서울大學校 師大論叢 第24輯 (1982. 6. 20)

Symmetry Breaking Patterns in SO(14)

Kwang Sup Soh
Dept. of Physics Education, College of Education
Seoul National University, Seoul 151, Korea

Abstract

Explicit realization of spontaneous symmretry breaking of $SO(14)$ to $SU(3)_c \times U_{em}$ is made by finding vacuum expectation values of Higgs fields. Renormalization group calculation of coupling constants shows that $SO(14) \rightarrow SU(7) \times U(1)$ is compatible with experiments.

I. Introduction

Symmetry in a Lagrangian may not be realized in a vacuum, which produces symmetry breaking in the physical system. Such a phenomenon is called spontaneous symmetry breaking. Particulary in locally gauge symmetric fields such symmetry breaking induces mass of the gauge bosons by the Higgs mechanism.[1]

The successful unification of electromagnetic and weak interaction by Weinberg and Salam[2] was achieved by the spontaneous symmetry breaking of $SU(2)_W \times U(1)_Y$ to $U(1)_{em}$. After this many attempts have been made to unify three fundamental interactions (strong, weak, and electromagnetic). Among these the Georgi-Glashow $SU(5)$ model[3] is the simplest yet most successful one up to now. However, the $SU(5)$ theory itself can not be the complete one, because it does not include all the known fermions. Out of three lepton families and two and half quark familes only one family is included in the $SU(5)$ model. This is called flavor problem.

The $SU(7)$ model[4] is proposed to solve the flavor problem. It includes all the known fermions and some exotic fermions, which should be observed in the future if the theory is correct.

The group $SO(14)$ contains both $SU(7)$ and $SU(2)_L \times SU(2)_R \times SO(10)$ as subgroup, which leads to a natural synthesis of the Pati-Salam idea[5] of lepton number as a fourth color with the $SU(7)$ flavor unified model. It provides a left-right symmetry and a quark-lepton symmetry which indicates a realistic quark-lepton unification. All the fermions are included in one irreducible representation, which is asthetically very attractive.

While Γ symmetry is a global one in the $SU(7)$ model, it is part of the local gauge symmetry of $SO(14)$. This means symmetry brcaking is done spontaneously rather than soft breaking method, which is another attractive point.[6]

In Sec. II we introduce spinor representations and particle assignments, and diagonal generators of interesting subgroups. In Sec. III representation of Higgs fields are given in real base and in complex $SU(7)$ base. Their mutual unitary transformation is explicitly written, and used in break up of representations. In Sec. IV the spontaneous symmetry breaking pattern is studied in detail by giving concrete vacuum expectation values of Higgs fields in each step. Running coupling constants are briefly reviewed. In the last section summary is presented.

II. The Spinor Representation and the Fermion Assignments

We begin with the Clifford algebra by introducing notations.[7] The fourteen Γ-matrices are

$$\Gamma_1=\sigma_1{}^{(7)}$$

$$\Gamma_{2i}=\sigma_1{}^{(7-i)}\times\sigma_3\times I^{(i-1)},\quad i=1,\cdots,6$$

$$\Gamma_{2i+1}=\sigma^{(7-i)}\times\sigma_2\times I^{(i-1)},\quad i=1,\cdots,6$$

$$\Gamma_{14}=\sigma_2\times I^{(6)} \tag{2-1}$$

where $\sigma_i's$ are the Pauli matrices, and I is the 2×2 identity metrix. The tensor product can be understood by following two examples:

$$\sigma_1{}^{(3)}=\sigma_1\times\sigma_1\times\sigma_1 \text{ and } \sigma_1\times A=\begin{pmatrix}0 & A\\ A & 0\end{pmatrix}$$

It is straightforward to show that these fourteen $\Gamma's$ satisfy the Clifford relations

$$\Gamma_i\Gamma_j+\Gamma_j\Gamma_i=2\delta_{ij},\quad i,j=1 \text{ to } 14 \tag{2-2}$$

The ninetyone generators of $SO(14)$ are

$$S_{ij}=\frac{1}{4i}(\Gamma_i\Gamma_j-\Gamma_j\Gamma_i),\quad i\neq j \tag{2-3}$$

which has the Lie Algebra relations,

$$[S_{ij},\ S_{kl}]=i(\delta_{ik}S_{jl}+\delta_{jl}S_{ik}-\delta_{il}S_{jk}-\delta_{jk}S_{il}). \tag{2-4}$$

Diagonal generators among these are $S_{2i-1,2i}$, $i=1$ to 7, whose linear combinations form physical observables such as Q, Y and Γ. The $SU(7)$ singlet generator is

$$I_0=\sum_{i=1}^{7} S_{2i-1,\,2i} \tag{2-5}$$

and the $SU(7)$ Cartan subalgebra is made by

$$M_i=S_{2i+1,\,2i+2}-S_{2i-1,\,2i} \quad i=1 \text{ to } 7 \tag{2-6}$$

The third component of $SU(2)_W$ is

$$\begin{aligned} T_L^{(3)}&=\frac{1}{2}(S_{9,\,10}-S_{7,\,8}) \\ &=\frac{1}{2}M_4 \end{aligned} \tag{2-7}$$

The electric charge is given by

$$\begin{aligned} Q&=\frac{-1}{3}(M_1+2M_2+3M_3)+M_6 \\ &=\frac{1}{3}(S_{1,\,2}+S_{2,\,4}+S_{5,\,6})-S_{7,\,8}-S_{11,\,12}-S_{13,\,14} \end{aligned} \tag{2-8}$$

and the electroweak hyper charge is

$$\begin{aligned} Y&=Q-T_L^{(3)} \\ &=\frac{1}{3}(S_{1,\,2}+S_{3,\,4}+S_{5,\,6})-\frac{S_{7,\,8}+S_{9,\,10}}{2}-S_{11,\,12}+S_{13,\,14}. \end{aligned} \tag{2-9}$$

The quantum number which plays a pivotal role in the assignment of fermions is defined by

$$\Gamma=\frac{2}{3}(S_{12}+S_{34}+_{56})+2(S_{13,\,14}-S_{11,\,12}) \tag{2-10}$$

Fermions are assigned on 64-dim. spinor representation. First, we clasify the {64} by $SO(10)\times SU(2)_L\times SU(2)_R$,

$$\zeta=\{64\}=\{1,2,16\}+\{2,1,16^*\} \tag{2-11}$$

Then we denote $\{16\}=\{A,B,C,D\}^t$ and $\{16^*\}=\{A^*,B^*,C^*,D^*\}^t$ where superscript t stands for "transposed", and A,B,⋯, etc., represent $SU(4)_c$ quartet of Pati-Salam. The {64}

Table I $\zeta=\{64\}$ with quantum numbers and fermion assignments.

{64}	$\zeta_1\sim\zeta_{16}$				$\zeta_{17}\sim\zeta_{32}$			
$SO(10)$	A	C^*	A	C^*	D^*	B	D^*	B
$SU(2)_L$	•	↑	•	↓	↓	•	↑	•
$SU(2)_R$	↑	•	↓	•	•	↓	•	↑
$SU(3)_C$	(1, 3*)	(1, 3)	(1, 3*)	(1, 3)	(1, 3*)	(1, 3)	(1, 3*)	(1, 3)
Q_0	(0, −2/3)	(0, 2/3)	(1, 1/3)	(−1, −1/3)	(1, 1/3)	(1, 5/3)	(2, 4/3)	(0, 2/3)
Γ	(1, −1/3)	(−1, 1/3)	(1, −1/3)	(−1, 1/3)	(3, 5/3)	(1, 7/3)	(3, 5/3)	(1, 7/3)
particles	(ν^c, u^c)	(ν, c)	(e^c, d^c)	(u, s)	(M^c, b^c)	(τ^c, y)	(T^c, X^c)	(ν, t)

{64}	$\zeta_{33}\sim\zeta_{48}$				$\zeta_{49}\sim\zeta_{64}$			
$SO(10)$	C	A^*	C	A^*	B^*	D	B^*	D
$SU(2)_L$	$\cdot$	$\uparrow$	$\cdot$	$\uparrow$	$\uparrow$	$\cdot$	$\uparrow$	$\cdot$
$SU(2)_R$	$\uparrow$	$\cdot$	$\uparrow$	$\cdot$	$\cdot$	$\uparrow$	$\cdot$	$\downarrow$
$SU(3)_C$	$(1, 3^*)$	$(1, 3)$	$(1, 3)$	$(1, 3)$	$(1, 3^*)$	$(1, 3)$	$(1, 3^*)$	$(1, 3)$
Q	$(0, -2/3)$	$(0, 2/3)$	$(1, 1/3)$	$(-1, -1/3)$	$(-1, -5/3)$	$(-2, -1/3)$	$(0, -2/3)$	$(-1, -1/3)$
Γ	$(1, -1/3)$	$(-1, 1/3)$	$(1, 1/3)$	$(-1, 1/3)$	$(-1, -7/3)$	$(-3, -5/2)$	$(-1, -7/3)$	$(-3, -5/3)$
particles	(ν^c, c^c)	(ν, u)	(μ^c, s^c)	(e, d)	(z, y^c)	(T, X)	(ν, t^c)	(M, b)

*all fermions are left-handed.

is shown in the table I, where $\downarrow$, $\uparrow$, . show up, down and singlet states, respectively, with respect to $SU(2)$. All fermions are left handed chiral.

III. Representation of Higgs fields.

We consider scalar fields of the following representations: $\phi_\mu\{14\}$, $\phi_{\mu\nu}\{91\}$, $\phi_{\mu\nu\rho}\{364\}$, $\phi_{\mu\nu\rho\delta}\{1001\}$. They are totally antisymmetric in their indices and the number inside paranthesis denotes dimensions of each representation.

All these fields are real valued. The action of $SO(14)$ on these scalar fields are given by

$$\delta\phi_{\mu_1\cdots\mu_k}=\sum_{i=1}^{k} R_{\mu_i\sigma}\phi_{\mu_1\cdots\sigma\cdots\rho_k} \tag{3-1}$$

where $R_{\mu\nu}$ is the antisymmetric ninetyone generators of $SO(14)$. Two examples may illustrate the points clearly:

$$\delta\phi_\mu=R_{\mu\sigma}\phi_\sigma$$

$$\delta\phi_{\mu\nu}=R_{\mu\sigma}\phi_{\sigma\nu}+R_{\nu\sigma}\phi_{\mu\sigma}$$

It is convenient to rearrange the indices such that the $SU(7)$ subgroup properties are manifest in the formulation. For this the indices $1\sim14$ are divided into two categories, namely, odd $(1, 3, \cdots\cdots 13)$ and even $(2, 4, \cdots\cdots 14)$. Then, we introduce a unitary transformation U defined as

$$\Phi=UH \tag{3-2}$$

where

$$\Phi_\mu=(\phi_1, \phi_3, \cdots, \phi_{13}\ ;\ \phi_2, \phi_4, \cdots\phi_{14})^t \tag{3-3}$$

$$H_\alpha=(H^1, H^2, \cdots H^7\ ;\ H_1, H_2, \cdots H_7)^t \tag{3-4}$$

and

$$U=\frac{[i\sigma_1+I]}{\sqrt{2}}=\frac{1}{\sqrt{2}}\begin{pmatrix} I & i \\ i & I \end{pmatrix} \tag{3-5}$$

Since ϕ_μ is real, we have

$$H^a=\frac{1}{\sqrt{2}}(\phi_{2a-1}-i\phi_{2a})\,;\quad H_a=\frac{-i}{\sqrt{2}}(\phi_{2a-1}+i\phi_{2a}) \tag{3-6}$$

$$H_a=-iH^{a*},\ a=1,\cdots,\ 7 \tag{3-7}$$

The unitary transformation of R is naturally induced. For this we rewrite $R_{\mu\nu}$ as block matrices,

$$R_{\mu\nu}=\begin{bmatrix} A & B \\ -B^t & D \end{bmatrix},\ A=-A^t,\ D=-D^t \tag{3-8}$$

It can be very easily checked that the dimension of R is 91. A and D has dim 21, each and B has 49 dimension. By the unitary transformation we obtain

$$\begin{aligned} T&=U^+RU \\ &=\frac{1}{2}\begin{bmatrix} (A+D)+i(B+B^t), & (B-B)^t+i(A-D) \\ (B-B^t)-i(A-D), & (A+D)-i(B+B^t) \end{bmatrix} \\ &=\begin{bmatrix} X & Y \\ Y^* & X^* \end{bmatrix} \end{aligned} \tag{3-9}$$

with the constraint conditions

$$X=-X^+,\ \ Y=-Y^t \tag{3-10}$$

Since the $SU(7)$ generators are

$$T_7=\begin{bmatrix} X & 0 \\ 0 & X^* \end{bmatrix},\ X=-X^+,\ trX=0 \tag{3-11}$$

the corresponding subalgebra in $SO(14)$ is

$$R_7=\begin{bmatrix} A & B \\ -B & A \end{bmatrix},\ B=B^t,\ A=-A^t \tag{3-12}$$

The diagonal elemcnts of $SO(14)$ generators are

$$R_d=\begin{bmatrix} 0 & d \\ -d & 0 \end{bmatrix},\ d=\text{diag}\cdot(d_1,d_2,\ldots,d_7) \tag{3-13}$$

and corresponding $U(7)$ expression is

$$T_d=\begin{bmatrix} id & 0 \\ 0 & -id \end{bmatrix}. \tag{3-14}$$

The bases of $SU(7)$ diagonal generator is made by traceless conditions:

$$\begin{aligned} &M_1=\text{diag.}(1,-1,0,0,0,0,0), \\ &M_2=\text{diag.}(0,1,-1,0,0,0,0), \\ &\quad\vdots \\ &M_6=\text{dig.}(0,0,0,0,0,1,-1). \end{aligned} \tag{3-15}$$

An arbitrary diagonal generator of $SU(7)$ is

$$T_7=\begin{pmatrix} iM & 0 \\ 0 & -iM \end{pmatrix}, \tag{3-16}$$

where M is a linear combinations of $M_2,\cdots M_6$. The isomorphism between $R_{\mu\nu}$ and T_7 can be easily checked to be

$$M_i = R_{2i-1,\,2i} - R_{2i+1,\,2i+2} \tag{3-17}$$

The invariants made by Higgs fields can be obtained by contraction. For example, the simplest one is

$$\sum_{a=1}^{14} \phi'_a \phi_a = \tilde{\Phi}', \ \Phi = \Phi'^{+}\Phi = H'^{+}H \tag{3-18}$$

$$= (H'^1 \ldots H'^7, H_1' \ldots H_7')^* \begin{pmatrix} H^1 \\ \vdots \\ H^7 \\ H_1 \\ \vdots \\ H^7 \end{pmatrix}$$

$$= (H'^{a*}H^a + H'_a{}^* H_a) \tag{3-19}$$

Another example is

$$\sum_{\mu\nu=1}^{14} \phi_{\mu\nu}' \phi_{\mu\nu} = T_r \Phi'^{\prime} \Phi$$

$$= T_r H'^{+} H. \tag{3-20}$$

Let us turn to decomposition of totally antisymmetric tensor fields with respect to $SU(7)$, which will be useful in the next section. Here, the necessary notation[8] is

$$H_\alpha = (H^a, H_a), \ a = 1, \cdots, 7; \ \alpha = 1, \cdots 14 \tag{3-21}$$

$$H_\alpha\{14\} = H^a\{7\} + H_a\{7^*\}$$

$$H_{\alpha\beta}\{91\} = H^{ab}\{21\} + H_{ab}\{21^*\} + H_b{}^a\{48+1\} \tag{3-22a}$$

$$H_{\alpha\beta\gamma}\{364\} = H^{abc}\{35\} + H_{abc}\{35^*\} + H_c{}^{ab}\{7+140^*\} + H_{bc}{}^a\{7^*+140\} \tag{3-22b}$$

$$H_{\alpha\beta\gamma\delta}\{1001\} = H^{abcd}\{35\} + H_{abcd}\{35\} + H_d{}^{abc}\{21+224\} \tag{3-22c}$$

$$+ H_{bcd}{}^a\{21^*+224\} + H_{cd}{}^{ab}\{1+48+392\}. \tag{3-22d}$$

IV. The Spontaneous Symmetry Breaking Patterns

There can be many possible patterns of symmetry breaking in the $SO(14)$. To determin which one is correct one phenomenologically, we compute the following parmeters for each pattern by renormalization group methods: strong coupling constant α_c, electromagnetic fine structure constant α_{em}, Weinberg-angle θ_w and proton life time. We had shown in earlier works[6] that most of the symmetry breaking patterns are not compatible with presently available experimental data

but one, namely, $SO(14) \to SU(7) \times U(1)$. The full steps of breakings are

$$SO(14) \underset{M}{\downarrow} SU(7) \times U(1)_x \underset{M_1}{\downarrow} SU(4)_s \times SU(3)_w \times U'(1) x U(1)_x \underset{M_2}{\downarrow}$$

$$SU(4)_s\times SU(3)_w\times U(1)_{T}\underset{M_3}{\downarrow} G_w \underset{M_w}{\downarrow} G_e \tag{4-1}$$

where G_w and G_e denote $SU(3)_c \times SU(2)_L \times U(1)_Y$ and $SU(3)_c \times U(1)_{e.m.}$, respectively. The mass scale of each symmetry breaking stage is indicated at the head of each downward arrow.

For the first step $SO(14)\to SU(7)\times U_x(1)$, we note that $H_{\alpha\beta}$ is a good candidate to induce the breaking because it includes $SU(7)$ singlet· Indeed, the $H_a{}^a$ component is not only a singlet with respect to $SU(7)$ but also neutral under $U(1)_x$, because the upper and the lower indices carry oppotite $U(1)_x$ charge. Therefore, we can write the v.e.v. of $H_{\alpha\beta}$ as

$$\langle H_{\alpha\beta}\rangle=\tilde{v}\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix} \tag{4-2}$$

For this v.e.v. we can explicitly show that $SO(14)$ is broken to $SU(7)\times U(1)_x$ by computing the transformation of $\langle H_{\alpha\beta}\rangle$ under T. The invariance condition of $\langle H_{\alpha\beta}\rangle$ is

$$\begin{aligned}\delta\langle H_{\alpha\beta}\rangle&=0\\ &=T_{\alpha\sigma}\langle H_{\sigma\beta}\rangle+T_{\beta\sigma}\langle H_{\alpha\sigma}\rangle\\ &=\tilde{v}\begin{bmatrix}X & Y\\ Y^* & X^*\end{bmatrix}\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}+\tilde{v}\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\begin{bmatrix}X^t & Y^{*t}\\ Y^t & X^{*t}\end{bmatrix}\\ &=\tilde{v}\begin{bmatrix}Y^t & X+X^t\\ 0 & Y^*\end{bmatrix}\end{aligned} \tag{4-3}$$

The above equation cleary shows that $X=(-)X^t$, $Y=0$ is the invariant subgroup condition of $\langle H_{\alpha\beta}\rangle$ and, of course, this is nothing but $SU(7)\times U_x(1)$.

For the next stage we need another antisymmetric tensor field $H_{\alpha\beta}'$ which has $SU(4)_s\times SU(3)_w\times U'(1)\times U(1)_x$ singlet component. The desired v.e.v. is

$$\langle H'_{\alpha\beta}\rangle=\begin{bmatrix}0 & V_1\\ 0 & 0\end{bmatrix} \tag{4-4}$$

$$V_1=\text{diag.}(3v_1, 3v_1, 3v_1, -4v_1, -4v_1, 3v_1, -4v_1).$$

Once again we can compute the invariant group,

$$\begin{aligned}\delta\langle H'_{\alpha\beta}\rangle&=0\\ &=\begin{bmatrix}X & 0\\ 0 & X^*\end{bmatrix}\begin{bmatrix}0 & V_1\\ 0 & 0\end{bmatrix}+\begin{bmatrix}0 & V_1\\ 0 & 0\end{bmatrix}\begin{bmatrix}X^t & 0\\ 0 & X^t\end{bmatrix}\\ &=\begin{bmatrix}0 & XV_1+V_1X^t\\ 0 & 0\end{bmatrix}\end{aligned} \tag{4-5}$$

This leads to $0=[X, V_1]$ which, in turn, means $SU(4)_s\times SU(3)_w\times U'(1)\times U(1)_x$ leaves V_1 invariant.

This third step is realized by $\langle H_{\mu\nu\lambda\sigma}\rangle=\{H^{1236}=v_2$, otherwise zero$\}$. It is manifest that

H^{1236} is singlet under $SU(4)_s \times SU(3)_w$ but not with respect to $U'(1) \times U(1)_x$. However, it is neutral with respect $U(1)_\Gamma$ because

$$\Gamma(H^{1236})=0 \tag{4-6}$$

The Γ breaking fourth stage is obtained by $\langle H_{\mu\nu\lambda}\rangle=(H_6^{45}=v_3,$ others zero). H_6^{45} is singlet with respect to $SU(3)_c \times SU(2)_w \times U(1)_Y$ but it breaks Γ

$$\Gamma(H_6^{45}) \neq 0 \tag{4-7}$$

For the last case we employ $\langle H_\alpha\rangle=\{H_5=v_w,$ others zero$\}$. H_5 is singlet with respect to $SU(3)_c \times U(1)_{em}$, but breaks $SU(2)_w \times U(1)_Y$.

At each stage gauge bosons of broken generators gain mass. For the purpose of illustration, we show how the gauge bosons of the first stage become massive and omit other lower levels because they are very similar. From the Lagrangian of Φ fields we get the mass term,

$$M=\sum_{\alpha\beta}(W_{\alpha c}^* \ V_{c\beta}+W^*_{\beta c}V_{\alpha c})(W_{\alpha b}V_{b\beta}+W_{\beta b}V_{\alpha b}) \tag{4-8}$$

$$=Tr(W^*V+VW^{*t})^t(WV+VW^t).$$

When $W=\begin{bmatrix} X & Y \\ Y^* & X^* \end{bmatrix} \qquad V=\bar{v}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$WV+VW^t=\bar{v}\begin{bmatrix} Y^t & X+X^+ \\ 0 & Y^* \end{bmatrix}=\bar{v}\begin{bmatrix} Y^t & 0 \\ 0 & Y^* \end{bmatrix}$$

we have $M=\bar{v}^2\, Tr(Y^{t+}Y_t+Y^tY^{t+})=2\bar{v}^2\, Tr(YY^+)$. (4-9)

This clearly shows that the broken generators Y gain masses proportional to $(v.e.v.)^2$ This is called Higgs mechanism.

The evolution of the running coupling constants is studied in detail elsewhere[6]. We quote the result briefly. From the coupling constants evaluated at M_w we find three phenomenological parameters α_c, α_{em} and $\sin^2\theta_w$

$$\frac{1}{4\pi\alpha_c}=g_{3c}^{-2}(M_w),\ \frac{\sin^2\theta_w}{4\pi\alpha_{em}}=g_{2w}^{-2}(M_w),\ \frac{3}{17}\ \frac{\cos^2\theta_w}{4\pi\alpha_{em}}=g_y^{-2}(M_w). \tag{4-10}$$

Detail computations show that

$$\frac{1}{4\pi\alpha_c}-\frac{\sin^2\theta_w}{4\pi\alpha_{em}}=\left(\frac{-11}{24\pi^2}\right)\ln\frac{M_1}{M_w} \tag{4-11}$$

$$\frac{1}{4\pi\alpha_c}-\frac{3}{17}\cdot\frac{\cos^2\theta_w}{4\pi\alpha_{em}}=\left(\frac{-11}{24\pi^2}\right)\left\{\frac{-42}{17}\ln\frac{M_1}{M_3}+3\ln\frac{M_1}{M_w}\right\} \tag{4-12}$$

Denoting $\xi=\frac{6\pi}{11}\left(\frac{1}{\alpha_c}-\frac{\sin^2\theta_w}{\alpha_{em}}\right),\ \eta=\frac{6\pi}{11}\left(\frac{1}{\alpha_c}-\frac{3}{17}\ \frac{\cos^2\theta_w}{\alpha_{em}}\right),$

we obtain

$$\ln\frac{M_1}{M_w}=-\xi,\ \ln\frac{M_1}{M_3}=\frac{17}{42}(\eta-3\xi). \tag{4-13}$$

If we take $\sin^2\theta_w(M_w)\simeq 0.20$ which is the standard $SU(5)$ prediction, $\alpha_{em}=1/128.5$[9], $\alpha_c=0.23$[10] we get $M_1\simeq 6.43\times 10^{17} G_eV$, $M_3=531 G_eV$ when $M_w=100 G_eV$. Since M_1 is much greater than 10^{14} G_eV, there will be no observable proton decay. The grand unification mass M must be larger than M_1 but could be of order M_1.

V. Summary

By one irreducible unitary representation of $SO(14)$, we include all the known light fermions with few unobserved exotic particles. The exotic families of fermions carry weird Γ quantum number: (x, b), 5/3; (t, y), 7/3; (T, M), 3.

The representation of Higgs fields are considered both in real base and in $SU(7)$ base. The latter turns out especially useful for the symmetry breaking pattern study. For the pattern $SO(14)\rightarrow SU(7)\times U(1)$ we showed in detail the necessary vacuum expectation value to induce the symmetry breaking. The mass of gauge bosons generated by Higgs mechanism is of order 10^{17} GeV, which is responsible for baryon number violation. So in this model proton is effectively stable.

In the $SU(7)$ model, the breaking of Γ symmetry was made by soft breaking in the Higgs sector to avoid the appearance of a Goldstone boson. In the present theory we don't worry about the Goldstone particle because Γ is a local gauge symmetry. It is simply broken spontaneously.

The full symmetry breaking steps are realized in sequence, by $\langle H_{\alpha\beta}\rangle$, $\langle H_{\alpha\beta}'\rangle$, $\langle H_{\mu\nu\lambda\sigma}\rangle$, $\langle H_{\mu\nu\lambda}\rangle$, $\langle H_{\alpha}\rangle$.

References

1. P.W. Higgs, Phys. Rev. Lett. **12**, 132 (1964).
2. S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symoposium No. 8), edited by N. Svrtholm (Almquist and Wiksells, Stockholm, 1968), p367.
3. H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974)
4. J.E. Kim, Phys. Rev. Lett. **45** 1916 (1980); Phys. Rev. **D23**, 2706 (1981).
5. J.C. Pati and A. Salam, Phys. Rev. **D10**, 275(1974).
6. K.S. Soh, J.E. Kim, H.S. Song, S.N.U. preprint (1982).

7. Y. Fujimoto, ICTP *preprint* (1981); H. Georgi and D.V. Nanopoulos, Nucl. Phys. **B155** 52 (1979)

8. D.B. Lichtenberg, Unitary Symmetry and Elementary Particles p84 (A.P. New York 1978)

9. W.J. Marciano, Phys. Rev. **D20**, 274 (1979)

10. S. Ting, Talk presented in 1981 Coral Gables conference, Miami, Jan. 1980.

Acknowledgment

This research has been supported in part by the grants from the Ministry of Education through the College of Education, S.N.U.

초 록

SO(14)에서 대칭성의 깨어짐

蘇 光 燮
(物理敎育科)

$SO(14)$군이 $SU(3)_c \times U(1)_{em}$으로 나누어지는 과정을 구체적으로 히그스장의 진공기대값으로써 보였다. 그리고 재규격화군에 의한 작용상수들의 변화를 계산하여 $SO(14) \to SU(7) \times U(1)$이 실험사실들과 부합됨을 고찰했다.