Outsourcing versus In-house Production: The Case of Product Differentiation with Cost Uncertainty

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The paper examines the relationship between the choice of outsourcing versus in-house production and product differentiation. We analyze the situation where a manufacturer decides between in-house production and outsourcing when faced with cost uncertainty and competition with a rival manufacturer in a differentiated goods market. We show that the degree of product differentiation does not affect the choice between in-house production and outsourcing. The result suggests that regardless of the intensity of competition, the manufacturer should decide on outsourcing if the degree of the cost efficiency of outsourcing exceeds certain thresholds.

Keywords: In-house production, Outsourcing, Asymmetric information, Cost uncertainty, Differentiated-goods duopoly

JEL Classification: D43, D82, L13

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I. Introduction

In recent decades, in light of the increasing fragmentation of economies expedited by rapid globalization, outsourcing has become a trend in organizational forms for modern firms in the entire industries. A recent article in the *Economist* (2008) reports on the exceptionally fast growth of outsourcing of late. For example, in the 1940s, outsourcing was estimated to yield only 20% of the GDP of the US, but in the 1990s, the proportion tripled to 60%. In the automobile industry in the 1990s, successful firms, such as Toyota, Honda, and Chrysler, were outsourcing around 70% of value-added in contrast with the least profitable firms, such as General Motors, that were outsourcing only 30%. In the context of business strategy, most managers have to decide on whether to produce their products in their own plants or subcontract them to the outsourcer. Through the development of Information and Communication Technology, outsourcing entails not only production activities but almost all other areas of firm activities, including human resource management and research and development.

One of the reasons for choosing outsourcing is technological efficiency. When the outsourcer possesses advanced technology, the manufacturer can share more profit with the outsourcer even if it cannot monopolize profit. Nevertheless, outsourcing has disadvantages in that production activities become a black box for manufacturers. If manufacturers are not aware of the technological information outsourcers possess, they are required to pay the outsourcers rent for the extra information. Therefore, manufacturers face a trade-off between gain from the efficient technology and loss from paying the information rent. When a firm decides on in-house production instead of outsourcing, although a manufacturer can obtain the entire profit, it loses the gain from the cost efficiency of outsourcing. When outsourcing is selected, although a manufacturer can acquire the gain from cost efficiency, it is required to share the gain with the outsourcer.

Considering this trade-off in this paper, we examine the form of production wherein a manufacturer chooses between in-house production and outsourcing when faced with cost uncertainty and competition with a rival manufacturer in a differentiated goods market. When the management decides on selecting organizational forms, technological uncertainty on production activities often ensues. Thus, a manufacturer faces uncertainty when choosing between in-house production and outsourc-
ing. Introducing the uncertainty of production cost into the model, we investigate how the degree of cost efficiency in outsourcing affects the choice of the form of production by a manufacturer with cost uncertainty. Moreover, because almost all modern firms are in a competitive position, they have to choose organizational forms and take the strategic effect of its decision on other rival firms into consideration. We examine how the choice of outsourcing versus in-house production is affected by the degree of product differentiation by addressing the market competition between manufacturers in conjunction with cost uncertainty.

Outsourcing is a key aspect of modern industrial production, and it has sparked considerable interest in the economic literature. Thus, there is a huge bulk of existing literature on outsourcing under different market conditions. As one of the leading papers, Grossman and Helpman (2005) present a framework based on which firms decide where to outsource in a general equilibrium model. In their model, outsourcing is related to relationship-specific investments governed by incomplete contracts. They clarify the determinants of the location of outsourcing. Grossman and Helpman (2004) compare in-house production and outsourcing within the context of a moral hazard wherein a firm is constrained by the nature of the offered contract to an outsourcer. They clarify the relationship between the productivity of the firm and the choice of organizational forms. Recently, Grossman and Rossi-Hansberg (2008) develop a tractable model of offshoring/outsourcing based on global tradable tasks.

Several papers deal with the choice of outsourcing versus in-house production for firms. Kamien et al. (1989) analyze the situation where firms compete in a Bertrand duopolistic competition in the first stage and have the option to choose subcontracting in the second stage. They examine how subsequent subcontracting production influences the initial price competition. Alyson (2006) examines the choice of in-house production and outsourcing by multinationals using a general equilibrium model by focusing on the varying availabilities of skilled labor among countries. He clarifies that the relative amount of highly skilled labor in the country affects the choice between in-house production and outsourcing. Nickerson and Bergh (1999) explore a duopoly model with Cournot competition to analyze the choice of organizational form by competing firms. They investigate how strategic interaction and governance costs affect organizational choice. Alvarez and Stenbacka (2007) find that an increase in market uncertainty leads to a higher proportion of partial outsourcing by applying a real options approach.
A prevailing view posits that outsourcing is fostered by the intensification of competition brought about by globalization. There are several articles that present the positive relationship between the intensifying competition and increase in outsourcing. Shy and Stenbacka (2003) focus on the strategic aspect of the design of organizational forms and show how competition in the input market affects production efficiency in a differentiated final goods market. They clarify the relationship between the intensity of competition in the input market and choice of organizational forms. In a closely related article, Shy and Stenbacka (2005) investigate the outsourcing decisions of firms when production requires a large number of inputs and find the optimal proportion of partial outsourcing, which is regarded as the equilibrium fraction of outsourced inputs. They show that intensified competition in a final goods market enlarges the set of outsourced components because the advantage of marginal monitoring costs by outsourcing is increased due to intensifying competition.

In contrast to Shy and Stenbacka (2003) that analyzed the relationship between outsourcing and the intensity of competition in the input market, our paper focuses on competition in the final goods market. Shy and Stenbacka (2003, 2005) and Alvarez and Stenbacka (2007) also analyze partial outsourcing to examine the optimal proportion of in-house production and outsourcing. In particular, Shy and Stenbacka (2005) demonstrate how intensifying competition enlarges the proportion of partial outsourcing. To justify the analysis of partial outsourcing, the existing literature presumes that firms can choose the parameters for organizational forms continuously. However, as the theory of organizational design suggests, firms often have to choose the parameters for organizational forms from discrete variables. Roberts (2004) introduces “non-convexity” in the set of available choices, implying that choices are not infinitely divisible as a key concept for organization design. For example, Roberts (2004) states that the firm cannot have a fractional number of plants; it either enters a market or does not. However, indivisibilities abound in the set of alternatives when firms decide. Similarly, in the case of decision making on organizational forms of firms, the choice between in-house production and outsourcing cannot be made at an intermediate level. Thus, partial outsourcing is not an option for firms. In accordance with the idea of non-convexity, this paper compares in-house production with outsourcing as a discrete choice.

Contrary to existing literature on outsourcing, this paper explores a model that can be utilized in deciding which organizational forms to
use with asymmetric information along with uncertainty. The seminal articles analyze the situation where the outsourcing firm is aware of the ability of the outsourcer in advance before offering the outsourcing contract. In contrast, this paper analyzes the case of asymmetric information between a manufacturer and outsourcer. Thus, our paper attempts to examine how the expected profit of the manufacturer is affected by the information rent paid to an outsourcer when the manufacturer is unaware of the ability of the outsourcer. This paper explores a duopoly model to address not only the uncertainty in the choice of organizational forms but also the asymmetric information between a manufacturer and an outsourcer regarding the marginal cost of outsourcing. Contrary to existing literature where the monitoring cost of outsourcing is provided exogenously as a strictly increasing convex function regarding the number of outsourcers, the present study analyzes the monitoring cost as the information rent paid explicitly to the outsourcer.

Our paper is closely related to the literature on delegation, as in the seminal papers by Katz (1991) and Fershtman et al. (1991). They focus on the strategic delegation under which the owner offers an incentive contract to the manager to determine the intensity of competition. However, in most of the existing literature, the delegating owner is already aware of the ability of the manager in advance before offering the contract and the hidden information of a delegated agent is not addressed. In the model, hidden information is a key difference of outsourcing from in-house production. Thus, we analyze what will occur if there is asymmetric information between a manufacturer and its outsourcer. Recently, Martimort and Piccolo (2010) analyze the strategic value of quantity forcing contracts in the competing manufacturer-retailer hierarchies. They show that manufacturers may leave contracts to retailers incomplete. However, the information structure in our paper differs from that in their paper. They analyze the situation where the private information for retailers is perfectly correlated. In contrast, our paper deals with the situation where the private information possessed by outsourcers is not correlated. Moreover, we focus on the strategic effect caused by hidden information alone during outsourcing.

1 Using a setting similar to that used in this paper, Hamada (2005) deals with the relationship between the profitability of the manufacturer and its decision on whether to observe the hidden information of a delegated agent in the delegation game. However, the choice of production form by the manufacturer is not considered.
Endogenizing the monitoring cost by introducing asymmetric information into the model, our paper shows that the degree of product differentiation does not affect the choice between in-house production and outsourcing. If the degree of product differentiation has an inverse relation to the intensity of competition, the result suggests that, regardless of the intensity of competition, the manufacturer decides on outsourcing if the degree of the cost efficiency of outsourcing exceeds certain thresholds.

The remainder of this paper is organized as follows. Section II introduces the model that allows each manufacturer to choose its organizational form of production when faced with cost uncertainty and competition with a rival manufacturer in a differentiated goods market. Section III derives the quantity and expected profit in the Cournot equilibrium. Section IV compares the expected profits of in-house production and outsourcing and presents the main results regarding the relationship between the degree of product differentiation and the choice of production forms. Section V presents the concluding remarks.

II. The Model

Consider that in a differentiated goods market, there are two manufacturers whose products have brand royalty. The manufacturer, \( M_i \) (\( i = 1, 2 \)), deliberates on whether to opt for in-house production or outsourcing to produce a good. If in-house production is the preferred option, \( M_i \) realizes the production activities for itself. If outsourcing is decided on, \( M_i \) hires an outsourcer and delegates to the outsourcer, \( O_i \), the responsibility of producing the brand product exclusively.

The product is produced with a constant marginal cost, regardless of the form of production chosen. To analyze the choice under cost uncertainty, we assume that at the time of choosing the production form between in-house production and outsourcing, \( M_i \) is unaware of not only the marginal cost of the outsourcer but also of its own. After choosing the production form, when in-house production is chosen, \( M_i \) duly recognizes its own marginal cost \( \theta_i^M \). When outsourcing is the preferred option, \( M_i \) searches for an outsourcer from several potential outsourcers before it learns of its own marginal cost. In outsourcing, a manufacturer possessing brand royalty with regard to differentiated goods delegates production activities to an outsourcer exclusively. After an outsourcer is chosen but before the contract is offered, the chosen \( O_i \),
determines its marginal cost through the preparation of production. When \( M_i \) offers a delegating contract to \( O_i \), \( M_i \) is unaware of the marginal cost of the delegated outsourcer, \( \theta_i^O \), while \( O_i \) knows its own marginal cost, \( \theta_i^O \). That is, \( \theta_i^O \) constitutes the private information for \( O_i \). We refer to \( \theta_i^O \) as \( O_i \)'s type. We denote the situation where \( M_i \) (resp. \( M_i' \)) chooses in-house production or outsourcing by \( \omega \) (resp. \( \omega' \)) \( \in \{M, O\} \), where \( M \) (resp. \( O \)) denotes in-house production by the manufacturer (resp. outsourcing).

\( M_i \) is unaware of the true value of marginal cost when choosing the production form; hence, at the decision stage, \( M_i \) forecasts that \( \theta_i^\omega; \omega \in \{M, O\} \) follows a certain ex ante probability distribution. For analytical simplification, we assume that both manufacturers face an identical probability structure; that is, \( \theta_i^M \) and \( \theta_i^O \) follow identical and independent distributions. However, it should be noted that the probability distribution when in-house production is chosen differs from that when outsourcing is chosen. \( \theta_i^M \) and \( \theta_i^O \) follow independent but different probability distributions.

The distribution lies in \( \theta_i^M \in [0, \bar{\theta}] \) and \( \theta_i^O \in [0, s\bar{\theta}] \); \( 0 < \bar{\theta} < 1, 0 < s < 1 \). As \( \bar{\theta} \) (resp. \( s\bar{\theta} \)) denotes the interval of the uncertain marginal cost \( \theta_i^M \) (resp. \( \theta_i^O \)), \( s \) represents (the inverse of) the degree of cost efficiency of outsourcing. As \( s \) decreases, outsourcing becomes more cost efficient than in-house production evaluated at the ex ante stage where there is cost uncertainty. \( f^\omega(\theta_i^\omega) \) and \( F^\omega(\theta_i^\omega) \) denote the density and cumulative probability functions, respectively. The probability structure constitutes common knowledge for both manufacturers (and if they exist, outsourcers). \( f^\omega(\theta_i^\omega) \) is assumed to be a twice continuously differentiable function. Moreover, we assume that the monotone hazard rate condition (MHRC) is satisfied for \( \theta_i^O \), that is, \( d[F^O(\theta_i^O)/f^O(\theta_i^O)]/d\theta_i^O > 0 \). We define the virtual type of \( O_i \) by \( V(\theta_i^O) = \theta_i^O + F^O(\theta_i^O)/f^O(\theta_i^O) \).

\( M_i \) in in-house production or \( O_i \) in outsourcing faces duopolistic competition. Thus, when both manufacturers choose outsourcing, each outsourcer competes with the rival outsourcer as a delegated agent in a differentiated goods market. The competition occurs in a Cournot fashion. The variables \( q_i \) and \( p_i \) denote the quantity and price of good \( i \), respectively. The inverse demand function of good \( i \) is given by

\[
p_i(q_i, q_{-i}) = 1 - q_i - \gamma q_{-i},
\]

where \( \gamma \in (0, 1) \) represents the degree of product differentiation between
two product-differentiated goods. We assume the conditions wherein all types of marginal costs ensure positive quantity (which will be discussed later). The ex post total profit is represented as follows:

\[ \pi_i(q_i, q_{-i}; \theta_i^o) = (p_i(q_i, q_{-i}) - \theta_i^o)q_i = (1 - \theta_i^o)q_i - q_i^2 - \gamma q_i q_{-i}. \]  

(2)

As \( M_i \) knows its own marginal cost, \( \theta_i^M \), after in-house production is selected, \( M_i \) maximizes its own profit with regard to quantity. When the contract is offered after outsourcing is selected, \( M_i \) is unaware of \( \theta_i^O \). \( M_i \) is required to offer the contract to \( O_i \), such that the incentive compatibility constraint is satisfied to induce the true information on \( \theta_i^O \). The offered contract depends on the quantity level \( q_i \), which is assumed to be verifiable. In accordance with the revelation principle, we concentrate on the direct truth-telling mechanism, \( (q_i(\hat{\theta}_i^O), t_i(\hat{\theta}_i^O)) \), where the quantity level and transfer \( (q_i, t_i) \) are self-selected depending on the type reported to \( M_i \) by \( O_i \). \( \hat{\theta}_i^O \). The contract is represented as a function from the reported type \( \hat{\theta}_i^O \) to \( (q_i, t_i) \). Thus, when \( O_i \) reports its type \( \hat{\theta}_i^O \) to \( M_i \), the quantity level \( q_i(\hat{\theta}_i^O) \) is implemented, and \( O_i \) pays \( M_i \) the transfer \( t_i(\hat{\theta}_i^O) \) as brand royalty depending on \( \hat{\theta}_i^O \). \( O_i \) decides whether to accept the offered contract. If the contract is accepted, \( O_i \) supplies the brand product for \( M_i \) in a differentiated goods market. If the contract is rejected, \( O_i \) obtains the reservation payoff, which is normalized to 0.

\( M_i \) is required to commit to the contract in advance. It is assumed that the rival counterpart cannot observe this contract \( t_i(q_i) \) when offering the contract, and \( M_i \) cannot write the contract based on \( q_{-i} \) implemented by \( M_{-i} \) because the information on the rival cannot be verified. The ex post total profit is denoted by \( \pi_i(q_i, q_{-i}; \theta_i^O) \). In outsourcing, \( M_i \)’s profit is the transfer paid from \( O_i \), \( M_i \) and \( O_i \) cannot know the rival’s cost, \( \theta_{-i} \), when the contract is offered; thus, \( M_i \) maximizes the expected value of the transfer \( t_i \). \( O_i \) maximizes the expected value of \( (\pi_i - t_i) \). In the offered contract, \( O_i \) is guaranteed more than the reservation payoff because individual rationality has to be satisfied.

The timing of the model is as follows. In the first stage, \( M_i \) chooses in-house production or outsourcing. In this stage, \( M_i \) does not know its own type, \( \theta_i^M \). If outsourcing is chosen, \( M_i \) chooses an outsourcer from several potential outsourcers. Before offering the contract in the next stage, the chosen \( O_i \) knows its type \( \theta_i^O \), although \( M_i \) is unaware of \( \theta_i^O \). In the second stage, if in-house production is chosen in the first stage, \( M_i \) recognizes \( \theta_i^M \) and decides the quantity level to maximize the ex-
expected profit. If outsourcing is chosen in the first stage, \( M_i \) offers the outsourcing contract to \( O_i \). \( O_i \) decides whether to accept the contract. If the contract is rejected, \( O_i \) obtains the reservation payoff. However, if the contract is accepted, \( M_i \) implements the quantity level \( q_i(\hat{\theta}_i^O) \) and receives the brand royalty \( t_i(\hat{\theta}_i^O) \) following the reported type by \( O_i \).

Regardless of the form of production chosen, the quantity levels \((q_i, q_{-i})\) are decided upon simultaneously and non-cooperatively. The solution follows the perfect Bayesian equilibrium.

### III. The Quantity and Expected Profit in Equilibrium

#### A. In-house Production

\( M_i \) maximizes the profit, \( \pi_i(q_i(M), E_{-i}q_{-i}(\theta_{-i}^O); \theta_i^M) \), with regard to \( q_i \), as \( M_i \) recognizes \( \theta_i^M \). Using the first-order condition (FOC), the reaction function is obtained as follows:

\[
q_i = q_i(E_{-i}q_{-i}(\theta_{-i}^O); \theta_i^M) = 1 - \theta_i^M - \frac{\gamma E_{-i}q_{-i}(\theta_{-i}^O)}{2}.
\]

The expectation of the reaction function with regard to \( \theta_i \) is as follows:

\[
E_i q_i(E_{-i}q_{-i}(\theta_{-i}^O); \theta_i^M) = \frac{1 - E\theta_i^M - \gamma E_{-i}q_{-i}(\theta_{-i}^O)}{2},
\]

where \( E\theta_i^M = E_i \theta_i^M \).

#### B. Outsourcing

The direct truth-telling contract must satisfy the incentive compatibility constraint for \( O_i \), that is, \( \theta_i^O \in \arg \max_{\theta_i^O} [E_i \pi_i(q_i(\theta_i^O), q_{-i}(\theta_{-i}^O); \theta_i^O) - t_i(\hat{\theta}_i^O)] \). \( O_i \)'s (expected) information rent is defined by \( U_i(\theta_i^O) = E_i \pi_i(q_i(\theta_i^O), q_{-i}(\theta_{-i}^O); \theta_i^O) - t_i(\hat{\theta}_i^O) \). It is assumed that \( q_i(\theta_i^O) \) and \( t_i(\theta_i^O) \) are continuously differentiable. Through standard techniques used to derive the optimal contract in the hidden information, the first- and second- order local conditions for incentive compatibility are \( \dot{U}_i(\theta_i^O) = -q_i(\theta_i^O) \leq 0 \) and \( \ddot{q}_i(\theta_i^O) \leq 0 \), respectively. \( O_i \)'s information rent decreases in \( \theta_i^O \).

For the sake of tractability of analysis, we replace \( t_i(\theta_i^O) \) with \( U_i(\theta_i^O) \).

\( E_i \) denotes the operator of expectation with regard to \( \theta_{-i}^O \).
The optimal contract is obtained by solving the following problem:

$$\max_{\{q_i(\theta^O_i), q_{-i}(\theta^O_{-i}); \theta^O \}} E_i \left[ E_x \pi_i(q_i(\theta^O_i), q_{-i}(\theta^O_{-i}); \theta^O) - U_i(\theta^O_i) \right],$$

(5)

s.t. $\dot{U}_i(\theta^O_i) = -q_i(\theta^O_i)$, \hspace{1cm} (6)

$\dot{q}_i(\theta^O_i) \leq 0$, \hspace{1cm} (7)

$U_i(\theta^O_i) \geq 0, \quad \forall \theta^O_i \in [0, \tilde{s}]$. \hspace{1cm} (8)

We assume that $\dot{q}_i(\theta^O_i) < 0$ (which is verified ex post). Using (6), $O_i$’s information rent is strictly decreasing in $\theta^O_i$ and (8) binds only at $s\tilde{\theta}$, that is, $U_i(s\tilde{\theta}) = 0$. Using integration by parts, $O_i$’s information rent is $E_iU_i(\theta^O_i) = -E_i[\dot{U}_i(\theta^O_i) [F^O(\theta^O_i)/f^O(\theta^O_i)] - E_i[q_i(\theta^O_i) [F^O(\theta^O_i)/f^O(\theta^O_i)]]].$ $M_i$ solves the following relaxed program:

$$\max_{\{q_i(\theta^O_i), q_{-i}(\theta^O_{-i}); \theta^O \}} E_i \left[ E_x \pi_i(q_i(\theta^O_i), q_{-i}(\theta^O_{-i}); \theta^O) - q_i(\theta^O_i) \frac{F^O(\theta^O_i)}{f^O(\theta^O_i)} \right].$$

(9)

Through the FOC, the reaction function is obtained as follows:

$$q_i = q_i(E_xq_{-i}(\theta^O_{-i}); \theta^O_i) = \frac{1 - V(\theta^O_i) - \gamma E_xq_{-i}(\theta^O_{-i})}{2},$$

(10)

Note that (10) is obtained by replacing $\theta^M_i$ with $V(\theta^O_i)$ in (3). The expectation of the reaction function with regard to $\theta^O_i$ is as follows:

$$E_iq_i(E_xq_{-i}(\theta^O_{-i}); \theta^O_i) = \frac{1 - EV^O - \gamma E_xq_{-i}(\theta^O_{-i})}{2},$$

(11)

where $EV^O = E_iV(\theta^O_i)$.

C. Equilibrium Quantity

We derive the equilibrium quantity in all cases where (i) both $M_i$
choose in-house production; (ii) both $M_i$ choose outsourcing; and (iii) $M_i$ chooses in-house production and $M_{-i}$ chooses outsourcing. All contingent cases are denoted by $(\omega, \omega')\in\{M, O\}^2$.

Solving the reaction functions for $i=1, 2$, (3) and (10), we obtain the equilibrium quantity in all cases for all $\theta_i^M\in[0, \bar{\theta}]$ and $\theta_i^O\in[0, s\bar{\theta}]$.

\[ q_{i,M}^M(\theta_i^M) = \frac{2 + \gamma E\theta_i^M}{2(2 + \gamma)} - \frac{\theta_i^M}{2}, \tag{12} \]

\[ q_{i,M}^O(\theta_i^M) = \frac{2(2 - \gamma) + \gamma(2EV^O - \gamma E\theta_i^M)}{2(2 + \gamma)(2 - \gamma)} - \frac{\theta_i^M}{2}, \tag{13} \]

\[ q_{i,O}^M(\theta_i^O) = \frac{2(2 - \gamma) + \gamma(2E\theta_i^M - \gamma EV^O)}{2(2 + \gamma)(2 - \gamma)} - \frac{V(\theta_i^O)}{2}, \tag{14} \]

\[ q_{i,O}^O(\theta_i^O) = \frac{2 + \gamma EV^O}{2(2 + \gamma)} - \frac{V(\theta_i^O)}{2}. \tag{15} \]

The equilibrium quantity strictly decreases in $\theta_i^\omega$ by the MHRC. It should be noted that $q_{i,O}^O(\theta_i^O)$ is obtained by replacing $\theta_i^M$ with $V(\theta_i^O)$ in (12) and $q_{i,M}^{M,O}(\theta_i^O) = q_{i,M}^{O,M}(\theta_i^O)$ \(\forall\theta_i^O = \theta_i^O\).

From (12)-(15), we obtain the following lemma.

**Lemma 1.** If $EV^O \geq E\theta_i^M$, $q_i^{M,M}(\theta_i^M) \leq q_i^{M,O}(\theta_i^M)$ and $q_i^{O,M}(\theta_i^O) \leq q_i^{O,O}(\theta_i^O)$.

All proofs can be found in Appendix A. $q_i(\theta_i^\omega)$ is strictly decreasing; thus, a sufficient condition for all types to ensure positive quantities is obtained by Lemma 1 as follows: If $EV^O > E\theta_i^M$ (resp. $EV^O < E\theta_i^M$), $q_i^{M,M}(\bar{\theta}) > 0$, $q_i^{O,M}(s\bar{\theta}) > 0$ (resp. $q_i^{M,O}(\bar{\theta}) > 0$, and $q_i^{O,O}(s\bar{\theta}) > 0$) constitute a sufficient condition for a positive quantity. In the following analysis, we assume that the above condition is satisfied.

**D. Manufacturer’s Expected Profit in the Equilibrium**

To derive $M_i$’s expected profit in the equilibrium, we define $M_i$’s expected profit by $\Pi_i^{\omega,\omega'}(\omega, \omega')\in\{M, O\}^2$. We obtain the following lemma.

**Lemma 2.** Regardless of whether $M_i$ chooses in-house production or outsourcing, $M_i$’s expected profit satisfies the following equation: $\Pi_i^{\omega,\omega'}$
TABLE 1

<table>
<thead>
<tr>
<th>Choice Game between In-House Production and Outsourcing</th>
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</thead>
<tbody>
<tr>
<td>in-house production</td>
</tr>
<tr>
<td>$\Pi_i^{M,M}$, $\Pi_i^{M,O}$</td>
</tr>
</tbody>
</table>

Substituting (12)-(15) into $\Pi_i^{\omega,\omega'} = E_i[q_i^{\omega,\omega'}]$, we derive $\Pi_i^{\omega,\omega'}$ by Lemma 2, we derive $M_i$’s expected profit in the equilibrium as follows:

$$\Pi_i^{M,M} = \frac{(1 - E\theta_i^M)^2}{(2 + \gamma)^2} + \frac{\sigma^2}{4},$$ (16)

$$\Pi_i^{M,O} = \frac{(2(1 - E\theta_i^M) - \gamma(1 - EV_i^M))^2}{(2 + \gamma)^2} + \frac{\sigma^2}{4},$$ (17)

$$\Pi_i^{O,M} = \frac{(2(1 - EV_i^M) - \gamma(1 - E\theta_i^M))^2}{(2 + \gamma)^2} + \frac{\sigma^2}{4},$$ (18)

$$\Pi_i^{O,O} = \frac{(1 - EV_i^O)^2}{(2 + \gamma)^2} + \frac{\sigma^2}{4}.$$ (19)

$\sigma^2 = E[q_i^{\omega,\omega'} - E\theta_i^{\omega,\omega'}]^2$ and $\sigma^2 = E[V(\theta_i^O) - EV_i^O]^2$ denote the variances of $\theta_i^M$ and $V(\theta_i^O)$, respectively.

IV. Choice between In-house Production and Outsourcing

In the first stage, $M_i$ faces the choice game between in-house production and outsourcing. The normal-form representation of this game is shown in Table 1.

In order for outsourcing to constitute a unique Nash equilibrium in the game, it is necessary that the decision to outsource by a manufacturer must constitute the dominant strategy in the game. Therefore, in order for $M_i$ to choose outsourcing for certain, $\Pi_i^{M,M} < \Pi_i^{O,M}$ and $\Pi_i^{M,O} < \Pi_i^{O,O}$ have to be satisfied. Comparing the expected profits, we obtain the following theorem.
Theorem 1. Suppose that $q_{i}^{o,ω}(θ_{i}^{o}) > 0 \forall θ_{i}^{M}$. The sufficient condition for outsourcing to be selected as the unique Nash equilibrium is that the following inequality should be satisfied:

$$σ_{V}^{2} - σ_{θ}^{2} > \frac{16(EV_{O} - Eθ_{i})((1 - Eθ_{i}^{M}) + (1 - γ)(1 - EV_{O}))}{(2 + γ)^{2}(2 - γ)}.$$

(20)

As shown in Theorem 1, under the general probability distribution with regard to cost uncertainty, whether outsourcing is preferred depends on the relative size of expectation and variance on $θ_{i}^{M}$ and $V(θ_{i}^{O})$, that is, $(Eθ_{i}^{M}, \sigma_{θ}^{2})$ and $(EV_{O}, \sigma_{V}^{2})$. This condition depends on the difference in the variances of costs associated with in-house production and the virtual cost function $V$. Under the general distribution function, (20) also depends on the degree of product differentiation.

We consider the case wherein the manufacturer faces identical cost uncertainty in in-house production and outsourcing as a specific case. That is, $θ_{i}^{M}$ and $θ_{i}^{O}$ follow the same probability distribution, $f_{M}(\cdot)=f_{O}(\cdot)$ and $s=1$. In the case of identical cost uncertainty, we obtain the following proposition.

Proposition 1. If the degree of cost uncertainty of outsourcing is identical to that of in-house production, then outsourcing is never chosen.

When the degree of cost uncertainty is identical, whether the rival chooses in-house production or outsourcing, the manufacturer expects to gain more profit by selecting in-house production, $Π_{i}^{M,ω'} > Π_{i}^{O,ω'}$, $ω' \in \{M, O\}$. When outsourcing has no cost efficiency, the dominant strategy for the manufacturer is to choose in-house production in the choice game. Proposition 1 suggests that in order for the manufacturer to choose outsourcing, the decision has to be supported by cost efficiency.

The manufacturer is required to provide information rent to the outsourcer when it chooses outsourcing; thus, it appears at first glance that if the degree of cost uncertainty is identical, the result of Proposition 1 is obviously satisfied. However, this is not obvious because the decrease in quantity required to reduce information rent has the secondary effect of mitigating market competition under strategic substitutes. Proposition 1 implies that the negative effect of the loss by paying information rent to the outsourcer always exceeds the positive effect of mitigated competition under strategic interaction.
In Theorem 1, the condition under which outsourcing is opted for by
the manufacturer depends on the functional form of the distribution.
However, we cannot characterize the interpretation of the condition (20)
in a more detailed way under the general distribution function. Any
density function can be approximated with the uniform distribution if
the support of uncertainty is sufficiently small. Hence, for tractability,
we consider the approximation of any density function through the
uniform distribution with regard to uncertainty in the following analysis.
In the uniform distribution, the density and cumulative func-
tions of \( \theta_i^M \) (resp. \( \theta_i^O \)) are represented by
\[
\begin{align*}
  f_M(\theta_i^M) &= \frac{1}{\theta_i^\bar}, \\
  F_M(\theta_i^M) &= \theta_i^M/\theta_i^\bar, \\
  f_O(\theta_i^O) &= \frac{1}{s \theta_i^\bar}, \\
  F_O(\theta_i^O) &= \theta_i^O/s \theta_i^\bar).
\end{align*}
\]

In the uniform distribution, a sufficient condition for positive quanti-
ties is assumed as follows:

**Assumption 1:**
\[
\bar \theta < \frac{4(2-\gamma)}{8-\gamma^2-4\gamma s} \text{ if } s < \frac{1}{2}, \quad \text{and} \quad \bar \theta < \frac{2(2-\gamma)}{(8-\gamma^2)s-\gamma} \text{ if } s \geq \frac{1}{2},
\]
which is derived in Appendix B. From the sufficient condition (20) in
Theorem 1, we obtain the following proposition.

**Proposition 2.** Suppose that \( \theta_i^\omega \) is uniformly distributed and Assump-
tion 1 is satisfied. If \( s < 1/2 \), regardless of the degree of substitutability,
outsourcing is chosen in the equilibrium. That is, if \( s < 1/2 \), \( \Pi_i^{O,\omega} > \Pi_i^{M,\omega} \),
\( \omega \in \{M, O\} \forall \gamma \in [0, 1] \).

Proposition 2 implies that in the uniform distribution, if the degree of
cost efficiency of outsourcing exceeds certain thresholds, that is, \( s < 1/2 \), a manufacturer selects outsourcing regardless of the degree of
substitutability. The choice between in-house production and out-
sourcing does not depend on the degree of substitutability of brand products
in a differentiated goods market. In the model, the degree of sub-
stitutability can be interpreted to have an inverse relation to the intensity
of market competition because when \( \gamma = 0 \), the market is monopolistic,
and when \( \gamma = 1 \), the market is duopolistic. Thus, Proposition 2 affirms
that whether or not outsourcing is more profitable than in-house pro-
duction bears no relationship to the intensity of competition. In other
words, this proposition suggests that product market competition does
not affect the manufacturer’s decision of outsourcing versus in-house
production, which is based only on cost efficiency.

The intuition behind this result is as follows. The cost under out-
sourcing is different from that under in-house production because the former includes the information rent of the outsourcer. The manufacturer enters into an exclusive contract with the outsourcer whose outside opportunities do not depend on market competition. Therefore, as market competition does not affect the information rent of the outsourcer, it does not affect the cost under outsourcing relative to that under in-house production.

It should be noted that although Proposition 2 is obtained under a uniform distribution, if cost uncertainty is sufficiently low, a similar result is obtained under the general distribution. Any general distribution can be approximated by the uniform distribution, that is, \( f_M(\theta_i) \approx \frac{1}{\overline{\theta}} \) and \( f_O(\theta_i) \approx \frac{1}{s\overline{\theta}} \) when \( \overline{\theta} \) is sufficiently small. Hence, \( \Pi_{i,\omega'}^O > \Pi_{i,\omega'}^M \) is approximately satisfied if \( s < 1/2 \).

In Proposition 2, \( s < 1/2 \) is the condition according to which outsourcing accrues greater expected profit than in-house production. Under this condition, \( s\overline{\theta} < E\theta_i^M = \overline{\theta}/2 \) is satisfied. Therefore, this condition implies that the upper bound of the uncertain cost in outsourcing is always less than the expectation of the uncertain cost in in-house production. As \( s \) represents (the inverse of) the cost efficiency of outsourcing, the fact that the degree of cost efficiency in order for outsourcing to be selected does not depend on \( \gamma \) implies that the degree of product differentiation and, as a consequence, the intensity of competition do not affect the decision of the manufacturer to outsource. Therefore, the result in Proposition 2 suggests that even if market competition intensifies, the manufacturer should decide whether to choose outsourcing under cost uncertainty by evaluating only the degree of cost efficiency of outsourcing.

The reason why the decision to outsource does not depend on \( \gamma \) is explained as follows. When \( s = 1/2 \), the expectation and variance of \( \theta_i^M \) are equivalent to those of \( \mathbb{V}(\theta_i^O) \), that is, \( E\theta_i^M = E\theta_i^O \) and \( \sigma_{\theta^2} = \sigma_{\theta^2} \), respectively. When \( s \geq 1/2 \), \( E\theta_i^O - E\theta_i^M \geq 0 \) and \( \sigma_{\theta^2} - \sigma_{\theta^2} \geq 0 \) are satisfied. Thus, \( (E\theta_i^O - E\theta_i^M) \) has the same sign as \( \sigma_{\theta^2} - \sigma_{\theta^2} \). However, because variance is affected by the square of the size of cost uncertainty in contrast with expectation, the size of the difference in variance is sufficiently greater than that in the case of expectation. Therefore, whatever the value of \( \gamma \) is, the impact of variance always exceeds that of expectation, and the relative size of the effects of variance and expectation determines the expected profit.

Finally, we examine how the expected total profit, which is denoted by \( E_iE_{-i} \pi_{\omega',\omega'} \), affects the degree of cost efficiency. If in-house pro-
duction is decided upon, as no outsourcer is hired, a manufacturer acquires the entire total expected profit. If outsourcing is decided upon, as a manufacturer is required to pay information rent to an outsourcer, the expected total profit is shared by the manufacturer and outsourcer. That is, \( E_i E_i \pi^M, \omega = \Pi_i^M, \omega \) and \( E_i E_i \pi^O, \omega' = \Pi_i^O, \omega' + E_i U_i \) are satisfied. Clearly, although the manufacturer is not different in both in-house production and outsourcing when \( s = 1/2 \), the expected total profit, which adds the information rent to \( \Pi_i^O, \omega' \), is always greater in outsourcing than in-house production. By comparing \( E_i E_i \pi^M M \) and \( E_i E_i \pi^O O \),\(^4\) we can clarify the expected total profits that are larger.\(^5\) For example, in the case of monopoly \( (\gamma = 0) \), \( E_i E_i \pi^M M < E_i E_i \pi^O O \) if \( s \in (0, (3+\sqrt{3})/6) \). In the case of duopoly \( (\gamma = 1) \), if \( s \in (0, 9/20) \) or \( (18/35, 1] \), \( E_i E_i \pi^M M < E_i E_i \pi^O O \). If \( s \in (9/20, 18/35) \), whichever is larger of the two, \( E_i E_i \pi^M M \) or \( E_i E_i \pi^O O \), depends on the relative sizes of \( s \) and \( \bar{\theta} \) (as derived in Appendix C). Unlike the comparison of \( M_i \)’s expected profit, whether in-house production and outsourcing yields a larger expected total profit depends on \( \gamma \). In sum, although the choice by a manufacturer between in-house production and outsourcing is not affected by \( \gamma \), the size of the expected total profit is affected by \( \gamma \).

V. Concluding Remarks

In this paper, we examined the relationship between the choice of outsourcing versus in-house production and product differentiation in a Cournot model with cost uncertainty and asymmetric information under outsourcing. If there is asymmetric information between a manufacturer and an outsourcer, the manufacturer has to pay the information rent to the outsourcer when outsourcing is chosen. We demonstrated that the degree of product differentiation and the intensity of competition do not affect the choice of a manufacturer between in-house production and outsourcing. This result is in sharp contrast to that in the existing literature.

Our result differs from that in the existing literature on account of the difference between the models with regard to the decision of a firm in choosing between in-house production and outsourcing. Partial out-

\(^4\) Even if we compare other pairs of the expected total profits, we obtain a similar result.

\(^5\) We did not derive this result comprehensively because the derivation is too complicated to be calculated.
sourcing has been analyzed in the existing literature. The analysis of partial outsourcing presumes implicitly that firms can choose the optimal proportion of outsourcing by adjusting the variables affecting the organizational structure continuously. For example, Shy and Stenbacka (2005) explore the model with a continuum of inputs that enable partial outsourcing to be analyzed by producing a certain portion of inputs for in-house and outsourcing other portions. Moreover, monitoring the cost of outsourcing is assumed to be strictly convex in the proportion of outsourcing. In the paper, firms have to choose either in-house production or outsourcing without any intermediate forms of production. Furthermore, in the paper, contrary to the existing literature, monitoring the cost of outsourcing has no diseconomy of scale and is determined by the information rent to the outsourcer. Therefore, if the choice between in-house production and outsourcing is made in a non-convex set of alternatives and monitoring cost is decided by the differences between the information structure of organizational forms, the result of this paper will be more appropriate than that in the existing articles.

Following the prevailing view that outsourcing is fostered by intensifying competition caused by globalization, the management emphasizes specialization on the division that has core competence with competitive advantage and withdrawal from the non-core division that is vulnerable to harsh competition. If this view is correct, as a market becomes more competitive, a firm loses its core competence in the market. Thus, firms are forecast to prefer outsourcing to in-house production in the midst of intense global competition. However, we present a view different from the prevailing one. The result in this paper suggests that based on the views on core competence, when the management decides on whether to outsource or not, it should distinguish between technological and competitive advantages. The decision to outsource is affected only by technological advantage, such as cost efficiency, and not by competitive advantage through proper market positioning, such as the degree of product differentiation.

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Appendix A

Proof of Lemma 1. \( q_i^{M,O}(\bar{\theta}) - q_i^{M,M}(\bar{\theta}) = q_i^{O,O}(\bar{\theta}) - q_i^{O,M}(\bar{\theta}) = \frac{\gamma(EV^O - E\theta^M)}{4 - \gamma^2} \)

\( \leq 0 \) if and only if \( EV^O \leq E\theta^M \).

Proof of Lemma 2. As \( M_i \) is not aware of its own type \( \theta_i^M \), when choosing in-house production, \( M_i \) expects \( \Pi_i^{M,\alpha^M} = E_i e_i \pi_i(q_i(\theta_i^M), q_i(\theta_i^\alpha); \theta_i^M) \) to be obtained. As \( E_i e_i \pi_i = p_i(q_i, E_i q_i) - q_i^M q_i \) by (2) and \( p_i(q_i, E_i q_i) - \theta_i^M - q_i \) by (1) and (3), we obtain \( \Pi_i^{M,\alpha^M} = E_i[q_i^{M,\alpha^M}]^2 \). When choosing outsourcing, \( M_i \) must provide information rent to \( O_i \). As \( O_i \)'s expected information rent is \( E_iU_i(\theta_i^O) \), \( \Pi_i^{O,\alpha^O} = E_i e_i \pi_i(q_i(\theta_i^O), q_i(\theta_i^\alpha); \theta_i^O) - E_iU_i(\theta_i^O) \) to be obtained. Combining (1) and (10), we obtain \( E_i e_i \pi_i(q_i(\theta_i^O), q_i(\theta_i^\alpha); \theta_i^O) = E_iU_i(\theta_i^O) \). As \( E_iU_i = E_i[q_i^{O,\alpha^O}(F/f)] \) is satisfied by integrating by parts, \( \Pi_i^{O,\alpha^O} = E_i[q_i^{O,\alpha^O}]^2 \) is satisfied.

Proof of Theorem 1. \( \Pi_i^{MM} < \Pi_i^{O,\alpha} \) and \( \Pi_i^{MO} < \Pi_i^{O,O} \) if and only if \( \sigma_i^2 - \sigma_\theta^2 > 16(EV^O - E\theta^M)A/(2 + \gamma)^2(2 - \gamma)^2 \) and \( \sigma_i^2 - \sigma_\theta^2 > 16(EV^O - E\theta^M)B/(2 + \gamma)^2(2 - \gamma)^2 \). (2) is satisfied. If and only if \( A \leq B \), \( (EV^O - E\theta^M)B/(2 + \gamma)^2(2 - \gamma)^2 > 16(EV^O - E\theta^M)A/(2 + \gamma)^2(2 - \gamma)^2 \) is satisfied regardless of the sign of \((EV^O - E\theta^M)\). Thus, the sufficient condition for outsourcing to be the dominant strategy is \( \sigma_i^2 - \sigma_\theta^2 > 16(EV^O - E\theta^M)B/(2 + \gamma)^2(2 - \gamma)^2 \).

Proof of Proposition 1. When \( f^M(\cdot) = f^O(\cdot) = f(\cdot) \) and \( s = 1 \), \( \theta_i^M \) and \( \theta_i^O \) are distributed in the same interval \([0, \bar{\theta}]\). Define \( E[F/f] = E[f(F/f)] \). As \( q_i^{M,M}(\theta) - q_i^{M,O}(\theta) = q_i^{O,O}(\theta) - q_i^{O,M}(\theta) = [\gamma^2 E[F/f] + (4 - \gamma^2)|F(\theta)/f(\theta)|]/2(4 - \gamma)^2 > 0 \), \( q_i^{M,M}(\theta) > q_i^{O,O}(\theta) \) and \( q_i^{M,O}(\theta) > q_i^{O,M}(\theta) \) are satisfied for all \( \theta \in [0, \bar{\theta}] \). Applying \( \Pi_i^{O,\alpha^O} = E_i[q_i^{O,\alpha^O}(\theta)]^2 \) by Lemma 2, we obtain \( \Pi_i^{MM} > \Pi_i^{O,\alpha^O} \) and \( \Pi_i^{MO} > \Pi_i^{O,O} \).

Proof of Proposition 2. As \( E\theta^M = \bar{\theta}/2 \), \( EV^O = s\bar{\theta} \), \( \sigma_\theta^2 = \bar{\theta}^2/12 \), and \( \sigma_i^2 = s^2\bar{\theta}^2/3 \) in the uniform distribution, the left-hand and right-hand sides of (20) in Theorem 1 are calculated as \((2s + 1)(2s - 1)\bar{\theta}^2/12\) and \(8(2s - 1)\bar{\theta}(1 - (\bar{\theta}/2) + (1 - \gamma)(1 - s\bar{\theta}))/(2 + \gamma)^2(2 - \gamma)^2 \). respectively. When \( s = 1/2 \), both sides are equal to zero and \( \Pi_i^{M,O} - \Pi_i^{O,O} \) is satisfied. If and only if \( s \leq 1/2 \), \( (2s + 1)\bar{\theta}^2/12 \geq 8(1 - (\bar{\theta}/2) + (1 - \gamma)(1 - s\bar{\theta}))/(2 + \gamma)^2(2 - \gamma)^2 \). Representing this inequality with regard to \( s \), we obtain \( s \geq s \equiv 96(2 - \gamma) -
for all \( \bar{s} > 1 \) and only if \( \bar{s} < 3/2(2 - \gamma)/\{(4 - \gamma)^2 + 32(1 - \gamma) + 16\} \). As \( 32(2 - \gamma)/\{(4 - \gamma)^2 + 32(1 - \gamma) + 16\} \geq 1 \), \( \bar{s} > 1 \) is satisfied. Thus, (20) is satisfied for all \( s \in (0, 1/2) \) and not satisfied for all \( s \in (1/2, 1) \).

\[ \text{(21)} \]

\textbf{Appendix B}

\textbf{Derivation of Assumption 1.} By Lemma 1, a sufficient condition for positive quantities is as follows. If \( EV^O > E\theta^M \) (resp. \( EV^O < E\theta^M \)), \( q_{iM}^{MM}(\bar{\theta}) > 0 \) and \( q_{iO}^{OM}(s\bar{\theta}) > 0 \) (resp. \( q_{iO}^{MO}(s\bar{\theta}) > 0 \) and \( q_{iO}^{OM}(s\bar{\theta}) > 0 \)). In the uniform distribution, \( EV^O \approx E\theta^M \) if and only if \( s \equiv 1/2 / \gamma + \gamma \). \( q_{iM}^{MM}(\bar{\theta}) - q_{iO}^{OM}(s\bar{\theta}) = q_{iM}^{MO}(\bar{\theta}) - q_{iO}^{OM}(s\bar{\theta}) = (8 - \gamma)(2s - 1)/\{(8 - \gamma)s - \gamma\}. \) If \( s > 1/2 \) (resp. \( s < 1/2 \)), \( q_{iM}^{MM}(\bar{\theta}) > q_{iO}^{OM}(s\bar{\theta}) \) (resp. \( q_{iO}^{OM}(s\bar{\theta}) > q_{iM}^{MO}(\bar{\theta}) \)). Thus, the sufficient condition is as follows: \( q_{iO}^{OM}(s\bar{\theta}) > 0 \) if \( s \equiv 1/2 \) and \( q_{iM}^{MO}(\bar{\theta}) > 0 \) if \( s < 1/2 \). \( q_{iO}^{OM}(s\bar{\theta}) > 0 \) if and only if \( \bar{s} < 4(2 - \gamma)/(8 - \gamma^2 - 4\gamma s) \).

\textbf{Appendix C}

\textbf{Comparison of expected total profits.} \( E_i E_i \pi^{MM} \approx E_i E_i \pi^{OO} \) if and only if \( \Pi_i^{MM} = \Pi_i^{OO} + E_i\{q_{iO}^{OM}(\bar{\theta})|f^{O}(\bar{\theta})|/ f^{O}(\bar{\theta})\} \}. \) Substituting (15), (16), and (19) into the above inequality and arranging them under a uniform distribution, we obtain the following inequality:

\[ \gamma \bar{s}^2 + (2 - \gamma)s - \frac{48 - [(2 + \gamma)^2 + 12\bar{\theta}]}{24} < 0. \]  

(21)

When \( \gamma = 0 \), Assumption 1 can be represented as \( \bar{s} < 1 \) if \( s < 1/2 \), and \( \bar{s} < 1/2s \) if \( s \geq 1/2 \). (21) is calculated as \( s < 1 - \bar{s}/3 \). If \( s < 1/2 \), (21) is always satisfied. If \( s \geq 1/2 \), in order for (21) and \( \bar{s} < 1/2s \) to be satisfied, \( 6s^2 - 6s + 1 < 0 \) must be satisfied, which is replaced by \((1/2 \leq s \leq (3 + \sqrt{3})/6 \approx 0.789 \). Thus, when \( \gamma = 0 \), \( E_i E_i \pi^{MM} < E_i E_i \pi^{OO} \) if \( s \in (0, 3 + \sqrt{3})/6 \). When \( \gamma = 1 \), Assumption 1 can be represented as \( \bar{s} < 4/(7 - 4s) \) if \( s < 1/2 \), and \( \bar{s} < 2/(7s - 1) \) if \( s \geq 1/2 \). (21) is calculated as \( \bar{s}s^2 + s - [16 - 7\bar{s}]/6 < 0 \), which is equivalent to \( s_1 < s < s_2 \), where \( s_1 \equiv \{-3 - \sqrt{6\bar{s}(16 - 7\bar{s}) + 9]/6\bar{s} < 0 \) and \( s_2 \equiv \{-3 + \sqrt{6\bar{s}(16 - 7\bar{s}) + 9]/6\bar{s} > 0 \). If \( s \in (0, s_2) \). (21) is satisfied. \( s_2 \) is the strictly decreasing function with regard to \( \bar{s} \). It is satisfied if \( s \in (0, 9/20) \) or \( (18/35, 1) \). Otherwise, \( s_2 < 1 \). Thus, in the case of \( \gamma = 1 \), if \( s \in (0, 9/20) \) or \( (18/35, 1) \),
$E_i E^{-i} \pi^{MM} < E_i E^{-i} \pi^{OO}$. If $s \in (9/20, 18/35)$, the larger value depends on the relative size of $s$ and $\theta$. If $s < s_2$, $E_i E^{-i} \pi^{MM} < E_i E^{-i} \pi^{OO}$.

References


