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Spin Poincare Symmetry and Chirality of Neutrino

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INTRODUCTION

No right-handed neutrinos have been yet found, and neutrinos seem to be massless even though there are much research activities on their possible massiveness. It is puzzling because there is no compelling reason for their left-handedness and masslessness, for example no gauge principle or symmetry appears to work here. Neither is the Goldstone-neutrino idea helpful.[1] Would not nature look more symmetric if left and right-handed fermions pair each other? The purpose of this article is to show that the chiral left-handed world is in fact more symmetric than the one with both left and right-handed neutrinos. The reason is that there exist a natural symmetry which would be broken if there were right-handed neutrinos. This is surprisingly the translational symmetry as a part of the Poincare symmetry, but realized in a spinor space: transformation of spinors with respect to Lorentz group can be extended to the full Poincare group.

SPIN POINCARE ALGEBRA

We use Itzykson and Zuber notation for representation of Dirac matrices with metric signature(+ − − −).[2]

$$\gamma^0 = -\sigma^1 \otimes I,\ \gamma^i = i\sigma^2 \otimes \sigma^i \tag{1}$$
$$\Sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\ \gamma^\nu\right],\ \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \sigma_3 \otimes I$$

The spinorial Lorentz symmetry algebra is realized as

$$\left[\Sigma^{\mu\nu},\ \Sigma^{\lambda\tau}\right] = 2i\left\{\eta^{\nu\lambda}\Sigma^{\mu\tau} - (\mu \leftrightarrow \nu) - (\lambda \leftrightarrow \tau) + (\mu \leftrightarrow \nu,\ \lambda \leftrightarrow \tau)\right\} \tag{2}$$

which we extend to the Poincare algebra by including the followings:

$$\left[\Sigma^{\mu\nu},\ \gamma_L^\lambda\right] = 2i\left\{\eta^{\nu\lambda}\gamma_L^\mu - (\mu \leftrightarrow \nu)\right\} \tag{3a}$$
$$\left[\gamma_L^\mu,\ \gamma_L^\nu\right] = 0 \tag{3b}$$

where

$$\gamma_L^\mu \equiv \frac{1}{2}\gamma^\mu (I - \gamma_5) \tag{4}$$

The commutativity of γ_L^μ is trivial because $\gamma_L^\mu \gamma_L^\nu = 0$, that is, they are nill-potent. We will call them spin-translation generators. This spinor realization of the Poincare algebra can be further enlarged to a conformal group by including γ_R^μ and γ_5 which we will not consider because it is not a symmetry of fermion equations.

Since $\left[\gamma_5,\ \gamma_L^\mu\right] = 2\gamma_L^\mu \neq 0$, chirality is not conserved under spin-translation. In order to see this more concretely let us consider a four-component Dirac spinor in terms of two-component spinors r and l as

$$\Psi = \left[\frac{r}{l}\right] \tag{5}$$

The spin-translation acts on them as

$$\Psi' = \exp\left(-it_\mu \gamma_L^\mu\right)\Psi = \left(1 - it_\mu \gamma_L^\mu\right)\Psi = \left[\frac{r - \left(it_\mu \sigma^\mu\right)l}{l}\right] \tag{6}$$

where

$$t_\mu \sigma^\mu = -t_o I + t_i \sigma^i \tag{7}$$

This clearly shows that the l-part is invariant, but the r-part is translated by the amount $(-it_\mu \sigma^\mu)l$.

INVARIANCE OF NEUTRINO EQUATIONS

The action of left-handed neutrinos is invariant under local spin-translations. Let ψ_ν be a four-component Dirac spinor whose Lagrangian is

$$L_N = \overline{\Psi}_\nu \gamma_L^\mu \left(i\partial_\mu\right)\Psi_\nu \tag{8}$$

which describes neutrino fields. It does not change under the transformation

$$\Psi_\nu' = \exp\left(-it_\mu \gamma_L^\mu\right)\Psi_\nu \tag{9}$$

because the translation affects only on the right-handed portion while the Lagrangian involves only the left-handed. Notice thet $\gamma_L^\mu \cdot i\partial_\mu$ is a $P_{spin}^\mu \cdot p_\mu$ type operator whereas the operator $\gamma_R^\mu i\partial_\mu$ is a $K_{spin}^\mu \cdot p_\mu$ type, where

$$\gamma_R^\mu = \frac{1}{2}\gamma^\mu\left(1+\gamma_5\right) \tag{10}$$

and K^μ is a conformal translation generator. Due to this mismatch it is not an invariant operator, i.e.,

$$L_R = \overline{\Psi}\gamma_R^\mu\left(i\partial_\mu\right)\Psi \neq invariant \tag{11}$$

with respect to the spin-translation. Hence electron equations are not invariant either:

$$L_E = \overline{\Psi}\gamma^\mu\left(i\partial_\mu\right)\Psi \neq invariant \tag{12}$$

It is also straightforward to show that inclusion of gauge fields would not alter the invariance properties, for example,

$$L = \overline{\Psi}_\nu \gamma_L^\mu\left(i\partial_\mu + W_\mu\right)\Psi_\nu \tag{13}$$

is invariant for any gauge field W. It is, however, remarkable that Yukawa coupling or mass terms do not violate the spin-translation:

$$L_y = \Phi\overline{\Psi}'\Psi' = \Phi\overline{\Psi}\Psi \tag{14}$$

This implies that what keeps neutrinos massless is not this symmetry itself, rather the lack of their right-handed partners because of the spin-translation invariance. This also means that the fermion mass generating mechanism used in the Glashow-Weinberg-Salam theory [3] is not affected by the introduction of the spin-translation. Nambu-Jona-Lasino technique [4] of dynamical mass generation, and spontaneous breaking of chiral symmetry are not conflicting with the new symmetry, but $<\overline{\Psi}\gamma_5\Psi>$ is not allowed unless the spin-translation is broken.

DISCUSSION

The presence of local spin-translation symmetry in neutrino fields does not mean that there exists a corresponding gauge field, because it can not couple minimally to neutrinos as

$$\overline{\Psi}\gamma_L^{\mu}\left(\left(e_{\mu}\right)_{\lambda}\gamma_L^{\lambda}\right)\Psi=0 \tag{15}$$

due to the nilpotency of γ_L^{γ}. Here $\left(e_{\mu}\right)_{\lambda}$ is the hypothetical gauge field. In the case of charged fermions the symmetry is broken explicitly in their Lagrangian, therefore there is no need for such fields.

The spin-translation symmetry does not necessarily exclude a possible existence of right-handed fermions. Because fermions of the form

$$\Psi_R=\begin{pmatrix} r \\ 0 \end{pmatrix} \tag{16}$$

is invariant under the translation. However, it can not be related to a left-handed partner, so it will be completely independent of physical left-handed neutrinos. As a remark we note that Majorana neutrinos violate the spin-translation symmetry, which suggests that physical neutrinos are not of this kind.

Looking back at the translation equ. (6) we note a formal analogy with Galilean boost. Let us compare them:

$$x'=x+\left(v/c\right)ct, \quad ct'=ct \tag{17}$$

$$r'=r+\left(-it_{\mu}\sigma^{\mu}\right)l, \quad l'=l \tag{18}$$

The spin-translation looks similar to the boost in classical space-time, whose possible generalization or significance we do not know. One thing it suggessts is, however, that there appears a new length scale in the spin-translation formula, whose determination may be of fundamental importance.

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APPENDIX

In the above text we showed that Dirac equation is not invariant under the spin-translation. It is, however, possible to construct a Lagrangian so that the full Poincare symmetry including the spin-translation is preserved. The desired on is

$$L = \overline{\Psi}\exp\left(-ix_\mu \gamma_L^\mu / \lambda\right)\ \gamma^\mu p'_\mu \exp\left(ix_\mu \gamma_L^\mu / \lambda\right)\Psi + m\overline{\Psi}\Psi \qquad \text{(A1)}$$

which is translation invariant because the spin-translation of Ψ is cancelled by the space-time translation: $x'_\mu = x_\mu + \lambda t_\mu$. For neutrino fields the modification has no effects. Here λ is a new length scale which can be determined by comparing predictions of the equation with experimental data. In order to see physical contents of the modification we can expand the modified Dirac opreator as

$$\begin{aligned}
&\exp\left(-ix_\mu \gamma_L^\mu / \lambda\right)\ \overline{P}\ \exp\left(ix_\mu \gamma_L^\mu / \lambda\right) \qquad \text{(A2)}\\
&= \overline{P} - i\frac{1}{\lambda}\left(\overline{xp}\gamma_R - \overline{px}\gamma_L\right) + \frac{1}{\lambda^2}\left(\overline{xpx}\right)\gamma_L\\
&= \overline{P} - i\frac{1}{2\lambda}\left\{\left(\overline{xp} - \overline{px}\right) + \left(\overline{xp} + \overline{px}\right)\gamma_5\right\} + \frac{1}{\lambda^2}\overline{xpx}\gamma_L
\end{aligned}$$

where $\overline{P} = \gamma^\mu P_\mu$, $\overline{x} = x_\mu \gamma^\mu$. It is worth mentioning that the $\frac{1}{\lambda}$ order terms include spin-orbit coupling and scale operation $x \cdot p$.

For a specific application we consider quark fields with gluon gauge fields, for which we simply replace P_{μ} by $p_{\mu}+G_{\mu}$ where is G_{μ} the color gauge fields. We may consider the length scale to be of a typical hadron size. Then there will be spin-translation effects in hadron properties including spectra, especially for low-lying mesons because the effect is distance dependent. This kinematical distance-dependent effect may be also related to the so-called quark-confinement.

We conjecture that strong interaction preserves the full Poincare symmetry by the above modification, and weak interaction keeps this symmetry by picking only the left-handed fields at the cost of parity symmetry. It is the electromagnetic interaction which breaks the spin-translation symmetry, which might be connected with properities of a photon. It is, of course, possible to imagine the Dirac equation is modified here also, but then the length scale must be very large compared with a typical atomic size.

As a closing remark we state that

$$\left\{\exp\left(-ix_{\mu}\gamma_{L}^{\mu}/\lambda\right)\ \overline{P}\ \exp\left(ix_{\mu}\gamma_{L}^{\mu}/\lambda\right)\right\}^{2}=\pi^{\mu}\pi_{\mu} \tag{A3}$$

where $\pi^{\mu}=P^{\mu}-\gamma_{L}^{\mu}/\lambda$, and this equation shows that the modified Dirac operator is a square-root of the modified Klein-Gordon equation. It also implies that (A4)

$$\left(p^{2}-m^{2}\right)^{2}\Psi=0$$

요약

스핀 포앙카레 대칭성과 뉴트리노의 좌선성

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스피노 공간에서 포앙카레 대칭성의 나툼을 구성하였는데, 우선성 스피노는 평행 이동을 하고, 좌선성 스피노는 불변이다. 이 평행 이동 대칭성에 의하여 뉴트리노가 질량이 없고 좌선성임을 설명할 수 있다. 스핀 평행이동에 불변인 디락 방정식의 변형이 고려된 바, 강입자의 성질을 통하여 시험될 수 있다.