Monotonicity and Maximum Principle for Multi-dimensional Hyperbolic Conservation Laws

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ABSTRACT

The present paper deals with an efficient and accurate multi-dimensional limiting strategy for hyperbolic conservation laws on structured and unstructured grids. The multi-dimensional limiting process (MLP) which was successfully proposed on structured grid system is extended to unstructured grids. The MLP condition satisfies the maximum principle, and various numerical results show that MLP is effective to prevent unwanted oscillations on multi-dimensional flow, which is a favorable feature of monotonicity.

One of the most difficult parts of solving the hyperbolic conservation laws is to deal with the non-monotonic discontinuities due to unwanted numerical oscillations. To prevent this phenomenon, many oscillation-free schemes have been developed, but most of them, including TVD, TVB, ENO and WENO, are based on mathematical analysis of one-dimensional convection equations. Though these schemes may work in some cases, numerical oscillations are usually occurred near shock discontinuity in multi-dimensional flow.

Due to the limitation of TVD condition on multiple dimensions, other complementary condition ensuring the monotonicity is needed, and the maximum principle is a good alternative. Based on this principle, remarkable progresses have been made, and one of the notable results is Spekreije's multi-dimensional monotonic condition [1]. However, it does not seem to be good enough to restrict oscillations on multi-dimensional shock discontinuity.

In order to find out a suitable criterion for oscillation-control in multiple dimensions, the one-dimensional monotonic condition was extended to multi-dimensional space and successfully formulated the multi-dimensional limiting process (MLP), which showed remarkably enhanced accuracy and convergence for numerous inviscid and viscous computations on structured grids [2, 3]. In the present work, this strategy has been also extended to unstructured grids, on which the multi-dimensional limiter is more important due to the geometric complexity (or multi-dimensionality) of non-aligned meshes. In addition, it is shown that the MLP on structured grids and unstructured grids satisfies the maximum principle.