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Supermatrix Formulation for Quantum Electrodynamics

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Massless QED: We present a new notation to describe both photon and electron fields in QED, which not only formulates the theory very succinctly but also suggests alternative views on the theory, and an extension of Lorentz symmetry. The new unifying field is

$$D_0 = \begin{bmatrix} A & \frac{1}{\sqrt{\mu}}\Psi \\ \frac{1}{\sqrt{\mu}}\overline{\Psi} & 0 \end{bmatrix}, \tag{1}$$

where $A = A_\mu \gamma^\mu$ is a 4×4matrix, Ψ is a 4-component Dirac spinor, $\overline{\Psi} = \Psi + \gamma^0$, and μ is a parameter of mass-dimension. Do is a 5×5 supermatrix, A is a bosonic variable, $\Psi(\overline{\Psi})$ is Grassmannian.(We consider $\Psi(\overline{\Psi})$ as anticommuting fields as in the path-integral quantization of QED.) With this Do we can write the Lagrangian for the massless QED as

$$L_0 = str\left\{\frac{1}{8}(\nabla D_0)^2 + \frac{e\mu}{3}(D_0)^3\right\}, \tag{2}$$

where

$$\nabla = \begin{bmatrix} \partial_\mu \gamma^\mu & 0 \\ 0 & -i4\mu \end{bmatrix} \tag{3}$$

and str denotes supertrace which is the trace of upper bosonic block minus the trace of lower bosonic block.

In the conventional notations the Lagrangian (2) is

$$L_0 = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 + i\overline{\Psi}\partial_\mu \gamma^\mu \Psi - e\overline{\Psi} A_\mu \gamma^\mu \Psi \tag{4}$$

We notice that $\frac{1}{2}\left(\partial_\mu A^\mu\right)^2$ is a gauge fixing term of opposite sign from the Feynman gauge, and the parameter μ is absent in the last expression.

For the path integral quantization we introduce an external source J

$$J = \begin{bmatrix} j_\mu \gamma^\mu & \xi \\ \overline{\xi} & 0 \end{bmatrix}, \tag{5}$$

such that

$$strJD_0 = 4 j^\mu A_\mu - \overline{\xi}\Psi - \overline{\Psi}\xi, \tag{6}$$

and a generating functional can be written as

$$Z[J] = \int d[D_0] \ \exp\ i\left\{\int d^4 x L + str\ \ JD_0\right\} \tag{7}$$

Famil Extension and Massive QED; It is rather trivial to include muon and tau fields by extending the dimension of D_0 as

$$\overline{\Psi}_1 \cdots \ D_0 = \begin{bmatrix} A_\mu \gamma^\mu & \frac{1}{\sqrt{\mu}}\Psi_1 & \frac{1}{\sqrt{\mu}}\Psi_2 & \frac{1}{\sqrt{\mu}}\Psi_3 \\ \frac{1}{\sqrt{\mu}}\overline{\Psi}_1 & & & \\ \frac{1}{\sqrt{\mu}}\overline{\Psi}_2 & & \mathbf{0} & \\ \frac{1}{\sqrt{\mu}}\overline{\Psi}_3 & & & \end{bmatrix}, \tag{8}$$

where Ψ_f, $f = 1,2,3$ are electron, muon, and tau fields, respectively. The D_0 is a 7×7 matrix field, whose first 4-index represents spinors and the rest three indices denote family. It is a kind of dimensional extension from the spinor space to the family space. The Lagrangian (2) does now formulate compactly the dynamics of photon, electron, moun, and tau fields.

For the massive QED we modify the D_0-field (1) to include a mass term b as

$$D_0 = \begin{bmatrix} A & \frac{1}{\sqrt{\mu}}\Psi \\ \frac{1}{\sqrt{\mu}}\overline{\Psi} & b \end{bmatrix}, \tag{9}$$

with which the Lagrangian becomes

$$\begin{aligned} L &= str\left[\frac{1}{8}(\nabla D)^2 + \frac{e\mu}{3}(D)^3\right] \\ &= \frac{-1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 + i\overline{\Psi}\partial_\mu\gamma^\mu\Psi - e\overline{\Psi}A_\mu\gamma^\mu\Psi - eb\overline{\Psi}\Psi + \left(2\mu^2 b^2 - \frac{e\mu}{3}b^3\right) \end{aligned} \tag{10}$$

For a constant b the electron mass is simply

$$m_e = eb \tag{11}$$

and the parameter μ plays no role in the dynamics.

The extension to include other families of charged leptons is again straightforward.

Discussions: Just for the sake of interest we may consider *b(x)* as a non-propagating auxilliary field, and remove it using field equations

$$b(x) = \frac{2\mu}{e}\left(1 \pm \sqrt{1 - \frac{e^2}{4\mu^3}\sigma}\right) \tag{12}$$

where $\qquad \sigma \equiv \overline{\Psi}\Psi \qquad (13)$

Then the Lagrangian becomes

$$\begin{aligned} L_b &\equiv -eb\overline{\Psi}\Psi + \left(2\mu^2 b^2 - \frac{e\mu}{3}b^3\right) \\ &= \frac{16}{3}\frac{\mu^4}{e^2} - 2\mu\sigma \pm \left(\frac{16\mu^4}{3e^2} - \frac{4\mu}{3}\sigma\right)\sqrt{1 - \frac{e^2}{4\mu^3}\sigma} \end{aligned} \tag{14}$$

where the local minimum of the potential occurs at the upper(+) sign. In this case we can show the electron mass is $m_e \approx 4\mu$. Even though this Lagrangian (14) is highly nonlinear, it is reminiscent of the chiral dynamics and might be of some use.[1]

It is possible to extend the D-field (8) to include not only photon, electron and its families but all the known leptons, quarks and vector bosons. Grand unified models based upon global SO $(4)_{L+R} \times SO(6)_c$ symmetry [2] and SO(10) group [3] have been proposed. [4] The Lagrangian (2) now reproduces conventional GUT Lagrangians except a major difference: the gauge field is only abelian *i.e.*

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \tag{15}$$

which is, of course, a very serious defect. Therefore this line of generalization seems unsuccessful. Another plausible generalization is to extend the Lorentz symmetry which is expressed here as

$$\overline{\Psi}\ D' = \begin{bmatrix} S & O \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_\mu \gamma^\mu & \frac{1}{\sqrt{\mu}}\Psi \\ \frac{1}{\sqrt{\mu}}\overline{\Psi} & 0 \end{bmatrix} \begin{bmatrix} S^{-1} & 0 \\ 0 & 1 \end{bmatrix}, \tag{16}$$

where S is the spinor representation of the Lorentz transformation $\Psi' = S\Psi$.

The extension to the form

$$\begin{pmatrix} S & O \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} S & T \\ U & V \end{pmatrix} \tag{17}$$

seems quite natural, but it requires to transform ∇ similarly in order to keep the Lagrangian invariant. Since this is different from currently fashionable supersymmetry it may worth further studies. [4]

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요약

양자전자기학의 초행열적 표현

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광량자와 전자를 하나로 통합하는 초행열을 도입하여 양자전자기학의 수식체계를 대단히 간략하게 표현하였다. 라그란지안은 아주 간략하게 $L = str\left(\frac{1}{8}(\nabla D)^2 + \frac{e\mu}{3}D^3\right)$으로 표현된다. 여러개의 전자같은 입자를 포함하도록 쉽게 일반화를 할 수 있으며, 비 아벨군으로 확장도 가능하다.