Joint Common Phase Error and Channel Estimation for OFDM-based WLANs in the Presence of Wiener Phase Noise and Residual Frequency Offset

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Abstract— In orthogonal frequency-division multiplexing (OFDM)-based wireless local area networks (WLANs), the phase noise (PHN) and a residual frequency offset (RFO) can cause the common phase error (CPE) and the inter-carrier interferences (ICI), which seriously degrade the performance of systems. In this paper, we propose a combined pilot symbol assisted and decision-directed scheme based on the least-squares (LS) and maximum-likelihood (ML) algorithms. In the proposed scheme, the CPE estimator is derived using the pilot symbols embedded in each OFDM symbol. Then, data symbols and channel response including the CPE are estimated using the CPE estimator. Simulation results present that the proposed scheme significantly improves the performance of OFDM-based WLANs.

Keywords- Orthogonal frequency-division multiplexing (OFDM), quadrature amplitude modulation (QAM), channel estimation, phase noise (PHN), wireless local area networks (WLANs).

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has recently received considerable attention for its robustness against frequency selective channels. It has been adopted by the IEEE 802.11a standard as the transmission technique for high-rate wireless local area networks (WLANs) [1]. A packet of OFDM-based WLANs consists of an OFDM packet preamble, a SIGNAL field, and an OFDM DATA field. The packet preamble includes ten short training symbols and two long training symbols which are used to perform the timing/frequency synchronization and channel estimation. The OFDM DATA field is composed of multiple OFDM symbols whose data bits are convolutionally encoded to exploit frequency diversity in frequency selective channels. Hong and Cheon examined the performance degradation due to channel estimation errors in OFDM-based WLANs [2]. Deneire et al. developed low-complexity maximum-likelihood (ML) channel estimation schemes in a pilot symbol assisted system (PSAM) and a combined PSAM决策-directed (DD) system [3].

However, as the literatures have reported, the OFDM system is very sensitive to the phase noise (PHN) [4]-[8] and a residual frequency offset (RFO) due to imperfect synchronization [2], [4], [8]. The PHN and a RFO cause both the common phase error (CPE) and the inter-carrier interferences (ICI), both of which impair the accurate channel estimation. For the PHN compensation, Bar-Ness and Wu proposed the phase noise suppression schemes with ideal channel state information [6], [7]. For both the PHN and the RFO compensation, Polydoros and Nikitopoulos proposed the decision-directed scheme with ideal channel state information [8].

Modeling PHN as a random process, two approaches are investigated in the literature. One assumes this random process as a zero-mean, stationary and finite-power random process, which can be obtained by measuring a real tuner with a phase-locked loop (PLL) [5]. The other does the typical PHN model of a free-running oscillator, which can be modeled as the complex exponent of a zero-mean, nonstationary and infinite power Wiener process [4], [5], [7], [8].

In this paper, we adopt the typical PHN model of a free-running oscillator for the PHN process. Moreover, we consider that the coarse frequency synchronization and fine frequency synchronization are performed using ten short training symbols and two long training symbols [9]. Assuming that one packet is composed of one long training symbol and OFDM data symbols, we propose the joint CPE and channel estimation scheme based on least-squares (LS) and the ML algorithms. Since it is hard to obtain the joint ML estimation of the CPE and the channel response, we treat the CPE estimation and the channel estimation problems separately. In the proposed scheme, the CPE estimator based on the LS algorithm is derived using the pilot symbols embedded in each OFDM symbol. Then, the channel response including the CPE is estimated based on the ML algorithm in the DD manner.

The rest of this paper is organized in the following order. Section II describes the system model of an OFDM system with the PHN and a RFO. Section III describes the proposed joint CPE and channel estimation scheme. Section IV presents the computer simulation results in terms of symbol error rate (SER) performance of the proposed scheme over a frequency selective fading channel. Some conclusion remarks are given in section V.
II. SYSTEM MODEL

We consider an OFDM system with \( N \) sub-carriers composed of a data sub-carrier set \( S_D \) with \( N_D \) sub-carriers, a pilot sub-carrier set \( S_P \) with \( N_P \) sub-carriers and a null sub-carrier set \( S_N \) with \( N_N \) sub-carriers [1]. The time-domain OFDM symbol is generated by \( N \)-point IDFT and a cyclic prefix (CP) of length \( N_{CP} \) is appended at the beginning of each time-domain OFDM symbol.

Then, the received signal vector \( \mathbf{r}_m \) at the \( m \)th OFDM symbol over frequency selective fading channel, after CP removal, can be expressed as

\[
\mathbf{r}_m = \mathbf{F}_{\theta_m} \mathbf{X}_m \mathbf{H}_m + \mathbf{w}_m \tag{1}
\]

where \( m = 0, 1, \cdots, N_{\text{packet}} - 1 \) is the OFDM symbol index and both the PHN \( \phi_m^{\text{PHN}} \) and the RFO \( \phi_m^{\text{RFO}} \) are represented by the matrix \( \mathbf{F}_{\theta_m} \) given as

\[
\mathbf{F}_{\theta_m} = \text{diag}\{ e^{j\phi_m^{\text{PHN}}(0)}, e^{j\phi_m^{\text{PHN}}(1)}, \cdots, e^{j\phi_m^{\text{PHN}}(N-1)} \}. \tag{2}
\]

The PHN \( \phi_m^{\text{PHN}} \) in (2) can be modeled as a discrete Wiener process by [8]

\[
\phi_m^{\text{PHN}}(n) = \phi_m^{\text{PHN}}(N-1) + \sum_{t=-N_{CP}}^{n} u(t) \tag{3}
\]

where \( 0 \leq n < N \) and \( \phi_m^{\text{PHN}}(n) = \sum_{t=-N_{CP}}^{n} u(t) \). In (3), \( u(t) \) denotes a mutually independent Gaussian random variable with zero mean and variance \( \sigma_u^2 = 2\pi T |N| \), where \( T \) is the OFDM symbol period and \( \beta \) represents the two-sided 3-dB linewidth of Lorentzian power density spectrum of the oscillator [7]. The RFO \( \phi_m^{\text{RFO}} \) in (2) can be modeled as an additional rotation due to the normalized RFO \( \nu \) by

\[
\phi_m^{\text{RFO}}(n) = \phi_m^{\text{RFO}}(N-1) + \frac{2\pi \nu (N_{CP} + n)}{N} \tag{4}
\]

where \( 0 \leq n < N \) and \( \phi_m^{\text{RFO}}(n) = 2\pi \nu (N_{CP} + n)/N \).

In (1), \( \mathbf{X}_m = \text{diag}\{X_m(k)\} \) for \( k \in S_D \cup S_P \) denotes a diagonal transmitted symbol matrix and \( \mathbf{F}_{\theta_m} \) does the IDFT matrix where \( 0 \leq n < N \) and \( k \in S_D \cup S_P \). Assuming ideal timing synchronization and sufficient CP, the channel frequency response can be given as \( \mathbf{H}_m = \mathbf{Dh}_m \) where \( \mathbf{h}_m = [h_m(0) \cdots h_m(L-1)]^T \) denotes the channel impulse response (CIR) with \( L \) multipaths and \( |\mathbf{D}|_{kj} = \exp\left(-j \frac{2\pi k j}{N}\right) \) for \( k \in S_D \cup S_P \), \( 0 \leq i < L \) and \( L \leq N_{CP} \). Moreover, the complex-valued additive white Gaussian noise vector \( \mathbf{w}_m = [w_m(0) \cdots w_m(N-1)]^T \) has the covariance matrix of \( \sigma_w^2 \mathbf{I}_N \) where \( \mathbf{I}_N \) denotes an \( N \times N \) identity matrix.

The frequency-domain received signal vector can be given as

\[
\mathbf{R}_m = \mathbf{F}^H \mathbf{r}_m = \mathbf{e}_m \mathbf{X}_m \mathbf{Dh}_m + \mathbf{n}_{ICL} + \mathbf{W}_m \tag{5}
\]

where \((\cdot)^H\) and \( \mathbf{W}_m = \mathbf{F}^H \mathbf{w}_m \) denote the conjugated transpose and the frequency response of AWGN, respectively. It is seen that the common phase error (CPE) \( \mathbf{e}_m \) in (5) affects all sub-carriers constantly, which is defined by

\[
\mathbf{e}_m = \frac{1}{N} \sum_{n=0}^{N-1} e^{j[\phi_m^{\text{PHN}}(n)-\phi_m^{\text{RFO}}(n)]} \tag{6}
\]

Moreover, \( \mathbf{n}_{ICL} \) in (5) implies the inter-carrier interferences (ICI) due to both the PHN and a RFO defined by

\[
\mathbf{n}_{ICL} = (\mathbf{F}^H \mathbf{F}_{\theta_m} \mathbf{F} - \mathbf{e}_m \mathbf{I}_{N_D+N_P}) \mathbf{X}_m \mathbf{Dh}_m. \tag{7}
\]

The autocorrelation of the CPE between adjacent OFDM symbols \( \Phi_e \) can be defined as

\[
\Phi_e = E[\mathbf{e}_m \mathbf{e}_{m+1}^+] = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} e^{j[\phi_m^{\text{PHN}}(q)-\phi_m^{\text{PHN}}(p)]} E\left[ e^{j[\phi_m^{\text{PHN}}(q)-\phi_m^{\text{PHN}}(p)]} \right] \tag{8}
\]

Since the PHN is a discrete Wiener process, the difference \( \phi_m^{\text{PHN}}(q) - \phi_m^{\text{PHN}}(p) \) in (8) is a zero-mean Gaussian random variable with variance equal to \( \sigma_u^2 |N + N_{CP} + q - p| \). Thus

\[
\Phi_e = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} e^{\frac{2\pi \nu (N + N_{CP} + q - p)}{N}} \left( \frac{\pi^2}{2} \left( N + N_{CP} + q - p \right) \right) \tag{9}
\]

After some algebraic manipulations, (9) can be given by

\[
\Phi_e = e^{\frac{\pi^2 \nu}{2} N} \times \left( \sum_{i=1}^{N} i \cosh \left( \frac{\pi^2 \nu}{N} \right) - \frac{\pi^2 \nu}{2} N \right). \tag{10}
\]
The $\epsilon_m$ manifests mainly as a phase rotation $\theta_{\epsilon_m} = \angle(\epsilon_m)$ for slow PHN and RFO processes [8]. Therefore, it can be expressed as $\epsilon_m = \epsilon_m^{(\text{res})} \epsilon_m^{(1)}$ where $\epsilon_m^{(\text{res})}$ is the multiplication factor to imply a residual CPE term, which defines the relation between adjacent OFDM symbols [8].

The transmitted symbol $X_m(k)$ is assumed as a random variable with zero mean and variance $\sigma_x^2 = E[|X(k)|^2]$. For the sake of calculation simplicity, and without restriction of generality, we can assume that the auto-correlation of the channel frequency response $E[|H_m(k)|^2]$ is equal to 1. The ICI vector $n_{IICI_m}$ in (7) can be approximated as a zero-mean Gaussian vector of the covariance matrix of $\sigma_{IICI_m}^2 I_{N_0+N_p}$, which is given by [2], [6]

$$\sigma_{IICI_m}^2 I_{N_0+N_p} = \sigma^2 \left( \frac{\pi \nu I (N-N_K)^2}{3N} + \frac{(\pi \nu)^2}{3} \right) I_{N_0+N_p}.$$  (11)

Here, we define the ICI-plus-noise vector as $V_m = n_{IICI_m} + W_m$ and assume that $V_m$ is a zero mean Gaussian vector with the covariance matrix of $\sigma_{V_m}^2 I_{N_0+N_p} = (\sigma_{IICI_m}^2 + \sigma_n^2) I_{N_0+N_p}$.

### III. PROPOSED SCHEME

#### A. Channel estimation at the preamble

The channel can be estimated using a long training symbol in the packet preamble. By defining a new effective CIR vector $h_m^{\text{eff}} = \epsilon_m h_m$, (5) is rewritten by

$$R_m = X_m D h_m^{\text{eff}} + V_m = X_m H^{\text{eff}} + V_m.$$  (12)

where $H^{\text{eff}} = D h_m^{\text{eff}}$ is the effective channel frequency response vector. Considering, without loss of generality, $X_m = \text{diag}[1.0]$, (12) becomes

$$R_m = D h_m^{\text{eff}} + V_m.$$  (13)

Thus, the ML based effective CIR vector estimate is given by [10]

$$\hat{h}_m^{\text{eff}} = (D^H D)^{-1} D^H R_m = h_m^{\text{eff}} + (D^H D)^{-1} D^H V_m = h_m^{\text{eff}} + V_m.$$  (14)

where $V_m' = (D^H D)^{-1} D^H V_m$ is a zero-mean Gaussian noise with the covariance matrix of $C_{V_m'} = \sigma_n^2 (D^H D)^{-1}$.

Note that the CIR length $L$ should be known in order to construct $(D^H D)^{-1} D^H$ in (14). The exact value of $L$ is usually not available and we assumed that the receiver has the estimate of $L$, which can be treated as a predetermined parameter considering the maximum expected CIR length [11]. Here, we simply assume that the estimate of $L$ is equal to the CIR length.

#### B. Joint CPE and channel estimation

When the variation of the CIR between adjacent OFDM symbols is negligible compared with $\Phi_m$, (12) can be approximated as

$$R_m = \epsilon_m^{(\text{res})} X_m H_m^{\text{eff}} + V_m.$$  (15)

Assuming that the effective channel frequency response estimate for the previous OFDM symbol is obtained, the phase of the $\epsilon_m^{(\text{res})}$ in (15) can be estimated by the least-squares (LS) method as

$$\hat{\theta}_{\epsilon_m^{(\text{res})}} = \angle[(X_m,P H_{m-1,P}^{\text{eff}})^H R_{m,P}]$$  (16)

where $R_{m,P}$, $X_{m,P}$ and $H_{m-1,P}^{\text{eff}}$ denote the received signal vector at the pilot sub-carriers, the diagonal pilot symbol matrix and the effective channel frequency response vector composed of only pilot sub-carriers, respectively. Then, the effective channel frequency response estimate can be obtained by

$$\hat{H}_m^{\text{eff}} = \exp(j \hat{\theta}_{\epsilon_m^{(\text{res})}}) \hat{H}_m^{\text{eff}}.$$  (17)

Assuming that the initial effective channel frequency response estimation from (17) leads to reliable tentative decisions, the DD scheme is proposed in order to obtain the accurate estimate of the effective CIR. Given the tentative symbol matrix estimate $\hat{X}_m$, which is assumed to be the same as the transmitted symbol matrix $X_m$, the normalized received vector $\hat{R}_m$ from (12) can be defined by

$$\hat{R}_m = (\hat{X}_m)^{-1} R_m = \hat{D}_m^{\text{eff}} + (\hat{X}_m)^{-1} V_m.$$  (18)

The normalized received vector $\hat{R}_m$ is Gaussian with mean $\hat{D}_m^{\text{eff}}$ and covariance matrix $C_{\hat{R}_m} = \sigma_n^2 E[|X_m^{-1}|^2]$. Thus, the ML based effective CIR estimate is given by [10]

$$\hat{h}_m^{\text{eff}} = (D^H C_{\hat{R}_m}^{-1} D)^{-1} D^H C_{\hat{R}_m}^{-1} \hat{R}_m = (D^H C_X^{-1} D)^{-1} D^H C_X^{-1} \hat{R}_m.$$  (19)

3042
where $C_{X^{-1}} = E[|X_m^{-1}|^2]$ is the diagonal matrix consists of $E[|1/X_m(k)|^2]$ for $k \in S_D$ and 1.0 for $k \in S_P$. When the average symbol energy $\sigma_s^2$ is normalized to unity, $E[|1/X(k)|^2]$ can be shown to have values of 1.0 for PSK signaling, 1.8889 for 16-QAM signaling and 2.6854 for 64-QAM signaling.

The proposed joint CPE and channel estimation scheme is implemented by the following steps.

Step 1) Exploiting a training symbol in the preamble, estimate the effective CIR $\hat{h}_{m}^{\text{eff}}$ by (14). Then, the effective channel frequency response vector can be estimated as $\hat{H}_{m}^{\text{eff}} = D\hat{h}_{m}^{\text{eff}}$. Increase $m$ by 1.

Step 2) Estimate $\hat{H}_{m}^{\text{eff}}$ by (17) using $\hat{H}_{m-1}^{\text{eff}}$ and $\hat{\theta}_{e_{m-1}}$, obtained by (16).

Step 3) Demodulate the OFDM symbol using the current effective channel frequency response vector $\hat{H}_{m}^{\text{eff}}$ and make the tentative symbol matrix $\hat{X}_{m}$.

Step 4) Obtain the normalized received vector $\hat{R}_{m}$ by (18).

Step 5) Estimate the effective CIR vector $\hat{h}_{m}^{\text{eff}}$ using (19).

Step 6) Perform the effective channel frequency response estimation according to $\hat{H}_{m}^{\text{eff}} = D\hat{h}_{m}^{\text{eff}}$.

Step 7) Re-estimate the transmitted symbol matrix $\hat{X}_{m}$ using the current effective channel frequency response vector $\hat{H}_{m}^{\text{eff}}$ from Step 6).

Step 8) Go to Step 2) with $m \leftarrow m - 1$.

Assuming that $(D^H C_{X^{-1}} D^{-1})^{-1} D^H C_{X^{-1}}$ is precomputed, $2(N_D+N_P)\times L$ complex products are required to evaluate $\hat{H}_{m}^{\text{eff}} = D\hat{h}_{m}^{\text{eff}}$ in Step 6) from $\hat{R}_{m}$. Further complexity reduction can be obtained by using FFT pruning or transform decomposition [12].

Regarding the accuracy of the effective channel frequency response estimator, we define the mean square error (MSE) at the OFDM symbol as $\hat{E}(\hat{H}_{m}^{\text{eff}} - H_{m}^{\text{eff}})^2$. The MSE of Step 6) with perfect tentative decisions can be analytically calculated as

$$
MSE_{H_{m}^{\text{eff}}} = E[\text{Tr}\{DZ\overline{V}_{m}V_{m}^H(X_{m}^{-1})^H Z^H D^H\}]
= \sigma_s^2 \text{Tr}\{D((D^H C_{X^{-1}} D^{-1})^{-1})^H D^H\} 
$$

where $\text{Tr}\{\cdot\}$ indicates the trace of a matrix and $Z = (D^H C_{X^{-1}} D^{-1})^{-1} D^H C_{X^{-1}}$. We will provide the MSE performance of the proposed scheme in the next section.

IV. SIMULATION RESULTS

The system parameters of the simulation environments correspond to the IEEE 802.11a physical layer standard, which is summarized as follows [1].

- The DFT size $N$ is 64, the number of data sub-carriers $N_D$ is 48, the number of pilot sub-carriers $N_P$ is 4, and the number of null sub-carriers $N_N$ is 12.
- The CP length $N_{CP}$ is 16.
- Each BPSK modulated pilot sub-carrier is located at $k = 21, 7, 7, 21$.
- The OFDM data sub-carriers are modulated using 16-QAM.
- The OFDM symbol period $T$ is $4.0 \mu s$.

We consider frequency-selective Rayleigh fading channels with exponential power delay profile given by $E[|h(l)|^2] = e^{-l/4} / \sum_{i=0}^{L} \frac{e^{-l/4}}{l}$ where $0 \leq l < L$ and $L=4$. We use the PHN with $2\pi \beta T = 0.01$ and the normalized RFO $\nu=0.01$. In the simulation, the average signal-to-noise ratio (SNR) is defined as $\frac{52\sigma_s^2}{64\sigma_n^2}$. The ZF represents the zero-forcing equalization (one-tap equalization) exploiting the long training symbol in the packet preamble [2]. In this case, the channel can be estimated as $\hat{H} = X^{-1}R$.

At first, we consider static channels for fixed wireless environments, which indicate that the CIR is constant over a packet but varies independently from one packet to the next. Fig. 1 shows the MSE performance of the effective channel frequency response estimators. It is seen that the proposed scheme outperforms the ZF with the phase noise suppression (PNS) scheme [6] and the ZF with the ML phase estimator (MLPH) scheme [8] in all ranges of SNR. Especially for SNR
greater than 20 dB, the MSE of the proposed scheme can achieve the simulated MSE with perfect tentative decisions. In Fig. 2, we present the SER performance versus SNR. The performance of the proposed scheme is also compared with the ideal performance without the PHN and a RFO, the performance with the ideal effective CIR, the performance of the PNS scheme with ideal CIR, the performance of the MLPH scheme with ideal CIR, that of the ZF with the PNS scheme and that of the ZF with the MLPH scheme. It can be concluded that it achieves almost the performance of the PNS scheme with the ideal CIR and is found to prevent remarkably the performance degradation due to the PHN, a RFO and channel estimation errors.

Fig. 3 shows the SER degradation due to the packet length of $N_{\text{packet}}$ in slowly time-varying channels. In our simulation, it is assumed that slowly time-varying channels follow the Jakes’ model [13], the carrier frequency is set to be 5.8GHz, and the user terminal is moving at speed 3 m/s. From this figure, when the packet length of $N_{\text{packet}}$ increases, the SER degradation of the ZF with the MLPH scheme increases due to channel estimation errors. However, the proposed scheme over slowly time-varying channels is robust with the increase of the packet length of $N_{\text{packet}}$.

V. CONCLUSION

In this paper, we proposed a joint CPE and channel estimation scheme for OFDM-based WLANs. At the packet preamble, the proposed scheme obtains the effective CIR from the received signals. Then, the proposed scheme estimates the CPE, data symbols, and the effective channel frequency response using the received signals and the effective channel frequency response for the previous OFDM symbol. The proposed scheme significantly compensates the performance degradation due to channel estimation errors, the PHN and a RFO. Moreover, the proposed scheme is robust in static or slowly time-varying channels compared with other schemes.

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