covariance matrix satisfying this objective is obtained by QR
decomposition of $F$ with appropriate power allocation.

Proof: The problem may be formulated as finding a
transmit covariance matrix set, $\{\Sigma_k\}_{k=1}^K$, such that

$$\{\Sigma_k\}_{k=1}^K = \arg \max_{\{\Sigma_k\}_{k=1}^K} R_{BM}(\{\Sigma_k\}_{k=1}^K)$$  (7)

where

$$R_{BM}(\{\Sigma_k\}_{k=1}^K) = \sum_{k=1}^K \log \frac{\|I + H_k (\Sigma_{k+1} + \cdots + \Sigma_K) H_k^H\|_F}{\|I + H_k (\Sigma_{k+1} + \cdots + \Sigma_K) H_k^H\|_F},$$  (8)

subject to the unknown-interference free constraint

$$H_i\Sigma_k = F_i\Sigma_k = 0, \quad i = 1, \ldots, K, \quad k < i$$  (9)

and the sum power constraint

$$\sum_{k=1}^K \text{tr}(\Sigma_k) \leq P, \quad x^H \Sigma_k x \geq 0, \forall x \in C^r, \forall k.$$  (10)

The constraint of (9) ensures that no interference to user $k$
can be caused by the users from $k + 1$ to $K$, noting that inter-
ference from other users (i.e. $1, \ldots, k - 1$) is presubtracted
by known interference cancellation (i.e. DPC encoding). Let
$\tilde{V}_k^T$ consist of the right singular vectors corresponding to
the zero singular values of $H_k$, which is equivalently obtained
by SVD of $\tilde{F}_k = [F_1^H, F_2^H, \ldots, F_{k-1}^H]^H$ as follows

$$\tilde{F}_k = \begin{bmatrix} \tilde{D}_k & 0 & 0 \\ 0 & V_k^1 & V_k^0 \end{bmatrix}^H$$  (11)

where $\tilde{D}_k$ is a diagonal matrix. Thus, constructing $\Sigma_k$ with
$V_k^0$ satisfies the constraint in (9), i.e., $\Sigma_k = V_k^0 \Sigma_k^0 V_k^0$ where
$\Sigma_k^0$ is design as follows. When the constraint in (9) is satisfied,
the system throughput is simply derived as

$$R_{BM}^* = \max_{\{\Sigma_k\}_{k=1}^K} R_{BM}(\{\Sigma_k\}_{k=1}^K).$$

Each element of $\{\Sigma_k\}_{k=1}^K$ is a positive semidefinite matrix,
since it is a covariance matrix as defined in Sect. 2.
MIMO BC, key functions. Firstly, the rotated channel vectors described in [16]. In our proposed method, we present two.

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selected rotated channels with $\frac{r_j}{L}$.

Moreover, we also take into account the approach that each user selects and feeds back $L$ active channel vectors out of $\min(t, r_j)$ rotated channel vectors corresponding to the $L$ largest eigenmodes, where eigenmode $\lambda_j$ is the power of the $j$th diagonal element of $D_k = \text{diag}(d_{k1}, \ldots, d_{k\min(t, r_j)})$. This approach offers further reduction in the feedback amount by a factor of $\min(t, r_j)/L$. Note that although the feedback amount is significantly reduced by user-side selection, the loss of throughput performance in MCSD is negligible as analyzed in Theorem 3.

5. Performance Analysis

In this section, the performance analysis is presented. Throughout this paper, the entries of $H_k$ are assumed to be i.i.d. zero-mean complex-Gaussian random variables.

Lemma 1: Assume that CSIT of all user $j$ ($\neq k$), i.e., $\{H_j\}_{j \neq k}$, is not known to user $k$. That is, CSIT of all user $j$ is not delivered to user $k$ from the transmitter as well as not exchanged between users. In this case, each user $k$ can estimate the achievable throughput by using $\tilde{F}_k = U_k^H H_k$, assuming that interference from all user $j$ is averaged out.

Proof: By applying the duality of MIMO BC and MAC [10], the sum capacity of an MIMO BC, which is equivalent to that of a dual MIMO MAC, with perfect channel knowledge at the transmitter is given by

$$C_{BC} = \max_{\Sigma_k \in \mathbb{S}_+^K} f_k(S_k)$$

where $S_k$ is the transmit covariance matrix of user $i$ in a dual MIMO MAC. The cost function $f_k(S_k)$ is given by

$$f_k(S_k) = \log_2 \left| I + \sum_{i=1}^{K} \tilde{H}_i^H S_i H_i \right|$$

where $\tilde{H}_k = H_k \Phi_{1/k}$ and $\Phi_{1/k}$ is the corresponding submatrix of $\Phi(A)$.

The maximum sum-rate of this system is then given by

$$R_{MCSD} = \max_A f_k(A)$$

$$= \max_A \left| \tilde{H}_k \right|$$

The cost function $f_k(A)$ is defined as

$$f_k(A) = \log_2 \left| \frac{P}{|A|} \Phi_{A \cup |A|} \right|$$

where $\Phi_{A \cup |A|}$ is averaged out.

1For example, the transmit covariance matrix of user $k$ in (4) is given by $\Sigma_{k, mmse} = P_k w_k w_k^H$ when $r_k = 1$. 

4.3 Greedy Multi-Channel Selection Diversity

Multiuser diversity is the promising solution to achieve the sum capacity of the multiuser channel. In the proposed scheme, we select the strong rotated channel vectors among available multi-user rotated channel vectors. Greedy MCSD is processed through the greedy-type ordering and selection of the channel vectors for active users. When channel vectors are selected and ordered, diversity gain is achieved with the increase of the number of users and antennas therein. The similar approach for MISO case was greedy ZF-DP as described in [16]. In our proposed method, we present two key functions. Firstly, the rotated channel vectors $\{f_j\}$ are exploited instead of the channel vectors $\{h_j\}$, since we consider multi-channel selection diversity but not just multiuser selection. Secondly, to further reduce the feedback overhead, a part of the channel selection process is performed at the user-side as well.

Let $A \subset \{1, \ldots, r\}, |A| \leq \frac{1}{2}t(t + 1)$ be a subset of the rotated channel vector indices that the base station selects for transmission using MCSD, and $F(A)^H = [F_1^A, \ldots, F_K^A]$ be the corresponding submatrix of $F$, where $F_k^A$ is the $A$th column vector of $F(A)^H$. The $t \times |A|$ unitary beamforming matrix $W_{tx}(A) = [w_1, \ldots, w_A]$ in (5) must now be obtained by MMSE QR decomposition of submatrix $F(A)$. The maximum sum-rate of this system is then given by

$$R_{MCSD} = \max_A f_k(A)$$

$$\leq R_{BM}(\Sigma_k \Box_{i=1}^K).$$

(14)

The cost function $f_k(A)$ is defined as

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and as in (4), $w_j = \sqrt{v_j}/\sqrt{\|v_j\|}$, $\tilde{w}_j = \Phi_{A \cup |A|}^{-1} f_j$. The equality in (14) holds if and only if $|A|$, ordering, and power allocation of the selected channels are optimized, whereas power is allowed to be equally distributed among the selected rotated channels with $\frac{r_j}{L}$ in (14). Moreover, we also take into account the approach that each user selects and feeds back $L$ active channel vectors out of $\min(t, r_j)$ rotated channel vectors corresponding to the $L$ largest eigenmodes, where eigenmode $\lambda_j$ is the power of the $j$th diagonal element of $D_k = \text{diag}(d_{k1}, \ldots, d_{k\min(t, r_j)})$. This approach offers further reduction in the feedback amount by a factor of $\min(t, r_j)/L$. Note that although the feedback amount is significantly reduced by user-side selection, the loss of throughput performance in MCSD is negligible as analyzed in Theorem 3.

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(17)

where $\tilde{H}_k = H_k \Phi_{1/k}$ and $\Phi_{1/k}$ is the corresponding submatrix of $\Phi(A)$.

Because of the assumption that user $k$ is not allowed to know CSIT of all other users (except $E[\Phi(A)] = a I$ where $a = 1 + \frac{k-1}{K}$ is a scalar), $\tilde{H}_k$ is transformed to $G_k = H_k E[\Phi_{1/k}]^{-1} = \frac{1}{a} H_k$. Hence, the maximum achievable rate can be derived by applying SVD to $G_k$, where $H_k$ is rotated by receive beamforming such that $F_k = U_k^H H_k$.

Lemma 2: We compare the performance of block MMSE-DP with (non-block) MMSE-DP in terms of the achievable

throughput. In block MMSE-DP, receive beamforming is performed by the left unitary matrix of the corresponding
channel, whereas MMSE-DP is known to be optimal for non-cooperative reception across receive antennas. For average throughput, block MMSE-DP outperforms MMSE-DP in an MIMO BC.

**Proof:** The problem in question can be seen as the comparison between two different scenarios: beamforming with $W_{rx} = U_k^H$ and beamforming with $W_{rx} = I$. Since SVD-based processing is optimal for the single-user case, the capacity of the dual MAC subject to $W_{rx} = U_k^H$ is larger or equal to that subject to $W_{rx} = I$, as follows

$$\max_{\Lambda \geq 0} \log \det(I + R_{U,\Lambda}(\Lambda)) \geq \max_{\Lambda \geq 0} \log \det(I + R_{U,\Lambda}(\Lambda))$$

(18)

where $R_{U,\Lambda}(\Lambda) = H_k^H U_k \Lambda U_k^H H_k = H_k^H A \Lambda_k$, $R_{U,\Lambda}(\Lambda) = H_k^H A H_k$, and $\Lambda$ is constrained to be a diagonal matrix. In the following, we show that the single-user inequality in (18) leads to the proof of the given Lemma. Applying the duality principle as in (16), the sum capacity obtained based on non-cooperative reception in the $k$th user, i.e., $W_{rx} = I$, can be represented as

$$R_k = \max_{\Lambda \geq 0} \log \det(I + R_{U,\Lambda}(\Lambda) + \sum_{i \neq k} H_i^H Q_i H_i).$$

(19)

where the maximization is subject to $\text{tr}(\Lambda) + \sum_{i \neq k} \text{tr}(Q_i) \leq P$ and $\Lambda, Q_i \geq 0$. Similarly, the maximum sum-rate obtained with the unitary beamforming at the receiver is expressed as

$$R_{U} = \max_{\Lambda \geq 0} \log \det(I + R_{U,\Lambda}(\Lambda) + \sum_{i \neq k} H_i^H Q_i H_i).$$

(20)

Observing the inequality in (18) and the same term added in both (19) and (21), we see that the average maximum sum-rate $E[R_{U}]$ is larger or equal to the average sum capacity for non-cooperative reception $E[R_k]$. Applying this result to the problem, it follows that

$$E[R_{BM}] \geq E[R_{MMSE-DP}]$$

(21)

which completes the proof. Note that it might not be true for instantaneous throughput. □

**Theorem 2:** In MIMO BC, the system with beamforming at each receiver using the left singular matrix offers the average throughput that is no worse than using any fixed unitary matrix beam at all receivers.

**Proof:** The proof is easy. We prove Theorem 2 based on Lemmas 1 and 2. In terms of minimizing interference, the fixed beam scheme performs better than the proposed beamforming in certain channel realizations or vice versa. Both cases are equally probable, i.e., $p_1 = p_2$ where $p_1$ and $p_2$ represent the probability of each case, respectively. This follows from the fact that $H_k$ is i.i.d. over index $k$. In other realizations with probability $p_3 = 1 - (p_1 + p_2)$, the proposed scheme always outperforms the other because of the signal-to-noise ratio (SNR) advantage. Thus, the average performance of the fixed beam scheme is no better than the proposed scheme as derived in (18). Note that the equality holds only when $p_3 = 0$. □

**Theorem 3:** In the proposed MCSD scheme, we select the strong effective (sub) channels, of which the number is larger or equal to one. The sum-rate achievable with the selected channels is almost equal to the sum capacity of total MISO channels $\{F_k\}_{k=1}^K$ in (23).

**Proof:** This channel selection exploits the fact that similar eigen-vectors, or sub channels, can exist in multiple MISO channels. Since the eigen-vectors inside a single user MIMO channel are not similar to one another, i.e., they are orthogonal, the problem in this theorem is different from the water-filling problem in single-user MIMO. Hence, we prove Theorem 3 by showing that in the multiple MISO channels, optimal power allocation is obtained based on solely the maximum eigen-values of each group if eigen-vectors in each group are nearly the same. Let $|\nu_{i,m}| = |\lambda_l|$ if $||\nu_i - \nu_l||^2 \leq ||\nu_i - \nu_l||^2$ for all $n \neq l$, $l = 1, \ldots, L$, where $\lambda_l$ and $\nu_i$ are the $i$th eigen-value and the $i$th eigen-vector, respectively. We also assume that $L$ is large enough so that $||\nu_i - \nu_l||^2$ is sufficiently small for all $l$. The sum capacity is then given by

$$R_{BM}(\Sigma_{i=1}^K) = \max_{P_k \geq 0, \sum P_k \leq P} \log \det(I + \sum \nu_i P_i \nu_i^H)$$

$$\approx \max_{P_k \geq 0, \sum P_k \leq P} \log \det(I + \sum \nu_i P_i \nu_i^H)$$

(22)

where $\nu_i = \nu_i \max_{\nu_i \in \Sigma_{i=1}^K}$, and $v_i \max = \max_n |\nu_{i,m}|$. In (22), we use the property of multiuser diversity [27], in which the sum rate is maximized by allocating no transmit power to certain sub channels if there is any other sub channel with the same direction and higher gain. □

6. **Numerical Results**

In this section, numerical results are presented. In Figs. 3 and 4, we compare the ergodic sum-rate performance of different MIMO downlink strategies. The SNR is assumed to be 10 dB. Given the number of users, TDMA-MIMO achieves the maximum sum-rate corresponding to the largest single-user capacity, which shows relatively a small gain in proportion to the number of users. When the number of the active channel vectors is equal to the number of the rotated channel vectors with the number of users being one, the performance of the proposed novel scheme is the same as that of TDMA-MIMO since in both cases receivers feed back the rotated channel matrix $F_k = D_k V_k^H$, instead of the full channel matrix $H_k$. In both figures, the performance of the novel scheme with full rotated channel vectors (i.e. $L = 2, 4$, respectively) is extremely close to the sum capacity driven by SP-IWF, while the performance with the active (partial rotated) channel vector (i.e. $L = 1$)
reaches the sum-rate with full vectors when the number of users is equal to 5 and 10, respectively. Both figures show sum-rate improvement of 2 bps/Hz over MMSE-DP scheme with full channel feedback and the gap of 0.4 bps/Hz from SP-IWF.

Furthermore, both figures illustrate the behaviors of sum-rate corresponding to different feedback overheads. In Fig. 3, each user has two eigenmodes, i.e., two rotated channel vectors, available since four transmit and two receive antennas are assumed. The sum-rate of the novel scheme with feedback of one active channel vector (one eigenvector multiplied by the corresponding eigenvalue that is the largest one) gets tightly close to the performance having feedback of two active channel vectors when the number of users is five. Contrastingly, TDMA-MIMO with one vector never becomes close to TDMA-MIMO with two vector. Four transmit and four receive antennas are considered in Fig. 4, where two feedback signaling (i.e., one, four active channel vectors) are examined for the novel and TDMA-MIMO schemes. Both figures show that the novel scheme with reduced feedback, i.e., with the fewer active channel vectors, achieves slightly lower rate performance with small number of users compared to the scheme with full rotated channel vector, and however, the performance approaches extremely close to the upper bound as the number of users increases. Note that when the feedback bandwidth is fixed, the reduction of active channel vectors may improve the quality of feedback signaling, of which the level can be represented based on the Shannon distortion-rate function \( D(R) \propto 2^{-2\alpha_1 R} \) where \( R \) is the given feedback rate and \( \alpha_1 = L/\min(t, r_k) \). Therefore, in the proposed scheme feedback of active channel vectors is shown to have the equivalent sum-rate performance with feedback of full rotated channel vectors, resulting in the outstanding feedback robustness. That is, the feedback signaling per user can be significantly reduced with the increase of the number of users.

7. Reconfigurable Implementation

Wireless MIMO systems would be capable of operating in a wide variety of scenarios with respect to the associated parameters, e.g., the number of users and the degrees of knowledge of CSIT. Reconfigurability, namely adaptability, is an important design principle that can be considered for MIMO systems in a diverse environment. Software defined radio, as an example of reconfigurable systems, attempts to achieve the best performance in each respective case whereas systems without reconfigurability maintain reasonable performance, which is considered to be robust in a varying environment. The study of adaptability in MIMO systems is motivated on a fundamental tradeoff between the number of active eigenmodes and the required feedback rate as discussed in Sect. 6. The reconfigurable system chooses the best number of active eigenmodes in terms of throughput performance given a constraint on the required amount of feedback. Although the TDMA-MIMO performance strongly depends on the trade-off, reconfigurability with respect to the number of users offers almost no performance benefit for the TDMA-MIMO system as shown in Fig. 4. However, a significant performance advantage is observed for greedy MCSD with block MMSE-DP.

Figure 5 compares a reconfigurable system against a robust system with the predetermined parameters in terms of sum rate. It is assumed that \( t = r_k = 4 \) for all \( k \). Based on the aforementioned MIMO fundamental trade-off, the reconfigurable MCSD operates with the variable number of active vectors: \( L = 4 \) for \( K = 1 \), \( L = 2 \) for \( K = 2 \), and \( L = 1 \) for \( K = 4 \). Note that the total amount of feedback stays the same for all three cases, i.e. \( KL = 4 \). For robust systems, however, we consider MCSD with the fixed number of active vectors for all \( K \), e.g., \( L = 1, 2, 4 \), since it does not allow operating with variable parameters. Thus, the required amount of feedback increases by the number of active vectors as a single user is added to the system. The figure represents that the reconfigurable system has the throughput performance two times higher than that of the robust system with \( L = 1 \) at \( K = 1 \), since it selects \( L = 4 \) for \( K = 1 \). On the other hand, the reconfigurable system chooses \( L = 1 \) for \( K = 4 \) and decreases the required feedback rate four times lower than that of the robust system with \( L = 4 \) and \( K = 4 \).
In this paper, we have proposed a multiuser MIMO transmission scheme that is efficient in terms of computational complexity and feedback overhead while obtaining near the maximum sum-rate of BC. Our novel scheme has employed block MMSE-DP at the transmitter, which reduces the computational complexity of designing transmit covariance matrices. Using MCSD in combination with block MMSE-DP, the proposed scheme with partial CSIT has still achieved the near-optimal sum capacity, which was not observed in TDMA-MIMO. Numerical results have shown that the gain of sum-rate is 2 bps/Hz over the conventional MMSE-DP scheme with full channel feedback and the gap from SP-IWF is negligibly small (i.e. 0.4 bps/Hz). Moreover, the proposed scheme has been shown to be appropriate for reconfigurable implementation and to reduce a considerable amount of feedback by adaptively selecting the number of active vectors without losing noticeable performance.

8. Conclusion

In this paper, we have proposed a multiuser MIMO transmission scheme that is efficient in terms of computational complexity and feedback overhead while obtaining near the maximum sum-rate of BC. Our novel scheme has employed block MMSE-DP at the transmitter, which reduces the computational complexity of designing transmit covariance matrices. Using MCSD in combination with block MMSE-DP, the proposed scheme with partial CSIT has still achieved the near-optimal sum capacity, which was not observed in TDMA-MIMO. Numerical results have shown that the gain of sum-rate is 2 bps/Hz over the conventional MMSE-DP scheme with full channel feedback and the gap from SP-IWF is negligibly small (i.e. 0.4 bps/Hz). Moreover, the proposed scheme has been shown to be appropriate for reconfigurable implementation and to reduce a considerable amount of feedback by adaptively selecting the number of active vectors without losing noticeable performance.

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References

The algorithm for greedy MMSE-DP is outlined as follows:

**Appendix: Greedy MMSE-DP Algorithm**

1. The base station selects the index pairs of user and its receive antenna, \( \{k_j, i_j\}, \; j \leq \frac{1}{2}(t + 1), \; j \leq r \) [29], [30] corresponding to the downlink channel matrices \( \mathbf{H}_{k} \) where \( \mathbf{H}_{k} = [\mathbf{h}_{k,1}, \ldots, \mathbf{h}_{k,K}] \). Before starting the procedure, we take as initial value \( m = 0 \).
2. Let \( m = m + 1 \), and if \( m > K \), stop the procedure. Otherwise, calculate the received SNR as a function of \( m \) for all \( (k, i) \) as follows:
   \[
   \gamma_{k,i}(m) = \mathbf{h}_{k,i}^H \left( \sum_{j=1}^{m-1} \mathbf{h}_{k,j,i} \mathbf{h}_{k,j,i}^H + \frac{m}{P} \right)^{-1} \mathbf{h}_{k,i}, \quad (A-1)
   \]
determine the \( m \) th selected index pair of user and its antenna as follows:
   \[
   (k_m, i_m) = \arg \max_{k \in \{1, \ldots, K\}; \; i \in \{1, \ldots, r_i\}} \gamma_{k,i}(m), \quad (A-2)
   \]
and calculate the sum-rate achievable with \( \gamma_{k_1,i_1}, \ldots, \gamma_{k_m,i_m} \) as follows:
   \[
   R_m = \sum_{j=1}^{m} \log_2 (1 + \gamma_{k_j,i_j}(m)). \quad (A-3)
   \]
3. Repeat Step 2 until \( R_m \leq R_{m-1} \).
4. Transmit
   \[
   \mathbf{x} = \frac{P}{m_o} \mathbf{W}_t \hat{\mathbf{s}} \quad (A-4)
   \]
where \( m_o = m - 1 \), \( \hat{\mathbf{s}} = [\hat{s}_{k_1}, \hat{s}_{k_2}, \ldots, \hat{s}_{k_{m_o}}]^T \), and \( \mathbf{W}_t = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{m_o}] \). The transmit basis \( \mathbf{w}_j \) is given by
   \[
   \mathbf{w}_j = \frac{\tilde{\mathbf{w}}_j}{\|\tilde{\mathbf{w}}_j\|}, \quad \tilde{\mathbf{w}}_j = \left( \sum_{l=1}^{m-1} \mathbf{h}_{k_l,i_l} \mathbf{h}_{k_l,i_l}^H + \frac{m}{P} \right)^{-1} \mathbf{h}_{k_j,i_j}, \quad (A-5)
   \]

Here, \( \hat{s}_{kj} \) is the output of the DPC encoding equation parameterized as a function of \( \{\hat{s}_{k}, \mathbf{h}_{k,j}^H, \mathbf{w}_j\}_{j=1}^{m} \) where the signal \( s_{kj} \) is associated to the \( k \)th user [31]. Note that the total throughput of this scheme is given by
   \[
   R_{\text{MMSE-DP}} = R_{m-1}.
   \]

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