

Decomposing G7 Business Cycle

Sungjin Cho · Jangryoul Kim

In this paper, we have estimated a model that incorporates two key features of business cycles, co-movement among economic variables and switching between regimes of expansion and recession, to aggregate quarterly data for the G7 countries. Two common factors, interpreted as reflecting the permanent and transitory components of the business cycle in the region, and estimates of turning points from one regime to the other were extracted from the data by using the Kalman filter and maximum likelihood estimation approach of Kim(1994). Estimation results confirm a fairly typical stylized fact of business cycles - recessions are steeper and shorter than recoveries, and both co-movement and regime switching are found to be important features of the business cycle in those countries as a whole. The two common factors produce sensible representations of the trend and cycle, and the estimated turning points agree quite well with independently determined chronologies. It also turns out that the degree of synchronization between the G7 and the Korean economy has significantly increased after the Asian currency crisis of 1997.

Keywords: Business cycles, Asymmetry, Co-movement, Permanent and transitory components

1. Introduction

Two main features of business cycle, i.e., co-movement among economic variables through the cycle and asymmetry in the evolution of the cycle, have been extensively examined in literature since the work of Burns and Mitchell(1946). Recent development in econometric methodology has prompted a resurgence of interest in and explicit analysis of those features by modern time series techniques.

With a view to capturing the asymmetric nature of business cycle, two types of models were proposed. The first kind, pioneered by Hamilton(1989), divides the business cycle into two phases, negative trend growth and positive trend growth, between which the stochastic trend output switches according to a latent state variable. This two phase business cycle

implies that, since the regime switch occurs in the growth rate of the trend or permanent component of output, a negative state results in an output loss that is permanent, even if output switches back to the expansion growth phase. The second kind, having its roots in the work of Friedman(1964, 1993) and formally developed by Kim and Nelson(1999), characterizes recessions as a temporary “pluck” down of output by large negative transitory shocks. That being the case, recessions are entirely transitory deviations from trend, not movements in the trend itself, and the resulting output loss is temporary.

The other regularity of business cycle, co-movement, has also been formally investigated since the work of Stock and Watson(1991). The basic insight of the dynamic factor model proposed by Stock and Watson is that the business cycles are measured by a common unobserved factor extracted from key variables reflecting economic activity. By employing four monthly coincident indicator series used to construct the Department of Commerce(DOC) composite index, they show that the common factor implied by the model corresponds closely to the DOC index.

In more recent studies, the two features of business cycle are analyzed simultaneously since the work of Diebold and Rudebusch(1996). They provide empirical and theoretical support for co-movement and asymmetry as important features of business cycle and suggest that the two features should be analyzed simultaneously. Along this line of research, many regime switching common factor models are developed, such as in Chauvet(1998), Kim and Yoo(1995), and Kim and Nelson(1998) to name a few. In those “synthetic” models, the common factor is defined as an unobserved variable that summarizes the common cyclical movements of a set of coincident macroeconomic variables, as in Stock and Watson(1991). Meanwhile, it is also subject to discrete shifts so that it can capture the asymmetric nature of business cycle phases, as in Hamilton(1989).

Despite the success in obtaining much sharper inferences on the state of the economy, most of the literature employing synthetic model focuses on either the Hamilton or Friedman types of asymmetry. Two exceptions would be Kim and Murray(2002) and Cerra and Saxena(2003), who incorporate both types of asymmetry by subjecting both the common permanent and transitory factors to respective regime switching.⁽¹⁾ Using US monthly data,

(1) Kim and Piger(2002) also consider the two types of asymmetry, but they assume that the regime shifts in permanent and transitory factors are governed by a single latent variable.

Kim and Murray(2002) find that most of variations of the US coincident variables during recessions is due to the common transitory components. Cerra and Saxena(2003) also introduce two factors and examine by how much the output of six Asian countries recovered from the currency crisis in 1997.

In this paper, we construct a model that incorporates the following three features: co-movement among economic variables, switching between regimes of booms and slump, and the recessions of both permanent and transitory nature. Most previous research using the synthetic models has typically used data from the US, and few studies of other economies have been undertaken.⁽²⁾ In view of the economic and financial turmoil the G7 countries are undergoing, it is worthwhile to understand the nature of business cycle in those countries. We estimate, therefore, the model to quarterly data of the whole G7 countries, and examine the relative importance of the permanent and transitory components of the business cycle in those countries as a whole. Estimation results confirm a fairly typical stylized fact of business cycles - recessions are steeper and shorter than recoveries, and both co-movement and regime switching are found to be important features of the business cycle in the region. The two common factors produce sensible representations of the trend and cycle, and the estimated turning points agree well with independently determined chronologies.

This paper is organized as follows. Section 2 presents the regime-switching dynamic factor model, which allows for a common peak-reverting component that switched independently of the common growth component. Section 3 discusses the empirical results. Finally, section 4 offers concluding remarks.

2. Model Specification

We follow the dynamic factor model developed by Kim and Murray(2002) and applied to Asian countries by Cerra and Saxena(2003). We assume that each individual time series Y_{it} , for $i = 1, \dots, N$ could be represented as

(2) Mills and Wang(2003) examined the asymmetry of the UK business cycle, but they introduce only the common permanent factor which is subject to the Hamilton-type asymmetry.

$$(2.1) \quad Y_{it} = \gamma_i C_t + v_{it} + \lambda_i x_t + \omega_{it}$$

where C_t and x_t are common permanent and transitory components, v_{it} and ω_{it} are idiosyncratic permanent and transitory innovation terms, (γ_i, λ_i) are factor loadings for permanent and transitory factors, respectively.

Following Kim and Nelson(1999) and Kim and Murray(2002), we difference the variables to handle the integration problem of the observed series and write the model in the following differenced mean-deviation form:

$$(2.2) \quad y_{it} = \gamma_i \Delta c_t + \lambda_i \Delta x_t + \eta_{it}$$

$$(2.3) \quad \Delta c_t = \phi \Delta c_{t-1} + \mu_0 + \mu_1 S_{1t} + v_t, \quad \mu_0 > 0, \quad \mu_1 < 0$$

$$(2.4) \quad x_t = \psi_1 x_{t-1} + \psi_2 x_{t-2} + \tau S_{2t} + u_t, \quad \tau < 0$$

$$(2.5) \quad \eta_{it} = \psi_{i1} \eta_{i,t-1} + \psi_{i2} \eta_{i,t-2} + e_{it}$$

where c_t is the demeaned common permanent factor. We put $v_t \sim iidN(0, \sigma_v^2)$, $u_t \sim iidN(0, \sigma_u^2(S_{2t}))$, and $e_{it} \sim iidN(0, \sigma_i)$. Note that the transitory innovation u_t is allowed to be state-dependent, i.e., $\sigma_u^2(S_{2t}) = \sigma_{u0}^2(1 - S_{2t}) + \sigma_{u1}^2 S_{2t}$.⁽³⁾ Equation (2.3) reproduces Hamilton(1989) regime switching of the common permanent component in which μ_0 determines the growth rate of the permanent component during expansion (i.e., $S_{1t} = 0$) and $\mu_0 + \mu_1$ determines the growth rate of the permanent component during contraction (i.e., $S_{1t} = 1$). Equation (2.4) models the Friedman(1964, 1993) regime switching of the common transitory component whose mean is zero during expansion (i.e., $S_{2t} = 0$) and negative ($\tau < 0$) during contraction (i.e., $S_{2t} = 1$). The latent variables S_{1t} and S_{2t} follow mutually independent two-state Markov processes, taking on value zero in expansion and one in contraction. Transition probability matrices for S_{1t} and S_{2t} are P_1 and P_2 , respectively, given by:

(3) Variance of permanent innovations (σ_v^2) and that of transitory innovations in the contraction regime (σ_{u2}^2) are fixed at one so that the factor loading coefficients are identified.

$$(2.6) \quad P_1 = \begin{bmatrix} p_{00} & 1-p_{11} \\ 1-p_{00} & p_{11} \end{bmatrix}, \quad P_2 = \begin{bmatrix} q_{00} & 1-q_{11} \\ 1-q_{00} & q_{11} \end{bmatrix}.$$

The assumption that the common idiosyncratic factor η_{it} follows an AR(2) process in equation (2.5) is the same as in Cerra and Saxena(2003).⁽⁴⁾ In determining the AR lag lengths for two common factors, we then follow Kim and Murray(2002) to consider four cases in which each factor is specified as either AR(1) or AR(2). After running a few diagnostic checks, we settled with the specification above.

The model comprising equations (2.1)–(2.6) allows us to investigate the role of the permanent and transitory components as well as idiosyncratic shocks over the business cycle. To estimate the model parameters as well as the unobserved components, we cast the model into a state-space representation with Markov-switching and use the approximate maximum likelihood algorithm in Kim(1994). Appendix presents the detailed description of the state representation of our model.

3. Empirical Results

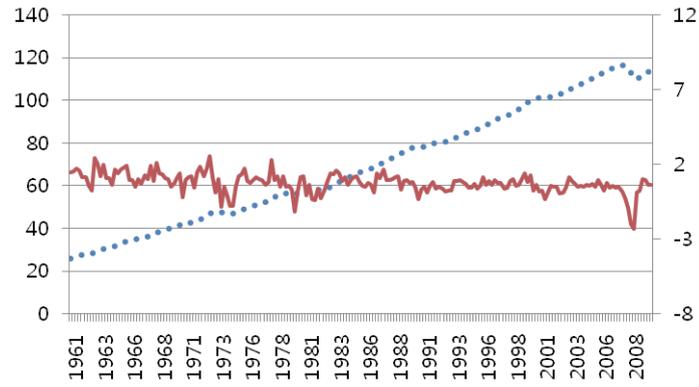
3.1. Data and Estimation

We use three quarterly time series on output, consumption, and investment, expecting that they are representative of aggregate economic conditions in the G7 countries. These series are real GDP, real private final consumption, and gross private fixed capital formations.⁽⁵⁾ All series are seasonally adjusted volumes indexes with the year 2000 as the base year. The sample period is from 1960Q1 to 2010Q4. Graphs of the three series are shown in <Figure 1>, where the dotted lines represent the log-levels of variables and the solid lined are for the log-differenced multiplied by 100.

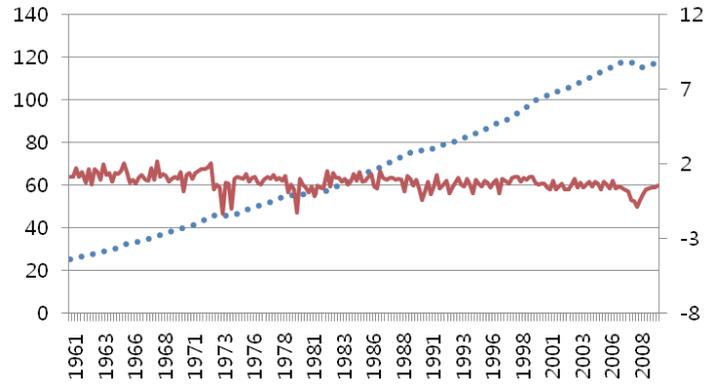
We need to check if the three series are individually integrated. Although we do not report the details here, the test results provide strong evidence that each series contains a unit root.

(4) We also used the AR(1) specification for idiosyncratic factors, but the results were qualitatively the same.

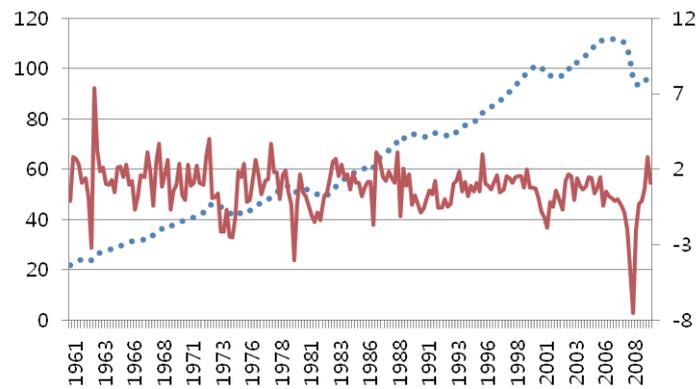
(5) Data series were obtained from the quarterly national account section of the OECD statistical database. The exact link is: <http://stats.oecd.org/Index.aspx?DataSetCode=QNA>.



(a) GDP



(b) Consumption



(c) Investment

<Figure 1> Time Series of the Three Variables

<Table 1> Estimates of Dynamic Factor Model

common permanent factor						
φ	μ_0	μ_1	p_{00}	p_{11}	σ_v^2	
0.969	1.197	-2.923	0.954	0.960	1	
(0.001)	(0.105)	(0.175)	(0.003)	(0.002)	-	
common transitory factor						
ψ_1	ψ_2	τ	q_{00}	q_{11}	σ_{u0}^2	σ_{u1}^2
1.412	-0.480	-1.191	0.923	0.734	1	3.302
(0.042)	(0.036)	(0.313)	(0.024)	(0.077)	-	(0.114)
idiosyncratic component						
	ψ_{i1}	ψ_{i2}	γ_i	λ_i	σ_i^2	
Δy_{1t}	-0.301	0.384	0.012	0.248	0.134	
	(0.149)	(0.142)	(0.001)	(0.011)	(0.016)	
Δy_{2t}	-0.139	-0.086	0.012	0.145	0.314	
	(0.038)	(0.039)	(0.001)	(0.008)	(0.008)	
Δy_{3t}	-0.156	-0.125	0.019	0.559	0.760	
	(0.041)	(0.053)	(0.002)	(0.025)	(0.026)	
Log-likelihood value		-430.399				

Note: The order of the variables in y_{it} is GDP, private consumption, and investment. Standard errors are in parentheses.

Therefore, we use the first differences of the variables in logs (multiplied by one hundred) as is implied by the model set out in Equations (2.1) to (2.6). Also, as in the model, all series are demeaned by subtracting the sample mean from each difference.

Standard stochastic growth models with capital accumulation imply that output, consumption, and investment share a common stochastic trend. Based on this insight along with the permanent income hypothesis, Kim and Piger(2002) identify consumption with the pure stochastic trend (without transitory variations) and impose the cointegrating restrictions among the three variables. In this paper, we follow Cerra and Saxena(2003) and do not impose this restriction in order to allow for possible liquidity constraints that would render at least some fraction of consumption dependent upon transitory income.

The estimation results are presented in <Table 1>. The estimated model seems successful in

extracting information about fluctuations in economic activity. The results support the presence of asymmetric business cycles that switch between two different states. For the common permanent component, the state 0 has positive and significant mean while the state 1 has a significantly negative mean. Similar results are obtained for the common transitory factor, in that the state 0 has mean zero and the state 1 has significantly negative mean, respectively. These results imply that both the Friedman and Hamilton type regime switches are important in explaining the business cycles in the data we consider.

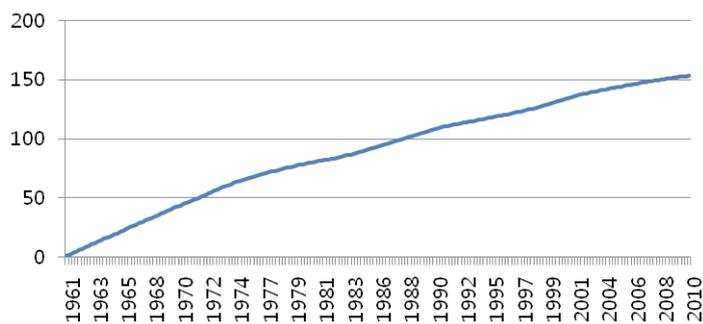
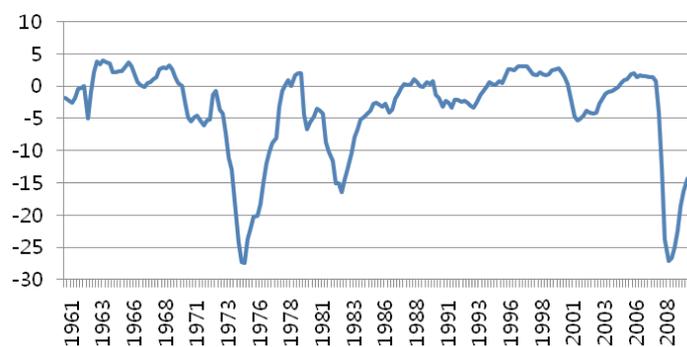
The transition probabilities associated with the expansion and contraction regimes of the common permanent factor are 0.954 and 0.960, respectively, which imply that the average durations of the two regimes for the permanent component are 4-5 years. For the common transitory factor, however, the results are quite different: the average duration of expansion phase is about 13 quarters, while that of contraction regime is less than one year. Overall, the estimates of transition probabilities and state-dependent means imply the following: over the business cycle, contractions are on average both steeper and shorter than expansions, which is consistent with the findings in the literature (e.g., Kim and Nelson(1998) for the US and Mills and Wang(2003) for the UK).⁽⁶⁾ However, the switches between regimes for the permanent factor are infrequent.

Moving to the bottom panel of <Table 1>, the negative AR coefficients for investment and consumption indicate that the idiosyncratic components of these series exhibit negative serial correlation, while the GDP series behaves differently with positive idiosyncratic autocorrelation (i.e., $\psi_{131} + \psi_{12} > 0$) with a lag. Estimates of factor loadings show that all three variables are pro-cyclical with positive factor loadings for both the permanent and transitory components, in agreement with conventional views of the business cycle. Of the three series, investment has the highest weighting on the two common factors, suggesting that this series is the most sensitive coincident variable.

If the factor loadings for the transitory component are all zero, our model collapses to a dynamic factor model which relegates all variations in data to a regime-switching common growth component. As we cannot test the joint hypothesis that these transitory factor loadings are all zero due to the non-standard nature of the problem,⁽⁷⁾ we test whether the factor

(6) Note that $q_{00} > q_{11}$ and $|\tau| > 0$ for the transitory component.

(7) Under the null hypothesis that $\lambda_i = 0$ for all i , the parameters associated with common transitory

(a) Common Permanent Factor, C_t (b) Growth Rate of Common Permanent Factor, Δc_t (c) Common Transitory Factor, x_t **<Figure 2>** Extracted Common Permanent Factor

component are not identified. Since the distribution of the test statistic in the presence of such nuisance parameters that exist only under the alternative hypothesis is unknown for the state space model we are dealing with, we test instead the individual hypothesis that $\lambda_i = 0$ for *one* i .

loadings for the transitory component are individually significant. The asymptotic t-ratios for these parameters indicate that they are all individually significant at the 1% level. This confirms that the common transitory factor should not be ignored in explaining the data.⁽⁸⁾

Figure 2(a) and 2(b) plot the extracted common permanent factor in levels and first differences, respectively.⁽⁹⁾ We argue that the permanent factor series reflects the movements of potential GDP (up to the factor loading). That being the case, the panel 2(b) exhibits four periods of major slowdowns: the two oil crises in 1973 and 1979, the 1999-2001 recession, and the subprime financial crisis in late 2007, during which the growth rate of permanent factor sharply decreases. Other findings in panel 2(b) are the secular slowdown in potential growth over the 70s and the stagnation of potential growth since then, except the late 90s in the midst of the ‘new economy.’

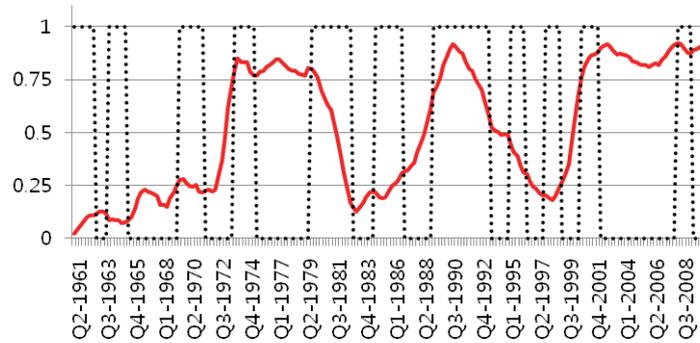
Figure 2(c) plots the extracted Markov switching common transitory factor, which can be interpreted as cyclical or trend-deviating component of the business cycle. This series clearly shows the high volatility of the 1980s and the relative stability of the 1990s, reflecting ‘the Great Moderation’ in the 1990s. Also, the plots for the transitory factor identify a brief recession around 1982. One possibly important finding is that a considerable, if not all, fraction of the impact on the G7 economy caused by the recent subprime crisis is transitory pluck-downs: the magnitude of the pluck-down in the transitory component is much larger than that in the permanent component, and all the more so if we consider the two factor loadings for GDP.

One way to measure the importance of the two common factors is to compare their ability to mimic actual business cycle peak-trough dates. <Figure 3> plots the filtered probabilities that the economy is in recessionary regimes, along with the actual business cycle chronology over the sample period.⁽¹⁰⁾ Panel (a) shows the recession probability $\Pr[S_{jt} = 1 | \Omega_T]$ or the

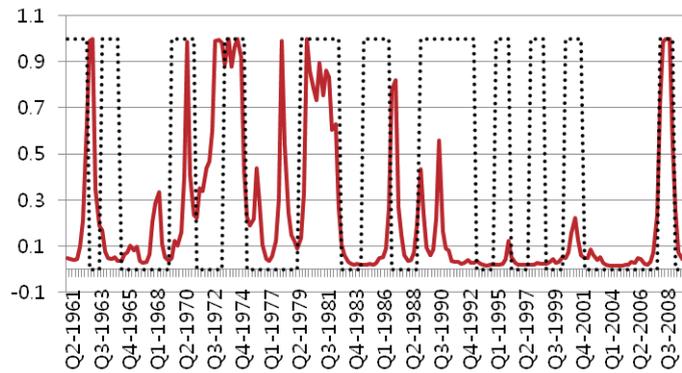
(8) One interesting finding is that the loading of transitory factor for consumption is significantly positive, albeit small, in contrast to the prediction of the permanent income hypothesis. As stated earlier, we view this as suggesting that liquidity constraints render at least some fraction of consumption dependent upon transitory income.

(9) Stock and Watson(1991) and Kim and Murray(2002) discuss details on how to construct the level of the common permanent factor.

(10) The contraction dates are: 60:Q3-63:Q1, 64:Q1-65:Q2, 69:Q3-71:Q2, 73:Q4-75:Q2, 79:Q2-82:



(a) Contraction in the Permanent Factor



(b) Contraction in the Transitory Factor

<Figure 3> Smoothed Probabilities of Contraction

permanent factor, while panel (b) plots that for the transitory factor. Overall, the plots show more frequent pluck-downs in the transitory factor than in the permanent factor: if we use the ‘0.5 rule’ to determine whether the economy is in contraction,⁽¹¹⁾ the estimated probabilities for the permanent factor calls 3 recessions while the transitory component gives 7. The three recessions from the mid-90s to 2001 in the reference cycle, however, are detected by neither the estimated recession probabilities for the permanent nor those for the transitory factors.

For a more formal comparison of the two factors, we again employ the ‘0.5’ rule to declare

Q4, 84:Q4-86:Q4, 89:Q2-93:Q3, 95:Q2-96:Q2, 98:Q1-99:Q1, 00:Q4-02:Q1.

(11) According to the ‘0.5 rule’, the economy is viewed in contraction if the smoothed probability of recession is greater than 0.5, i.e., $\Pr[S_{jt} = 1 \mid \Omega_T] > 0.5, j = 1, 2$.

model-determined recession periods and evaluate the resulting dating performance by the Quadratic Probability Score(QPS) of Diebold and Rudebusch(1989), defined as:

$$(3.1) \quad QPS = \frac{1}{T} \sum_t (\Pr[S_{jt} = 1 | \Omega_T] - D_t)^2, \quad j=1,2$$

where D_t is a (0,1) dummy for the OECD recession dating. The closer this measure is to zero, the better is the model fit to the business cycle chronology. We obtain a QPS score of 0.508 for the recession dates called by the permanent factor only, while the QPS for the transitory factor is much lower 0.377, thus confirming the additional importance of transitory component identifying business cycle phases.

3.2. Synchronization with the Korean Business Cycle

In this subsection, we examine the degree of business cycle synchronization between the G7 and the Korean economy. We first obtain the permanent and cyclical components of the Korean business cycle.⁽¹²⁾ We then measure the degree of synchronization between the two economies. For each common factor, we calculate two measures of synchronization: the correlation coefficients between the estimated components of the two economies, and those between the smoothed probabilities of contraction. In so doing, we can check if the degree of synchronization measured by correlation coefficients shows any significant changes across the pre- and post the Asian currency crisis in 1997. <Table 2> below reports the results.

Strikingly, it turns out that the degree of synchronization is much higher in the post-1998 era than before. For instance, when we focus on the changes in the permanent component, the correlation between Korean and the G7 region over the whole and the pre-1998 period are mere 0.362 and 0.383, respectively. In the post-1998 era, however, the correlation coefficient

<Table 2> Business Cycle Synchronization with Korea

	Correlation between	whole period	pre-1998	post-1998
Trend	Δc_t	0.362	0.383	0.620
growth rate	$\text{Prob}\{S_{1t} = 1\}$	0.351	-0.169	0.591
Cyclical	x_t	-0.001	-0.132	0.570
fluctuation	$\text{Prob}\{S_{2t} = 1\}$	0.184	0.173	0.713

(12) See Kim(2011) for the details of the model and data used for the Korean economy.

increases to 0.620, suggesting positive degree of synchronization after the Asian currency crisis. This finding is robust across all cases considered in <Table 2>, regardless of whether we calculate correlation between contraction probabilities or we focus on the synchronization of cyclical component. One conclusion is clear: the post-1998 increase in the business cycle synchronization between Korea and the G7 region, probably due to the structural reform pursued in Korea since then.

4. Conclusion

In this paper, we have estimated a model that incorporates two key features of business cycles, co-movement among economic variables and switching between regimes of expansion and recession, to aggregate quarterly data for the G7 countries. Two common factors, interpreted as reflecting the permanent and transitory components of the business cycle in the region, and estimates of turning points from one regime to the other were extracted from the data by using the Kalman filter and maximum likelihood estimation approach of Kim(1994).

Estimation results confirm a fairly typical stylized fact of business cycles - recessions are steeper and shorter than recoveries, and both co-movement and regime switching are found to be important features of the business cycle in the region. In particular, a considerable fraction of recessions are explained by movements in the common transitory component. As a whole, the two common factors combined produce sensible representations of the trend and cycle, and the estimated turning points agree well with independently determined chronologies. As an application of the results, we examine the degree of business cycle synchronization with Korea. It turns out that the degree of synchronization, measured by correlation coefficients between common components or contraction probabilities, has significantly increased since the Asian currency crisis.

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Appendix

The model in equations (2.2)-(2.6) can be expressed in state-space representation, comprising the measurement and transition equations as follows:

Measurement Equation

$$\begin{bmatrix} \Delta y_t \\ \Delta i_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & \lambda_1 & -\lambda_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & \lambda_2 & -\lambda_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_3 & \lambda_3 & -\lambda_3 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta n_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{1t-1} \\ z_{2t} \\ z_{2t-1} \\ z_{3t} \\ z_{3t-1} \end{bmatrix}$$

Transition Equation:

$$\begin{bmatrix} \Delta n_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{1t-1} \\ z_{2t} \\ z_{2t-1} \\ z_{3t} \\ z_{3t-1} \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 S_{1t} \\ \tau S_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_1 & \psi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_{11} & \psi_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_{21} & \psi_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{31} & \psi_{32} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta n_{t-1} \\ x_{t-1} \\ x_{t-2} \\ z_{1t-1} \\ z_{1t-2} \\ z_{2t-1} \\ z_{2t-2} \\ z_{3t-1} \\ z_{3t-2} \end{bmatrix} + \begin{bmatrix} v_t \\ u_t \\ 0 \\ e_{1t} \\ 0 \\ e_{2t} \\ 0 \\ e_{3t} \\ 0 \end{bmatrix}$$

Covariance Matrix of the Disturbance Vector:

$$E \begin{pmatrix} \begin{bmatrix} v_t \\ u_t \\ 0 \\ e_{1t} \\ 0 \\ e_{2t} \\ 0 \\ e_{3t} \\ 0 \end{bmatrix} \begin{bmatrix} v_t \\ u_t \\ 0 \\ e_{1t} \\ 0 \\ e_{2t} \\ 0 \\ e_{3t} \\ 0 \end{bmatrix}' \end{pmatrix} = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_u^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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