Comparison of Two FFT Structures for Fractionally-Spaced Frequency Domain Equalizer

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Abstract – **The use of a fractionally-spaced frequency domain equalizer (FS-FDE) needs to transform the received signal into the frequency domain. We consider the two fast Fourier transform (FFT) schemes, 2*N*-point FFT and two *N*-point FFTs, for transform of oversampled *N* symbol data in the FS-FDE receiver. It is shown that the two receiver schemes are equivalent in terms of the characteristics of the FFT output, per-bin SNR after the maximum-ratio combining, computational complexity and residual out-of-band noise rejection property. These properties are verified by computer simulation in additive white Gaussian noise (AWGN) and typical multi-path channel.**

I. INTRODUCTION

As the demand for data communications increases with explosive growth of the Internet access and interactive multimedia services, broadband wireless access techniques are emerging as an attractive and economical alternative to the wireline access technologies [1,2]. The use of multi-level QAM signal has been considered to maximize the spectrum efficiency, providing data rates of tens of Mbps. There exists, however, a fundamental challenge in transmitting at high data rate over the radio channel; the receiver must overcome the inter-symbol interference (ISI) due to multi-path propagation.

The ISI problem can be mitigated by employing an equalizer in the receiver. The performance and complexity of the equalizer significantly depends on the span of the ISI. In a typical radio channel with an RMS delay spread of $1\mu s$, the ISI has a span of about 3~4 symbols when the symbol rate is 1 MBaud. The use of a conventional decision-feedback equalizer (DFE) with a moderate tap size can provide acceptable equalization performance [3]. However, when the symbol rate is larger than 10 MBaud, the use of a conventional DFE may not be practical due to large implementation complexity and long initialization time.

The use of a single-carrier transmission scheme with a frequency-domain equalizer (SC-FDE) has been considered as an alternative low-complexity scheme. [4-8]. The SC-FDE transmits the information data in a block for ease of processing in the frequency domain. The FDE is known to be simpler to implement and easier to initialize than the time-domain equalizer [7]. Early works on the SC-FDE assumed baud-rate sampling in the receiver front-end [4-6]. However, the use of a fractionally-spaced equalizer is preferred due to its robustness to the sampling phase and the residual out-of-band noise rejection property [9]. Recently, there have been proposed two types of fractionally-spaced FDEs (FS-FDEs) [7,8]. It was reported that both schemes can provide good equalization performance with computational complexity considerably lower than the time-domain equalizer. When each block comprises N data symbols, the receiver in [7] employs a single $2N$-point FFT and the receiver in [8] uses two N-point FFTs to transform the oversampled $2N$ samples into the frequency domain. However, no analytic result has been reported on the comparison of the two FS-FDE receivers.

This paper compares these two FFT structures for the FS-FDE, particularly in terms of the characteristics of the FFT outputs, per-bin SNR after the maximum-ratio combining (MRC), computational complexity and residual out-of-band noise rejection property. Following Introduction, we describe a wireless transmission system that employs the FS-FDE in Section 2. The structures of the two receivers are described and compared in Section 3. Finally, conclusions are summarized in Section 4.

II. Single Carrier Modulation with FS-FDE

We consider a wireless transceiver as depicted in Fig. 1. The QAM symbol sequence $\{a_n\}$ is formatted into a block format comprising N symbols. As in the OFDM transceiver, each block contains a cyclic prefix of size N_g symbols. The duration $N_g T$ of the cyclic prefix should be larger than the channel delay spread, where T is the symbol time interval. The data symbol sequence is pulse-shaped by the transmit filter $g_T(t)$ with roll-off factor α and up-converted into the radio frequency for transmission.

The radio channel can be characterized by a time-varying linear filter whose impulse response is represented by

$$f(\tau) = \sum_l f_l(t)\delta(\tau - \tau_l) \tag{1}$$

where $f_l(t)$ and τ_l are the complex channel gain and the delay of the l-th path, respectively, and $\delta(\cdot)$ is the Kronecker delta function. Assuming that the channel has very slow fading, the channel gain can be assumed unchanged during the transmission of each block. Thus, the time-dependency of $f_l(t)$ can be omitted without loss of generality.

The baseband received signal after the receive filter $g_R(t)$ can be represented as

$$r(t) = \sum_{n=-\infty}^{\infty} a[n]h(t-nT) + n(t) \quad (2)$$

where $h(t)$ is the overall impulse response of the channel given by

$$h(t) = g_T(t) * f(t) * g_R(t) \quad (3)$$

Here * denotes the convolution process. The received signal oversampled at a rate of $2/T$ can be represented as

$$r[n] = \sum_k h[k]a[n-k] + n[n] \quad (4)$$

where $h[n]$ represents $h(t)$ sampled at $t = nT/2$ and $n[n]$ is the sampled zero mean AWGN with autocorrelation function $R_{nn}[k] = N_o \delta[k]$.

The sampled received signal $r[n]$ is transformed by the FFT for equalization in the frequency domain. The output of the FDE is transformed back to the time-domain by the inverse FFT (IFFT). Note that the SC-FDE receiver performs equalization in the frequency domain and detection in the time domain, while the OFDM scheme performs both equalization and detection in the frequency domain.

III. Two FFT Structures for FS-FDE

We consider the two FS-FDE receivers proposed in [7,8], whose block diagram is depicted in Fig. 2. One processes $2N$ samples using a single $2N$-point FFT and the other two N-point FFTs. We call the single $2N$-point FFT and two N-point FFTs schemes "type-$2N$" and "type-N", respectively, for ease of description.

In the type-$2N$ receiver, the $2N$ samples are transformed into the frequency domain by the $2N$-point FFT. Let $R[k]$ be the output of the $2N$-point FFT,

$$\begin{aligned} R[k] &= \sum_{n=0}^{2N-1} r[n]\exp[-j\frac{2\pi kn}{2N}] \\ &= A[k]H[k] + N[k],\ k = 0,1,\ldots,2N-1 \end{aligned} \quad (5)$$

where $A[k]$, $H[k]$ and $N[k]$ are the discrete Fourier transform (DFT) of $a[n]$, $h[n]$ and $n[n]$, respectively,

$$\begin{aligned} A[k] &= \sum_{n=0}^{2N-1} a[n]\exp[-j\frac{2\pi kn}{2N}] \\ H[k] &= \sum_{n=0}^{2N-1} h[n]\exp[-j\frac{2\pi kn}{2N}] \\ N[k] &= \sum_{n=0}^{2N-1} n[n]\exp[-j\frac{2\pi kn}{2N}] \end{aligned} \quad (6)$$

Note that, since $a[n] = 0$ for an odd n, $A[k+N] = A[k]$. Thus, $R[k]$ and $R[k+N]$ contain the same data information although the channel gains are different.

In the type-N receiver, the $2N$ samples are divided into the even and odd phase terms and then transformed into the frequency domain using two N-point FFTs. Let $R_e[k]$ and $R_o[k]$ be the output of the N-point FFT of the even and odd phase samples, respectively. Then $R_e[k]$ and $R_o[k]$ can be written as

$$\begin{aligned} R_e[k] &= \sum_{n=0}^{N-1} r[2n]\exp[-j\frac{2\pi kn}{N}],\ k = 0,1,\ldots,N-1 \\ R_o[k] &= \sum_{n=0}^{N-1} r[2n+1]\exp[-j\frac{2\pi kn}{N}],\ k = 0,1,\ldots,N-1 \end{aligned} \quad (7)$$

Note that $R_e[k]$ and $R_o[k]$ contain the same data information. It can be shown that

$$\begin{aligned} R[k] &= R_e[k] + W_{2N}^k R_o[k] \\ R[k+N] &= R_e[k] - W_{2N}^k R_o[k] \end{aligned} \quad (8)$$

or

$$\begin{aligned} R_e[k] &= A[k]\frac{H[k]+H[k+N]}{2} + \frac{N[k]+N[k+N]}{2} \\ R_o[k] &= A[k]\frac{H[k]-H[k+N]}{2W_{2N}^k} + \frac{N[k]-N[k+N]}{2W_{2N}^k} \end{aligned} \quad (9)$$

where $W_{2N} = \exp(-j/2N)$. It can be seen that the output of the $2N$-point FFT is uniquely determined by the output of the N-point FFT and vice versa. That is, both FFT outputs contain the same information on the received signal.

We consider the per-bin SNR after the MRC. The MRC operation combines the two frequency bins containing the same data information, that is, $R[k]$ and $R[k+N]$ in the type-$2N$ receiver and $R_e[k]$ and $R_o[k]$ in the type-N receiver. The weighting coefficient in the MRC is the complex conjugate of the channel coefficient of each frequency bin. For ease of description, we assume that the channel coefficients are estimated perfectly using a periodically inserted pilot signal.

The equalizer (EQ) operation divides the MRC output by the channel gain. In the type-$2N$ receiver, the output of EQ is

$$Y[k] = \frac{H^*[k]R[k] + H^*[k+N]R[k+N]}{|H[k]|^2 + |H[k+N]|^2} \quad (10)$$

when the zero-forcing (ZF) criterion is used and

$$Y[k] = \frac{H^*[k]R[k] + H^*[k+N]R[k+N]}{|H[k]|^2 + |H[k+N]|^2 + N_o/E\{|a[n]|^2\}} \quad (11)$$

when the minimum mean-squared error (MMSE) criterion is used, where $E\{X\}$ denotes the ensemble average of X. The output of the EQ in the type-N receiver can be calculated in a similarly way.

After the MRC, the k-th frequency bin can be represented by

$$\begin{aligned} U[k] &= H^*[k]R[k] + H^*[k+N]R[k+N] \\ &= A[k]\left(|H[k]|^2 + |H[k+N]|^2\right) \\ &\quad + H^*[k]N[k] + H^*[k+N]N[k+N] \end{aligned} \quad (12)$$

in the type-2N receiver and

$$\begin{aligned} V[k] &= \left(H^*[k]+H^*[k+N]\right)R_e[k] \\ &\quad + W_{2N}^k\left(H^*[k]-H^*[k+N]\right)R_o[k] \\ &= A[k]\left(|H[k]|^2+|H[k+N]|^2\right) \\ &\quad + H^*[k]N[k]+H^*[k+N]N[k+N] \\ &= U[k] \end{aligned} \tag{13}$$

in the type-N receiver. Since both receivers produce the same signal and noise components after the MRC, they provide the same SNR at the decoder input.

The computational complexity of the two FDE schemes can be compared in terms of the number of multiplications/divisions for each symbol. There are five major operations in the receiver: FFT, MRC, channel estimation, equalization and IFFT.

Considering the FFT, the 2N-point FFT and two N-point FFTs require $8N\log_2 2N$ and $8N\log_2 N$ multiplications. The complexity difference between the two FFT schemes is small compared to the whole computational complexity of the FFT when $N \geq 64$.

Since the out-of-band frequency bins can be excluded in the combining process, the type-2N requires less computation than the type-N for the MRC and channel estimation. For the MRC, the type-2N and type-N require $4(1+\alpha)N$ and $8N$ multiplications, respectively. The computational complexity for the channel estimation is largely dependent on the employed estimation algorithm. When the least-square (LS) algorithm is used, which is usually referred as one of the lowest implementation complexity [10], the type-2N and type-N require $4(1+\alpha)N$ and $8N$ multiplications, respectively.

Both schemes require the same one tap equalization and N-point IFFT. The equalization requires N divisions and the IFFT requires $4N\log_2 N$ multiplications. Table 1 summarizes the number of multiplications/divisions per symbol required by the two receivers. The computational complexity per symbol is depicted in Fig. 3 as a function of N, when the roll-off factor of the shaping filter is 0.2, 0.4 and 0.6. It can be seen that the type-N receiver requires computational complexity slightly less than the type-2N receiver. It can also be seen that the difference increases as the roll-off factor increases.

The use of oversampling enables both receivers to reject the out-of-band noise that cannot be removed by the receive filter. Since the frequency bins with index $N(1+\alpha)/2 < k < N(3-\alpha)/2$ contain only the noise component, they can be excluded explicitly by the MRC in the type-2N receiver. The out-of-band noise component $N[k+N]$, $k > N(1-\alpha)/2$, can be cancelled in the combining process and only the in-band signal and noise terms remain in the type-N receiver. Thus, the use of the FS-FDE does not require a receive filter with sharp cut-off response.

To evaluate the performance of the two FDE structures, computer simulation is performed in AWGN and indoor multi-path channels. The parameters of the SC-FDE systems are determined similar to those of the OFDM system specified in the IEEE 802.11a [8] as summarized in Table 2. The length of the FFT block and cyclic prefix are 64 and 12. The multi-path channel has a RMS delay spread of 50ns, which covers most of the indoor propagation environment [11].

To see the residual out-of-band noise rejection property, we consider the use of 65-tap or 26-tap receive filter, whose frequency response is depicted in Fig. 4. Fig 5 depicts the BER performance when the two MMSE FS-FDE schemes are applied to 16-QAM and 64-QAM with no channel coding. Perfect channel estimation is assumed in the MRC. It can be seen that the two schemes provide similar performance. It can also be seen that the performance does not degrade when the 26-tap receive filter is used instead of the 65-tap receive filter.

IV. Concluding Remarks

This paper has investigated the characteristics of the two FS-FDE receivers proposed in [7,8]. They are compared in terms of the property of the FFT output, per-bin SNR after the MRC, computational complexity and residual out-of-band noise rejection property. Both receivers provide the same BER performance regardless of the equalization scheme and have the residual out-of-band noise rejection property. The type-N scheme requires computational complexity slightly less than the type-2N scheme. Since the two schemes are equivalent in the BER performance and require almost the same computational complexity, they can be equally applied to the transmission of single-carrier QAM signals.

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Table 1. Number of multiplications/divisions per symbol.

Operation	Type A	Type B
FFT	$8\log_2 2N$	$8\log_2 N$
MRC	$4(1+\alpha)$	8
Channel Estimation	$4(1+\alpha)$	8
Equalization	2	2
IFFT	$4\log_2 N$	$4\log_2 N$

Table 2. System parameters.

Symbol rate (Mbaud)	14.25
Block length (μs)	4.4912
Cyclic Prefix Length (μs)	0.8421
Data symbol per block	64

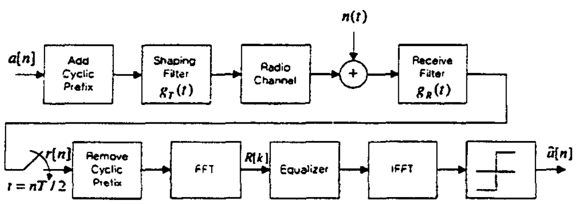
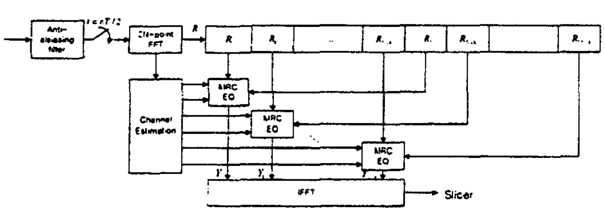
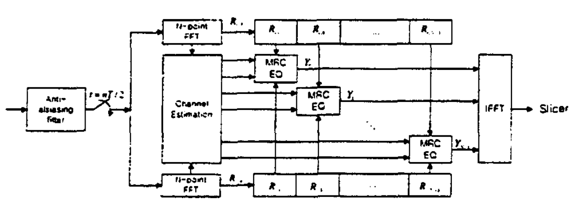


Fig. 1. System model.



(a) type-2N receiver



(b) type-N receiver

Fig. 2 Block diagram of the FS-FDE.

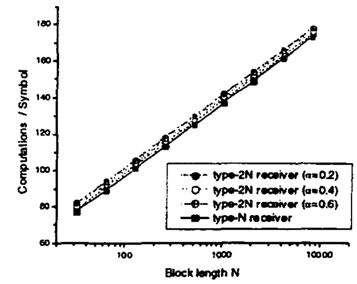


Fig. 3 Comparison of the computational complexity.

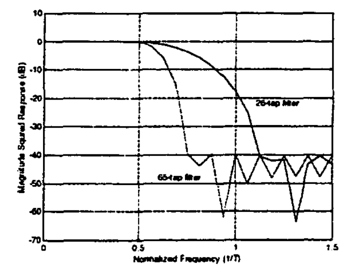
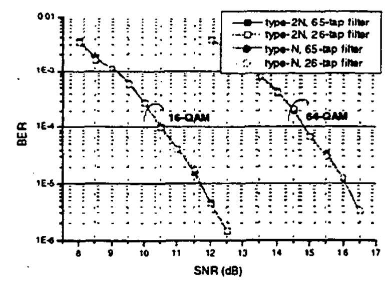
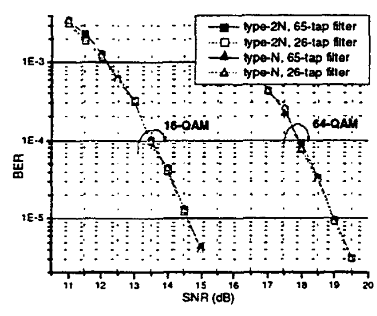


Fig. 4 Frequency response of the receive filter.



(a) AWGN channel



(b) Multi-path channel

Fig. 5 BER peformance.