Random Beamforming in Spatially Correlated Multiuser MISO Channels

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Abstract - In this paper, we propose a new beamforming scheme that generates beams by exploiting the spatial correlation in multiuser environments. The proposed scheme generates random beams using eigenvectors corresponding to dominant eigenvalues of the channel covariance matrix, increasing the multiuser diversity gain. Moreover, it can obtain an additional SNR gain by transmitting the signal with a beam weight coherent to the effective channel corresponding to dominant eigenvectors of the channel covariance matrix of a selected user. Simulation results show that the proposed scheme is quite effective in correlated channel environments.

I. INTRODUCTION

The capacity of multiuser wireless systems has significantly been increased with the development of multi-antenna techniques [1]. Recently, it has been shown that dirty paper coding (DPC) achieves a capacity region of Gaussian multi-input multi-output (MIMO) broadcast channel [2]. Moreover, it can be implemented with much reduced computational complexity by means of suboptimum techniques [3], [4]. However, these schemes require full channel state information (CSI) of all users at the transmitter. In practice, however, such information may not easily be available at the transmitter due to limited capacity of the feedback signaling link, especially when the number of users is large.

The use of partial CSI has been considered to reduce the feedback signaling overhead. Opportunistic beamforming [5] is a multi-antenna technique that can work with partial CSI (e.g., effective signal-to-noise ratio (SNR)) in multiuser environments. The use of different random beams at each time makes the overall channel gain in fast and large fluctuation, enabling to achieve a large multiuser diversity (MUD) gain. Moreover, the opportunistic beamforming can provide a coherent beamforming configuration as the number of users increases to infinity. However, when the number of users is not sufficiently large, it may not be feasible to find a user matched to a randomly generated beam, suffering from noticeable performance degradation. This problem can be alleviated by employing a coherent opportunistic beamforming technique [6], where the BS transmits data to a selected user with a coherent beam weight rather than a randomly generated one. Although the coherent opportunistic beamforming achieves both the

beamforming and MUD gain [6], the use of coherent beam weight may increase the feedback signaling overhead.

Recent experimental results indicate that multi-antenna channels are often correlated in real environments [7]. In spatially correlated environments, most of the channel power is concentrated on a few eigen-dimensions of the channel covariance matrix. Exploiting this property, opportunistic eigen-beamforming schemes maximize the SNR in an average sense with a marginal feedback overhead [8]. However, as the spatial correlation decreases, the opportunistic eigen-beamforming may suffer from performance loss due to decreased channel gain of the principle eigenvector.

In this paper, we consider the generation of random beams using eigenvectors corresponding to the largest $L(\leq M)$ eigenvalues of the channel covariance matrix, where M is the number of transmit antennas. As L increases, the proposed scheme can increase the beamforming gain at the expense of MUD gain reduction, and vice versa. Considering this tradeoff, we analyze the performance in the presence of imperfect CSI due to the use of quantized beam weight and optimize the value of L for a given operation condition (e.g., the number of users and the spatial correlation).

The rest of the paper is organized as follows: The system model in consideration is described in Section II. The proposed scheme is described and analyzed considering the effect of beam weight quantization in Section III. The performance of the proposed scheme is verified by computer simulation in Section IV. Finally, conclusions are summarized in Section V.

II. SYSTEM MODEL

Consider a MISO downlink environment, where the BS has M transmit antennas and each of K users has a single receive antenna. For ease of description, we assume that users have a same average SNR of $\bar{\gamma}$ and experience independent fading. We also assume that each user can perfectly estimate the CSI and the BS can get the CSI from users through a feedback signaling link. Letting $\mathbf{x}$ be an M -dimensional transmit signal vector, the received signal of user k can be represented as

$$y_k = \mathbf{h}_k^* \mathbf{x} + n_k \quad (1)$$

where $\mathbf{h}_k$ denotes the M -dimensional channel vector of user k whose elements are zero mean complex Gaussian random variables with unit variance, but not independent, n_k is additive white Gaussian noise (AWGN) of user k , and the

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superscript $*$ denotes conjugate transpose. Assuming that the data symbol s is transmitted using an M -dimensional beam weight $\mathbf{w}$, the transmit signal $\mathbf{x}$ can be represented as

$$\mathbf{x} = \mathbf{w}s \quad (2)$$

where the Frobenius norm of the beam weight is equal to one (i.e., $\|\mathbf{w}\|=1$) and $E\{|s|^2\}=P$, resulting in an average transmit power of P .

The channel covariance matrix of user k defined by

$$\mathbf{R}_k \triangleq E\{\mathbf{h}_k \mathbf{h}_k^*\} \quad (3)$$

is a positive definite Hermitian matrix and thus can be decomposed as

$$\mathbf{R}_k = \mathbf{Q}_k \mathbf{\Sigma}_k^2 \mathbf{Q}_k^* \quad (4)$$

where $\mathbf{Q}_k = [\mathbf{q}_{k1} \cdots \mathbf{q}_{kM}]$ is an M -dimensional unitary matrix whose columns $\{\mathbf{q}_{ki}\}$ are eigenvectors of $\mathbf{R}_k$ and $\mathbf{\Sigma}_k$ is an M -dimensional diagonal matrix whose diagonal elements $\{\sigma_{k1}, \dots, \sigma_{kM}\}$ are descending-ordered non-negative real values, i.e., $\sigma_{k1} \geq \dots \geq \sigma_{kM} \geq 0$. Since each element of $\mathbf{h}_k$ has unit variance, it can be shown that $\sum_{i=1}^M \sigma_{ki}^2 = M$. The channel $\mathbf{h}_k$ can be represented as [9]

$$\mathbf{h}_k = \mathbf{R}_k^{1/2} \tilde{\mathbf{h}}_k = \mathbf{Q}_k \mathbf{\Sigma}_k \mathbf{Q}_k^* \tilde{\mathbf{h}}_k \quad (5)$$

where $\tilde{\mathbf{h}}_k = [\tilde{h}_{k1} \cdots \tilde{h}_{kM}]^T$ is an M -dimensional vector whose entries are independent and identically distributed (i.i.d.) zero mean complex Gaussian random variables with unit variance, $\mathbf{R}_k^{1/2}$ is the square-root of $\mathbf{R}_k$ (i.e., $\mathbf{R}_k^{1/2} = \mathbf{Q}_k \mathbf{\Sigma}_k \mathbf{Q}_k^*$), and the superscript T denotes transpose. Since the unitary transformation does not change the isotropicity of an i.i.d. channel, $\mathbf{Q}_k^* \tilde{\mathbf{h}}_k$ is identically distributed with $\tilde{\mathbf{h}}_k$. Thus (5) can be rewritten as

$$\mathbf{h}_k = \mathbf{Q}_k \mathbf{\Sigma}_k \tilde{\mathbf{h}}_k. \quad (6)$$

III. PROPOSED BEAMFORMING SCHEME

When the channel is correlated, most of the channel power is concentrated on a few eigenvalues of the spatial covariance matrix, say L eigenvalues. In this case, it may be sufficient to consider a subspace spanned by eigenvectors corresponding to such dominant eigenvalues. This correlation property is fully considered in the proposed scheme where each user calculates the SNR assuming the use of a beam weight randomly generated by eigenvectors corresponding to the largest L eigenvalues of its spatial covariance matrix and then reports it to the BS. The BS chooses a user based on the reported SNR and then the selected user sends its optimum beam weight generated by the same L eigenvectors to the BS. Finally, the BS transmits the data to the selected user with the reported beam weight.

The eigenspace corresponding to the largest L eigenvalues of user k can be represented by an $(M \times L)$ -dimensional matrix $\mathbf{Q}_{k,L}$ whose columns are the first L columns of $\mathbf{Q}_k$, i.e., $\mathbf{Q}_{k,L} = [\mathbf{q}_{k1} \cdots \mathbf{q}_{kL}]$. Note that $\mathbf{Q}_{k,L}$ maximizes the term $E\{\|\mathbf{h}_k^* \mathbf{Q}_{k,L}\|^2\}$ among all $(M \times L)$ -dimensional matrices with orthonormal columns, which is directly related to the beamforming gain of user k [12]. Let $\mathbf{h}_{k,L} (\triangleq \mathbf{Q}_{k,L}^* \mathbf{h}_k)$ be a dimension-reduced channel of user k . With the use of an L -dimensional random beam weight $\tilde{\mathbf{w}}_L$, the received signal of user k can be rewritten as

$$y_k = \mathbf{h}_{k,L}^* \tilde{\mathbf{w}}_L s + n_k. \quad (7)$$

The corresponding SNR $\gamma_{k,L}$ is given by

$$\gamma_{k,L} = \bar{\gamma} \|\mathbf{h}_{k,L}^* \tilde{\mathbf{w}}_L\|^2. \quad (8)$$

Assuming that the BS selects a user $\hat{k}$ having the maximum SNR as

$$\hat{k} = \arg \max_{k \in \{1, \dots, K\}} \bar{\gamma} \|\mathbf{h}_{k,L}^* \tilde{\mathbf{w}}_L\|^2, \quad (9)$$

the proposed scheme with the use of beam weight $\mathbf{w}_L = \mathbf{h}_{\hat{k},L} / \|\mathbf{h}_{\hat{k},L}\|$ yields an SNR represented as

$$\gamma_{\text{prop}} = \bar{\gamma} \|\mathbf{h}_{\hat{k},L}^* \mathbf{w}_L\|^2 = \bar{\gamma} \left\| \mathbf{h}_{\hat{k},L}^* \frac{\mathbf{h}_{\hat{k},L}}{\|\mathbf{h}_{\hat{k},L}\|} \right\|^2 = \bar{\gamma} \|\mathbf{h}_{\hat{k},L}\|^2. \quad (10)$$

The performance of the proposed scheme can be analyzed by representing the dimension-reduced channel $\mathbf{h}_{k,L}$ as

$$\mathbf{h}_{k,L} = \mathbf{Q}_{k,L}^* \mathbf{h}_k = \mathbf{Q}_{k,L}^* \mathbf{Q}_k \mathbf{\Sigma}_k \tilde{\mathbf{h}}_k = \mathbf{\Sigma}_{k,L}^* \tilde{\mathbf{h}}_k \quad (11)$$

where $\mathbf{\Sigma}_{k,L}$ is an $(M \times L)$ -dimensional rectangular matrix whose L diagonal elements are the same as the first L diagonal elements of $\mathbf{\Sigma}_k$ and whose off-diagonal elements are zero. Then $\mathbf{h}_{k,L}$ can be represented as

$$\mathbf{h}_{k,L} = \mathbf{\Sigma}_{k,L}^* \tilde{\mathbf{h}}_k = [\tilde{h}_{k1} \sigma_{k1} \cdots \tilde{h}_{kL} \sigma_{kL}]^T = \tilde{h}_{k1} \sigma_{k1} \mathbf{1}_1 + \cdots + \tilde{h}_{kL} \sigma_{kL} \mathbf{1}_L \quad (12)$$

where $\mathbf{1}_i$ denotes an L -dimensional unit norm vector whose i -th element is one and the other elements are zero. The unit norm vector $\mathbf{1}_i$ can be represented by a set of orthonormal vectors $\{\mathbf{o}_1, \dots, \mathbf{o}_L\}$ randomly chosen from an L -dimensional unit sphere as

$$\mathbf{1}_i = \alpha_{i1} \mathbf{o}_1 + \cdots + \alpha_{iL} \mathbf{o}_L \quad (13)$$

where the coefficients $\{\alpha_{i1}, \dots, \alpha_{iL}\}$ are identically distributed zero mean random variables satisfying $\sum_{l=1}^L |\alpha_{il}|^2 = 1$ (i.e., $E\{|\alpha_{il}|^2\} = 1/L$ for $i=1, \dots, L$, and $l=1, \dots, L$). Then, the

dimension-reduced channel can be rewritten as

$$\begin{aligned}\mathbf{h}_{k,L} &= \tilde{h}_{k1}\sigma_{k1}\mathbf{1}_1 + \cdots + \tilde{h}_{kL}\sigma_{kL}\mathbf{1}_L \\ &= \sum_{l=1}^L \tilde{h}_{kl}\sigma_{kl}(\alpha_{l1}\mathbf{o}_1 + \cdots + \alpha_{lL}\mathbf{o}_L) \\ &= \left(\sum_{l=1}^L \tilde{h}_{kl}\sigma_{kl}\alpha_{l1}\right)\mathbf{o}_1 + \cdots + \left(\sum_{l=1}^L \tilde{h}_{kl}\sigma_{kl}\alpha_{lL}\right)\mathbf{o}_L.\end{aligned}\quad (14)$$

Letting $\beta_{kl} \triangleq \sum_{l=1}^L \tilde{h}_{kl}\sigma_{kl}\alpha_{li}$, it can easily be seen that $\{\beta_{k1}, \dots, \beta_{kL}\}$ are identically distributed zero mean random variables with variance given by

$$E\{|\beta_{kl}|^2\} = \sum_{l=1}^L E\{|\tilde{h}_{kl}|^2\} E\{|\alpha_{li}|^2\} \sigma_{kl}^2 = \frac{1}{L} \sum_{l=1}^L \sigma_{kl}^2. \quad (15)$$

Assuming that the random beam generated by user k is $\mathbf{o}_1$ (i.e., $\tilde{\mathbf{w}}_L = \mathbf{o}_1$) without loss of generality, (8) can be rewritten as

$$\gamma_{k,L} = \bar{\gamma} \|\mathbf{h}_{k,L}^* \tilde{\mathbf{w}}_L\|^2 = \bar{\gamma} \left| (\beta_{k1}\mathbf{o}_1 + \cdots + \beta_{kL}\mathbf{o}_L)^* \mathbf{o}_1 \right|^2 = \bar{\gamma} |\beta_{k1}|^2. \quad (16)$$

Since the BS selects a user $\hat{k}$ having the maximum SNR (i.e., $\hat{k} = \arg \max_{k=1,\dots,K} \bar{\gamma} |\beta_{k1}|^2$), it can be seen that $|\beta_{\hat{k}1}|^2 = \max_{k=1,\dots,K} |\beta_{k1}|^2$. The resulting SNR with the use of the optimum beam weight can be represented as

$$\gamma_{\text{prop}} = \bar{\gamma} \|\beta_{\hat{k}1}\mathbf{o}_1 + \cdots + \beta_{\hat{k}L}\mathbf{o}_L\|^2 = \bar{\gamma} \left(|\beta_{\hat{k}1}|^2 + \cdots + |\beta_{\hat{k}L}|^2 \right) \quad (17)$$

whose expected value is given by

$$E\{\gamma_{\text{prop}}\} = \bar{\gamma} \left(E\{|\beta_{\hat{k}1}|^2\} + E\{|\beta_{\hat{k}2}|^2\} + \cdots + E\{|\beta_{\hat{k}L}|^2\} \right) \quad (18)$$

where $E\{|\beta_{\hat{k}l}|^2\} = \sum_{l=1}^L \sigma_l^2 / L$ for $l=2, \dots, L$ from (15).

For ease of analysis, assume that $\{\beta_{11}, \dots, \beta_{K1}\}$ are Gaussian random variables and that $\sigma_{1l} = \cdots = \sigma_{Kl} = \sigma_l$ for $l=1, \dots, M$. Then $|\beta_{\hat{k}1}|^2$ can be represented by the maximum of K i.i.d. exponential random variables with mean $\sum_{l=1}^L \sigma_l^2 / L$, yielding [10]

$$E\{|\beta_{\hat{k}1}|^2\} \approx \frac{1}{L} \sum_{l=1}^L \sigma_l^2 \sum_{k=1}^K \frac{1}{k} \quad (19)$$

where $\approx$ is due to the Gaussian approximation of $\{\beta_{11}, \dots, \beta_{K1}\}$. Then it can be shown that

$$\begin{aligned}E\{\|\mathbf{h}_{\hat{k},L}\|^2\} &= \bar{\gamma} \left(E\{|\beta_{\hat{k}1}|^2\} + E\{|\beta_{\hat{k}2}|^2\} + \cdots + E\{|\beta_{\hat{k}L}|^2\} \right) \\ &\approx \bar{\gamma} \frac{1}{L} \sum_{l=1}^L \sigma_l^2 \left(\sum_{k=1}^K \frac{1}{k} + L - 1 \right).\end{aligned}\quad (20)$$

Here, the term $\sum_{i=1}^K 1/k$ denotes the MUD gain which increases with the number of users. Note that the MUD gain increases by an additional amount of $L-1$ due to the use of optimum beam weight and also by a factor of $L^{-1} \sum_{l=1}^L \sigma_l^2$ due to the generation of random beams based on the long-term CSI. In fact, the conventional schemes are special cases of the proposed beamforming, i.e., $L=M$ for the coherent opportunistic beamforming [6] and $L=1$ for the opportunistic eigen-beamforming [8].

Based on the number of users and the spatial correlation, the optimum value of L of user k can be determined as

$$\begin{aligned}L^*(k) &= \arg \max_{L \in \{1, \dots, M\}} \bar{\gamma}_{\text{prop}} \\ &\approx \arg \max_{L \in \{1, \dots, M\}} \frac{1}{L} \sum_{l=1}^L \sigma_l^2 \left(\sum_{k=1}^K \frac{1}{k} + L - 1 \right).\end{aligned}\quad (21)$$

In practice, the eigenvalues of the channel covariance matrix may be somewhat different for each user. In this case, $L^*(k)$ can be determined using its own eigenvalues $\{\sigma_{k1}, \dots, \sigma_{kL}\}$ as

$$L^*(k) \approx \arg \max_{L \in \{1, \dots, M\}} \frac{1}{L} \sum_{l=1}^L \sigma_{kl}^2 \left(\sum_{k=1}^K \frac{1}{k} + L - 1 \right). \quad (22)$$

The beam weight is often reported in a quantized form due to limited capacity of the feedback signaling link. Assume that the beam weight is reported by a B -bit index of a pre-determined codebook $C = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{2^B}\}$ [11]. Then, for a given $\mathbf{h}_{\hat{k},L}$, the user can determine the beam weight maximizing the SNR among 2^B entries in the codebook as

$$\mathbf{w}_L = \arg \max_{\mathbf{v} \in C} |\mathbf{h}_{\hat{k},L}^* \mathbf{v}|. \quad (23)$$

The effect of the quantization on the SNR performance can be analyzed by representing $\mathbf{w}_L$ as [13]

$$\mathbf{w}_L = \alpha \frac{\mathbf{h}_{\hat{k},L}}{\|\mathbf{h}_{\hat{k},L}\|} + \sqrt{1-|\alpha|^2} \mathbf{u} \quad (24)$$

where $\mathbf{u}$ is an L -dimensional unit norm vector isotronically distributed over a space orthogonal to $\mathbf{h}_{\hat{k},L}$ and α ($|\alpha| < 1$) is a random variable related to the quantization noise. Since the codebook is generated independently of channel realizations, α can also be assumed to be independent of $\mathbf{h}_{\hat{k},L}$. Then, the SNR due to the use of the quantized beam weight can be represented as

$$\gamma_{\text{prop}} = \bar{\gamma} \left| \mathbf{h}_{\hat{k},L}^* \left(\alpha \frac{\mathbf{h}_{\hat{k},L}}{\|\mathbf{h}_{\hat{k},L}\|} + \sqrt{1-|\alpha|^2} \mathbf{u} \right) \right|^2 = \bar{\gamma} |\alpha|^2 \|\mathbf{h}_{\hat{k},L}\|^2. \quad (25)$$

Thus, we have

$$E\{\gamma_{\text{prop}}\} \approx \bar{\gamma} \bar{\alpha}_{B,L}^2 \frac{1}{L} \sum_{l=1}^L \sigma_l^2 \left(\sum_{k=1}^K \frac{1}{k} + L - 1 \right) \quad (26)$$

where $\bar{\alpha}_{B,L}^2 \triangleq E\{|\alpha|^2\}$ denotes the amount of gain reduction due to the use of a B -bit quantized L -dimensional beam weight. Note that $\bar{\alpha}_{B,L}^2$ can be found off-line from (23) and (24) as

$$\bar{\alpha}_{B,L}^2 = E \left\{ \left| \mathbf{e}^* \left(\arg \max_{\mathbf{v} \in \mathcal{C}} |\mathbf{e}^* \mathbf{v}| \right) \right|^2 \right\} \quad (27)$$

where $\mathbf{e}$ is an unit-norm random vector isotropically distributed over an L -dimensional unit sphere.

Assuming the use of a random codebook whose entries are randomly chosen from an L -dimensional unit sphere, it can be shown that [14]

$$\bar{\alpha}_{B,L}^2 = 1 - 2^{-B} \cdot \beta \left(2^B, \frac{L}{L-1} \right) \quad (28)$$

where $\beta(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the Beta function and $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the Gamma function. Using an approximation [13]

$$\bar{\alpha}_{B,L}^2 \approx 1 - 2^{-\frac{B}{L-1}}, \quad (29)$$

the average SNR can be rewritten as

$$E\{\gamma_{\text{prop}}\} \approx \bar{\gamma} \left(1 - 2^{-\frac{B}{L-1}} \right) \frac{1}{L} \sum_{l=1}^L \sigma_l^2 \left(\sum_{k=1}^K \frac{1}{k} + L - 1 \right). \quad (30)$$

It can be seen that when B is small, the gain reduction $1 - 2^{-B/(L-1)}$ due to the quantization becomes significant as L increases, implying the use of a small L is desirable and vice versa. The optimum value $L^*(k)$ for user k can be determined as

$$L^*(k) \approx \arg \max_{L \in \{1, \dots, M\}} \bar{\gamma} \bar{\alpha}_{B,L}^2 \frac{1}{L} \sum_{l=1}^L \sigma_{kl}^2 \left(\sum_{k=1}^K \frac{1}{k} + L - 1 \right). \quad (31)$$

IV. PERFORMANCE EVALUATION

The design and analysis of the proposed scheme is verified by computer simulation. It is assumed that the BS uses four transmit antennas and each user uses a single receiver antenna in Rayleigh fading channel whose covariance matrix of user k is given by [15]

$$\mathbf{R}_k = \begin{bmatrix} 1 & \zeta_k & \zeta_k^2 & \zeta_k^3 \\ \zeta_k^* & 1 & \zeta_k & \zeta_k^2 \\ \zeta_k^{2*} & \zeta_k^* & 1 & \zeta_k \\ \zeta_k^{3*} & \zeta_k^{2*} & \zeta_k^* & 1 \end{bmatrix} \quad (32)$$

where the correlation magnitude $|\zeta_k|$ is the same for all users, while its phase value is chosen randomly and independently from $[0, 2\pi]$. Note that this correlation model provides users with the same eigenvalues (i.e., $L^*(1) = \dots = L^*(K)$). It is also assumed that the beam weight is quantized using a random codebook and reported to the BS through an error-free feedback signaling link and that the BS chooses a user having the largest SNR unless stated otherwise.

Fig. 1 depicts the effect of spatial correlation on the SNR performance of the proposed beamforming when $B = 6$, $K = 8$ and $\gamma = 10$ dB. It can be seen that the proposed beamforming and the coherent opportunistic beamforming provide almost the same performance when the channel is in low correlation (e.g., when $|\zeta| \leq 0.1$). However, as the spatial correlation increases, the proposed scheme begins to outperform the coherent opportunistic beamforming scheme by using a smaller L , hence obtaining a larger MUD gain with less quantization noise. When the spatial channel is sufficiently correlated (e.g., when $|\zeta| \geq 0.5$), the proposed scheme provides the same performance as the opportunistic eigen-beamforming because $L^* = 1$. Adaptively choosing L^* , the proposed scheme can outperform the conventional schemes in a wide range of spatially correlated channel environments. It can also be seen that the analytic results agree well with the simulation ones despite the use of Gaussian approximation in (19).

Fig. 2 illustrates the effect of the beam weight quantization on the achievable SNR when $K = 8$, $|\zeta| = 0.3$ and $\gamma = 10$ dB. When the number of feedback bits is small (e.g., $B = 2$), the proposed beamforming and the opportunistic eigen-beamforming have the same performance because $L^* = 1$. This implies that when the constraint on the feedback signaling is strict, the use of only long-term CSI is highly desirable. On the other hand, when B is large, the proposed scheme performs almost the same as the coherent opportunistic beamforming due to the use of a large L . It can be seen that the proposed beamforming scheme noticeably outperforms the conventional beamforming schemes in a practical range of B (e.g., $3 \leq B \leq 7$).

Fig. 3 depicts the achievable capacity when $B = 6$, users equally have an average SNR of 5, 10, 15 or 20 dB and a root mean square (RMS) of the angle of departure (AoD) chosen independently and randomly from -60 degrees to 60 degrees. It is assumed that users are selected by a proportional fair scheduler at a time constant of 1000 slots [16], where one slot duration is 5 ms. It is also assumed that four transmit antennas are equally separated by four wavelengths in a row and correlated according to the tapped delay-line parameters of the Urban Macro Scenario in [17] with different AoD for each user. and the temporal correlation is generated by the Jake's model [18] in 2.4 GHz with user mobility of 5 km/h. It is assumed that users report their effective SNR to the BS at time t , the selected user reports its optimum beam weight to the BS at time $t + \tau_1$, and finally the data is delivered to the selected user at time $t + \tau_2$ where $\tau_1 = \tau_2 / 2 = 2$ ms. For reference, the

performance of the opportunistic beamforming [5] is also depicted. It can be seen that the proposed scheme outperforms the other schemes; noticeable capacity improvement over the opportunistic eigen-beamforming for small K (e.g., $K \leq 8$) and over the coherent opportunistic beamforming for large K (e.g., $K \geq 16$). The opportunistic beamforming provides the worst performance mainly because it exploits only the SNR information of users.

V. CONCLUSIONS

We have proposed a new beamforming scheme in spatially correlated MISO channel environments. The proposed scheme improves the MUD gain as well as the beamforming gain over the conventional schemes by effectively reducing the dimension of the channel. The proposed scheme adjusts the effective dimension of the channel according to the spatial correlation of the channel and the number of users. The simulation results show that the proposed beamforming scheme is quite effective in correlated channel environments.

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Fig. 1. SNR associated with $|\zeta|$ when $B = 6$, $K = 8$ and $\gamma = 10$ dB.

Fig. 2. SNR according to B when $K = 8$, $|\zeta| = 0.3$ and $\gamma = 10$ dB.

Fig. 3. Performance according to K with the use of proportional fair scheduling.