Performance Analysis of Binary-CDMA systems in Multi-Path Fading Channel

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Abstract - Binary CDMA (B-CDMA) is a new multiple access communication scheme that employs a constant envelope modulation scheme. By quantizing the envelope of multiple users' CDMA signal into a small number of levels, the B-CDMA can reduce the peak-to-average power ratio, while preserving the advantages of CDMA signaling such as the soft capacity and robustness to interference. In this paper, we analyze the performance of B-CDMA systems in multi-path channel assuming that the spreading factor is not too small. Finally, the analytic results are verified by computer simulation.

1. Introduction

In CDMA systems, multiple codes are used for data transmission of multiple users. However the sum of multiple codes causes a large peak-to-average power ratio (PAPR), requiring the use of linear power amplifiers with a large back-off. Binary CDMA (B-CDMA) is a new modulation method that quantizes multiple users' CDMA signal into a small number of levels for constant envelope modulation [1]. Thus, the B-CDMA scheme can reduce the power amplifier burden, while preserving the advantages of CDMA signaling such as the soft capacity and robustness to interference. The B-CDMA system is quite suitable for wireless transmission systems that require low cost and/or low power consumption. For example, it can effectively be applied to wireless home networking and satellite communications [2] [3].

The B-CDMA signal can be generated by various methods, such as the pulse width (PW), multi-phase (MP) and code selection (CS) methods [1]. The PW B-CDMA signal can be obtained by transforming the magnitude of multi-level signal into a finite number of pulse widths. The MP B-CDMA is generated by transforming the magnitude of multi-level signal into a finite number of phases. The CS B-CDMA can be obtained by selecting the spreading code according to the data bits. Then, the selected code is modulated using a PSK modulation scheme. In the PW B-CDMA system, the transmission bandwidth increases as the quantization level increases. In practice, the signal can be quantized into two levels to accommodate the transmission bandwidth [1]. Note that the two-level PW B-CDMA is a special case of the MP B-CDMA.

Nonlinear quantization in the B-CDMA signaling makes it difficult to analytically evaluate the performance. Most of previous results were obtained by computer simulation in additive white Gaussian noise (AWGN) channel [4,5]. The performance of PW B-CDMA system was evaluated under a special condition when the magnitude of the signal after the despreading is constant [5]. The performance of MP B-CDMA system was analyzed by calculating the error probability considering all possible combinations of the signal [6]. However this method may not be applicable when the spreading gain is large.

In this paper, we analyze the performance of MP B-CDMA system in multi-path fading channel assuming that the spreading factor and the number of user are not too small. Since the CS B-CDMA is the same as the MP B-CDMA except the code selection block [7], the analytic results can also be applied to the analysis of CS B-CDMA system.

2. System model

In the MP B-CDMA system, the sum of multiple users’ data is quantized into a finite number of levels and then modulated using a PSK modulation scheme. Fig. 1 depicts the transceiver structure of a baseband-equivalent MP B-CDMA system, where \( b_i \) denote the data bit of the \( j \)-th user and \( d = [d_1, d_2, \cdots, d_N] \) denotes the sum of multiple users’ data, given by

\[
d_i = \sum_{k=1}^{N_c} b_k c_i^k
\]

Here, \( N_c \) is the number of users and \( N \) is the spreading factor and \( c_i^k \) is the \( i \)-th chip of spreading code \( c_i = [c_i^1, c_i^2, \cdots, c_i^N] \) for the \( j \)-th user with unit power (i.e., \( \sum_{l} |c_i^l|^2 = 1 \)). The sum of multiple users’ data is quantized at the chip-level as

\[
s_i = f_q(d_i)
\]

where \( s_i \) is the \( i \)-th chip of quantized signal \( s = [s_1, s_2, \cdots, s_N] \) and \( f_q(x) \) denotes the quantization function that maps the signal \( x \) in quantization region \( \Phi_v \) (i.e., \( x \in \Phi_v \)) onto signal point \( m_v \). The quantized signal \( s_i \) can also be represented as

\[
s_i = d_i + q_i
\]

where \( q_i \) denotes the quantization noise. The quantized signal \( s_i \) is PSK-modulated at the chip level as

\[
x_i = f_{map}(s_i)
\]
where \( \chi \) is the \( i \)-th chip of PSK modulated signal \( \mathbf{x} = [x_1, x_2, \cdots, x_n]^T \) and \( f_{\text{map}}(s) \) denotes the mapping function that maps the quantized signal point \( s = m_c \) onto a PSK constellation \( \mu_i \) as shown in Fig. 2. Note that the guard phase is required in the MP B-CDMA to reduce fatal errors between the signal points with the largest distance, such as \( \mu_1 \) and \( \mu_{\ell} \) [8].

Since the B-CDMA is mainly intended for low cost and low power applications in mild channel environment, we consider the transmission of B-CDMA signal over a wireless channel with a line-of-sight (LOS). The B-CDMA signal can be detected without using a rake receiver, reducing the implementation complexity. In this case the LOS path term is used for data detection and non-LOS path terms behave as the interference. Since the performance depends only on total amount of interference power, we can replace multiple non-LOS paths as a single one with the same interference power. Thus, we assume a two multi-path channel with LOS path gain \( h_L \) and non-LOS path gain \( h_N \) represented as

\[
h_L = A + d_i + j d_Q = g_L e^{j \theta}
\]

\[
h_N = h_I + jh_Q = g_N e^{j \phi}
\]

where \( d_i \) and \( d_Q \) are statistically independent zero mean Gaussian random variables with the same variance \( \sigma^2 \). \( A \) is the LOS component, \( h_I \) and \( h_Q \) are statistically independent zero mean Gaussian random variables with the same variance \( \sigma^2 \), \( g_L \) and \( g_N \) denote the amplitude gain of the LOS and non-LOS path, respectively, and \( \theta \) and \( \phi \) denote the phase gain of the LOS and non-LOS path, respectively. It can be shown that the probability density function (pdf) of \( g, \alpha \) and \( \varphi \) can be represented as [10]

\[
p_L(g_L) = \frac{g_L}{\sigma_L} \exp \left( -\frac{g_L^2 + A^2}{2\sigma_L^2} \right) I_0 \left( \frac{g_L A}{\sigma_L^2} \right)
\]

\[
p_N(g_N) = \frac{g_N}{\sigma_N} \exp \left( -\frac{g_N^2}{2\sigma_N^2} \right)
\]

\[
p_\varphi(\varphi) = \frac{1}{2\pi}, \quad 0 \leq \varphi < 2\pi
\]

where \( I_0(x) \) is the modified Bessel function of the first kind of the zero order

\[
I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp(x \cos \psi) d\psi
\]

The received signal can be represented as

\[
\mathbf{r} = [r_1, r_2, \cdots, r_n]^T
\]

where \( r_i \) denotes the \( i \)-th chip signal

\[
r_i = h_i x_i + h_{i-\Delta} n_i
\]

Here, \( n_i \) denotes the noise term in the \( i \)-th chip and \( \Delta \) denote the amount of delay of the non-LOS path.

The PSK demodulator transforms the phase information of \( r_i \) into the magnitude,

\[
y_i = f_{\text{map}}^{-1}(r_i)
\]

where \( y_i \) is the PSK demodulated \( i \)-th chip signal represented as

\[
y_i = s_i + e_i
\]

Here \( e_i \) denotes the error signal due to multi-path interference and additive noise.

The detection variable \( z_j \) for the \( j \)-th user can be obtained

\[
z_j = \mathbf{y}^T \mathbf{c}^j
\]

Since PSK demodulated signal \( \mathbf{y} \) can be written as

\[
\mathbf{y} = \sum_{k=1}^{N} b_k \mathbf{c}^k + \mathbf{q} + \mathbf{e}
\]

\( z_j \) can be rewritten as

\[
z_j = b_j (\mathbf{c}^j)^T \mathbf{c}^j + \sum_{k=1, k \neq j}^{N} b_k (\mathbf{c}^j)^T \mathbf{c}^k + \mathbf{q}^T \mathbf{c}^j + \mathbf{e}^T \mathbf{c}^j
\]

where \( \mathbf{e} = [e_1, e_2, \cdots, e_N]^T \) and \( \mathbf{q} = [q_1, q_2, \cdots, q_N]^T \). Since \( (\mathbf{c}^j)^T \mathbf{c}^j = N^2 \) and the spreading codes are orthogonal to each other, it can be shown that

\[
z_j = \mathbf{N} b_j + \sum_{i=1}^{N} t_i c_i^j + \sum_{i=1}^{N} e_i c_i^j
\]

where the first term is the desired signal, and the second and third terms denote the quantization noise and the PSK demodulation error, respectively. Finally, the binary data is detected by

\[
\hat{b}_j = \begin{cases} 
1, & \text{if } z_j \geq 0 \\
-1, & \text{if } z_j < 0 
\end{cases}
\]

3. Performance analysis

A. Quantization noise

Since the quantization introduces no correlation between the chips, we can assume that \( q_i c_i^j, i = 1, 2, \cdots, N \), are statistically independent and identically distributed (iid) random variables. Thus, the quantization noise \( \sum q_i c_i^j \) can be approximated as a zero mean Gaussian random variable with variance \( N\sigma_q^2 \) by the central limit theorem, where \( \sigma_q^2 \) is the variance of \( q_i c_i^j \).

The variance \( \sigma_q^2 \) can be calculated as
where $p_d(x)$ is the pdf of the sum of multiple codes $d_i$ at the i-th chip. The variance of quantization noise $\sigma_q^2$ highly depends on $m_v$ and $\Phi_v$. The optimum $m_v$ and $\Phi_v$ minimizing $\sigma_q^2$ can be obtained using the Lloyd-max algorithm [9]. Assuming that the number of multi-codes is not too small, the sum of multiple codes can be approximated as a Gaussian random variable. Since the spreading code of each user has unit power, the variance of $d_i$ is equal to the number of multi-codes. Thus, the pdf of $d_i$ can be approximated as

$$p_d(x) \cong \frac{1}{\sqrt{2\pi N_c}} \exp \left(-\frac{x^2}{2N_c}\right)$$  \hspace{1cm} (21)

**B. PSK detection error**

The PSK detection error $e_i c_i^j$ has correlation with the transmitted data bit $b_j$.

Denoting $\Gamma_{g_L,g_S} = E \left[ e_i c_i^j \mid g_L, g_S \right]$, the mean of PSK detection error $E \left[ e_i c_i^j \mid g_L, g_S \right]$ for given $b_j$, $g_L$ and $g_S$ can be represented as (refer to Appendix)

$$E \left[ e_i c_i^j \mid b_j, g_L, g_S \right] = b_j \Gamma_{g_L,g_S} \hspace{1cm} (22)$$

The variance $\sigma_{e_i c_i^j}^2$ of $e_i c_i^j$ for given $g_L$ and $g_S$ can be represented as

$$\sigma_{e_i c_i^j}^2 = E \left[ e_i c_i^j \mid g_L, g_S \right] - \left( \Gamma_{g_L,g_S} \right)^2 \hspace{1cm} (23)$$

where

$$E \left[ e_i c_i^j \mid g_L, g_S \right] = \sum_{m_v} \sum_{m_l} \sum_{m_n} p(m_v, m_l, m_n \mid g_L, g_S) \hspace{1cm} (24)$$

where $p(m_v, m_l, m_n \mid g_L, g_S)$ is the probability that signal $m_v$ is transmitted and signal $m_l$ is detected for given channel gain $g_L$ and $g_S$ (refer to Appendix). Since we can assume that $e_i c_i^j$, $j = 1, 2, \cdots, N_c$ are statistically iid, PSK detection error $\sum e_i c_i^j$ after the despreading can be approximated as a Gaussian random variable with mean $N b_j \Gamma_{g_L,g_S}$ and variance $N \sigma_{e_i c_i^j}^2$.

Considering the bias term $N b_j \Gamma_{g_L,g_S}$, the detection variable $z_j$ for the j-th user can be rewritten as

$$z_j = N \left[ 1 + \Gamma_{g_L,g_S} \right] b_j + Q + D \hspace{1cm} (25)$$

where $Q$ and $D$ are independent zero mean Gaussian random variables with variance $N \sigma_q^2$ and $N \sigma_{e_i c_i^j}^2$, respectively. Note that $\Gamma_{g_L,g_S}$ has a negative value, degrading the detection performance.

The bit error probability (BER) for given $g_L$ and $g_S$ can be represented as

$$P_{e_i c_i^j} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{N^2 \left( 1 + \Gamma_{g_L,g_S} \right)^2}{N \sigma_q^2 + N \sigma_{e_i c_i^j}^2}} \right) \hspace{1cm} (26)$$

where $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$.

Finally, the BER can be obtained by

$$P_e = \int \frac{1}{2} \text{erfc} \left( \sqrt{\frac{N \left( 1 + \Gamma_{g_L,g_S} \right)^2}{\sigma_q^2 + \sigma_{e_i c_i^j}^2}} \right) p(g_L) p(g_S) dg_L dg_S \hspace{1cm} (27)$$

**4. Performance evaluation**

To verify the analysis, the performance of two B-CDMA systems with $N = 128$ and $N = 64$ is evaluated by computer simulation in two-multipath fading channel: One is 8-PSK system with $L = 7$ with one guard phase and the other is QPSK system with $L = 4$ without guard phase. We use Walsh-Hadamard codes as the spreading code. The channel condition can be characterized in terms of $K$ and $\gamma$ defined as

$$K = 10 \log_{10} \left( \frac{A^2}{2 \sigma_L^2} \right) \hspace{1cm} (28)$$

$$\gamma = 10 \log_{10} \left( \frac{1}{2 \sigma_N^2} \right) \hspace{1cm} (29)$$

We assume that the Rician path has unit power, i.e., $A^2 + 2 \sigma_L^2 = 1$.

Fig. 3 and Fig. 4 depict the BER performance of MP B-CDMA systems associated with different quantization levels and interference power. It can be seen that the analytic results agree well with the simulation results but they are slightly better than the actual ones. This is mainly due to Gaussian approximation of error terms. In Fig. 3 (a), the B-CDMA with $L=4$ has floored BER performance at high SNR larger than 30dB unlike the DS-CDMA system, mainly due to the quantization noise. The effect of quantization noise can also be seen in Fig. 3 (b). Fig. 4 depicts the BER performance of B-CDMA with $L=7$. It can be seen that the performance of B-CDMA systems is improved as $L$ increases. As a result, the BER flooring effect appears at a higher SNR with the use of $L=7$ compared to the use of $L=4$. It may be desirable to properly choose the quantization level according to the operation condition.

Fig. 5 depicts the performance of B-CDMA system for different values of $N$ and $N_c$. The discrepancy between the analysis and simulation results is not negligible when $N$ and $N_c$ are small. This is mainly due to fact that the Gaussian approximation for the quantization noise and PSK detection error is not accurate for small values of $N$ and $N_c$. But it can be seen that the difference is still in affordable range for small values of $N$ and $N_c$. 

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5. Conclusion

In this paper, the performance of B-CDMA systems has been analyzed in terms of the BER in multi-path fading channel. Quantization noise and PSK detection error have been analyzed using Gaussian approximation assuming that the spreading factor is not too small. The analytic results have been verified by computer simulation. The B-CDMA can provide BER performance a few dB inferior to the DS-CDMA, while significantly reducing the implementation complexity. Numerical results indicate that the B-CDMA is quite applicable to real applications operating requirements. The analytic results can also be applied to other B-CDMA systems.

APPENDIX : Derivation of \( E\{c_c | g_L, g_N\} \)

Since \( e_c \) and \( b_j \) have zero mean, the covariance of \( e_c \) and \( b_j \) for given \( g_L \) and \( g_N \) is equal to

\[
\text{COV}(e_c, b_j | g_L, g_N) = E\{e_c | b_j | g_L, g_N\} \tag{A.1}
\]

where \( E\{x\} \) denotes the expectation of \( x \). Considering all the signal points, it can be seen that

\[
E\{e_c b_j | g_L, g_N\} = \sum_{i=1}^{N} \sum_{j} p(m_i, m_j | g_L, g_N) E\{e_c b_j | m_i, m_j\} = p(m_i, m_j | g_L, g_N) E\{e_c b_j | m_i, m_j\} = \sum_{i|1}^{N} \sum_{j} p(m_i, m_j | g_L, g_N) E\{e_c \} = \sum_{i|1}^{N} \sum_{j} p(m_i, m_j | g_L, g_N) E\{e_c \} \tag{A.2}
\]

where \( p(m_i, m_j | g_L, g_N) \) is the probability that signal \( m_i \) is transmitted and signal \( m_j \) is detected for given channel gain \( g_L \) and \( g_N \).

It can be shown that \( E\{e_c b_j | g_L, g_N\} \) has a negative value. When a transmitted signal point \( m_i \) has a positive amplitude, it is more likely that the PSK detection error \( e_c \) has a negative value. Since

\[
m_i = \sum_{k=1}^{N} c_k b_k + q_i \tag{A.3}
\]

and \( q_i \) has zero mean and is uncorrelated with \( c_k b_j \), it is more likely that \( c_k b_j \) will be 1 rather than -1, causing \( E\{e_c b_j \} \) to have a negative value. Similarly, for negative values of \( m_i \), it can be shown that the detection error \( e_c \) has a positive value rather than negative value and \( c_k b_j \) would -1 rather than 1.

Using the conditional probability, \( p(m_i, m_j | g_L, g_N) \) can be rewritten as

\[
p(m_i, m_j | g_L, g_N) = p(m_i, g_L, g_N) p(m_j | g_L, g_N) \tag{A.4}
\]

Here, \( p(m_i) \) is the probability of PSK signal point \( m_i \), represented as

\[
p(m_i) = \int_{\xi}^{1} \sqrt{2 \pi N_c} \exp\left(-\frac{x^2}{2N_c}\right) dx \tag{A.5}
\]

and \( p(m_i, m_j, g_L, g_N) \) is the error probability that the PSK signal point is misdected to \( m_w \) for given \( m_i \), \( g_L \) and \( g_N \). It can be shown that [10]

\[
p(m_i | m_w, g_L, g_N) = \begin{cases} \int_{\xi}^{1} \sqrt{2 \pi N_c} \exp\left(-\frac{x^2}{2N_c}\right) dx & \text{for } m_i \neq m_w \\int_{\xi}^{1} \sqrt{2 \pi N_c} \exp\left(-\frac{x^2}{2N_c}\right) dx & \text{for } m_i = m_w \end{cases} \tag{A.6}
\]

where \( E \) is the chip energy, equal to the energy of the PSK signal point, \( \xi \) is the ratio of the chip energy to noise spectral density, equal to \( E/\eta^2 \), \( \eta^2 \) is the phase of the l-th PSK signal point, and \( \phi' \) is the sum of phase of \( x_{i-5} \) and \( \phi \) of the non-LOS path. It can be assumed that \( \phi' \) is uniformly distributed over \( [0, 2\pi] \).

Since \( c_k b_j \) does not depend on \( m_w \), we have

\[
p(c_k b_j = d | m_w, m_i) = p(c_k b_j = d | m_i) \tag{A.7}
\]

There are statistically \( p(c_k b_j = 1 | m_i)N_k \) number of ones and \( p(c_k b_j = -1 | m_i)N_k \) number of minus ones out of \( N_k \) multiple users' data \( (c_k b_j, k=1,2,\ldots, N_k) \) for given \( m_i \). Thus, it can be shown that

\[
m_i \equiv [p(c_k b_j = 1 | m_i) - p(c_k b_j = -1 | m_i)]N_k \tag{A.8}
\]

Since

\[
p(c_k b_j = 1 | m_i) + p(c_k b_j = -1 | m_i) = 1 \tag{A.9}
\]

and from (A.8) and (A.9) we have

\[
p(c_k b_j = 1 | m_i) \equiv \frac{N_k + m_i}{2N_k}, \quad p(c_k b_j = -1 | m_i) \equiv \frac{N_k - m_i}{2N_k} \tag{A.10}
\]

It can be seen that

\[
E\{e_c c_k b_j | g_L, g_N\} = \sum_{j=1}^{N} \sum_{d} \xi p(e_c | b_j = d, g_L, g_N) p(b_j = d) = \sum_{d} p(b_j = d) E\{e_c c_k b_j = d, g_L, g_N\} \tag{A.11}
\]

where \( p(b_j = d) = p(b_j = -d) = 0.5 \). Since the PSK detection error has zero mean,

\[
E\{e_c c_k | g_L, g_N\} = \sum_{j=1}^{N} E\{e_c c_k b_j = d, g_L, g_N\} p(b_j = d) = 0 \tag{A.12}
\]

From (A.11) and (A.12), we can obtain that

\[
E\{e_c c_k b_j = 1, g_L, g_N\} = E\{e_c c_k b_j | g_L, g_N\} \tag{A.13}
\]

and

\[
E\{e_c c_k b_j = -1, g_L, g_N\} = -E\{e_c c_k b_j | g_L, g_N\} \tag{A.14}
\]

Denoting \( E\{e_c c_k b_j | g_L, g_N\} = \Gamma_{g_L, g_N} \), it can be shown that

\[
E\{e_c c_k b_j | g_L, g_N\} = b_j \Gamma_{g_L, g_N} \tag{A.15}
\]
References