서울大學校 師大論叢 第25輯(1982. 12. 20)

Four Family Flavor Unification

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Flavor unification is attempted in $SU(7)$ and $SO(14)$. Three different models are presented for the unification of four families of quark and lepton which are assigned to the 64 dim. spinor representation of $SO(14)$. They are real with respect to $SU(3)\times SU(2)\times U(1)$ but complex with respect to $SU(3)\times SU(2)\times U(1)\times U(1)$. Salient points including proton decay of Georgi-Glashow $SU(5)$ model are accomodated in our model. An experimentally important prediction is an existence of new parity conserving neutral current about M_w.

1. Introducation

Non Abelian gauge theories invented by Yang and Mills[1] are renormalizable[2] and used for group theoretical generalizations of quantum electrodynamics. The most successful application of the Abelian field theories was made by Glashow, Weinberg and Salam[3] for unifying weak and electromagnetic interactions. The experimental verification of the theory lies chiefly in determining group structure of the interactions i.e., $SU(2)_w\times U(1)_Y$. Another important use of non Abelian gauge models is the strong interaction theory of color $SU(3)_c$ group. It is now widely accepted that low energy interaction symmetry is $SU(3)_c\times SU(2)_w\times U(1)_Y$[4].

A giant step forward in unifying all the three interactions by a simple group is made by Glashow and Georgi[5]. Their $SU(5)_{GG}$ theory is so simple and so powerful in understanding hitherto mysterious things that it becomes a prototype of grand unifying theories. However, it can not be a complete theory because it does not explain the old question of muon's existence, which is called a flavor problem. Another shortcoming of the theory is that it had no testable predictions except proton decay[6]. It says there is a vast gap between ordinary level ($M \simeq 100$GeV) and $SU(5)$ relevant energy level ($M \simeq 10^{15}$GeV). In other words, it does not have many experimental predictions.

In this paper we elaborate the $SU(7)$ [7] and the $SO(14)$ [8] models which not only resolve the flavor puzzle but also reveal interesting group structure around ordinary energy ($M \simeq 100$ GeV). Considering that the essential ingredient of Weinberg-Salam theory is its group nature, we believe that our prediction of new interaction, $SU(3)_c \times SU(2)_w \times U(1)_Y \times SU(2)_V$ will be utmost importance in future experiments. A confirmation of such structure will be a strong support of the grand unifying group even though it can only be directly tested at super high energy.

In sect. 2 we will present the methodology for searching the simplest possible grand unifying group and its results. In sect. 3 group theoretically allowed symmetry breaking patterns which are compatible with measured Weinberg angle[9] will be studied. In sect. 4 we elaborate the properties of new interaction $SU(2)_V$ (or $U(1)_J$) and in sect. 5 we discuss phenomenological implications. In the appendix notations of Clifford algebra are explained.

2. Search for Models

In his paper "Towards a Grand Unified Theory of Flavor" [10] Georgi formulated three guide lines which a good grand unified theory should satisfy.

The first law of *GU* (grand unification): the representation of the left handed (*LH*) fermions must be real with respect to the color $SU(3)_c$ subgroup.

The second law of *GU:* the representation of the *LH* fermions should be complex with respect to the $G_w = SU(3)_c \times SU(2)_w \times U(1)_Y$ subgroup.

The third law of *GU:* no irreducible representation should appear more than once in the representation of the *LH* fermions.

The first law is necessary to allow the fermions to have mass while the second law protects them from gaining very large mass. The third law is an asthetic claim. Besides these laws, a theory must be anomaly free to be renormalizable.

Georgi made a systematic investigation in $SU(N)$ groups and found that $SU(11)$ is the smallest group satisfying his laws. His result shows, as we interprete it, that if we keep the low energy symmetry(G_w) as simple as possible then the *GU* group must become large. We propose to examine the problem the other way around: How to modify G_w such that a GU group becomes as small as possible? For a systematic analysis in this direction we put forward few requirements(RQM).

RQM 1: the LH fermion representation is anomaly free and satisfies Georgi's first and third law.

RQM 2: it includes all the known light fermions.

RQM 3: it is complex with respect to G_V and the symmetry breaking mass scales of G_V and G_w are of the same order, where $G_V \supset G_w$.

The minimal group satisfying RQM 1 and 2 is $SU(7)$ and the next one is its extension to $SO(14)$. Fortunately we find that $SU(7)$ (and $SO(14)$) satisfies the third RQM. Now we proceed to construct models in detail. Since the spinor representation of $SO(14)$ includes $SU(7)$ anomaly free representation it is better to examine $SO(14)$, which automatically solves $SU(7)$ problems.

2.1. Four Possible Electromagnetic Charge Assignments

The 64 dim. spinor representation decomposes into $SU(7)$ representations,

$$(64) = \Psi_0 + \Psi_A + \Psi^{AB} + \Psi_{ABC}$$
$$= (\text{singlet}) + (\underline{7}^*) + (\underline{21}) + (\underline{35}^*). \tag{2-1}$$

The $SU(7)$ embedds trivially Georgi-Glashow $SU(5)$ which. in turn, has the color $SU(3)$ and the weak $SU(2)$ as subgroups. First three indices $A=1,2,3$ are color parts, $A=4,5$ are weak indices, and $A=6,7$ are named V-indices. The color indices and the weak indices are denoted by a,b,c and i,j, respectively.

The $(\underline{64})$ can be broken down to $SU(3)_c \times SU(2)_w$ representations. Among them we note that $(\underline{3},\underline{2}) = (\Psi^{ai}, \Psi_{abi})$ and $(\underline{3}^*,\underline{1}) = (\Psi_a, \Psi^{ab}, \Psi_{a45}, \Psi_{a67})$. On the other hand the color and the weak properties of fermions are well known. That is, $(u,d)_L$ and $(c,s)_L$ belong to $(\underline{3},\underline{2})$ while u_L^c, d_L^c, c_L^c, and s_L^c to $(\underline{3}^*,\underline{1})$.

Now by the RQM 2 the electromagnetic charge Q must be assigned to fit the experimentally observed values:

$$Q\{\underline{3},\ \underline{2}\} = (2/3,\ -1/3),$$
$$Q\{\underline{3}^*,\ \underline{1}\} = (-2/3 \text{ or } 1/3). \tag{2-2}$$

The most general charge operator Q in $SO(14)$ is

$$Q = a'(S_1+S_3+S_5) + b'(S_7) + (b'-1)\ S_9 + c'S_{11} + d'S_{13} \tag{2-3}$$

which can be expressed by $SU(7)$ generator

$$Q = \text{diag.}\ (a,a,a,b,b\text{-}1,c,d) + xI_0, \tag{2-4}$$

where $3a+2b+c+d-1=0$ and

$$I_0(\Psi_0)=1,\ I_0(\Psi_A)=-5/7,\ I_0(\Psi^{AB})=3/7,\ I_0(\Psi_{ABC})=-1/7, \tag{2-5}$$

$$a'=a-2x/7,\ b'=b=b-2x/7,\ c'=c-2x/7,\ d'=d-2x/7. \tag{2-6}$$

From eqs. (2-2) we derive the following equations.

$$Q\{(a4)\}=a+b+3x/7=2/3,$$

$$Q\{(ab4)^*\}=-2a-b-1x/7=-1/3, \tag{2-7a}$$

and $Q\{(a)^*\}=-a-5x/7,\ Q\{(ab)\}=2a+3x/7,$

$$Q\{(a45)^*\}=-a-2b+1-x/7 \text{ and } Q\{(a67)^*\}=-a-c-d-x/7 \tag{2-7b}$$

are matching with $(-2/3,\ -2/3,\ 1/3,\ 1/3)$.

There are two solutions, namely,

$$\text{case A, } Q=\text{diag. }(-1/3,\ -1/3,\ -1/3,\ 1,\ 0,\ c,\ -c), \tag{2-8a}$$

$$\text{case B, } Q=\text{diag. }(-1/21,\ -1/21,-1/21,\ 2/7,\ -5/7,\ c,\ d)+I_0,\quad c+d=4/7. \tag{2-8b}$$

After making use of quark sector we proceed to match lepton sector. The $SU(3)_c\times SU(2)_w=(\underline{1},\underline{2})$ representation in $(\underline{64})$ are $(\psi_i, \psi_{i67}, \psi^{i6}, \psi^{i7})$ while the known leptons are $(e,\nu_1)_L$, $(\mu,\nu_2)_L$ and $(\tau,\nu_3)_L$. The electromagnetic charges of ψ_i and ψ_{i67} are already determined

$$Q\{(4)^*\}=Q\{(467)^*\}=-1, \tag{2-9}$$

which enables us the identification,

$$\psi_4=e_L \text{ and } \psi_{467}=\mu_L. \tag{2-10}$$

It is fortunate we have one more lepton family which provides us valuable information to determine Q almost uniquely. We infer $Q\{(46)\}$ and $Q\{(47)\}$ must be either 1 or 0. From this we obtain two possible solutions for each case of A and B.

$$\text{Case AN: } Q=\text{diag. }(-1/3,\ -1/3,\ -1/3,\ 1,\ 0,\ 0,\ 0)$$

$$\text{Case AE: } Q=\text{diag. }(-1/3,\ -1/3,\ -1/3,\ 1,\ 0,\ 1,\ -1) \tag{2-11}$$

$$\text{Case BN: } Q=\text{diag. }(-1/21,\ -1/21,\ -1/21,\ 2/7,\ -5/7,\ 2/7,\ 2/7)+I_0,$$

$$\text{Case BE: } Q=\text{diag. }(-1/21,\ -1/21,\ -1/21,\ 2/7,\ -5/7,\ -5/7,\ 9/7)+I_0. \tag{2-12}$$

Here N and E stand for non-exotic and exotic, respectively. The table I displays the $SU(3)_c\times SU(2)_w$ contents and Q of the $(\underline{64})$. It clearly shows Case AN and Case BN have ordinary charge assignment, but Case AE and Case BE have exotic particles. Note that Case AN[7] and Case BN[8] were used in the flavor unifying theory, and Case AE was found by Jihn. E. Kim[11].

2.2. Definition of J_3

Since the four possible charge assignments make the fermion representation real with

Table I. Charge Assignments

$SU(7)$	$SU(3)\times SU(2)$	Q_{AN}	Q_{BN}	Q_{AE}	Q_{BE}
(ai)	$(3,2)$	$(2/3,-1/3)$	$(2/3,-1/3)$	$(2/3,-1/3)$	$(2/3,-1/3)$
$(abi)^*$	$(3,2)$	$(2/3,-1/3)$	$(2/3,-1/3)$	$(2/3,-1/3)$	$(2/3,-1/3)$
$(ai6)^*$	$(3^*,2)$	$(-2/3,\ 1/3)$	$(-2/3,\ 1/3)$	$(-5/3,-2/3)$	$(1/3,\ 4/3)$
$(ai7)^*$	$(3^*,2)$	$(-2/3,\ 1/3)$	$(-2/3,\ 1/3)$	$(1/3,\ 4/3)$	$(-5/3,-2/3)$
$(ab6)^*$	$(3,1)$	$2/3$	$-1/3$	$-1/3$	$2/3$
$(ab7)^*$	$(3,1)$	$2/3$	$-1/3$	$5/3$	$-4/3$
$(a6)$	$(3,1)$	$-1/3$	$2/3$	$2/3$	$-1/3$
$(a7)$	$(3,1)$	$-1/3$	$2/3$	$-4/3$	$5/3$
$(a)^*$	$(3^*,1)$	$1/3$	$-2/3$	$1/3$	$-2/3$
(ab)	$(3^*,1)$	$-2/3$	$1/3$	$-2/3$	$1/3$
$(a45)^*$	$(3^*,1)$	$-2/3$	$1/3$	$-2/3$	$1/3$
$(a67)^*$	$(3^*,1)$	$1/3$	$-2/3$	$1/3$	$-2/3$
$(i)^*$	$(1,2)$	$(-1,0)$	$(-1,0)$	$(-1,0)$	$(-1,0)$
$(i67)^*$	$(1,2)$	$(-1,0)$	$(-1,0)$	$(-1,0)$	$(-1,0)$
$(i6)$	$(1,2)$	$(1,0)$	$(1,0)$	$(2,1)$	$(0,-1)$
$(i7)$	$(1,2)$	$(1,0)$	$(1,0)$	$(0,-1)$	$(2,1)$
(45)	$(1,1)$	1	0	1	0
(67)	$(1,1)$	0	1	0	1
$(abc)^*$	$(1,1)$	1	0	1	0
$(456)^*$	$(1,1)$	-1	0	-2	1
$(457)^*$	$(1,1)$	-1	0	0	-1
$(6)^*$	$(1,1)$	0	-1	-1	0
$(7)^*$	$(1,1)$	0	-1	1	-2
singlet	$(1,1)$	0	1	0	1

respect to G_w, we should extend the group so that they become complex representations with respect to the enlarged group. Let us take a minimal extension to

$$G_V=SU(3)_c\times SU(2)_w\times U(1)_Y\times U(1)_J \tag{2-13}$$

and proceed to find J.

As a physical requirement we assume

$$J(u)=J(d)=J(c)=J(s)=J(e)=J(\mu)=0 \tag{2-14}$$

but J of other states may not be vanishing.

First by the group structure in eq. (2-13), must take a form given by

$$J=x(S_1+S_2+S_3)+y(S_7+S_9)+zS_{11}+tS_{13}, \tag{2-15}$$

where x, y, z and t are to be determined, J must not include Y as a component. We find that J's are uniquely determined by the eqs. (2-14) and (2-15) for each case. The followings are solutions for each case.

Case AN:

$$Q=-(S_1+S_3+S_5)/3+S_7$$
$$Y=-(S_1+S_3+S_5)/3+(S_7+S_9)/2$$
$$J=S_{11}-S_{13}. \tag{2-16}$$

Case BN:

$$Q=-(S_1+S_3+S_5)/3-S_9$$
$$Y=-(S_1+S_3+S_5)/3-(S_7+S_9)/2$$
$$J=S_{11}-S_{13}. \tag{2-17}$$

Case AE:

$$Q=-(S_1+S_3+S_5)/3+S_7+S_{11}-S_{13}$$
$$Y=-(S_1+S_3+S_5)/3+(S_7+S_9)/2+S_{11}-S_{13}$$
$$J=\text{non existence} \tag{2-18}$$

Case BE:

$$Q=-(S_1+S_3+S_5)/3-S_9-S_{11}+S_{13}$$

Table II. Fermion Assignments

$SU(7)$	$SU(3)\times SU(2)$	J_3	Q_{AN}	$SO(14)_T$	Q_{BN}	$SO(14)_F$
(ai)	(3, 2)	0	(2/3, −1/3)	$(u, d)_L$	(2/3, −1/3)	$(u, d)_L$
$(abi)^*$	(3, 2)	0	(−1/3, 2/3)	$(s, -c)_L$	(−1/3, 2/3)	$(s, -c)_L$
$(a)^*$	(3*, 1)	0	1/3	$d_L{}^c$	−2/3	$u_L{}^c$
$(ab7)^*$	(3*, 1)	0	1/3	$s_L{}^c$	−2/3	$c_L{}^c$
(ab)	(3*, 1)	0	−2/3	$u_L{}^c$	1/3	$d_L{}^c$
$(a45)^*$	(3*, 1)	0	−2/3	$c_L{}^c$	1/3	$s_L{}^c$
$(a6)$	(3, 1)	1/2	−1/3	b_L	2/3	t_L
$(ab7)^*$	(3, 1)	1/2	2/3	t_L	−1/3	b_L
$(a7)$	(3, 1)	−1/2	−1/3	h_L	2/3	f_L
$(ab6)^*$	(3, 1)	−1/2	2/3	f_L	−1/3	h_L
$(ai6)^*$	(3*, 2)	−1/2	(−2/3, 1/3)	$(t^c, b^c)/_L$	(−2/3, 1/3)	$(t^c, b^c)_L$
$(ai7)^*$	(3*, 2)	1/2	(−2/3, 1/3)	$(f^c, h^c)_L$	(−2/3, 1/3)	$(f^c, h^c)_L$
$(i)^*$	(1, 2)	0	(−1, 0)	$(e, \nu_1)_L$	(−1, 0)	$(e, \nu_1)_L$
$(i67)^*$	(1, 2)	0	(−1, 0)	$(\mu, \nu_2)_L$	(−1, 0)	$(\mu, \nu_2)_L$
(45)	(1, 1)	0	1	$e_L{}^c$	0	$\nu_{1L}{}^c$
$(abc)^*$	(1, 1)	0	1	$\mu_L{}^c$	0	$\nu_{2L}{}^c$
singlet	(1, 1)	0	0	$\nu_{1L}{}^c$	1	$e_L{}^c$
(67)	(1, 1)	0	0	$\nu_{2L}{}^c$	1	$\mu_L{}^c$
$(i6)$	(1, 2)	1/2	(1, 0)	$(\tau^c, \nu_3{}^c)_L$	(1, 0)	$(\tau^c, \nu_3{}^c)_L$
$(i7)$	(1, 2)	−1/2	(1, 0)	$(\epsilon^c, \nu_4{}^c)_L$	(1, 0)	$(\epsilon^c, \nu_4{}^c)_L$
$(6)^*$	(1, 1)	−1/2	0	ν_{3L}	−1	τ_L
$(456)^*$	(1, 1)	−1/2	−1	τ_L	0	ν_{3L}
$(7)^*$	(1, 1)	1/2	0	ν_{4L}	−1	ϵ_L
$(457)^*$	(1, 1)	1/2	−1	ϵ_L	0	ν_{4L}

$Y=-(S_1+S_3+S_5)/3-(S_7+S_9)/2-S_{11}+S_{13}$

$J=$non existence. (2-19)

In conclusion we obtained three possible grand unifying models, which are

$$SU(7):\ Q=\text{diag.}\ (-1/3,\ -1/3,\ -1/3,\ 1,\ 0,\ 0,\ 0) \tag{2-20}$$

$$SO(14)_T:\ Q=\text{diag.}\ (-1/3,\ -1/3,\ -1/3,\ 1,\ 0,\ 0,\ 0) \tag{2-21}$$

$$SO(14)_F:\ Q=\text{diag.}\ \left(\frac{-1}{21},\ \frac{-1}{21},\ \frac{-1}{21},\ \frac{2}{7},\ \frac{-5}{7},\ \frac{2}{7},\ \frac{2}{7}\right)+I_0. \tag{2-22}$$

Here $I_3=$diag. (0, 0, 0, 1/2, −1/2, 0, 0) and $J_3=$diag. (0, 0, 0, 0, 0, 1/2, −1/2) for all three cases. For the sake of simplicity we expressed in $SU(7)$ notations.

In table II the particle assignments of $SO(14)_T$ and $SO(14)_F$ are compared. The $SU(7)$ case is identical to $SO(14)_T$ except that $\nu_{1L}{}^c$ is missing. In the table $SU(3)\times SU(2)$ contents Q and J_3 are given. Y are not given because it can be read immediately. As far as particle assignments are concerned $SO(14)_T$ and $SO(14)_F$ are equivalent, but their subgroup structures are quite different.

3. Symmetry Breaking Pattern

Model 1. $SU(7)$

First we consider two possible symmetry branchings[7] given by

$$SU(7)\underset{M_7}{\downarrow} SU(5)_{GG}\times SU(2)_V\times U(1)\underset{M_G}{\downarrow} G_W\times H_V\underset{M_V}{\downarrow} G_W\underset{M_W}{\downarrow} G_e, \tag{3-1}$$

where $G_e=SU(3)_c\times U(1)_{em}$. The two patterns differ by the choice of H_V, namely, H_V can be either $SU(2)_V$ or $U(1)_J$. Note J_3 is a generator of $SU(2)_V$ and $U(1)_J$.

Since the Georgi-Glashow one family unifying $SU(5)_{GG}$ is separated from $SU(2)_V$, renormalization of coupling constants are independent each other. We note that the well known $SU(5)_{GG}$ results are simply reproduced here, i.e., [12]

$$\ln\frac{M_G}{M_W}=\frac{1}{2}\eta=\xi \tag{3-2}$$

where

$$\xi=6\pi\left(\frac{\sin^2\theta_w}{\alpha_{em}}-\frac{1}{\alpha_c}\right)/11 \tag{3-3}$$

and

$$\eta=\frac{6\pi}{\alpha_{em}}\left(\frac{3}{5}\cos^2\theta_w-\sin^2\theta_w\right)/11 \tag{3-4}$$

The coupling constants of $SU(2)_V$ $(U(1)_J)$ evolves as

$$g_V{}^{-2}=g_7{}^{-2}+d(-2+F)\ \ln\frac{M_7}{M_V}, \tag{3-5}$$

or

$$g_J{}^{-2}=g_7{}^{-2}+d(-2+F)\ \ln\frac{M_7}{M_6}+dF\ \ln\frac{M_G}{M_V}, \tag{3-6}$$

where $d=11/(24\pi^2)$, $F=16/11$. These equations show that V-interaction is weaker than ordinary weak interaction.

Another pattern available is

$$SU(7)\underset{M_7}{\downarrow}SU(3)_c\times SU(4)_{WV}\times U(1)\underset{M_1}{\downarrow}G_W\times H_V\underset{M_W}{\downarrow}G_e, \tag{3-7}$$

whose coupling constants give following relations:

$$\ln\frac{M_1}{M_W}=\frac{\eta}{2}-\frac{7}{24}(\eta\text{-}2\xi)$$

and

$$\ln\frac{M_7}{M_1}=\frac{5}{24}(\eta\text{-}2\xi). \tag{3-8}$$

Since for reasonable values of α_{em}, α_c, and Weinberg angles $\eta-2\xi\simeq0$, we have $M_7\simeq M_1\simeq M_{GG}$.

Model 2. $SO(14)_T$

There can be four principal types of symmetric patterns:

(2A) $SO(14)_T\longrightarrow SU(7)\times U(1)$

(2B) $SO(14)_T\longrightarrow SO(10)_G\times SO(4)_V$

(2C) $SO(14)_T\longrightarrow SO(10)_{SV}\times SO(4)_W$

(2D) $SO(14)_T\longrightarrow SO(6)_S\times SO(8)_{VW}$.

The case (2A) is essentially same as the $SU(7)$ case. The type (2B) is very similar to $SO(10)$ of Georgi-Nanopoulous[13] whose breaking patterns are studied extensively in the literature[14]. We omit reiterating details here.

The full stages of the pattern (2C) is given by

$$SO(14)_T\underset{M_{14}}{\downarrow}SO(10)_{SV}\times SO(4)_W\underset{M_{10}}{\downarrow}SO(6)_S\times SO(4)_V\times SO(4)_W\underset{M}{\downarrow}G_W\times H_V\underset{M_V}{\downarrow}G_W\underset{M_W}{\downarrow}G_e.$$

The renormalized coupling constants are related by

$$\ln\frac{M}{M_W}=\eta/2+(2\xi-\eta)/12,$$

$$3\ln\frac{M_{14}}{M_{10}}+\ln\frac{M_{10}}{M}=5(2\xi-7)/24. \tag{3-9}$$

For reasonable parameters we have $M_{14}\simeq M_{10}\simeq M=M_{GG}$.

The case (2D) evolves as

$$SO(14)_T \underset{M_{14}}{\downarrow} SO(6)_S \times SO(8)_{VW} \underset{M_6}{\downarrow} SO(6)_S \times SO(4)_V \times SO(4)_W \underset{M}{\downarrow} G_W \times H_V \underset{M_V}{\downarrow} G_W \underset{M_W}{\downarrow} G_e.$$

The corresponding relations are,

$$\ln\frac{M}{M_W}=\eta/2+(\xi-7)/12,$$

$$\ln\frac{M_{14}}{M_6}-\ln\frac{M_6}{M}=\frac{5}{24}(\eta-2\xi) \qquad (3\text{-}10)$$

Here again $M_{14}\simeq M_6\simeq M\simeq M_{GG}$.

Model 3. $SO(14)_F$

We have four interesting patterns which are given in sequel.

Case (3A):

$$SO(14)_F \underset{M_{14}}{\downarrow} SU(7)\times U(1) \underset{M_7}{\downarrow} G_W \times H_V \underset{M_V}{\downarrow} G_W \underset{M_W}{\downarrow} G_e.$$

An important point to note is that $U(1)$ carries electromagnetic charge and there is no intermediate step for $SU(5)_{GG}$. The coupling constants are related by

$$\ln\frac{M_7}{M_w}=\xi=\frac{\eta}{2}-\frac{34}{5}\ln\frac{M_{14}}{M_7} \qquad (3\text{-}11)$$

Case (3B):

$$SO(14)_F \underset{M_{14}}{\downarrow} SO(10)_{SV}\times SO(4)_W \underset{M_{10}}{\downarrow} SO(6)_S\times SO(4)_W\times SO(4)_V \underset{M}{\downarrow} G^W\times H_V \underset{M_V}{\downarrow} G_W \underset{M_W}{\downarrow} G_e.$$

This pattern was studied in detail ref. [8]. and the result is as usual $M_{14}\simeq M_{10}\simeq M\simeq M_{GG}$.

Case (3C):

$$SO(14)_F \underset{M_{14}}{\downarrow} SO(10)_S\times SO(4)_V \underset{M_{10}}{\downarrow} SO(6)_S\times SO(4)_W\times SO(4)_V \underset{M}{\downarrow} G_W\times H_V \underset{M_V}{\downarrow} G_W \underset{M_W}{\downarrow} G_e.$$

Case (3D):

$$SO(4)_F \underset{M_{14}}{\downarrow} SO(6)_S\times SO(8)_{WV} \underset{M_6}{\downarrow} SO(6)_S\times SO(4)_W\times SO(4)_V \underset{M}{\downarrow} G_W\times H_V \underset{M_V}{\downarrow} G_W \underset{M_W}{\downarrow} G_e.$$

The coupling constant relations of (3C) and (3D) are identiclal to (2C) and (2D), respectively.

$SU(7)$ does not have Pati-Salam type left-right symmetry[15], while $SO(14)$ models do. One of the major diffrences between $SO(14)_T$ (or $SU(7)$) and $SO(14)_F$ is that the latter does not go through $SU(5)_{GG}$. However, $SU(7)$ does not necessarily go through $SU(5)_{GG}$ either, as clearly shown in eq. (3-7).

There can be other variations of symmetry breaking patterns which differ in intermediate steps. In all cases we note that there is a large scale mass difference (100 GeV$\sim$ 10^{15}GeV) as in $SU(5)_{GG}$. Numerical details depend upon sign and magnitude of $2\xi-\eta$.

4. V-Interaction

The Georgi-Glashow $SU(5)$ model [5] is simple and very powerful to understand many things in a unified way: quantization of electric charge, fractional quark charge and integral lepton charge, only one chirality partion of quarks and leptons in the charged current weak interactions, the relative strength of color and electromagnetic interaction, and the Weinberg angle $\sin^2\theta \simeq 0.2$. Nevertheless its success is mostly theoretical because it is almost untestable due to a large mass scale (10^{15}Gev) except proten decay [6].

The present $SU(7)$ (or SO (14)) model does include all the theoretical successes of $SU(5)_{GG}$ and proton decay prediction. In addition to these we have experimentally verifiable consequences in the reasonable energy range. One obvious test will be the prediction of four families of fermions. However, the most important and direct proof will be the new group structure $G_V = SU(3)_C \times SU(2)_W \times U(1)_Y \times H_V$, where $H_V = SU(2)_V$ or $U(1)_J$ around $M_V \simeq M_W$.

The strength of V-interaction depends upon symmetry breaking pattern. Hence the measurement of α_V will fix the symmetry pattern almost uniquely. In $SU(7)$ the V-interaction can not be stronger than weak interaction, therefore, we named it very weak interaction.

V-bosons are electromagnetically neutral and parity conserving. This will make it hard to detect V-interaction because parity conserving neutral currents can easily be dominated by electromagnetic interaction. Furthemore, V doesn't mediate interaction between $J=0$ particles (two lighter families) and, therefore, we have to wait till heavier fermions are produced. If $SU(2)_V$ is broken at M_{GG} level, then only $U(1)_J$ will survive down to M_V level. The $U(1)_J$ boson can not change families i.e., $h \leftrightarrow b$, $t \leftrightarrow f$, $\epsilon \leftrightarrow \tau$ interactions are forbidden. By mixing between families there may appear very weak signal of V-interactions in the light families.

It is well known experimentally $\Delta S \neq 0$ neutral currents are suppressed. The present model satisfy Weinberg-Glashow theorem [16], therefore there is no flavor changing neutral currents modulo possible Higgs effects.

The name "family" to put particles into four sets does not go well with the group structure. Many efforts to unify families by horizontal symmetries [17] seemed to fail because they made use of the family concept. We do not use the term "horizontal symmetry" because there is not such a thing in our model. For instance, there is no horizontal interaction between (u, d) family and (c, s) family at low energy. We prefer the word 'very weak' interaction because it is quite likely that V-interaction strength is weaker than weak interaction.

5. Discussion

For the full investigation of the theory one is bound to study Higgs field effects. Hower, Higgs fields are essentially free parameters of the model which can be adjusted by hand. Therefore we decided not to include Higgs related things in the present article. Among the three models $SU(7)$, $SO(14)_T$ and $SO(14)_F$, we will take up $SU(7)$ only in the discussion because variations to $SO(14)$ is quite obvious, though their experimental implications are not necessarilly trivial.

Four families of quark and lepton with no exotic particles are agreeing with cosmological bounds on the number of neutrinos[18]. According to our theory we are supposed to see one more lepton ϵ, the sought after t-quark and another family (h, f). In $SU(7)$ there is no right handed electron neutrino, which implies electron neutrino is massless. The muon RH neutrino is self real, while ν_{3L} and ν_{4L} form a real pair with respect to G_V. Hencc these three neutrinos can develop super heavy masses if Higgs fields are included [19]. So, we expect muon LH neutrino can have mass, small it may be, unlike electron LH neutrino.

B-L is a global quantumber conserved in $SU(7)$, but is a broken local gauge genetator in $SO(14)$. The operator is defined as

$$B-L=I_0+4Y/5-4Y_a/7, \tag{5-1}$$

$Y_a=(1/5,\ 1/5,\ 1/5,\ 1/5,\ 1/5,\ -1/2,\ -1/2)$.

Proton and nucleon decay amplitudes will be very similar to those of $SU(5)_{GG}$[6]. However, cosmological implication could differ because at early stage of hot universe all the gauge bosons in $SU(7)$ will play equally important roles. So we expect that baryon asymmetry problems may be reexamined according to this new situation[20].

The μ-e universality is manifest in our particle assignment table. They can transform each other via exchange of gauge bosons which are super heavy, This means that muon lepton number and electron lepton number is separately conserved in the same level as baryon and lepton are conserved separately.

Recent results from CESR[21] show that B-meson decay is well described by Kobayashi-Masakawa[22] and is in conflict with topless models[23]. Decay modes of τ lepton agree with τ lepton number conservation and with the τ being a sequential lepton[24]. We find that these results are consistent with our models, but we should calculate amplitudes using Higgs because mixing is involved.

Finally we just mention that the gauge hierarchy problems is still unresolved and our models have the same magnetic monopole problem of Georgi-Glashow $SU(5)$[25].

Acknowledgment

We are benefitted from conversations with P.Y. Pac, H.S. Song, J. Kim and J.E. Kim. This research was supported in part by the grants from the Ministry of Education through the Research Institute of Basic Sciences, Seoul National University, and Korean Science and Engineering Foundation.

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Appendix

Notations of $SO(14)$ Spinor Representation

Our notation of the spinor representation is given by the following choice of the Clifford algebra matrices.

$$\Gamma_1=\sigma_1^{(7)}$$
$$\Gamma_{2i}=\sigma_1^{(7-i)}\times\sigma_2\times I^{(i-1)} \quad i=1,\ \cdots\cdots,\ 6$$
$$\Gamma_{2i+1}=\sigma_1^{(7-i)}\times\sigma_3\times I^{(i-1)} \quad i=1,\ \cdots\cdots,\ 6$$
$$\Gamma_{14}=\sigma_2\times I^{(6)} \qquad \text{(A-1)}$$

where σ_k's are the Pauli matrices, and I is the 2×2 identity matrix. The tensor product is illustrated by following two examples:

$$\sigma_1^{(3)}=\sigma_1\times\sigma_1\times\sigma_1 \text{ and } \sigma_1\times A=\begin{pmatrix}0 & A\\ A & 0\end{pmatrix}.$$

It is staright forward to show that these fourteen Γ's satisfy the Clifford relations

$$\Gamma_i\Gamma_j+\Gamma_j\Gamma_i=2\delta_{ij},\quad i,j=1 \text{ to } 14 \qquad \text{(A-2)}$$

The ninetyone generators of $SO(14)$ are

$$S_{ij}=\frac{1}{4i}(\Gamma_i\Gamma_j-\Gamma_j\Gamma_i) \quad i\neq j \qquad \text{(A-3)}$$

which have the Lie Algebra relations,

$$[S_{ij},\ S_{kl}]=i(\delta_{ik}S_{jl}+\delta_{jl}S_{ik}-\delta_{il}S_{jk}-\delta_{jk}S_{il}). \qquad \text{(A-4)}$$

Diagonal generators among these are $S_{2i-1,2i}\equiv S_{2i-1}$, $i=1$ to 7, whose linear combinations form physical observables such as Q, Y and J_3.

The $SU(7)$ singlet generator is

$$I_0=\sum_{i=1}^{7}S_{2i-1} \qquad \text{(A-5)}$$

and the $SU(7)$ Cartan subalgebra is made by

$$M_i=S_{2i+1}-S_{2i-1},\quad i=1 \text{ to } 7. \qquad \text{(A-6)}$$

4종류 입자군의 통일

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$SU(7)$과 $SO(14)$을 써서 입자군의 통일을 시도하였으며, 4종류의 구성자와 경입자들을 $SO(14)$의 스프노 나툼에 맞추었다. 이들은 $SU(3)\times SU(2)\times U(1)$에 대해서는 실수나툼이지만, $SU(3)\times SU(2)\times U(1)\times U(1)$에 대해서는 복소나툼이 된다. 양성자 붕괴를 포함한 $SU(5)$ 이론의 장점들을 포함하고 있으며, 실험적으로 중요한 의미를 가지는 우기성 보존의 중성류를 예시하고 있다.