Performance Comparison of FFH and MCFH Spread-Spectrum Systems with Optimum Diversity Combining in Frequency-Selective Rayleigh Fading Channels

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***Abstract*—In this letter, the performance of frequency-hopping spread-spectrum systems employing noncoherent reception and transmission diversity is analyzed for frequency-selective Rayleigh fading channels. Two different types of transmission diversity systems, a fast frequency-hopping (FFH) system and a multicarrier frequency-hopping (MCFH) system, are investigated. In order to combine received signals from transmit diversity channels, the optimum diversity combining rule based on the maximum-likelihood criterion is developed. Probability of error equations are derived, and utilized to evaluate the performance of the two systems. MCFH systems are found to outperform FFH systems when the channel delay spread is severe, while FFH systems are superior to MCFH systems when a channel varies rapidly. Furthermore, it is found that performance enhancement due to an increase of diversity order is more significant for MCFH systems than for FFH systems in frequency-selective fading channels. The effect of frequency-selective fading is also investigated in determining optimum frequency deviations of binary frequency-shift keying signals.**

***Index Terms*—Diversity methods, frequency-hop communication, frequency-selective Rayleigh fading, frequency-shift keying, spread-spectrum communication.**

I. Introduction

FREQUENCY-HOPPING spread-spectrum (FHSS) systems have been widely used in military communications. Demands for high data rate services in FHSS systems have been increasing. In high data rate systems, the effects of frequency-selective fading should be considered due to an increase in the ratio of delay spread to symbol duration. The effects of frequency-selective fading on an FHSS system employing orthogonal binary frequency-shift keying (BFSK) signals are investigated in [1] and [2] under the assumption that the frequency separation between two orthogonal BFSK signals is large enough for the correlation between two correlator outputs to be negligible. In practice, it is advantageous to use the minimum frequency separation in multiple-access environments to increase the number of frequency slots for a given total bandwidth [3]. When the minimum frequency separation is employed, the correlation between two correlator outputs as a result of frequency-selective fading and fast fading may be significant, and it is not assumed negligible in this letter.

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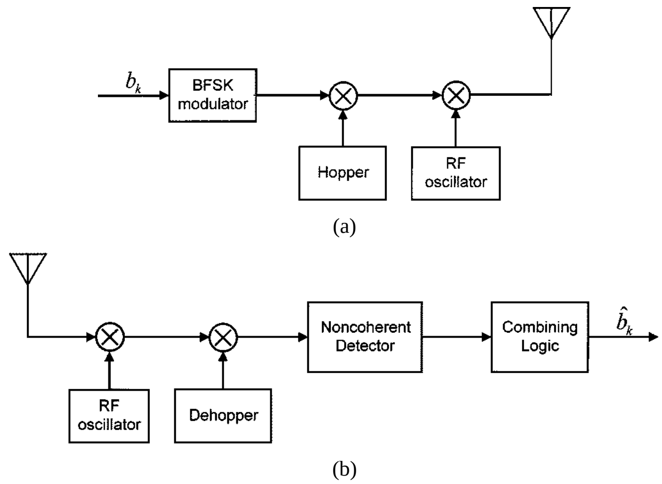
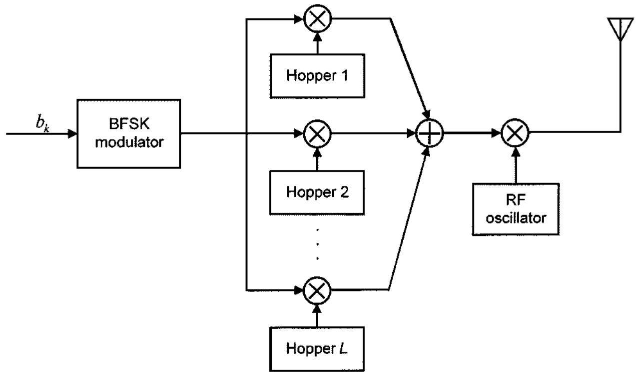
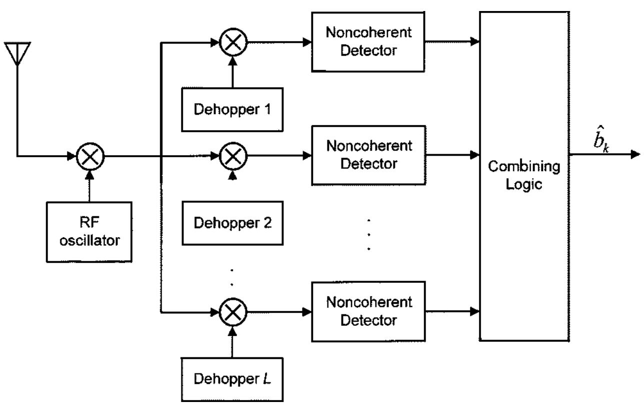


Fig. 1. FFH system block diagrams. (a) Transmitter. (b) Receiver.

Transmission diversity provides protection against jamming, multiple-access interference, and fading. For FHSS systems, the diversity may be realized in the form of fast frequency-hopping (FFH) and multicarrier transmission. FFH is a conventional diversity technique in FHSS systems; multicarrier transmission is an alternative diversity technique in FHSS systems. In an FFH system, diversity is obtained by changing a transmit frequency more than once over one symbol duration. The transmit frequency is selected from the entire transmit frequency band. In a multicarrier frequency-hopping (MCFH) system, the total frequency band is partitioned into several disjoint subbands on which replicas of the same signal are simultaneously transmitted. Each replica hops independently in its subband. FFH systems have attracted considerable interest and their performance has been widely studied over the past few decades [4], [5]. Recently, a multicarrier transmission technique has been proposed and the use of MCFH has been investigated for coherent FHSS systems employing binary phase-shift keying (BPSK) in [6]. However, in FHSS systems, coherent demodulation for BPSK signal is relatively difficult. Frequency-shift keying (FSK) modulation with noncoherent demodulation is typically employed in FHSS systems [1]–[5]. Hence, in this letter, BFSK modulation and noncoherent demodulation are assumed to be employed for both FFH and MCFH systems. Block diagrams of the FFH system and the MCFH system are shown in Figs. 1 and 2, respectively. MCFH systems require more devices than FFH systems. However, the devices including frequency synthesizer for FFH systems are required to operate more rapidly than those for MCFH systems. The use of FFH system may not be feasible for high data rate systems, due to its high speed requirements.



(a)



(b)

Fig. 2. MCFH system block diagrams. (a) Transmitter. (b) Receiver.

For systems employing transmission diversity, diversity receptions should be combined in some way in the receiver. A number of diversity combining schemes for FFH systems have been developed, and their performances have been studied [4], [5], [7]. These combining schemes may also be applied to MCFH systems. The optimum combining schemes based on the maximum-likelihood criterion have been developed only for static and frequency-nonselective slowly varying channels. For static channels with partial-band interference, the optimum combining is the sum of the logarithms of zeroth-order modified Bessel functions [5]. For slow and frequency-nonselective Rayleigh fading channels, the optimum combining rule, given that all of the diversity receptions have the same power spectral density (PSD) of background noise, is square-law equal-gain combining [7]. In this letter, the optimum combining rule is developed for frequency-selective fast varying Rayleigh fading channels with the background noise PSD of each diversity reception not being equal. This rule is also applicable to frequency-nonselective slowly varying channels.

Based on the developed optimum diversity combining rule, bit-error rate (BER) equations are derived for FFH and MCFH systems. These equations are utilized to compare the performances of these two systems, and to investigate the effects of diversity order. Furthermore, the effects of frequency-selective fading on the optimum frequency deviations for transmit FSK signals are investigated.

This letter is organized as follows. Section II describes the system and channel models. In Section III, the optimum diversity combining rule and equations for the probability of error are derived. In Section IV, performance evaluation is presented and performance comparisons between FFH and MCFH systems are made. Finally, conclusions are drawn in Section V.

II. System and Channel Models

The systems considered in this letter are FHSS systems with BFSK modulation, noncoherent detection, and diversity order L. Diversity order refers to the number of hops per symbol for FFH systems and the number of subbands for MCFH systems. Each transmit diversity channel is modeled as a frequency-selective Rayleigh fading process and is assumed to be independently faded. The maximum delay spread of each diversity reception is assumed to be smaller than one hop duration for FFH systems, which is smaller than the symbol duration. It is also assumed that one symbol is transmitted during one hop duration in MCFH systems, and adjacent symbols in time are transmitted in far distant frequency slots such that multipath interference from the previous symbol is negligible.

Transmitter block diagrams of FFH and MCFH systems are depicted, respectively, in Figs. 1(a) and 2(a). The complex baseband equivalent of the transmitted signal for each system can be represented as

$$s(t) = \begin{cases} \displaystyle\sum_{k=0}^{\infty}\sum_{\ell=0}^{L-1} \sqrt{2S} e^{j(2\pi(f_{\ell,k}+b_k f_d)t+\phi_{\ell,k})} \\ \qquad p_{T_h}(t-kT-\ell T_h); \ \text{FFH} \\ \displaystyle\sum_{k=0}^{\infty}\sum_{\ell=0}^{L-1} \sqrt{2S} e^{j(2\pi(f_{\ell,k}+b_k f_d)t+\phi_{\ell,k})} \\ \qquad p_{T_h}(t-kT_h); \ \text{MCFH} \end{cases} \tag{1}$$

where S is the transmit power of each diversity transmission, T is the symbol duration, and T_h is the hop duration. $f_{\ell,k}$ and $\phi_{\ell,k}$ are, respectively, the hop frequency and random phase for the ℓth diversity transmission of the kth symbol. $b_k \in \{-1, +1\}$ is the kth data symbol, and $p_\lambda(t) = 1$ for $t \in (0, \lambda)$ and zero, otherwise. The frequency deviation of a BFSK signal is denoted by $f_d = (h/2T_h = \Delta f/2)$, where h is the normalized frequency deviation and Δf is the frequency separation between two BFSK signals. When the total transmit power of $s(t)$ is S_t, the value of S in (1) for an FFH system is S_t and that of S for an MCFH system is S_t/L. Similarly, the value of T_h for an FFH system is T/L and that of T_h for an MCFH system is T. Correspondingly, the values of f_d and Δf would be different for the two systems.

The channel model is a wide-sense stationary uncorrelated scattering (WSSUS) model, described in [8] and [9]. The low-pass equivalent impulse response of the ℓth diversity channel may be written as

$$c_\ell(t;\tau) = \alpha_\ell(t;\tau)e^{j\zeta_\ell(t;\tau)}, \qquad \ell = 0, 1, \ldots, L-1 \tag{2}$$

where $\alpha_\ell(t;\tau)$'s are independent and identically distributed (i.i.d.) Rayleigh random processes and $\zeta_\ell(t;\tau)$'s are i.i.d. uniform random processes over $[0, 2\pi)$. The autocorrelation function of the WSSUS channel is given as [8]

$$\begin{aligned} R_c(\Delta t; \tau, \tau') &= \tfrac{1}{2} E\left[c^*(t;\tau)c(t+\Delta t;\tau')\right] \\ &= R_c(\Delta t;\tau)\delta(\tau-\tau') \end{aligned} \tag{3}$$

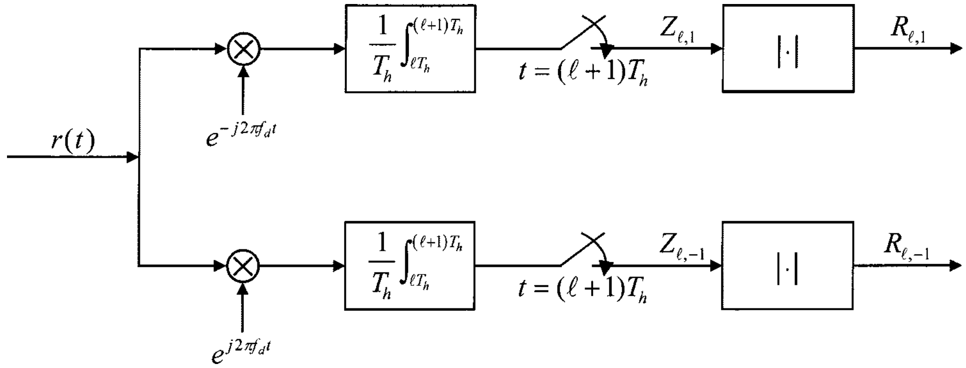


Fig. 3. Noncoherent detector for the ℓth diversity reception.

where $*$ denotes a complex conjugate operation. Since the channel response for each diversity transmission is assumed to be i.i.d., the autocorrelation of each channel is the same for all ℓ, so that the subscript ℓ is dropped in (3). If we let $\Delta t = 0$ in $R_c(\Delta t;\, \tau)$, the resulting autocorrelation function $R_c(0;\, \tau)$ is a multipath intensity profile, and denoted as $I_c(\tau)$. Assuming that the multipath intensity profile is time invariant, $R_c(\Delta t;\, \tau)$ may be represented as

$$R_c(\Delta t;\, \tau) = I_c(\tau)\phi_c(\Delta t) \tag{4}$$

where $\phi_c(\Delta t)$ is the autocorrelation function in the Δt variable normalized by $I_c(\tau)$ for all τ [8].

III. Performance Analysis

A. Correlator Outputs and Their Statistics

Receiver block diagrams are shown in Figs. 1(b) and 2(b). After down-converting and dehopping, the complex baseband equivalent of the received signal over the first symbol duration may be expressed as

$$r(t) = \begin{cases} \displaystyle\sum_{\ell=0}^{L-1}\int_0^{T_m} \sqrt{2S}\alpha_\ell(t;\, \tau)e^{j(2\pi b_0 f_d t + \theta_\ell(t;\,\tau))}d\tau \\ \quad \cdot p_{T_h}(t-\ell T_h) + n_\ell(t),\ t\in[0,T); \quad \text{FFH} \\ \displaystyle\sum_{\ell=0}^{L-1}\int_0^{T_m} \sqrt{2S}\alpha_\ell(t;\, \tau)e^{j(2\pi b_0 f_d t + \theta_\ell(t;\,\tau))}d\tau \\ \quad + n_\ell(t),\ t\in[0,T); \quad \text{MCFH} \end{cases} \tag{5}$$

where $\theta_\ell(t;\, \tau) = \zeta_\ell(t;\, \tau) + \phi_{\ell,0}$, and T_m is the maximum delay spread of each diversity channel. $n_\ell(t)$ represents a background noise and modeled as a low-pass equivalent additive white Gaussian noise (AWGN) process with PSD N_ℓ. We assume that data symbol b_0 is either $+1$ or -1 with equal probability. Without loss of generality, it is assumed that data symbol b_0 is $+1$ hereafter. Each diversity reception is demodulated by a noncoherent detector [3]. As shown in Fig. 3, a noncoherent detector consists of two branches of correlator followed by an envelope detector. We assume that the receiver is time synchronous to the first arriving signal (i.e., $\tau = 0$).

The two correlator outputs of the ℓth diversity reception are denoted, respectively, by $Z_{\ell,1}$ and $Z_{\ell,-1}$, and may be expressed as

$$\begin{aligned} Z_{\ell,1} = &\frac{1}{T_h}\int_0^{T_m}\int_\tau^{T_h} \sqrt{2S}\alpha_\ell(t;\, \tau)e^{j\theta_\ell(t;\,\tau)}\,dt\,d\tau \\ &+ \frac{1}{T_h}\int_0^{T_h} n_\ell(t)e^{-j2\pi f_d t}\,dt \end{aligned} \tag{6}$$

$$\begin{aligned} Z_{\ell,-1} = &\frac{1}{T_h}\int_0^{T_m}\int_\tau^{T_h} \sqrt{2S}\alpha_\ell(t;\, \tau)e^{j\theta_\ell(t;\,\tau)}e^{j2\pi\Delta f t}\,dt\,d\tau \\ &+ \frac{1}{T_h}\int_0^{T_h} n_\ell(t)e^{j2\pi f_d t}\,dt. \end{aligned} \tag{7}$$

In static environments, when symbol $+1$ is transmitted in the absence of noise, $Z_{\ell,-1}$ is zero if an orthogonal BFSK is employed. However, in fading environments, $Z_{\ell,-1}$ is not zero, since multipath signal components and signal variation over one hop duration may destruct orthogonality. This effect is represented as the first term of (7), which will be referred to, hereafter, as interference component in this letter. The second term in (6) and (7) represents an AWGN component. Since all the terms in (6) and (7) are zero-mean complex Gaussian random variables, $Z_{\ell,1}$ and $Z_{\ell,-1}$ are also zero-mean complex Gaussian random variables whose variances and correlation coefficient are given by

$$\begin{aligned} \sigma^2_{\ell,1} = \frac{1}{2}E\left[|Z_{\ell,1}|^2\right] = &\frac{2S}{T_h}\int_0^{T_m}\int_0^{T_h-\tau} \\ &R_c(t;\, \tau)\left(1-\frac{t+\tau}{T_h}\right)dt\,d\tau + \frac{N_\ell}{T_h} \end{aligned} \tag{8}$$

$$\begin{aligned} \sigma^2_{\ell,-1} = \frac{1}{2}E\left[|Z_{\ell,-1}|^2\right] = &\frac{2S}{T_h}\int_0^{T_m}\int_0^{T_h-\tau} \\ &R_c(t;\, \tau)\cos(2\pi\Delta f t)\left(1-\frac{t+\tau}{T_h}\right)dt\,d\tau + \frac{N_\ell}{T_h} \end{aligned} \tag{9}$$

$$\begin{aligned} \rho_\ell = &\ \tfrac{1}{2}E\left[Z^*_{\ell,1}Z_{\ell,-1}\right]\big/\sigma_{\ell,1}\sigma_{\ell,-1} \\ = &\left(\frac{S}{T_h^2}\int_0^{T_m}\int_\tau^{T_h}\int_\tau^{T_h} R_c(t_1-t_2;\, \tau)e^{j2\pi\Delta f t_1}\right. \\ &\left.\cdot\, dt_1\,dt_2\,d\tau + \frac{N_\ell}{T_h^2}\int_0^{T_h} e^{j2\pi\Delta f t}dt\right)\Bigg/\sigma_{\ell,1}\sigma_{\ell,-1}. \end{aligned} \tag{10}$$

As shown in Figs. 1(b), 2(b), and 3, decisions are made based on L pairs of noncoherent detector outputs, $R_{\ell,1} = |Z_{\ell,1}|$ and $R_{\ell,-1} = |Z_{\ell,-1}|$ for $\ell \in \{0, 1, \ldots, L-1\}$. They should be combined in some way to form decision statistics for the receiver.

B. Optimum Diversity Combining Rule

To find the optimum diversity combining rule based on the maximum-likelihood criterion, we should find the conditional joint probability density function (pdf) of noncoherent detector outputs, $R_{\ell,1}$ and $R_{\ell,-1}$ for $\ell \in \{0, 1, \ldots, L-1\}$, conditioned on a transmitted data symbol. This pdf is referred to as a likelihood function. Since each diversity reception is assumed to be independent of each other, the likelihood function for data symbol $b_0 = +1$ can be expressed as

$$\begin{aligned} p_R\,&(r_{0,1}, r_{1,1}, \ldots, r_{L-1,1}, r_{0,-1}, \\ &\quad r_{1,-1}, \ldots, r_{L-1,-1}\,|b_0 = +1) \\ &= \prod_{\ell=0}^{L-1} p_{R_\ell}\,(r_{\ell,1}, r_{\ell,-1}\,|b_0 = +1) \end{aligned} \tag{11}$$

where $p_{R_\ell}(r_{\ell,1}, r_{\ell,-1}|b_0 = +1)$ is the conditional joint pdf of the noncoherent detector outputs for the ℓth diversity reception. The joint pdf $p_{R_\ell}(r_{\ell,1}, r_{\ell,-1}|b_0 = +1)$ can be easily found using the joint Rayleigh distribution given in [7], if the variances of $Z_{\ell,1}$ and $Z_{\ell,-1}$ are the same. However, the variances of $Z_{\ell,1}$ and $Z_{\ell,-1}$ are different in our problem, as shown in (8) and (9). Hence, the results in [7] cannot be applied.

To find the joint pdf of $R_{\ell,1}$ and $R_{\ell,-1}$, the complex Gaussian random variables $Z_{\ell,1}$ and $Z_{\ell,-1}$ are expressed in terms of in-phase and quadrature components

$$Z_{\ell,1} = X_{\ell,1} + jY_{\ell,1} \qquad Z_{\ell,-1} = X_{\ell,-1} + jY_{\ell,-1} \tag{12}$$

where $X_{\ell,1}$, $X_{\ell,-1}$, $Y_{\ell,1}$, and $Y_{\ell,-1}$ are zero-mean jointly Gaussian random variables. The joint pdf of $X_{\ell,1}$, $X_{\ell,-1}$, $Y_{\ell,1}$, and $Y_{\ell,-1}$ conditioned on $b_0 = +1$ can be calculated as in (13), shown at the bottom of the page, where $\rho_{\ell r}$ and $\rho_{\ell i}$ are, respectively, the real and imaginary components of the complex correlation coefficient ρ_ℓ defined in (10). The pdf in (13) is expressed in terms of rectangular coordinate elements. A transformation may be made from the rectangular coordinate onto the polar coordinate via the change of variables

$$\begin{aligned} R_{\ell,i} &= \sqrt{X_{\ell,i}^2 + Y_{\ell,i}^2} \\ \Psi_{\ell,i} &= \tan^{-1}(Y_{\ell,i}/X_{\ell,i}), \qquad i = 1, -1. \end{aligned} \tag{14}$$

$\Psi_{\ell,1}$ and $\Psi_{\ell,-1}$ are uniform random variables over $[0, 2\pi)$. By averaging the conditional joint pdf $p_{R_\ell,\Psi_\ell}(r_{\ell,1}, r_{\ell,-1}, \psi_{\ell,1}, \psi_{\ell,-1}|b_0 = +1)$ over $\Psi_{\ell,1}$ and $\Psi_{\ell,-1}$, we can obtain the conditional joint pdf of $R_{\ell,1}$ and $R_{\ell,-1}$

$$\begin{aligned} p_{R_\ell}\,&(r_{\ell,1}, r_{\ell,-1}\,|b_0 = +1) \\ &= \frac{r_{\ell,1} r_{\ell,-1}}{\sigma_{\ell,1}^2 \sigma_{\ell,-1}^2 \left(1 - |\rho_\ell|^2\right)} \\ &\quad \cdot I_0\left(\frac{|\rho_\ell| \cdot r_{\ell,1} r_{\ell,-1}}{\sigma_{\ell,1}\sigma_{\ell,-1}\left(1 - |\rho_\ell|^2\right)}\right) \\ &\quad \cdot \exp\left[-\frac{\sigma_{\ell,-1}^2 r_{\ell,1}^2 + \sigma_{\ell,1}^2 r_{\ell,-1}^2}{2\sigma_{\ell,1}^2\sigma_{\ell,-1}^2\left(1 - |\rho_\ell|^2\right)}\right] \end{aligned} \tag{15}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. Similarly, the likelihood function for data symbol $b_0 = -1$ will be obtained from (11) and (15), by exchanging $\sigma_{\ell,1}$ and $\sigma_{\ell,-1}$ in (15).

After straightforward algebraic manipulation and extraction of common terms in the log-likelihood functions, the optimum decision rule is derived as

$$\sum_{\ell=0}^{L-1} \frac{\sigma_{\ell,1}^2 - \sigma_{\ell,-1}^2}{\sigma_{\ell,1}^2\sigma_{\ell,-1}^2(1 - |\rho_\ell|^2)} \cdot \left(R_{\ell,1}^2 - R_{\ell,-1}^2\right) \underset{\hat{b}_0=-1}{\overset{\hat{b}_0=+1}{\gtrless}} 0. \tag{16}$$

This equation indicates that the decision variable associated with $\hat{b}_0 = +1$ is constructed as the weighted sum of squares of $R_{\ell,1}$ for all ℓ, and the decision variable associated with $\hat{b}_0 = -1$ is constructed in a similar manner. These two variable values are compared to estimate a transmit symbol. Note that the combining rule in (16) is different from the combining rule in [5], which is developed for static channels. In (16), it can be shown that the ℓth weighting factor depends on the variances and the correlation coefficient of correlator outputs for the ℓth diversity reception. $\sigma_{\ell,1}^2$ is composed of signal and noise components, and $\sigma_{\ell,-1}^2$ interference and noise components. The numerator $\sigma_{\ell,1}^2 - \sigma_{\ell,-1}^2$ represents a difference between signal power and interference power, since the noise powers in $\sigma_{\ell,1}^2$ and $\sigma_{\ell,-1}^2$ are the same. $\sigma_{\ell,1}^2 - \sigma_{\ell,-1}^2$ is the same for all ℓ, when the transmit power is the same and fading process is i.i.d. for each diversity channel. The denominator $\sigma_{\ell,1}^2\sigma_{\ell,-1}^2(1 - |\rho_\ell|^2)$ represents that the weighting factor should be small when the noise power is large. The reason is that as the noise power increases, $\sigma_{\ell,1}^2$ and $\sigma_{\ell,-1}^2$ increase and $|\rho_\ell|$ decreases.

$$\begin{aligned} p_{X_\ell,Y_\ell}\,(x_{\ell,1}, x_{\ell,-1}, y_{\ell,1}, y_{\ell,-1}\,|b_0 = +1) &= \frac{1}{(2\pi)^2\sigma_{\ell,1}^2\sigma_{\ell,-1}^2\left(1 - |\rho_\ell|^2\right)} \\ &\quad \cdot \exp\left[-\frac{\sigma_{\ell,-1}^2\left(x_{\ell,1}^2 + y_{\ell,1}^2\right) + \sigma_{\ell,1}^2\left(x_{\ell,-1}^2 + y_{\ell,-1}^2\right)}{2\sigma_{\ell,1}^2\sigma_{\ell,-1}^2\left(1 - |\rho_\ell|^2\right)}\right] \\ &\quad \cdot \exp\left[\frac{\rho_{\ell r}\,(x_{\ell,1}x_{\ell,-1} + y_{\ell,1}y_{\ell,-1}) + \rho_{\ell i}\,(x_{\ell,1}y_{\ell,-1} - x_{\ell,-1}y_{\ell,1})}{\sigma_{\ell,1}\sigma_{\ell,-1}\left(1 - |\rho_\ell|^2\right)}\right] \end{aligned} \tag{13}$$

To compare the performance of FFH and MCFH systems and to evaluate the effects of diversity order in typical frequency-selective fading channels, the PSD N_ℓ of background noise for each diversity reception is assumed to be the same, i.e., $N_\ell = N_0$, where N_0 is the one-sided PSD of thermal noise. From this assumption, the variances and correlation coefficient of the correlator outputs, given in (8)–(10), are the same for all ℓ: $\sigma_{\ell,1}^2 = \sigma_1^2, \sigma_{\ell,-1}^2 = \sigma_{-1}^2$, and $\rho_\ell = \rho$ for $\ell \in \{0, 1, \ldots, L-1\}$. With this assumption, the optimum combining rule in (16) becomes square-law equal-gain combining, which is the same result as in [7], where orthogonality between BFSK signals is maintained.

C. Probability of Error

Based on (16) and the above assumption, the probability of error for the optimally combined signal may be expressed as

$$P_e = \int_{-\infty}^{0} p(D|b_0 = +1)\, dD \tag{17}$$

where D is the decision variable defined as $D = \sum_{\ell=0}^{L-1} D_\ell$, and $D_\ell = R_{\ell,1}^2 - R_{\ell,-1}^2$. $p(D|b_0 = +1)$ is the conditional pdf of D, given $b_0 = +1$. The conditional pdf $p(D|b_0 = +1)$ may be found using (11) and (15) with appropriate transformations of random variables. However, this work is unnecessarily involved and the solution is not concise. It can be shown that the decision variable D in (17) may be viewed as a special case of the general quadratic form investigated in [9, Appendix B], where the characteristic function-based approach is presented to obtain a simple closed-form expression for the probability of error. Equation (17) may be rewritten in terms of the characteristic function of D, which is denoted by $\Phi_D(jv)$, as

$$\begin{aligned} P_e &= \frac{1}{2\pi} \int_{-\infty}^{0} \int_{-\infty}^{\infty} \Phi_D(jv) e^{-jvD}\, dv dD \\ &= -\frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{\Phi_D(jv)}{v}\, dv. \end{aligned} \tag{18}$$

Since D is the sum of L i.i.d. random variables D_ℓ ($\ell = 0, 1, \ldots, L-1$), the characteristic function of D is simply the Lth power of that of D_ℓ, or, $\Phi_D(jv) = [\Phi_{D_\ell}(jv)]^L$. The characteristic function of D_ℓ is given as [9]

$$\Phi_{D_\ell}(jv) = \frac{v_1 v_2}{(v + jv_1)(v - jv_2)} \tag{19}$$

where v_1 and v_2 are defined as

$$\begin{aligned} v_1 \triangleq\ & -\frac{\sigma_1^2 - \sigma_{-1}^2}{4\sigma_1^2\sigma_{-1}^2(1-|\rho|^2)} \\ & + \sqrt{\left(\frac{\sigma_1^2 - \sigma_{-1}^2}{4\sigma_1^2\sigma_{-1}^2(1-|\rho|^2)}\right)^2 + \frac{1}{4\sigma_1^2\sigma_{-1}^2(1-|\rho|^2)}} \end{aligned} \tag{20}$$

$$\begin{aligned} v_2 \triangleq\ & \frac{\sigma_1^2 - \sigma_{-1}^2}{4\sigma_1^2\sigma_{-1}^2(1-|\rho|^2)} \\ & + \sqrt{\left(\frac{\sigma_1^2 - \sigma_{-1}^2}{4\sigma_1^2\sigma_{-1}^2(1-|\rho|^2)}\right)^2 + \frac{1}{4\sigma_1^2\sigma_{-1}^2(1-|\rho|^2)}}. \end{aligned} \tag{21}$$

Through the use of a conformal transformation from the v plane to the u plane via the change in variable $u = -(v_1/v_2) \cdot [(v - jv_2)/(v + jv_1)]$, and the binomial series expansion of a term, (18) may be expressed as

$$\begin{aligned} P_e = &\frac{1}{(1+\gamma)^{2L-1}} \sum_{\ell=0}^{2L-1} \binom{2L-1}{\ell} \\ &\cdot \gamma^\ell \frac{1}{2\pi j} \int_\Gamma \frac{1}{u^{L-\ell}(1-u)}\, du \end{aligned} \tag{22}$$

where Γ is a circular contour of radius less than unity that encloses the origin, and γ is defined as

$$\gamma \triangleq \frac{v_2}{v_1} = \frac{\sigma_1^2 - \sigma_{-1}^2 + \sqrt{\left(\sigma_1^2 + \sigma_{-1}^2\right)^2 - 4|\rho|^2\sigma_1^2\sigma_{-1}^2}}{\sigma_{-1}^2 - \sigma_1^2 + \sqrt{\left(\sigma_1^2 + \sigma_{-1}^2\right)^2 - 4|\rho|^2\sigma_1^2\sigma_{-1}^2}}. \tag{23}$$

For $\ell \geq L$, the contour integral is zero by Cauchy's theorem [10], since the integrand $1/u^{L-\ell}(1-u)$ is an analytic function in Γ. However, for $0 \leq \ell \leq L-1$, the contour integral should be calculated using Residue theorem [10]. Thus, the probability of error expression in (22) may be simplified to

$$P_e = \sum_{\ell=0}^{L-1} \binom{2L-1}{\ell} \frac{\gamma^\ell}{(1+\gamma)^{2L-1}} \tag{24}$$

which may be expressed in an alternative form

$$P_e = \sum_{\ell=0}^{L-1} \binom{L+\ell-1}{\ell} \frac{\gamma^\ell}{(1+\gamma)^{L+\ell}}. \tag{25}$$

The equivalence of (24) and (25) can be shown by repeatedly applying a basic formula $\binom{n}{r} = \binom{n+1}{r} - \binom{n}{r-1}$ to (25). It should be noted that when $\rho = 0$, (25) becomes the probability of error equation developed for frequency-nonselective slow Rayleigh fading channels [7].

IV. Performance Evaluation

The BER performance of FFH and MCFH systems is evaluated in this section using (23) and (24). The variances and correlation coefficient in (8)–(10) are required for (23), and calculated by Monte Carlo integration technique [11]. The autocorrelation function of a fading channel in (4) is assumed to be described by an exponential multipath intensity profile and Jakes' fading model [12]

$$R_c(\Delta t; \tau) = \frac{(\mu/T_m)(e^{-\mu\tau/T_m} - e^{-\mu})}{1-(1+\mu)e^{-\mu}} J_0(2\pi f_D \Delta t) \tag{26}$$

where μ is a decaying factor and set to 0.5 in this letter, f_D is the maximum Doppler spread, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. Orthogonal signaling ($h = 1.0$) is implied, unless explicitly specified.

Fig. 4 shows the performance of FFH and MCFH systems for several values of maximum delay spreads, when the normalized maximum Doppler spread $f_D T = 0.01$. Diversity order L is set to 3. The performance of FHSS systems is found to be significantly degraded in frequency-selective fading environments with delay spread. The performance degradation due to delay spread is found much more severe in FFH systems than in

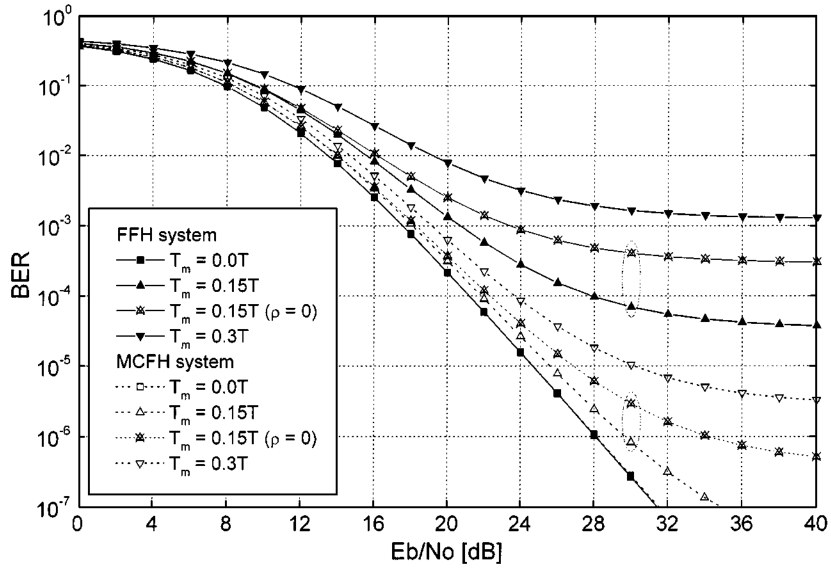


Fig. 4. BER performance of FFH and MCFH systems for various delay spreads ($L = 3,\ f_D T = 0.01$).

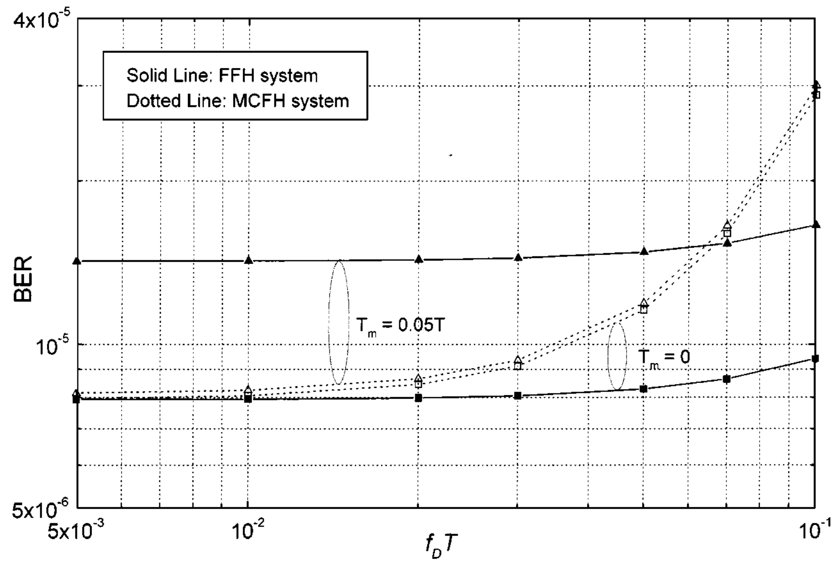


Fig. 5. BER performance of FFH and MCFH systems for various Doppler spreads ($L = 3,\ T_m = 0,\ 0.05T,\ E_b/N_0 = 25$ dB).

MCFH systems. This can be explained as follows. The probability of error may be proved to be a monotonically decreasing function of γ by differentiating (23) with respect to γ. From (8)–(10), and (23), γ is observed to be related to the ratio of T_m to T_h, which is defined as an effective delay spread in this letter. It can be shown that γ decreases with the effective delay spread, due to an increase in σ_{-1}^2 and a decrease in σ_1^2 and $|\rho|$. Thus, the value of γ is smaller for an FFH system than for an MCFH system, for a given delay spread, since the effective delay spread for an FFH system is L times larger than that of an MCFH system.

To investigate the effects of correlation between two correlator outputs, the performance of FFH and MCFH systems with the correlation ignored and $T_m = 0.15T$ are obtained by setting $\rho = 0$ in (23) and plotted in Fig. 4. The large differences between the correlation ignored and not-ignored cases indicate that the correlation should not be ignored.

Fig. 5 depicts how the BER performance varies with the normalized Doppler spread $f_D T$ for two delay spread values, when L =3 and $E_b/N_0 = 25$ dB. It is found that MCFH systems are more sensitive to the normalized Doppler spread $f_D T$ than FFH systems. In other words, the performance degradation due to an increase of $f_D T$ is more severe for MCFH systems than for FFH systems. The reason is that the hop duration of MCFH systems is L times larger than that of FFH systems. Energy loss is larger for larger hop duration in the correlator, when fading process varies rapidly during hop duration in Rayleigh fading environments. Hence, FFH systems are L times robust to an increase of Doppler spread than MCFH systems. It is shown in Fig. 5 that the BER performance of FFH systems for $f_D T = 0.1$ is almost the same as that of MCFH systems for $f_D T = 0.033\,(= 0.1/L)$, when $T_m = 0$. The combined effects of the delay spread and Doppler spread can be seen by comparing the BER performance of the two systems in Fig. 5, when $T_m/T = 0.05$. MCFH sys-

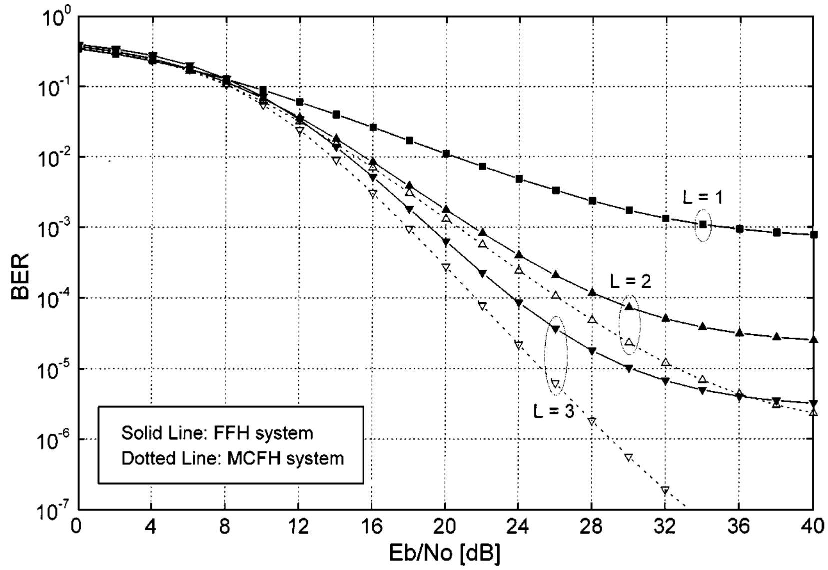


Fig. 6. BER performance of FFH and MCFH systems for various L's ($T_m = 0.1T,\ f_DT = 0.01$).

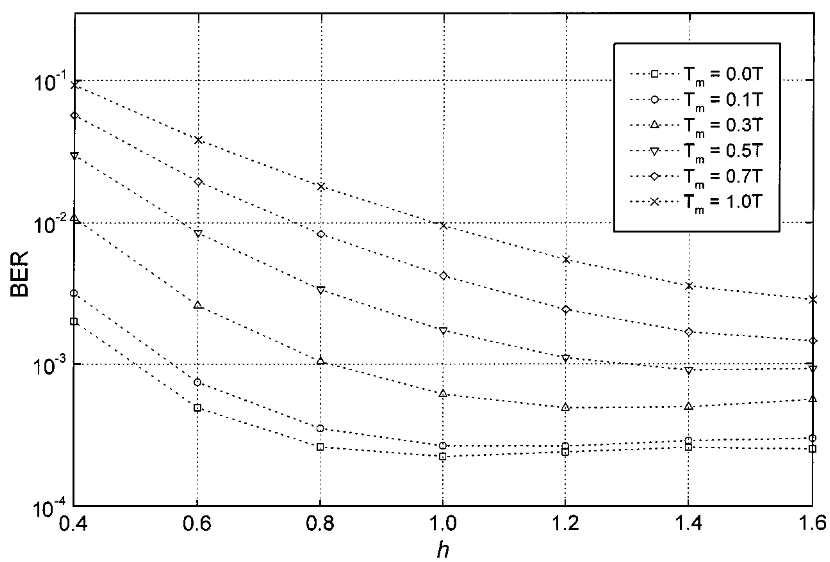


Fig. 7. BER performance of MCFH system for various h's ($L = 3,\ f_DT = 0.01,\ E_b/N_0 = 20$ dB).

tems are shown to outperform FFH systems for $f_DT \leq 0.063$; the opposite is true for $f_DT > 0.063$.

The effects of diversity order on the BER performance of FFH and MCFH systems are shown in Fig. 6. It is found that an increase in diversity order L from 1 to 3 improves the BER performance of FFH systems to a small extent, and that of MCFH systems to a large extent. For MCFH systems, the effective delay spread does not change with diversity order, since the hop duration does not change with diversity order. For FFH systems, the effective delay spread increases with diversity order. Hence, the performance improvement due to the increase in diversity order is smaller for FFH systems than that for MCFH systems.

It is well known that in static channels, the optimum h for correlator-based noncoherent detection of BFSK signals are integer values to satisfy the orthogonality condition, if the multiple-access interference is not considered [3], [9]. In multiple-access environments, the optimum h depends on the number of users in the network and the signal-to-noise ratio [3]. In fading channels, however, delay spread and channel variation also affect the optimum h. Thus, the optimum value varies with channel condition. Fig. 7 shows the BER performance of an MCFH system for various values of h ranging between 0.4–1.6, when $L = 3,\ f_DT = 0.01$, and $E_b/N_0 = 20$ dB. For a given practical range of h and delay spread, the optimum h is found to increase with delay spread. This can be explained as follows. Since the desired signal power in (8) is not affected by h, h is optimal when the interference power in (9) is minimized. It can be shown that h which minimizes the interference power increases with delay spread.

V. Conclusions

The BER performance of FFH and MCFH systems in frequency-selective Rayleigh fading channels is presented and compared in this letter. The optimum diversity combining rule based on the maximum-likelihood criterion is developed. It is found that the optimum combining is the weighted sum of the squares of noncoherent detector outputs. A weighting factor is shown to depend on the variances and correlation coefficient of correlator outputs for each diversity reception. Based on the developed optimum diversity combining rule, the expressions for the probability of error are derived and evaluated for various channel conditions. It is found that frequency-selective Rayleigh fading severely degrades the performance of FHSS systems. MCFH systems are found to outperform FFH systems in frequency-selective fading environments. On the other hand, it is found that FFH systems are superior to MCFH systems in fast fading environments. Although diversity improves the performance of both MCFH and FFH systems, the diversity gain is found to be greater for MCFH systems than for FFH systems. It is also shown that the optimum frequency deviations of transmit FSK signals increase with delay spread.

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