Students' colloquial and mathematical discourses on infinity and limit: The case of an American and a Korean student

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Abstract

The purpose of this study is on the relationships between students' uses and understandings of the notions of infinity and limit in both colloquial and mathematical discourses. How students' colloquial discourse on infinity and limit correlates with their mathematical discourse, in the case of an American and a Korean student, will be analyzed based on three distinctive features of mathematical discourses: mathematical uses of words, discursive routines, and endorsed narratives. According to the results of the current study, colloquial discourse seems to correlate with mathematical discourse because of certain clear relationships between the colloquial and mathematical discourses of the American and Korean students on infinity and limit.

Key words: colloquial discourse, mathematical discourse, infinity, limit

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Ⅰ. Introduction

The instructional aim in calculus is to develop the concepts of continuity, the derivative, and the integral. As Cauchy pointed out, a derivative, an integral, and an infinite series can be interpreted as a limit (Hairer & Wanner, 1996, p. 170). Thus, we need to consider the framework upon which the limit concept is based. Limits are tightly related, and dependent on, the notions of infinity. For instances, the infinitely small is fundamental in the notion of limit. Infinite processes, such as infinite sequences and series, can thus be a crucial part of understanding the limit process (Monaghan, 2001; Kleiner, 2001). So, in order to better serve their future learning of higher level concepts, we need to understand how students use the concepts of infinity and limit simultaneously.

Most students have extreme difficulties in acquiring the limit concept because of its abstract, formal definition and its precision (Mamona-Downs, 2001). The difficulties can come from not only inside of mathematics classrooms but also within non-school contexts. As many researchers (Smith, 2002; Davis, 2001; Duffin & Simpson, 2000; Moss & Case, 1999) point out, students' school learning is strongly influenced by the knowledge they bring with them to the classroom from out-of-school settings. Although Mamona-Downs (2001) addresses that the concept of infinity is never directly experienced by one's senses in the physical world, it is reasonable to think that students enter the classroom with preexisting conceptions of not only infinity but also limit from everyday language. Thus, it is important to know how students use the notions of infinity and limit in colloquial discourse to focus on how students learn infinity and limit in mathematical discourse.

Infinity and limit are the terms used loosely in everyday language, but they have a very specific meaning in mathematics. Therefore, students who have different experiences with language can have very different interpretations of the discourse that is used in their mathematics classrooms. The focus of this study is on the relationships between students' uses and understandings of the notions of infinity and limit in both mathematical and everyday discourses.

Ⅱ. Theoretical Background

Discourses are the acts of verbal and non-verbal communication with others or with oneself. Language and discourse are both tools and products of cognitive, social, and cultural practice (Vygotsky, 1978). Thinking can be regarded as a special case of the activity of communication (Ben-Yehuda et al., 2005). Mathematical activity can be seen as a form of communication. Thus, mathematics is a form of discourse. As Rogoff (1990) points out, young children come to their conceptual development as a result of social interactions with significant others. Thus, mathematics learning is the development of discourse as it evolves with those who are more knowledgeable about mathematics than itself.

The ways that students take up classroom or disciplinary discourses are shaped by the everyday discourses they bring to classroom (Moje et al., 2001). Thus, the use of a given concept in everyday language can have a critical impact upon students' future conceptual frameworks in mathematics. Many mathematical meanings are often selected by students arbitrarily from among many potential natural language interpretations (Epp, 2003). Therefore, the everyday discourses that surround students and those in which they participate can influence the way in which the concepts of infinity and limit are understood in the classroom. Knowing the cultural background of the students can help in designing situations in which concepts that have been historically difficult may be easier to understand for an individual in the classroom. For instances, experiences with technology extend our counting system and the speed of calculation. The pictures of fractal mathematics in computer may influence students' conceptions of infinity and limit. Thus, students' everyday discourses of infinity and limit in their cultures are crucial for their understanding of infinity and limit in mathematical contexts. Confusion between the way in which infinity and limit are used in everyday discourse and the way in which they are understood in mathematical discourse can be one of the main difficulties in acquiring the concept of infinity and limit.

The specific reason for the selection of native-English and native-Korean speakers is the discontinuity in Korean and the continuity in English between the colloquial and mathematical discourses on infinity and limit. In English, the words infinity and limit appear in colloquial language and students are likely to be familiar with the terms before they encounter them in a formal mathematical context. In contrast, the formal mathematical Korean words for infinity and limit learned in school or university do not originate in Korean colloquial discourse. This study is only a pilot for a future comprehensive project to understand students' colloquial and mathematical discourses of the words *infinity* and *limit* in English and Korean.

Ⅲ. Methodology

A. Research question

The purpose of this study, to characterize how students think about infinity and limit, led to the following research question: How does colloquial discourse correlate with mathematical discourse on infinity and limit in regards to an American and a Korean student?

B. Population

Two undergraduates, a U.S. American (U) and a Korean (K), were interviewed. The U.S. American (U) was an English speaker from the United States while the Korean (K) was a non-native English speaker from South Korea whose first language was therefore Korean. U was a 20-year-old sophomore who was majoring Pre-medication and taking the applied calculus course at a university. K was a 21-year-old sophomore enrolled at another university. She was not taking a calculus course.

C. Material

The Questionnaire for interview consists of eight categories (See Appendix). The first category of eleven problems and the second one of seven problems are intended for scrutinizing students' colloquial discourses on infinity and limit. In the first category, students are asked to create a sentence for a given word while they need to say the same thing without using the underlined word in the second category. The other words such as small, triangle and more in the first and second categories are intentionally embedded to look into students' colloquial discourses on infinity and limit more thoroughly. If students would think a sentence only in a mathematical context, the sentence is not appropriate to investigate their colloquial discourses. The third and fourth categories are directed at examining students' expressions on infinity by comparing two sets and counting the equivalent fractions. The fifth and sixth categories are intended to explore students' discourses on limit. The seventh and eighth categories aim for investigating students' mathematical discourses on both infinity and limit.

D. Procedures

The interview for each student was conducted for more than twenty minutes. The two interviews were conducted in the participants' first languages, and were based on 29 questions in the Questionnaire. Thus, the problem-set for the Korean student was translated into Korean by two Korean doctoral students. For each student to concentrate on a given problem, one small card for each problem was made and shown during the interview. For instance, the problems in the first category were given to students as a card with only one word written on it such as small, large, and others. All interviews were audio-taped, video-taped, and transcribed for further analysis. The interview for the American (U) took place in the interviewer's university office and the other interview (K) in the interviewer's home. Students' discourses were analyzed based on the transcripts made by using audio and video recordings.

E. Methods of analysis

Data were analyzed based on three distinctive features of mathematical discourses: mathematical uses of words, discursive routines, and endorsed narratives, to understand the impact of colloquial discourse to mathematical discourse on infinity and limit (Ben-Yehuda et al., 2005). Uses of words implies how do the participants use keywords of colloquial and mathematical discourse regarding infinity and limit. Discursive routines are the patterns of repetitive actions in students' discourses. Endorsed narratives are propositions that are accepted as facts in mathematics. Based on these three distinctive aspects, the degree of objectification in each student's discourse was also discussed.

Ⅳ. Selected Findings

A. Key words and their use

The American(U)'s keywords in her colloquial discourse on infinity are unlimited, everlasting, and illimitless, based on her six expressions. She probably can use illimitable instead of illimitless because there is no such word. K's keywords in her colloquial discourse on infinity are unlimited, uncountable, and unimaginable, based on her six responses. Although K didn't use unlimited directly, unlimited can be a keyword because she expressed that limited and infinite are opposite. Their responses are summarized in <Table 1>.

Although both U and K demonstrated a concept of unlimited in their uses of infinity, two differences were found between their colloquial discourses. First of all, U shows the countable and continuous processes without end in her expressions of infinity, while K expresses uncountable. Secondly, U expresses a more concrete conception of no limit in counting, while K shows a more abstract understanding of unimaginable number. These two differences can be summarized as endless counting processes in U's colloquial discourse and uncountable and unimaginable abstract in K's colloquial discourse.

Table 1. Students' colloquial discourses on infinity

Problems	Expressions	
	The American (U)	The Korean (K)
10	There are *infinite* ways if you think and say one thing	The limited one and the *infinite (mu-han-han)* one are opposite
11	I will never count to *infinity*	Do you want to be beaten by *infinity (mu-han-dae)*
14	There are *way* too many lawyers	There are *unlimitedly or uncountably (cel-soo-up-see)* too many lawyers
15	He has *unlimited* potential	He has *Unimaginable (sang-sang-hal-soo-up-neon)* potential
16	My love for you is *everlasting*	My love for you is *unimaginable*
17	*The quantity* of $\frac{1}{\chi}$ is 0 as x approaches *illimitless amount*	*The last number (ma-je-mak soo)* of $\frac{1}{\chi}$ is 0 as x approaches *unimaginable number*

We can easily see that their uses in colloquial discourse correlate with their mathematical definition based on the above characteristics. In the last problem, they defined infinity and limit. U defined infinity as follows in her mathematical discourse:

U: Infinity is just increasing my number every time. There is *an infinite amount of* numbers that we can utilize. I *don't* think infinity is *an object* as a number.

Here, U explicitly speaks of "an infinite amount of number" and her claim that infinity is not an object but a process in her mathematical definition. As we look at endless counting processes in her colloquial discourse, those conceptions are embedded in her mathematical definition as an infinite amount of numbers as processes. K also shows the close correlations between colloquial discourse and mathematical discourse. She defined infinity as follows:

K: What are *uncountable*. The infinite thing…

Here, K also explicitly mentions "what are uncountable" and something infinite in her mathematical definition. As we consider uncountable and unimaginable as abstract characteristics of her colloquial discourse, those aspects are also imbedded in her mathematical definition of what are uncountable and the abstract infinite thing.

U's keywords in her colloquial discourses on limit are poor and quantity based on her four expressions. K's keywords in her colloquial discourses on limit are low and the last number based on her three answers (See <Table 2>). She gave only three responses because she gave up one of the four questions after struggling for around 30 seconds.

Table 2. Students' colloquial discourses on limit

Problems	Expressions	
	The American (U)	The Korean (K)
7	My knowledge is limited in many areas of science	There is a limited amount here
8	There is no limit to what someone can do	No answer
13	Eyeglasses are for people with poor eyesight	Eyeglasses are for people with low eyesight
17	The quantity of $\frac{1}{\chi}$ is 0 as x approaches illimitless amount	The last number of $\frac{1}{\chi}$ is 0 as x approaches unimaginable number

Although the correlations between colloquial discourse and mathematical discourse on limit are not clear at first sight, if we would look thoroughly into their definitions of infinity and limit together, we can see some relationship because limits are tightly related to the notions of infinity. They defined limit as follows in their mathematical discourse:

U: Limit is *where I am only going to go to* as I keep going up and up my number. Limit is *the number* that function can go to.

K: Limit is what we *can reach* really *in the last*. Limit says the limitation of power.

Here, U overtly mentions "where I am only going to go to" and her claim that limit is the number. K also explicitly speaks of "what we can reach in the last". We can see three differences in their definitions. One of them is that there are ongoing processes in U's definition while there is a conception of being able to reach in the last in K's definition. Another difference is that there is a uni-directionality in U's definition that is not for K's because U used "up and up" and K made use of nothing about direction. The other is that U relates limit to function, while K connects it with a non-school context like power. We can see the first difference in their colloquial discourses on limit and infinity. The quantity in U's keywords in her colloquial discourse on limit is embedded in her mathematical discourse as number. It seems that everlasting processes in her colloquial and mathematical discourse on infinity are implanted into her mathematical discourse on limit as ongoing processes. The last number in K's keywords in her colloquial discourse seems to be interpreted in her mathematical definition as "in the last". It seems that there is no correlation between K's two definitions: one is what we can reach in the last and the other is limitation of power. It may be a confusion between the way in which limit are used in her everyday discourse and the way in which it is understood in mathematical discourse.

B. Routines

When U and K compared two infinite sets in problems 20, 21, and 22, they showed different patterns of repetitive actions in their discourses. When U compares two infinite sets, she considers one-to-one correspondence for her justification. For instance, when she compared odd numbers and even, she explained that they are the same because every other number's either odd or even (See <Table 3>). She is looking at the matching processes between odd numbers and even. U' routines in comparing two infinite sets can be closely related to her colloquial and mathematical discourses on infinity as endless processes.

Table 3. Routines in comparison

Problems	Expressions	
	The American (U)	The Korean (K)
Odd & even number	Every other number's either odd or even	If you would count from 1 to 10, there are five odd number and five even numbers
Grains of sand & the sky	It's never end. I have no idea	Because the whole is size of the sky and there are grains of sand only in some places
Odd number & Integers	Because odd numbers can be integers …	Because odd numbers are part …

When K tried to solve the same problems, she justified her answers with a part-whole conception. For instance, when she compared odd numbers and integers, she explained that integers are bigger than odd numbers because odd numbers are part of integers. She also pointed out that the size of the sky is bigger than the grains of sand in the world because the whole is the size of the sky and there are grains of sand only in some places

(See <Table 3>). The whole in her explanation seems to be the whole in the world. The conception of uncountable in her colloquial and mathematical discourses on infinity may influence the way in which she considered a finite case in comparing two infinite sets.

When U and K tried to find the limit of each function as x goes to infinity, they also demonstrated different patterns in their discourses. After finding the limit, U looked at infinite processes in the processes of limit for her justification. For instance, when she tried to find the limit of $\frac{\chi^2}{1+\chi}$, she explained that her limit is infinity because as she increases her number, her number is going to be bigger (See <Table 4>). The first her number means is x value and the second one is the value of function $\frac{\chi^2}{1+\chi}$. We can see a relationship between her colloquial and mathematical discourses on limit and her routines in finding the limit of a given function as ongoing infinite processes.

K's justifications are different from U's. She focused on a finite case in the processes of limit. For instance, she explained that the numerator value is bigger than the denominator value and the numerator value is less than the denominator value respectively in justifying the limits of $\frac{\chi^2}{1+\chi}$ and $\frac{\chi^2}{(1+\chi)^2}$ as zero and infinity (See <Table 4>). The abstract conception of uncountable and the last number in her colloquial and mathematical discourses on infinity and limit may also make her focus on a finite case in the processes of limit.

Table 4. Routines in limit-finding tasks

Problems	Expressions	
	The American (U)	The Korean (K)
$\frac{1}{\chi}$	I am gonna get smaller smaller number but I am never gonna get a negative number. So, zero is my imit. It's not [murmuring] looking like a limit.	It's decreasing and zero.
$\frac{\chi^2}{1+\chi}$	My limit is infinity because as I increase my number, my number is going to be bigger.	Infinity. Because the above is a square and the numerator value is bigger than the denominator value.
$\frac{\chi^2}{(1+\chi)^2}$	Zero. My number value bigger number every time. It's not gonna be the same as top.	It's gonna be zero because the numerator value is less than the denominator value.

C. Endorsed narratives

Endorsed narratives can be seen in U's and K's mathematical discourses on limit and infinity. In U's definition of infinity, infinity is an infinite amount of numbers as processes. In U's description, limit can be interpreted as ongoing processes. Infinity is what are uncountable and the abstract infinite thing in K's explanation. Limit is what we can reach really in the last to K. In terms of substantiating their definitions, there is a difference between U and K. U showed a way of substantiating in translating her definitions and understanding in colloquial and mathematical discourses into problems in comparing two infinite sets and finding the limit of a given function. In other words, there is a close relationship between uses of keywords and routines in her colloquial and mathematical discourses. However, K didn't show any evidence in translating from conceptions in her discourses to problem solving. Although K showed a close relationship between colloquial discourse and mathematical discourse on infinity and limit, there is no clear connection between uses of keywords and routines in her colloquial and mathematical discourses.

D. Objectification

The use of a noun counts as objectified if this noun is applied as if it refers to a self-sustained, discourse-independent entity. In U's colloquial and mathematical discourses on infinity, endless processes can be considered as a characteristic. Uncountable imagination can be thought of as an important property in K's colloquial and mathematical discourses on infinity. U's conception of infinity is more objectified than K's because U's conception can be used more concretely and precisely in understanding, problem solving with and applying infinity. While U used one-to-one correspondence in comparing two infinite sets, K used a part-whole conception. One-to-one correspondence is more objectified than a part-whole conception because the former is more generalized and can be used in the finite and infinite sets. Process could be seen in U's discourses on infinity and limit while K showed a finite conception in her discourses on infinity and limit. One of the important cognitive transitions from finite to infinite can be to understand appropriate processes as infinite.

Ⅴ. Discussion

A. Relationship between colloquial discourse and mathematical discourse

Although we can see clearly a deep relationship between two students' colloquial discourse and mathematical discourse, we can not say that only colloquial discourse influences mathematical discourse. Because understanding is a continuing process of organizing one's knowledge structures (von Glasersfeld, 1987), it's likely that colloquial discourse and mathematical discourse interconnect and influence each other. Therefore, the impact of colloquial discourse on mathematical discourse and the influence of mathematical discourse on colloquial discourse are likely to be interwoven in a continuing process of student understanding.

B. Difficulty of translation

One of the greatest difficulties in scrutinizing a Korean student's colloquial discourse is translation because English and Korean have such different language structures. Another is the relationship between a word in Korean and its counterpart in English. For instance, there is only one word for triangle in English. However, there are two words for representing triangle in Korean. One is usually used in colloquial discourse while the other is usually reserved for mathematical discourse. When K tried to say the same thing of "some arrow points have triangular forms" without using "triangular" in problem12, she just used triangular in colloquial discourse instead of using triangular in mathematical discourse.

Ⅵ. Conclusion

A more concrete conception as endless counting processes in U's colloquial discourse on infinity has influence on her mathematical discourse on infinity and limit. In addition, U's routines in infinity and limit are closely related to her colloquial and mathematical discourses on infinity and limit. The concrete conception as everlasting counting processes in U's colloquial discourse can impact on the degree of objectification in terms of substantiating her definitions in understanding, problem solving with and applying infinity and limit.

A more intangible conception as uncountable and unimaginable abstract in K's colloquial discourse on infinity correlates with her mathematical discourse on infinity. Although there is no correlation between K's two definitions, there is a close relationship between her colloquial and mathematical discourses on limit. It may be a confusion between the way in which limit are used in her everyday discourse and the way in which it is understood in mathematical discourse. The abstract conception of uncountable and the last number in her colloquial and mathematical discourses on infinity and limit may influence the way in which she considered a finite case in comparing two infinite cases and finding the limit of a given function.

Although the sample size of the current study is too small to allow for generalization, what was found in this study may serve as a basis for new hypotheses to be tested in a larger-scale project in the future. According to the result of the current study, colloquial discourse seems to correlate with mathematical discourse because of certain clear correlations between the colloquial and mathematical discourses on infinity and limit of the American and Korean students.

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Appendix : Questionnaire for interview

I. Create a sentence with the following word (term).

1. Small
2. Large
3. 158
4. – 2
5. Triangle
6. Triangular
7. Limited
8. Limit
9. Diagonal
10. Infinite
11. Infinity

II. Say the same thing without using the underlined word.

12. Some arrow points have **triangular** forms.
13. Eyeglasses are for people with **limited** eyesight.
14. There are **infinitely** too many lawyers.
15. He has **infinite** potential.
16. My love for you is **infinite**.
17. The **limit** of $\frac{1}{\chi}$ is 0 as x approaches **infinity**.
18. In box A, there are **more** matches than in box B.

III. Which is a greater amount and how do you know?

19. A: Your fingers B: Your toes
20. A: Odd numbers B: Even numbers
21. A: Grains of sand in the world B: Size of the sky
22. A: Odd numbers B: Integers

IV. $\frac{1}{4} = 0.25,\ \frac{2}{8} = 0.25,\ \frac{3}{12} = 0.25,\ \cdots$

How many such equalities can you write?

V. What do you think will happen later in this table? How do you know?

χ	$\frac{\sqrt{\chi+25}-5}{\chi}$
1.0	0.099020
0.5	0.099505
0.1	0.099900
0.05	0.099950
0.01	0.099990
0.005	0.099995
0.001	0.099999

VI. What is the limit of the following when x goes to infinity?

23. $\frac{1}{\chi}$

24. $\frac{\chi^2}{1+\chi}$

25. $\frac{\chi^2}{(1+\chi)^2}$

VII. Read aloud

26. $\lim_{\chi\to\infty}\frac{\chi^2+3\chi}{2\chi^2}=\frac{1}{2}$. Explain what it says.

VIII. What is limit?

What is infinity?

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