Joint Channel Estimation and Phase Noise Suppression for OFDM Systems

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***Abstract*— The joint channel estimation and phase noise suppression scheme for orthogonal frequency-division multiplexing (OFDM) systems is proposed for a case where channel estimation is needed symbol by symbol. In the proposed scheme, channel estimation and phase noise suppression are performed iteratively via the expectation-maximization (EM) algorithm. The proposed algorithm mitigates the performance degradation due to phase noise effectively while providing the accurate channel estimate with comparatively few pilot subcarriers so that the spectral efficiency of an OFDM system is improved.**

Keywords- Channel estimation, phase noise, EM, OFDM.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive technique to support high data rate transmission over frequency selective fading channels. However, it is known to be very sensitive to a channel estimation error and phase noise.

Several papers analyze the effects of phase noise in OFDM systems [1]-[5]. It is known that the phase noise, which is usually modeled as the Wiener process, causes both common phase error (CPE) and inter-carrier interferences (ICI) [1]. The ICI induced by phase noise is known to be difficult to be corrected due to its random nature but [6] introduces a cancellation coding to combat ICI signals. On the other hand, CPE leads to the constant phase rotation of the desired signals for all subcarriers. In [5], it is shown that the CPE is corrected by the phase rotation observed in the received pilot subcarrier signals embedded in each OFDM symbol. Moreover, the phase noise suppression (PNS) scheme proposed in [7] uses not only pilot subcarriers but also data subcarriers in decision-directed manner to estimate CPE. Note that these schemes are operated under known channel state information (CSI).

In this paper, we propose the joint channel estimation and phase noise suppression scheme. In the proposed scheme, pilot subcarriers in each OFDM symbol are employed to perform joint channel estimation and phase noise suppression iteratively via the expectation-maximization (EM) algorithm. Expectation and maximization steps provide channel and CPE estimates.

The rest of this paper is organized in the following order. Section Ⅱ describes the system model of an OFDM system with phase noise. Section Ⅲ describes the proposed joint channel estimation and PNS scheme based on the EM algorithm. Section Ⅳ presents the computer simulation results in terms of symbol error rate (SER) performance of the proposed algorithm over a frequency selective fading channel. Some conclusion remarks are given in section Ⅴ.

II. SYSTEM MODEL

Let N be the number of subcarriers and $M = 2N_\alpha+1$ be the number of modulated subcarriers. Note that $N - M$ subcarriers at the edges of the spectrum are not used. Then, the received signal at the k-th subcarrier can be written as [5]

$$r_k = \varepsilon \cdot s_k H_k + \xi_k + w_k \tag{1}$$

where s_k, H_k and w_k denote the transmitted symbol, the channel coefficient and the additive white Gaussian noise at the k-th subcarrier, respectively. Moreover, CPE and ICI caused by phase noise are represented by ε and ξ_k, respectively in (1). Thus we can define ε and ξ_k as

$$\varepsilon = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\theta_n}\ , \tag{2}$$

$$\xi_k = \sum_{\substack{m=-N_\alpha \\ m\neq k}}^{N_\alpha} \frac{s_m H_m}{N} \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi(m-k)n}{N}+\theta_n\right)} \tag{3}$$

where θ_n denotes the phase noise at the n-th sample in the OFDM symbol. The received signal vector $\mathbf{r} = \left[r_{-N_\alpha} \cdots r_{-1}\ r_0\ r_1 \cdots r_{N_\alpha}\right]^T$ can be written in the matrix form shown as

$$\mathbf{r} = \varepsilon \cdot \mathbf{SDh} + \boldsymbol{\xi} + \mathbf{w} \tag{4}$$

where $\mathbf{h} = \left[h_1\ h_2 \cdots h_L\right]^T$ presents the channel impulse response (CIR) with L multipaths and

$$\mathbf{S} = diag\left\{s_{-N_\alpha}, \cdots, s_{-1}, s_0, s_1, \cdots, s_{N_\alpha}\right\} \tag{5}$$

denotes a diagonal symbol matrix. The discrete Fourier transform (DFT) matrix $\mathbf{D}$ in (4) contains the entries of

$$[\mathbf{D}]_{k,l} = e^{-j\frac{2\pi k}{N}(l-1)} \qquad |k| \leq N_\alpha, \quad l = 1, 2, \ldots, L \quad (6)$$

The additive white Gaussian noise vector **w** in (4) has the covariance matrix of $\sigma_w^2 I$.

Moreover, it is known that the ICI term ξ shown in (4) has the nature of white noise [4], [5] and the variance of ICI term σ_ξ^2 can be approximated by $2\pi\beta T \cdot M / 3N$ [7] where T denotes the OFDM symbol period and the parameter β is well described in [1]. Here, we define the summation of additive noise and ICI terms as $\upsilon = \xi + \mathbf{w}$, which has zero mean and variance $\sigma_\xi^2 + \sigma_w^2$.

III. Joint Channel Estimation and Phase Noise Suppression

The EM algorithm is an iterative two-step algorithm that consists of the expectation and maximization steps. The EM algorithm iterates until estimates converge [8], [9].

A. Proposed Scheme

The log-likelihood function can be expressed as

$$\Lambda(\varepsilon, \mathbf{s}|\mathbf{h}) = \mathrm{Re}\{\varepsilon \cdot \mathbf{r}^H \mathbf{SDh}\} - \frac{1}{2}|\varepsilon|^2 |\mathbf{SDh}|^2 \quad (7)$$

where $(.)^H$ denotes a conjugated transpose. In the optimal application of the EM algorithm, given ε^i and $\mathbf{s}^i$, the expectation step at the $(i+1)$-th iteration evaluates the CIR estimate $\mathbf{h}^{i+1}$ shown as

$$\mathbf{h}^{i+1} = E[\mathbf{h}|\mathbf{r}, \varepsilon^i, \mathbf{s}^i] = \frac{(\varepsilon^i)^*}{|\varepsilon^i|^2} \mathbf{K}_h{}^i \mathbf{D}^H (\mathbf{S}^i)^H \mathbf{r} \quad (8)$$

where $\mathbf{K}_h{}^i$ is defined as

$$\mathbf{K}_h{}^i = \left[\frac{\sigma_\upsilon^2}{|\varepsilon^i|^2} \mathbf{R}_h^{-1} + \mathbf{D}^H (\mathbf{S}^i)^H \mathbf{S}^i \mathbf{D} \right]^{-1} \quad (9)$$

and $\mathbf{R}_h = \mathrm{E}[\mathbf{hh}^H]$. However, for QAM signals, $(\mathbf{S}^i)^H\mathbf{S}^i$ is not constant and $|\varepsilon^i|^2$ is variable according to phase noise conditions. Therefore, it is required to perform matrix inversion of (9) for $\mathbf{K}_h{}^i$ at every iteration, which increases the computational complexity. In this paper, we propose a suboptimal algorithm to overcome the computational complexity of optimal EM applications.

Assuming that the sequence estimate at the i-th iteration $\mathbf{s}^i$ is the same as the transmitted data symbols **s** at convergence, we define a normalized received signal vector $\mathbf{r}^i$ as

$$\mathbf{r}^i = (\mathbf{S}^i)^{-1}\mathbf{r} = \varepsilon \cdot \mathbf{Dh} + \upsilon' \quad (10)$$

where $\upsilon' = (\mathbf{S}^i)^{-1}\upsilon$. For an M-ary QAM scheme, υ' is an additive white Gaussian noise vector with variance

$$\sigma_{\upsilon'}^2 = \frac{1}{M}\sum_{m=1}^{M} \frac{\sigma_\upsilon^2}{|s_m|^2} = \gamma\sigma_\upsilon^2 \quad (11)$$

where s_m is the m-th possible symbol and γ is defined as a variance scaling factor [10]. The variance scaling factor can be shown to have the value of $\gamma = 1.8889$ for 16-QAM when the average symbol energy of a 16-QAM system is normalized to unity. Then, the expectation step in (8) and (9) is rewritten as

$$\mathbf{h}^{i+1} = \frac{(\varepsilon^i)^*}{|\varepsilon^i|^2} \mathbf{K}_h \mathbf{D}^H \mathbf{r}^i \quad (12)$$

where

$$\mathbf{K}_h = (\sigma_{\upsilon'}^2 \mathbf{R}_h^{-1} + \mathbf{D}^H\mathbf{D})^{-1} \quad (13)$$

and it is assumed that $|\varepsilon^i|^2 \approx 1$ in (13).

The maximization step, which is utilized to generate the $(i+1)$-th estimate ε^{i+1} and $\mathbf{s}^{i+1}$, can be written as

$$\mathbf{s}^{i+1} = \arg\max_{\mathbf{s}} \Lambda(\varepsilon^i, \mathbf{s}|\mathbf{h}^{i+1}) \quad (14)$$

$$\varepsilon^{i+1} = \arg\max_{\varepsilon} \Lambda(\varepsilon, \mathbf{s}^{i+1}|\mathbf{h}^{i+1}) \quad (15)$$

and the solution of (15) is given by

$$\varepsilon^{i+1} = \frac{1}{|\mathbf{S}^{i+1}\mathbf{Dh}^{i+1}|^2} (\mathbf{S}^{i+1}\mathbf{Dh}^{i+1})^H \mathbf{r} \quad (16)$$

Note that, in (12) and (16), it is seen that the proposed algorithm iteratively provides the CIR estimate in expectation step and the CPE estimate in maximization step via the EM algorithm.

B. Initialization

The initial estimate $\mathbf{h}^0$ can be given by the channel estimate in previous OFDM symbol. The initial CPE estimate ε^0 is obtained by using $\mathbf{h}^0$ and pilot subcarriers embedded in each OFDM symbol [5]. Then, the initial sequence estimate $\mathbf{s}^0$ can be evaluated by

$$\mathbf{s}^0 = \arg\max_{\mathbf{s}} \Lambda(\varepsilon^0, \mathbf{s}|\mathbf{h}^0) \quad (17)$$

IV. Simulation Results

OFDM system parameters for computer simulation correspond to IEEE 802.11a standard [11] summarized as follows.

- The DFT size N is 64.
- The number of modulated subcarriers equals to 53 excluding the subcarrier with zero index ($N_\alpha = 26$).
- The number of pilot subcarriers is 4.
- Each subcarrier is modulated by 16-QAM.
- Guard and IFFT/FFT intervals are 0.8μs and 3.2μs, respectively.

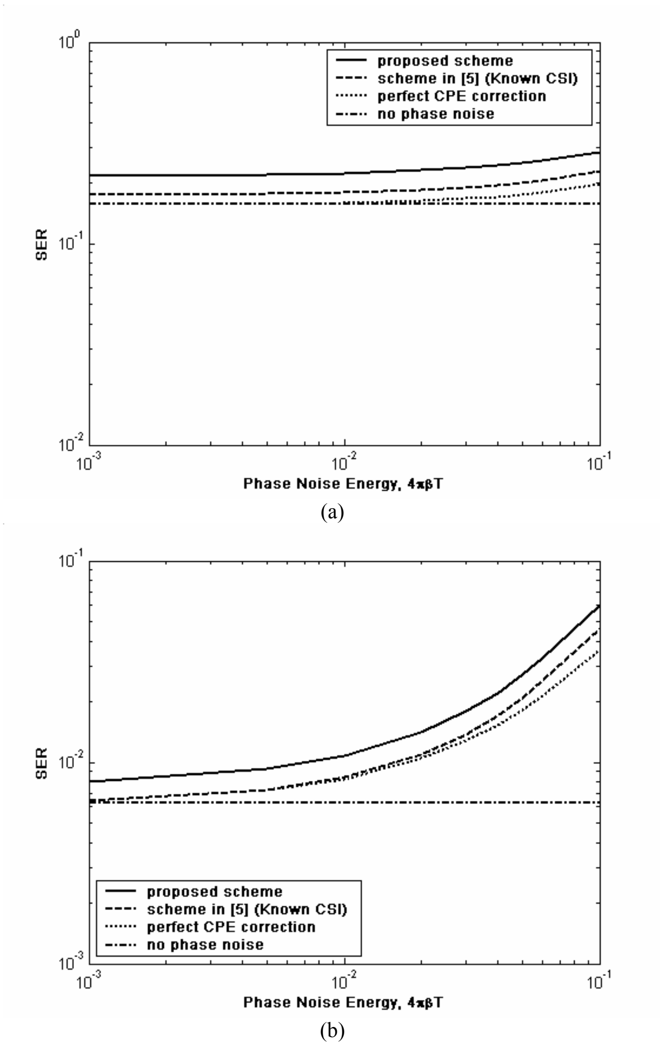

Figure 1. Comparisons of SER performance according to the different values of phase noise energy, (a) SNR = 15 dB, (b) SNR = 30 dB.

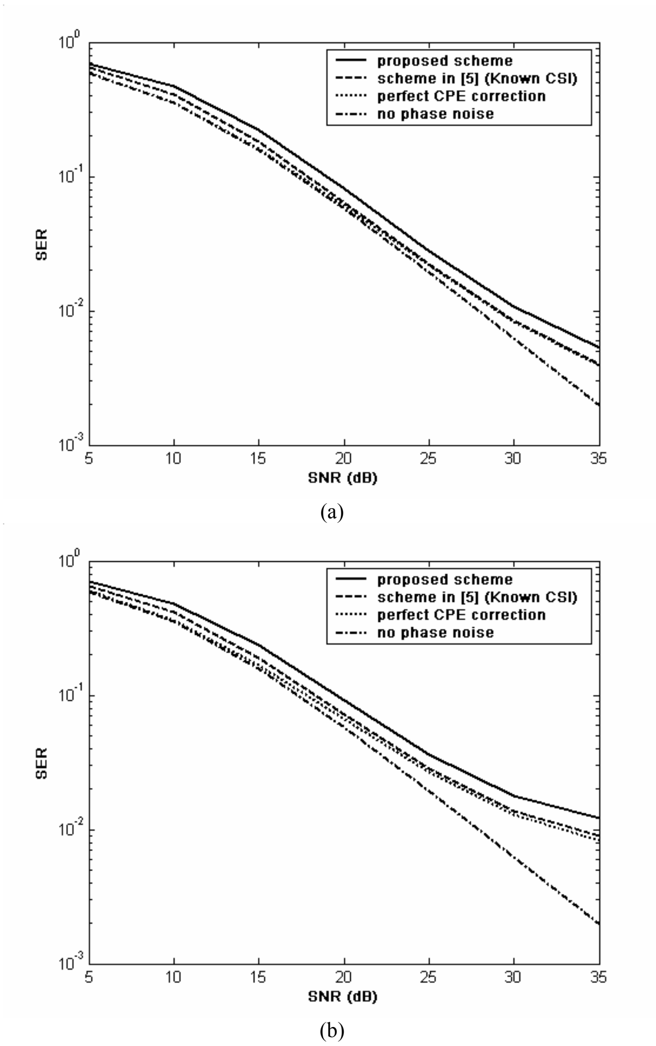

Figure 2. SER performance versus SNR, (a) phase noise energy $4\pi\beta T$ = 0.01 (rad^2), (b) phase noise energy $4\pi\beta T$ = 0.03 (rad^2).

- Channel variation is considered as normalized Doppler spread given as BT = 0.002 [9].

A channel environment for computer simulation assumed to be a frequency selective fading channel with 8 multipaths (L = 8), which is lager than the number of pilot subcarriers and has an exponentially decaying power delay profile. Each multipath is mutually independent and set to vary symbol by symbol according to the Rayleigh distribution [12]. Moreover, it is assumed that a frame is composed of 20 OFDM symbols and the first symbol in each frame is dedicated to a preamble symbol.

From simulation results, it is observed that two iterations are enough to improve the performance of the OFDM system so that the number of iterations of the proposed algorithm is fixed to two.

Figure 1 compares SER performance according to the different values of phase noise energy. It is seen that, for various phase noise conditions, the performance of the proposed channel and CPE estimation algorithm is comparable with that of the scheme in [5] under known CSI.

In Figure 2, it can be observed that, at high SNR, there is the performance degradation induced by ICI in the presence of phase noise although CPE is ideally corrected. Moreover, the proposed algorithm shows quite comparable performance with the scheme in [5] with known CSI and it has about 1 dB performance loss in all ranges of SER compared with that of perfect CPE correction. It can be concluded from Figure 2 that the proposed algorithm provides proper channel and CPE estimates.

V. Conclusion

In this paper, we proposed a joint channel estimation and PNS scheme for an orthogonal frequency-division multiplexing (OFDM) system. Channel and CPE estimations are performed iteratively via the expectation-maximization (EM) algorithm and the number of iterations is fixed to two for the advantage not only of a small but also fixed number of iterations. The symbol error rate (SER) performance of an OFDM system are presented to show that the proposed algorithm mitigates the performance degradation due to phase noise effectively while providing the accurate channel estimate. Therefore this scheme can be used for the OFDM systems in the place where the number of multipaths is larger than that of pilot subcarriers and the channel varies so rapidly with respect to time that channel estimation is needed symbol by symbol.

ACKNOWLEDGMENT

This work was supported by University IT Research Center Project.

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