Transmit Beamforming with Reduced Channel State Information in MIMO-OFDM Wireless Systems

Seung-Hyeon Yang, Jae-Yun Ko, and Yong-Hwan Lee
School of Electrical Engineering and INMC
Seoul National University
Seoul, Korea
Email: lis7@ttl.snu.ac.kr, paul@fruit.snu.ac.kr, ylee@snu.ac.kr

***Abstract*—The use of multiple antennas for diversity and array gain may require accurate channel state information (CSI) on each subcarrier in orthogonal frequency division multiplexing systems. Such CSI is usually reported from the receiver to the transmitter, causing a significant amount of feedback signaling burden. Recent work can noticeably reduce the feedback signaling burden by exploiting the channel correlation properties in the frequency domain, but it can be applied to the generation of a single beam [1]. In this paper, this idea is extended to the use of multiple beams for spatial multiplexing. Simulation results show that the proposed scheme can improve the performance using the same amount of feedback signaling burden or provide the same performance while noticeably reducing the amount of feedback signaling burden, compared to conventional schemes.**

Keywords-codebook; feedback; MIMO; OFDM system; precoding;

I. Introduction

The use of multi-input multi-output (MIMO) systems has been considered indispensable for the improvement of transmission throughput. The performance improvement with the use of multiple antennas is deeply associated with the accuracy of channel state information (CSI). When the transmitter has the CSI, it can achieve diversity gain as well as array gain by means of transmit beamforming, significantly improving the system performance [2]–[4]. Such beamforming techniques, however, require accurate CSI at the transmitter, which may not easily be achievable in addition to noticeable feedback signaling overhead.

Orthogonal frequency division multiplexing (OFDM) has been recognized as a key transmission technique in broadband wireless communication systems. Recently, combined use of MIMO and OFDM schemes has been applied to advanced wireless systems. To employ a beamforming technique in MIMO-OFDM systems, the transmitter may require the CSI of each subcarrier, increasing the amount of feedback signaling overhead linearly proportional to the number of subcarriers. This problem can be alleviated by employing a codebook method [5]–[9], where the beam weight is determined from a finite set of weight vectors, called codebook, and the index of the corresponding weight vector is reported to the transmitter through a feedback signaling channel in the uplink. The feedback signaling overhead can further be reduced by recursively encoding the codeword based on the previous ones [10]. A further simplified scheme, so-called clustering, can reduce the signaling overhead by combining a number of adjacent subcarriers into a cluster that uses the same beam weight [11]. Performance degradation due to the clustering can be alleviated by interpolating the beam weight in the frequency domain [11]. However, this may require additional phase information. Recently, the channel correlation in the frequency domain has been exploited to reduce the feedback signaling for the transmit beamforming [1]. The beam weight can be generated in a differential manner using the correlation characteristics between the adjacent subcarriers. When the channel of adjacent subcarriers is correlated, the beam weight for the current subcarrier can be represented in terms of the beam weight for the previous adjacent subcarrier and a term corresponding to the weight difference between the two subcarriers. However, it is applicable to the generation of a single beam.

In this paper, we consider the extension of the idea in [1] to multi-beam schemes for spatial multiplexing. For spatial multiplexing, the data streams are often precoded based on the CSI before the transmission [13]–[17]. We consider spatial multiplexing, where the beam weight (or precoding) is determined by a codebook comprising unitary matrices. The transmitter estimates the CSI from the feedback information and then generates the precoding weight from it.

This paper is organized as follows. Section II describes the system model in consideration. The conventional beamforming technique that considers a signal beam is briefly discussed in Section III. Section IV presents the proposed scheme that generates multiple beams for spatial multiplexing. The

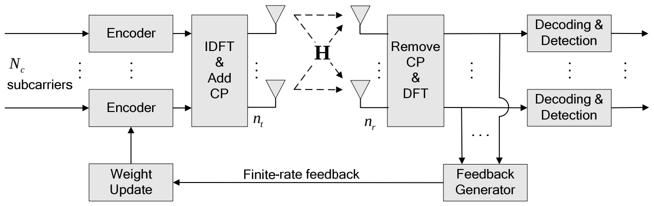


Figure 1. Block diagram of an OFDM based MIMO system with precoding.

performance is verified by computer simulation in Section V. Finally, Section VI summarizes conclusions.

Notation: Bold upper and lower letters denote matrices and vectors, respectively. $(\cdot)^T$ and $(\cdot)^*$ denote transpose and conjugate transpose, respectively. $E\{\cdot\}$ denotes the expectation operator. $\|\cdot\|$ and $tr[\cdot]$ denote the Frobenius norm and the trace of a matrix, respectively. $U(N,m,n)$ denotes the set of N $(m\times n)$-dimensional (D) matrices with orthonormal columns. $\lceil x \rceil$ denotes the smallest integer larger than or equal to x.

II. System Model

Consider a MIMO-OFDM based wireless system, where the base station (BS) uses n_t transmit antennas and the mobile station (MS) uses n_r receive antenna as illustrated in Fig. 1. Assume that the OFDM system utilizes N_c subcarriers and the k -th subcarrier transmits an n_s -D vector signal $\mathbf{s}(k)=[s_1(k)\ s_2(k)\ ...\ s_{n_s}(k)]^T$, where $n_s \le \min(n_t, n_r)$ and $s_i(k)$ is the i -th data stream. When $\mathbf{s}(k)$ is weighted by an ($n_t \times n_s$) precoding matrix $\mathbf{F}(k)$, the received signal vector $\mathbf{r}(k)=[r_1(k)\ r_2(k)\ ...\ r_{n_r}(k)]^T$ through subcarrier k can be represented as

$$\mathbf{r}(k)=\mathbf{H}(k)\mathbf{F}(k)\mathbf{s}(k)+\mathbf{n}(k), \quad k=0,...,N_c-1 \tag{1}$$

where $\mathbf{n}(k)=[n_1(k),n_2(k),...,n_{n_r}(k)]^T$ is the n_r -D additive white Gaussian noise (AWGN) vector, and $\mathbf{H}(k)$ is the ($n_r \times n_t$) channel matrix whose entries are independent identically distributed (i.i.d.) complex zero mean Gaussian random variables with unit variance. Define the correlation coefficient between adjacent subcarriers by

$$\rho=\frac{tr\left[E\left\{\mathbf{H}(k)\mathbf{H}^*(k-1)\right\}\right]}{\sqrt{E\left\{\|\mathbf{H}(k)\|^2\right\}E\left\{\|\mathbf{H}(k-1)\|^2\right\}}}\,. \tag{2}$$

In this paper, we assume that the CSI is perfectly estimated at the receiver and reported to the transmitter through a finite-rate feedback link.

III. Transmit Beamforming with a Single Beam [1]

For ease of description, we briefly describe the transmit beamforming with the use of a single beam by exploiting the channel correlation in the frequency domain [1].

Consider the use of a single receive antenna (i.e., multi-input single-output (MISO) scheme). Then, (1) can be rewritten as

$$r_1(k)=\mathbf{h}(k)\mathbf{f}(k)s_1(k)+n_1(k) \tag{3}$$

where $\mathbf{h}(k)$ is the first row of $\mathbf{H}(k)$ and $\mathbf{f}(k)$ is the first column of $\mathbf{F}(k)$. Unless the number of transmit antennas is too small, the beam weight of subcarrier k can be represented in terms of the beam weight of subcarrier $(k-1)$ as [1]

$$\mathbf{f}(k)\approx\rho^*\mathbf{f}(k-1)+\sqrt{1-|\rho|^2}\,\mathbf{z} \tag{4}$$

where $\mathbf{z}$ is an random vector isotropically distributed over an n_t -D unit sphere. In fact, the random vector $\mathbf{z}$ indicates the direction of $\mathbf{f}(k)$ from $\mathbf{f}(k-1)$. Since $\sqrt{1-|\rho|^2}\le 1$, for a given amount of information size, the term $\mathbf{z}$ can be quantized with a better accuracy compared to the quantization of $\mathbf{f}(k)$. Moreover, since the correlation coefficient ρ is not fast varying, the amount of additional information for the correlation coefficient is marginal.

The random vector $\mathbf{z}$ can be quantized as [1]

$$\hat{\mathbf{z}}=\arg\max_{\mathbf{z}\in C}\left|\mathbf{h}(k)\left(\frac{\rho^*\mathbf{f}(k-1)+\sqrt{1-|\rho|^2}\,\mathbf{z}}{\left\|\rho^*\mathbf{f}(k-1)+\sqrt{1-|\rho|^2}\,\mathbf{z}\right\|}\right)\right| \tag{5}$$

where C denotes a codebook comprising N $(n_t\times 1)$ unit norm vectors. The receiver reports a $\lceil \log_2 N \rceil$-bit index of $\hat{\mathbf{z}}$ to the transmitter. The transmitter recovers the beam weight for subcarrier k as

$$\mathbf{f}(k)=\frac{\rho^*\mathbf{f}(k-1)+\sqrt{1-|\rho|^2}\,\hat{\mathbf{z}}}{\left\|\rho^*\mathbf{f}(k-1)+\sqrt{1-|\rho|^2}\,\hat{\mathbf{z}}\right\|}\,. \tag{6}$$

IV. Transmit Beamforming for MIMO Systems

Consider the use of multi-beam for spatial multiplexing gain. In the presence of channel correlation between the adjacent OFDM subcarriers, the channel matrix of subcarrier k can be represented in terms of the channel matrix of subcarrier $(k-1)$ as

$$\mathbf{H}(k)=\rho\mathbf{H}(k-1)+\sqrt{1-|\rho|^2}\,\mathbf{E} \tag{7}$$

where $\mathbf{E}$ is an $(n_r\times n_t)$ matrix whose entries are i.i.d. complex zero mean Gaussian random variables with unit variance and independent of the entries of $\mathbf{H}(k-1)$. Unless the number of transmit antennas is too small, it can be shown from the law of large numbers that $\mathbf{E}\mathbf{E}^*\approx n_t\mathbf{I}_{n_r}$ [12]. Then, (7) can be rewritten as

$$\mathbf{H}(k) \approx \rho \mathbf{H}(k-1) + \sqrt{n_t}\sqrt{1-|\rho|^2}\mathbf{Z}^* \quad (8)$$

where $\mathbf{Z} = \mathbf{E}^* / \sqrt{n_t}$. The receiver calculates $\mathbf{Z}$ and quantizes it using a predefined codebook $\boldsymbol{C}'$ comprising N $(n_t \times n_s)$ matrices whose columns are orthonormal to each other.

The matrix $\mathbf{Z}$ can be quantized in various ways. For example, it can be quantized to maximize the capacity as [15]

$$\hat{\mathbf{Z}} = \arg\max_{\mathbf{Z}\in \boldsymbol{C}'} \left(\log_2 \det\left(\mathbf{I}_{n_s} + \frac{\varepsilon_s}{n_s N_0} \hat{\mathbf{F}}^*(k)\mathbf{H}^*(k)\mathbf{H}(k)\hat{\mathbf{F}}(k) \right) \right) \quad (9)$$

where ε_s is the total transmit power and $\hat{\mathbf{F}}(k)$ is defined by

$$\hat{\mathbf{F}}(k) = R_{1:n_s}\left(\rho \mathbf{H}(k-1) + \sqrt{n_t}\sqrt{1-|\rho|^2}\mathbf{Z}^* \right) \quad (10)$$

where $R_{1:L}(\mathbf{A})$ denotes the first L columns of the right singular matrix of $\mathbf{A}$. It can also be quantized to minimize the mean square error (MSE) as [15]

$$\hat{\mathbf{Z}} = \arg\min_{\mathbf{Z}\in \boldsymbol{C}'} tr\left(\frac{\varepsilon_s}{n_s}\left(\mathbf{I}_{n_s} + \frac{\varepsilon_s}{n_s N_0} \hat{\mathbf{F}}^*(k)\mathbf{H}^*(k)\mathbf{H}(k)\hat{\mathbf{F}}(k) \right)^{-1} \right). \quad (11)$$

The receiver reports a $\lceil \log_2 N \rceil$ -bit index of $\hat{\mathbf{Z}}$ to the transmitter. The transmitter estimates the CSI from the feedback information ρ and $\hat{\mathbf{Z}}$ as

$$\hat{\mathbf{H}}(k) = \rho \hat{\mathbf{H}}(k-1) + \sqrt{n_t}\sqrt{1-|\rho|^2}\hat{\mathbf{Z}}^* \quad (12)$$

and determines the precoding matrix $\mathbf{F}(k)$ as

$$\mathbf{F}(k) = R_{1:n_s}\left(\hat{\mathbf{H}}(k) \right). \quad (13)$$

This scheme can be applied to the clustering scheme by means of interpolation. Assume that K subcarriers are combined into a cluster with a representative subcarrier mK , where $m = 1,..., N_c / K - 1$. From the estimated channel matrices $\left\{ \hat{\mathbf{H}}(mK) \right\}$, the channel matrix for other subcarriers can be estimated as

$$\hat{\mathbf{H}}(mK+l) = \left(1 - \frac{l}{K} \right)\hat{\mathbf{H}}(mK) + \frac{l}{K}\hat{\mathbf{H}}(mK+K) \quad (14)$$

where m is the cluster index and $0 \le l \le K-1$. Finally, the transmitter determines the precoding matrix by the first n_s columns of the right singular matrix of $\hat{\mathbf{H}}(mK+l)$ (i.e, $\mathbf{F}(mK+l) = R_{1:n_s}\left(\hat{\mathbf{H}}(mK+l) \right)$. Note that as in the signal beam case, the proposed scheme does not require any additional information for the interpolation (e.g., the phase information in [11]).

The codebook can be generated as [1] and [15]

$$\boldsymbol{W} = \arg \max_{\mathbf{X}\in \boldsymbol{U}(\boldsymbol{N}, \boldsymbol{n_t}, \boldsymbol{n_s})} \delta(\mathbf{X}) \quad (15)$$

TABLE I. SIMULATION PARAMETERS

Number of subcarriers (N_c)	2048
Subcarrier spacing	48.8 kHz
Cluster size (K)	8
Channel model	Exponentially decaying model
RMS delay spread	167ns
Antenna configuration	4x2
Number of data streams (n_s)	2
Link adaptaion	Ideal (i.e., Shannon's capacity curve

where

$$\delta(\mathbf{X}) = \min_{1\le k \le l \le N} d(\mathbf{F}_k, \mathbf{F}_l). \quad (16)$$

Here $\mathbf{F}_n$ denotes the n-th entry of $\mathbf{X}$ and $d(\mathbf{A},\mathbf{B})\left(= \|\mathbf{A}-\mathbf{B}\|\right)$ is the Euclidean distance between matrices $\mathbf{A}$ and $\mathbf{B}$.

Assuming the use of two different codebooks known to the transmitter and receiver, the proposed algorithm can be realized as follows. First, the feedback information can be generated at the receiver in the following steps.

1. Quantize the channel matrix corresponding to the this subcarrier into a b_1 -bit codeword (or index) using a codebook comprising N_1 matrices.
2. Using another codebook comprising N_2 matrices, quantize the random matrix $\mathbf{Z}$ into a b_2 ($b_2 \le b_1$)-bit codeword (e.g., (9) or (11)).

After generating the feedback information, the receiver reports it to the transmitter through a feedback signaling link. Then, the transmitter generates the precoding matrices of each subcarrier using this feedback information as follows.

1. Generate the channel matrix corresponding to the representative subcarrier of each cluster by (12).
2. Generate the beam weight for other subcarriers by interpolation (e.g., by (14)).

V. PERFORMANCE EVALUATION

The performance of the proposed scheme is verified by computer simulation when applied to a system with four transmit antennas and two receive antennas (i.e., (4×2) MIMO configuration). The simulation condition is mostly from [18] and summarized in Table 1. It is assumed that the CSI is reported to the transmitter without error and delay.

Fig. 2 depicts the performance of the proposed scheme where b_1 and b_2 are quantized into 6 bits, respectively. For comparison, the performance of the clustering and the

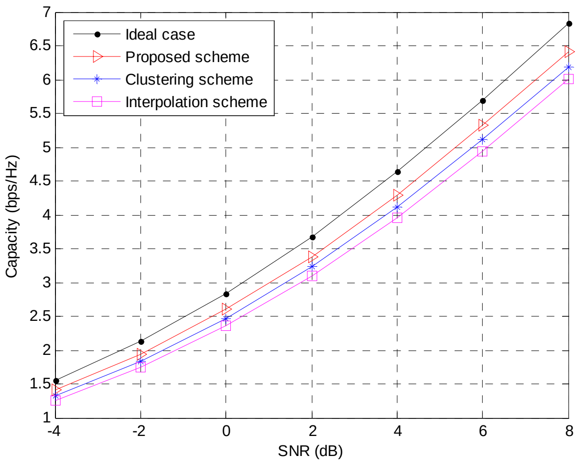
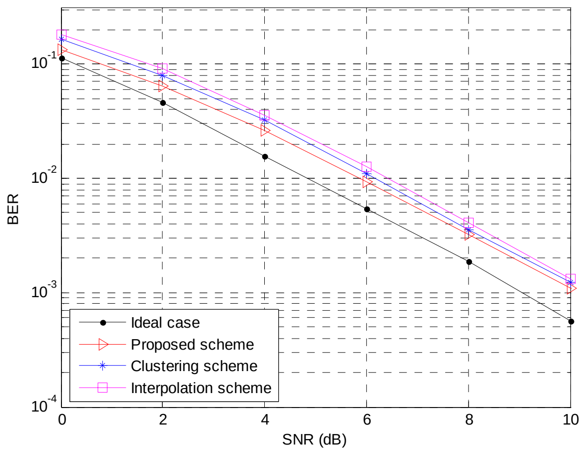


Figure 2. Capacity of (4×2) precoding schemes.



(a) RMS delay spread = 167ns

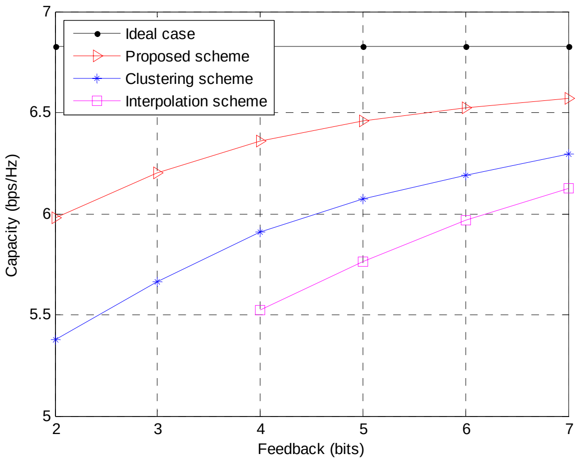
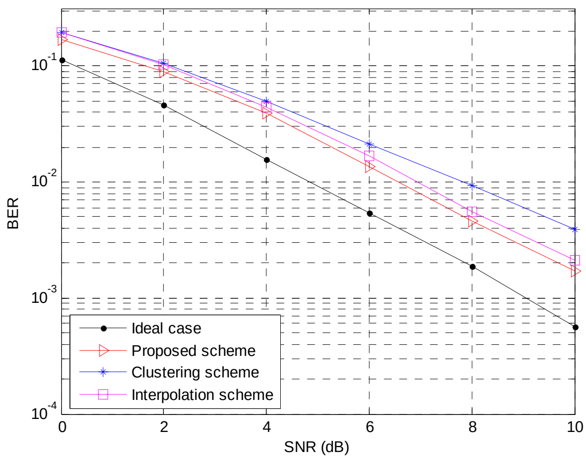


Figure 3. Capacity associated with the codebook size (SNR = 8dB).



(b) RMS delay spread = 436ns

Figure 4. BER performance of precoding schemes.

interpolation-based unitary precoding scheme is also depicted [15], [16], where the clustering scheme uses 6 bits for the codebook and the interpolation scheme uses 4 bits for the codebook and 2 bits for the phase information. Note that all the schemes except the ideal case use the same amount of feedback signaling for precoding. All schemes employ precoding whose coefficient is determined to maximize the system capacity. It can be seen that the proposed scheme provides best capacity performance while the interpolation-based scheme has the worst performance due to the need of additional phase information, therefore using fewer bits for the precoding matrix.

Fig. 3 depicts the capacity of the proposed scheme associated with the amount of feedback overhead, which represents the amount of feedback signals required for the information of each cluster after the initialization. The proposed scheme uses $b_1 = 7$ bits codewords for the initialization and the interpolation scheme uses 2 bits for the phase information. It can be seen that the proposed scheme can provide performance close to the ideal case even with the use of 6 bits for the beam weight. It can also be seen that the proposed scheme can reduce the amount of feedback signaling for the beam weight to provide the same performance as the other schemes.

Fig. 4 depicts the BER performance of the proposed scheme when the signal is transmitted using QPSK modulation with a 1/2-rate convolutional code. For comparison, the performance of the clustering and the interpolation-based unitary precoding scheme is also depicted [15], [16]. All schemes except the ideal case uses the same amount of feedback signaling, 6 bits per one cluster, and employ precoding whose coefficient is determined to minimize the mean square error. It can be seen that the proposed scheme noticeably outperforms the conventional schemes and that the interpolation-based scheme has better performance than the clustering scheme due to the diversity gain through the additional phase information.

VI. CONCLUSIONS

In this paper, we have proposed a new feedback reduction scheme in OFDM based wireless systems. The proposed scheme can reduce the amount of feedback signaling burden by generating the precoding matrix using the channel information on the previous subcarrier and frequency channel correlation. The simulation results show that the proposed scheme can estimate the channel information more accurately than conventional schemes for a given amount of feedback signaling burden, improving the system capacity or reducing the error probability. This implies that the proposed scheme can reduce the amount of feedback signaling burden to provide the same performance as conventional schemes.

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